Key words: BDU, LQR, Predictive Control, Robustness.

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LQR AND PREDICTIVE CONTROL TUNED VIA BDU

This work presents the BDU technique (Bounded Data Uncertainties) and the tuning of the linear quadratic regulator (LQR) via this technique, which considers models with bounded uncertainties. The BDU method is based on constrained game-type formulations, and allows the designer to explicitly incorporate a priori information about bounds on the sizes of the uncertainties into the problem statement. Thus, on the one hand the uncertainty effect is not over-emphasized, avoiding an overly conservative design and on the other hand the uncertainty effect is not under-emphasized, avoiding an overly sensitive to errors design. A feature of this technique consists of its geometric interpretation. The structure of the paper is the following, in the first section some problems about the least-squares method in the presence of uncertainty are introduced. The BDU technique is shown in the second section and the LQR controller in the third. After that a new guided way of tuning the LQR is offered, taking into account the uncertainties bounds via the BDU. The consequence of this method is that both recursive and algebraic Riccati equations are modified. Finally, some examples are shown and the main conclusiones and future work are commented.

1. LEAST-SQUARES IS SENSITIVE TO DATA ERRORS

The least-squares method is present in a big amount of identification and control theories, such as the Kalman filter, the LQR, LQG, GPC and so on. Its popularity is due to its easy statement and solution. So, given a noisy measurement vector $b \in \mathbb{R}^m$ that is related to an unknown vector $x \in \mathbb{R}^n$ via the linear model $Ax = b + v \approx b$ for some known matrix $A \in \mathbb{R}^{m\times n}$ (with $m \geq n$), x is estimated by solving $\|Ax-b\|_2$ denoting $\|\ldots\|_2$ the Euclidean norm of its vector argument (it will also be used to denote the maximum singular value of a matrix argument). The solution $\hat{x}$, denoting A^+ the pseudoinverse matrix of A, is

$$\hat{x} = \left[A^T A\right]^{-1} A^T b = A^+ b \tag{1}$$

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Partially supported by projects AGL-2002-04108-C02-01 and DPI2001-3106-C02-02 MCYT (SPAIN).

The method considers that all the errors and the uncertainties are present only in the vector b, and so the matrix A is exactly known. The vector v denotes a noise term that explains the mismatch between the measured vector b and the vector Ax. Usually $v \neq 0$, and so the vector b will not lie in the column span of A ($b \notin \Re(A)$), and the least-squares problem therefore seeks the vector $\hat{b} = A\hat{x} \in \Re(A)$ that is closest to b in the Euclidean norm sense. The main disadvantage of the least-squares method consists of the fact that the method is sensitive to data errors. More specifically, a least-squares design that is based on given data (A,b) can perform poorly if the true data happens to be a perturbed version of (A,b) say $(A+\delta A,b)$ for some unknown δA. Besides, perturbation errors in the data are very common in practice and they can be due to several factors including the approximation of complex models by simpler ones, the presence of unavoidable experimental errors when collecting data, or even due to unknown or unmodelled effects [9].

The regularized least-squares method, which is used to combat much of the ill-conditioning that arises in pure least-squares problems [3], [4], [10], improves the system robustness. Regularization involves choosing in advance a positive parameter λ and then selecting x by solving

$$\min_x \left[\|Ax-b\|_2^2 + \lambda \|x\|_2^2 \right] \tag{2}$$

So the matrix inversion results easier and more accurate. The solution $\hat{x}$ is now given by $\hat{x} = \left[A^T A + \lambda I\right]^{-1} A^T b$. The drawback is that an intelligent selection of the regularization parameter λ by the designer is required. So if λ results too much big (over-regularization) an overly conservative design is obtained and if λ results too much small (under-regularization) the design is sensitive to errors. The BDU method will select the parameter from the given data without user intervention and in a certain optimal manner.

2. THE BDU TECHNIQUE

2.1. THE BDU STATEMENT

The Bounded Data Uncertainties problem, BDU [1], [2], [5], [7], [8], so called Min-Max problem was proposed and solved, via the secular equation in [2]. The BDU problem seeks a solution $\hat{x}$ that performs 'best' in the worst-possible scenario inside a bounded region. That means

$$\min_x \max_{\substack{\|\delta A\|_2 \le \eta_A \\ \|\delta b\|_2 \le \eta_b}} \left\| (A+\delta A)x - (b+\delta b) \right\|_2 \tag{3}$$

where (A,b) represents the nominal model and $(A+\delta A, b+\delta b)$ the unknown perturbed model, because δA and δb are unknown but a bound of them is known, η_A and η_b, which define a region around the nominal model (A,b), $\|\delta A\|_2 \le \eta_A$ and $\|\delta b\|_2 \le \eta_b$.

It can be regarded as a constrained two-player game problem, with the designer trying to pick an x that minimizes the residual norm while the opponent δA tries to maximize the residual norm. The goal consists of determing the solution $\hat{x}$ whose maximum residual is the smallest possible. From the geometric interpretation [2] it is established the condition for the nonzero solution

$$\eta_A < \frac{\|A^T b\|_2}{\|b\|_2} \tag{4}$$

2.2. TRANSFORMING INTO A MINIMIZATION PROBLEM

Assuming that the previous condition holds, the constrained Min-Max problem defined in equation (3) is expressed as the following unconstrained minimization problem

$$\min_x J(x) = \min_x \left(\|Ax-b\|_2 + \eta_A \|x\|_2 + \eta_b \right) \tag{5}$$

2.3. THE MINIMIZATION

The cost function $J(x)$ is convex in x. Assuming $J(x)$ is differentiable ($x \ne 0$, $Ax-b \ne 0$), its gradient exists, and defining the positive parameter λ as

$$\lambda = \frac{\eta_A \|Ax-b\|_2}{\|x\|_2} \tag{6}$$

the global minimum is obtained

$$x = \left(A^T A + \lambda I\right)^{-1} A^T b \tag{7}$$

As the parameter λ depends on x, the solution is obtained by solving the nonlinear equations system formed by equations (6) and (7), where λ is expressed as a nonlinear equation depending on λ, A, b and η_A. Defining $F(\lambda)$ as the secular equation

$$F(\lambda) = \frac{\eta_A \|Ax-b\|_2}{\|x\|_2} - \lambda \tag{8}$$

a unique solution $\hat{\lambda} > 0$ exists so that $F(\hat{\lambda}) = 0$, which can be determined, for example, by employing a bisection-type algorithm to solve the nonlinear equation.

2.4. EXTENDED COST FUNCTION

Another statement of the BDU problem, which is very useful in the context of the linear quadratic regulator (LQR), LQG, Predictive Control, because it penalizes the control effort (in this case the variable x) is the following

$$\min_{x} \max_{\substack{\|\delta A\|_2 \le \eta_A \\ \|\delta b\|_2 \le \eta_b}} \left[\left\| (A+\delta A)x - (b+\delta b) \right\|_2^2 + \rho \|x\|_2^2 \right] \tag{9}$$

where ρ is the penalization parameter. On the one hand, if $\delta A = \delta b = 0$, the solution $\hat{x} = \left(A^T A + \rho I\right)^{-1} A^T b$ is obtained, being ρ the regularization parameter in the absence of uncertainty. On the other hand, when the uncertainty is present, a new regularization parameter λ is obtained. The maximization solution of the index (9) results in

$$\min_{x} \left[\left(\|Ax-b\|_2 + \eta_A \|x\|_2 + \eta_b \right)^2 + \rho \|x\|_2^2 \right] \tag{10}$$

being a convex function in x. So via $\nabla J(x) = 0$ the global minimum can be obtained

$$x = \left(A^T A + \lambda I\right)^{-1} A^T b \tag{11}$$

$$\lambda = \frac{\eta_A \|Ax-b\|_2}{\|x\|_2} + \frac{\rho \|Ax-b\|_2}{\|Ax-b\|_2 + \eta_A \|x\|_2 + \eta_b} \tag{12}$$

As before, the solution is obtained by solving the nonlinear equations system formed by the equations (11) and (12), being (12) the nonlinear secular equation which depends on λ, A, b, ρ, η_A and η_b. It is noticeable that if $\rho = 0$ the solution is the same as the one of the previous section.

3. THE LINEAR QUADRATIC REGULATOR

3.1. FINITE PREDICTION HORIZON

The primary objective of the linear quadratic regulator (LQR) is to regulate the state of a linear state-space model to zero while keeping the control cost low. The following simple one-dimensional state-space model (A and b are scalars) is considered

$$x_{i+1} = Ax_i + bu_i \tag{13}$$

where x_0 denotes the value of the initial state, and the $\{u_i\}$ denote the control sequence. In the LQR problem, a control sequence $\{u_i\}$ that solves

$$\min_{\{u_i\}} \left(px_{N+1}^2 + \sum_{i=0}^{N} \left[qx_i^2 + ru_i^2 \right] \right) \tag{14}$$

is sought for some given $r, p > 0$ and $q \geq 0$ and over an interval of time $0 \leq i \leq N$. This problem can be solved recursively by splitting the cost function into two terms, where only the second term, through the state-space equation (13) for x_{i+1}, is dependent on u_i. Minimizing over u_i, the following state-feedback control law over $0 \leq i \leq N$ is obtained

$$\begin{aligned} \hat{u}_i &= -k_i x_i \\ k_i &= \frac{Abp_{i+1}}{r + b^2 p_{i+1}} \\ p_i &= A^2 p_{i+1} - \frac{A^2 b^2 p_{i+1}^2}{r + b^2 p_{i+1}} + q = \frac{rA^2 p_{i+1}}{r + b^2 p_{i+1}} + q \\ p_{N+1} &= p \end{aligned} \tag{15}$$

These equations show that the optimal control at time i is a scaled multiple of the state at the same time instant i, and the gain k_i is defined in terms of the given model parameters $\{A, b, r\}$ and in terms of the cost p_{i+1}, which is propagated via the Riccati recursion with boundary condition $p_{N+1} = p$.

3.2. INFINITE PREDICTION HORIZON

Sometimes, if the prediction horizon is too much small the system can result unstable. Considering an infinite horizon ensures stability, if and only if, first, the pair (A, b) is controllable or at least stabilizable, that means the closed-loop system poles are inside the unit circle, second, the pair (A, T) is detectable, being $q = T^T T$, and third, $r > 0$.

The one-dimensional state-space model considered holds the previous conditions, so it is possible to state the LQR problem with infinite prediction horizon and to obtain the control law $u_i = -kx_i$, $\forall i$, being k a constant. So the recursive Riccati equation from the previous section transforms into the following algebraic Riccati equation

$$
\begin{aligned}
u_i &= -kx_i \\
k &= \frac{Abp}{r + b^2 p} \\
p &= \frac{rA^2 p}{r + b^2 p} + q
\end{aligned}
\tag{16}
$$

4. LQR TUNED VIA BDU

4.1. FINITE PREDICTION HORIZON

The following one-dimensional state-space model with parametric uncertainties δA and δb is considered

$$x_{i+1} = \left(A + \delta A\right) x_i + \left(b + \delta b\right) u_i \tag{17}$$

where x_0 denotes the value of the initial state, and the $\{u_i\}$ denote the control sequence. A bound of the uncertainty is known ($\|\delta A\|_2 \le \eta_A, \|\delta b\|_2 \le \eta_b$) so that the LQR problem can be transformed into a BDU problem

$$\min_{\{u_i\}} \max_{\substack{\|\delta A\|_2 \le \eta_A \\ \|\delta b\|_2 \le \eta_b}} \left(p x_{N+1}^2 + \sum_{i=0}^{N} \left[q x_i^2 + r u_i^2 \right] \right) \tag{18}$$

for some given $r, p > 0$ and $q \ge 0$ and over an interval of time $0 \le i \le N$. As in the previous section the problem can be solved recursively by splitting the cost function into two terms, where only the second term, through the state-space equation (17) for x_{N+1}, is dependent on u_N. So the second term is considered

$$\min_{u_N} \max_{\substack{\|\delta A\|_2 \le \eta_A \\ \|\delta b\|_2 \le \eta_b}} \left[p x_{N+1}^2 + q x_N^2 + r u_N^2 \right] \tag{19}$$

and x_{N+1} is substituted. After that x_N is removed (because it does not depend on u_N and so it does not affect the minimization result), and through some changes of variables the term is stated as a BDU problem of type (9)

$$\min_{u_N} \max_{\substack{\|\delta A\|_2 \le \eta_A \\ \|\delta b\|_2 \le \eta_b}} \left[\left\| \left(\hat{A} + \delta A\right) u_N - \left(\hat{b} + \delta b\right) \right\|_2^2 + r \left\| u_N \right\|_2^2 \right] \tag{20}$$

where

$$
\begin{aligned}
\hat{A} &= p^{\frac{1}{2}} b \qquad & \hat{b} &= -p^{\frac{1}{2}} A x_N \\
\tilde{\delta} A &= p^{\frac{1}{2}} \delta b \qquad & \tilde{\delta} b &= -p^{\frac{1}{2}} \delta A x_N \\
\tilde{\eta}_A &= p^{\frac{1}{2}} \eta_b \qquad & \tilde{\eta}_b &= p^{\frac{1}{2}} \eta_A \left\| x_N \right\|_2 = p^{\frac{1}{2}} \eta_A \left| x_N \right|
\end{aligned}
\tag{21}
$$

The solution will be nonzero if the following condition holds

$$
\tilde{\eta}_A < \frac{\left\| \hat{A}^T \hat{b} \right\|_2}{\left\| \hat{b} \right\|_2} \tag{22}
$$

which after the changes of variables transforms into the condition (being $p^{1/2} \geq 0$)

$$
\eta_b < \left\| b \right\|_2 \tag{23}
$$

Minimizing over the variable u_N results in

$$
\begin{aligned}
u_N &= \left(\lambda_N I + \hat{A}^T \hat{A} \right)^{-1} \hat{A}^T \hat{b} \\
\lambda_N &= \frac{\tilde{\eta}_A \left\| \hat{A} u_N - \hat{b} \right\|_2}{\left\| u_N \right\|_2} + \frac{r \left\| \hat{A} u_N - \hat{b} \right\|_2}{\left\| \hat{A} u_N - \hat{b} \right\|_2 + \tilde{\eta}_A \left\| u_N \right\|_2 + \tilde{\eta}_b}
\end{aligned}
\tag{24}
$$

which through the changes of variables results in

$$
\begin{aligned}
u_N &= -\frac{Abp}{\lambda_N I + b^2 p} x_N \\
\lambda_N &= \frac{1}{1 + \frac{\eta_A}{|A|}} \left[\frac{r}{1 - \frac{\eta_b}{|b|}} - p b^2 \left[\frac{\eta_A}{|A|} + \frac{\eta_b}{|b|} \right] \right]
\end{aligned}
\tag{25}
$$

On the other hand, via the substitution of the solution u_N in (19) the p_N cost is obtained

$$
p_N = p A^2 \left(\frac{\lambda_N + \eta_b p |b|}{\lambda_N + b^2 p} + \frac{\eta_A}{|A|} \right)^2 + \frac{r A^2 b^2 p^2}{\left(\lambda_N + b^2 p \right)^2} + q \tag{26}
$$

The control law at time instant N is defined by the equations (24), (25) and (26). The control law for the rest of time instants $0 \le i \le N$ results in

$$
\begin{aligned}
u_i &= -k_i x_i \\
k_i &= \frac{Abp_{i+1}}{\lambda_i I + b^2 p_{i+1}} \\
\lambda_i &= \frac{1}{1+\frac{\eta_A}{|A|}}\left[\frac{r}{1-\frac{\eta_b}{|b|}} - p_{i+1}b^2\left[\frac{\eta_A}{|A|}+\frac{\eta_b}{|b|}\right]\right] \\
p_i &= p_{i+1}A^2\left(\frac{\lambda_i + \eta_b p_{i+1}|b|}{\lambda_i + b^2 p_{i+1}} + \frac{\eta_A}{|A|}\right)^2 + \frac{rA^2b^2p_{i+1}^2}{\left(\lambda_i + b^2 p_{i+1}\right)^2} + q
\end{aligned}
\tag{27}
$$

where the cost p_i is propagated via the recursive Riccati equation modified via the BDU technique with boundary condition $p_{N+1} = p$.

The parameter λ_i must be positive, otherwise it should be set to zero and the control law would be $k_i = A/b$. If the necessary BDU condition (23) does not hold, the parameter λ_i evaluates to a negative value, but if it holds it is not expected the parameter λ_i to be a positive number. The condition which ensures the positive value of λ_i is

$$\frac{r}{1-\frac{\eta_b}{|b|}} > p_{i+1}b^2\left[\frac{\eta_A}{|A|}+\frac{\eta_b}{|b|}\right] \tag{28}$$

4.2. INFINITE PREDICTION HORIZON

The cost function of the LQR with infinite prediction horizon

$$\min_{\{u_i\}} \max_{\substack{\|\delta A\|_2 \le \eta_A \\ \|\delta b\|_2 \le \eta_b}} J = \min_{\{u_i\}} \max_{\substack{\|\delta A\|_2 \le \eta_A \\ \|\delta b\|_2 \le \eta_b}} \left(\sum_{i=0}^{\infty}\left[qx_i^2 + ru_i^2\right]\right) \tag{29}$$

can be splitted into two terms

$$J = J_1 + J_2 = \sum_{i=0}^{\infty} qx_i^2 + \sum_{i=0}^{\infty} ru_i^2 \tag{30}$$

If the control law is assumed to be $u_i = -kx_i$ and the initial state x_0, the state of the closed-loop system will therefore evolve along the trajectory

$$x_{i+1} = \left(\left(A+\delta A\right)-\left(b+\delta b\right)k\right)^{i} x_0 \tag{31}$$

So, the cost function can be expressed as

$$J = \lim_{i\to\infty} L_{i+1}x_0^2 + \lim_{i\to\infty} L'_{i+1}x_0^2 \tag{32}$$

where

$$\begin{aligned} L_{i+1} &= \left(\left(A+\delta A\right)-\left(b+\delta b\right)k\right)^{2} L_i + q \\ L'_{i+1} &= \left(\left(A+\delta A\right)-\left(b+\delta b\right)k\right)^{2} L'_i + rk^2 \end{aligned} \tag{33}$$

If the previous series converge (the cost function value is bounded), the system stability is ensured

$$J = \left[L_\infty + L'_\infty\right]x_0^2 = px_0^2 \tag{34}$$

and the algebraic Riccati equation is obtained

$$p = \left(\left(A+\delta A\right)-\left(b+\delta b\right)k\right)^{2} p + q + rk^2 \tag{35}$$

This equation depends on k, which can be calculated by minimizing J w.r.t. u_0 and so the control law $u_0 = -kx_0$ is obtained. The state x_0 can be removed from the minimization since it does not depend on u_0

$$J = \left(\left(A+\delta A\right)-\left(b+\delta b\right)k\right)^{2} px_0^2 + qx_0^2 + rk^2x_0^2 \tag{36}$$

and through the changes of variables the cost is stated as a BDU problem [6]

$$\min_{u_0} \max_{\substack{\|\delta A\|_2 \le \eta_A \\ \|\delta b\|_2 \le \eta_b}} \left[\left\| \left(\hat{A}+\delta A\right)u_0 - \left(\hat{b}+\delta b\right)\right\|_2^2 + r\left\|u_0\right\|_2^2 \right] \tag{37}$$

where

$$
\begin{aligned}
\hat{A} &= p^{\frac{1}{2}} b & \hat{b} &= -p^{\frac{1}{2}} A x_0 \\
\widehat{\delta A} &= p^{\frac{1}{2}} \delta b & \widehat{\delta b} &= -p^{\frac{1}{2}} \delta A x_0 \\
\hat{\eta}_A &= p^{\frac{1}{2}} \eta_b & \hat{\eta}_b &= p^{\frac{1}{2}} \eta_A \|x_0\|_2 = p^{\frac{1}{2}} \eta_A |x_0|
\end{aligned}
\tag{38}
$$

Minimizing over the variable u_0 the solution $u_0 = \left(\hat{A}^T \hat{A} + \lambda I\right)^{-1} \hat{A}^T \hat{b}$ is obtained, and through the changes of variables results in

$$
u_0 = -k x_0 = -\frac{Abp}{\lambda I + b^2 p} x_0 \tag{39}
$$

It has been proved that the constant gain k exists, so both variables p and λ are also constant. From equations (35) and (39) the control law u_i for $0 \le i \le N$ can be obtained

$$
\begin{aligned}
u_i &= -k x_i \\
k &= \frac{Abp}{\lambda I + b^2 p} \\
\lambda &= \frac{1}{1 + \frac{\eta_A}{|A|}} \left[\frac{r}{1 - \frac{\eta_b}{|b|}} - p b^2 \left[\frac{\eta_A}{|A|} + \frac{\eta_b}{|b|} \right] \right] \\
p &= p A^2 \left(\frac{\lambda + \eta_b p |b|}{\lambda + b^2 p} + \frac{\eta_A}{|A|} \right)^2 + \frac{r A^2 b^2 p^2}{\left(\lambda + b^2 p\right)^2} + q
\end{aligned}
\tag{40}
$$

As in the previous section, the condition which ensures that λ is a positive value is

$$
\frac{r}{1 - \frac{\eta_b}{|b|}} > p b^2 \left[\frac{\eta_A}{|A|} + \frac{\eta_b}{|b|} \right] \tag{41}
$$

5. EXAMPLE

This example shows how the use of the BDU technique for the LQR tuning can improve the system performance. The following system $x_{i+1} = \left(A + \delta A\right) x_i + \left(b + \delta b\right) u_i$ is considered, where $A = 0.9$, $b = 1$, $\eta_A = 0.2$ and $\eta_b = 0.27$ (it is assumed $\delta A = 0.2$ and $\delta b = -0.27$). The LQR parameters are $p = r = 1$ and $q = 0.04$, being $x_0 = 10$ and $N = 80$

(finite prediction horizon). The necessary BDU condition holds, so $0.27 = \eta_b < \|b\|_2 = 1$. In Figure 1 the closed-loop response with the LQR is shown. The system turns unstable (the closed-loop pole in steady-state tends to 1.02).

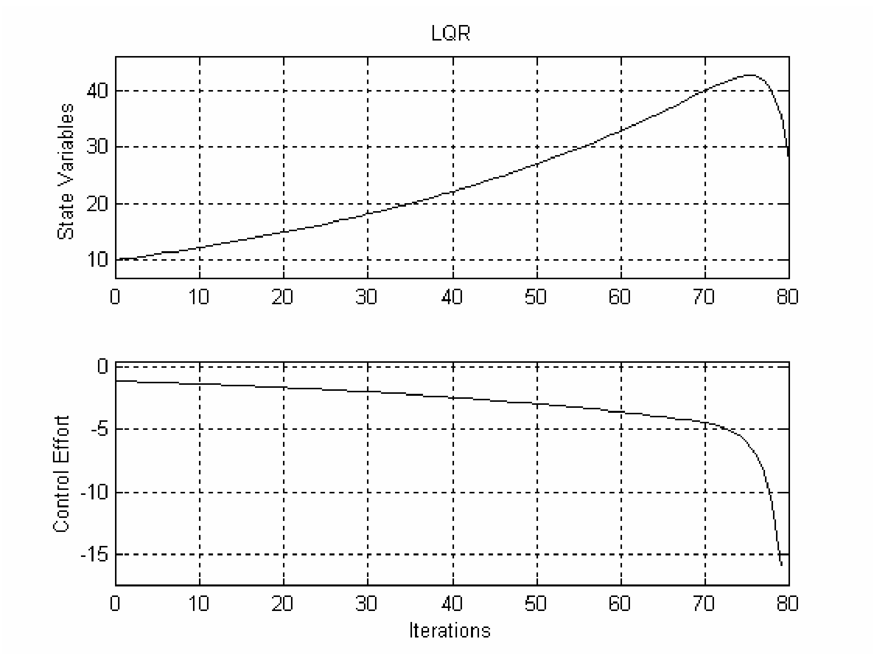

Fig. 1. LQR with finite horizon

If the LQR is tuned via the BDU technique the system turns stable (the closed-loop pole in steady-state tends to 0.84) as it can be seen in Figure 2.

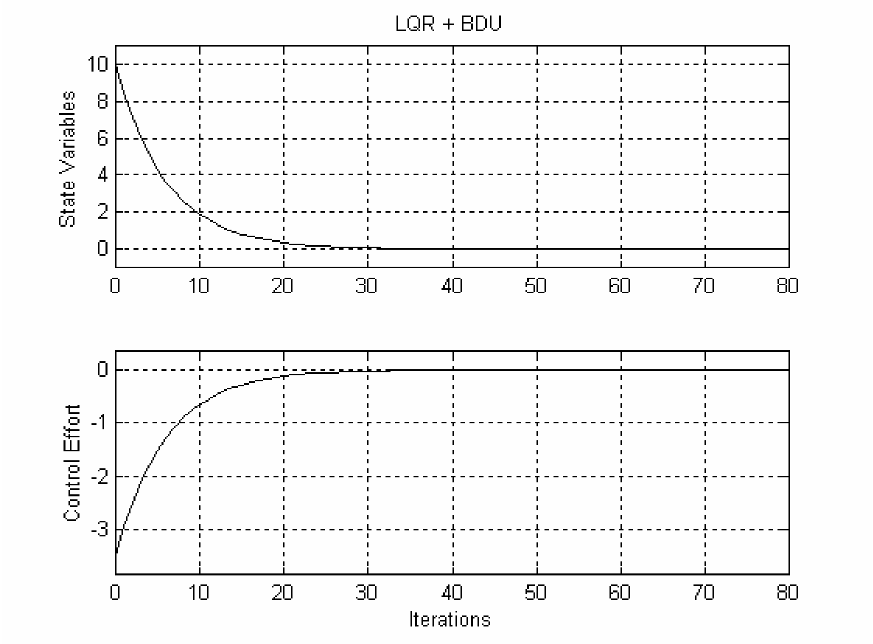

Fig. 2. LQR/BDU with finite horizon

6. CONCLUSIONS AND FUTURE WORK

The main conclusion of the paper consists of using the BDU technique for tuning the LQR, in one way that the system robustness is improved. So, the empirically tuned parameters p and r are modified taking into account the uncertainties bounds. It implies the modification of the recursive and algebraic Riccati equations.

As future work, it is desired the extension to multidimensional systems, which is not trivial, because in this case both the gain k_i and the term p_i depend on x_i, and the solution has the form of a two-point boundary value problem (TPBVP) and is obtained iteratively.

More future work consists of the application of the BDU method to other controllers, such as the Predictive Control. So, the penalization parameter λ of the control effort can be tuned taking into account the uncertainties bounds. The application is easy since the Predictive Controllers can be expressed in a least-squares format.

REFERENCES

[1] CHANDRASEKARAN S., GOLUB M., GU M., SAYED A.H., *Worst-case parameter estimation with bounded model uncertainties*, American Control Conference, 1997.

[2] CHANDRASEKARAN S., GOLUB M., GU M., SAYED A.H., *Parameter estimation in the presence of bounded data uncertainties*, SIMAX, 1998, No. 19, pp. 235-252.

[3] GOLUB G.H., VAN LOAN CH.F., *Matrix Computations*, Johns Hopkins University Press, 1996.

[4] LAWSON C.L., HANSON R.J., *Solving least-squares problems*, SIAM, 1995.

[5] NASCIMENTO V.H., SAYED A.H., *Optimal state regulation for uncertain state-space models*, American Control Conference, Vol. 1, San Diego 1999, pp. 419-424.

[6] RAMOS C., SANCHIS J., MARTÍNEZ M., HERRERO J.M., *Sintonizado del LQR y Control Predictivo mediante BDU*, XI Congreso Latinoamericano de Control, Preprints ISBN 959-237-117-2, 2004.

[7] SAYED A.H., NASCIMENTO V.H., *Design criteria for uncertain models with structured and unstructured uncertainties*, Robustness in Identification and Control, Vol. 245, Springer Verlag, London 1999, pp. 159-173.

[8] SAYED A.H., NASCIMENTO V.H., CHANDRASEKARAN S., *Estimation and control with bounded data uncertainties*, Linear Algebra and its Applications, Vol. 284, Elsevier, 1998, pp. 259-306.

[9] SKOGESTAD S., POSTLETHWAITE I., *Multivariable feedback control. Analysis and Design*, John Wiley and Sons, 1996.

[10] TIKHONOV A.N., *Regularization of incorrectly posed problems*, Soviet Math, Vol. 4, 1963, pp. 1624-1627.