

Shrinkage, Cracking and Deflection- the Serviceability of Concrete Structures

R.I. Gilbert

Professor and Head, School of Civil and Environmental Engineering
The University of New South Wales, Sydney, NSW, 2052
Email: i.gilbert@unsw.edu.au

ABSTRACT

This paper addresses the effects of shrinkage on the serviceability of concrete structures. It outlines why shrinkage is important, its major influence on the final extent of cracking and the magnitude of deflection in structures, and what to do about it in design. A model is presented for predicting the shrinkage strain in normal and high strength concrete and the time-dependent behaviour of plain concrete and reinforced concrete, with and without external restraints, is explained. Analytical procedures are described for estimating the final width and spacing of both flexural cracks and direct tension cracks and a simplified procedure is presented for including the effects of shrinkage when calculating long-term deflection. The paper also contains an overview of the considerations currently being made by the working group established by Standards Australia to revise the serviceability provisions of AS3600-1994, particularly those clauses related to shrinkage.

KEYWORDS

Creep; Cracking; Deflection; Reinforced concrete; Serviceability; Shrinkage.

1. Introduction

For a concrete structure to be serviceable, cracking must be controlled and deflections must not be excessive. It must also not vibrate excessively. Concrete shrinkage plays a major role in each of these aspects of the service load behaviour of concrete structures.

The design for serviceability is possibly the most difficult and least well understood aspect of the design of concrete structures. Service load behaviour depends primarily on the properties of the concrete and these are often not known reliably at the design stage. Moreover, concrete behaves in a non-linear and inelastic manner at service loads. The non-linear behaviour that complicates serviceability calculations is due to cracking, tension stiffening, creep, and shrinkage. Of these, shrinkage is the most problematic. Restraint to shrinkage causes time-dependent cracking and gradually reduces the beneficial effects of tension stiffening. It results in a gradual widening of existing cracks and, in flexural members, a significant increase in deflections with time.

The control of cracking in a reinforced or prestressed concrete structure is usually achieved by limiting the stress increment in the bonded reinforcement to some appropriately low value and ensuring that the bonded reinforcement is suitably distributed. Many codes of practice specify maximum steel stress increments after cracking and maximum spacing requirements for the bonded reinforcement. However, few existing code procedures, if any, account adequately for the gradual increase in existing crack widths with time, due primarily to shrinkage, or the time-dependent development of new cracks resulting from tensile stresses caused by restraint to shrinkage.

For deflection control, the structural designer should select *maximum deflection limits* that are appropriate to the structure and its intended use. The calculated deflection (or camber) must not

exceed these limits. Codes of practice give general guidance for both the selection of the maximum deflection limits and the calculation of deflection. However, the simplified procedures for calculating deflection in most codes were developed from tests on simply-supported reinforced concrete beams and often produce grossly inaccurate predictions when applied to more complex structures. Again, the existing code procedures do not provide real guidance on how to adequately model the time-dependent effects of creep and shrinkage in deflection calculations.

Serviceability failures of concrete structures involving excessive cracking and/or excessive deflection are relatively common. Numerous cases have been reported, in Australia and elsewhere, of structures that complied with code requirements but still deflected or cracked excessively. In a large majority of these failures, shrinkage of concrete is primarily responsible. Clearly, the serviceability provisions embodied in our codes do not adequately model the in-service behaviour of structures and, in particular, fail to account adequately for shrinkage.

The quest for serviceable concrete structures must involve the development of more reliable design procedures. It must also involve designers giving more attention to the specification of an appropriate concrete mix, particularly with regard to the creep and shrinkage characteristics of the mix, and sound engineering input is required in the construction procedures. High performance concrete structures require the specification of high performance concrete (not necessarily high strength concrete, but concrete with relatively low shrinkage, not prone to plastic shrinkage cracking) and a high standard of construction, involving suitably long stripping times, adequate propping, effective curing procedures and rigorous on-site supervision.

This paper addresses some of these problems, particularly those related to designing for the effects of shrinkage. It outlines how shrinkage affects the in-service behaviour of structures and what to do about it in design. It also provides an overview of the considerations currently being made by the working group established by Standards Australia to revise the serviceability provisions of AS3600-1994 [1], particularly those clauses related to shrinkage.

2. Designing for Serviceability

When designing for serviceability, the designer must ensure that the structure can perform its intended function under the day to day service loads. Deflection must not be excessive, cracks must be adequately controlled and no portion of the structure should suffer excessive vibration. Shrinkage causes time-dependent cracking, thereby reducing the stiffness of a concrete structure, and is therefore a detrimental factor in all aspects of the design for serviceability.

Deflection problems that may affect the serviceability of concrete structures can be classified into three main types:

- (a) Where excessive deflection causes either aesthetic or functional problems.
- (b) Where excessive deflection results in damage to either structural or non-structural element attached to the member.
- (c) Where dynamics effects due to insufficient stiffness cause discomfort to occupants.

Examples of deflection problems of type (a) include objectionable visual sagging (or hogging), and ponding of water on roofs. In fact, any deflection that prevents a member fulfilling its intended function causes a problem of this type. Type (a) problems are generally overcome by limiting the total deflection to some appropriately low value. The total deflection is the sum of the short-term and time-dependent deflection caused by the dead load (including self-weight), the prestress (if any), the expected in-service live load, and the load-independent effects of shrinkage and temperature changes. When the total deflection exceeds about span/200 below the horizontal, it may become visually unacceptable. The designer must decide on the maximum limiting value for the total deflection and

this limit must be appropriate for the particular member and its intended function. A total deflection limit of span/200, for example, may be appropriate for the floor of a carpark, but is inadequate for a gymnasium floor which may be required to remain essentially plane under service conditions.

Examples of type (b) problems include deflections resulting in cracking of masonry walls or other partitions, damage to ceiling or floor finishes, and improper functioning of sliding windows and doors. To avoid these problems, a limit must be placed on that part of the total deflection that occurs after the attachment of such elements. This *incremental deflection* is usually the sum of the long-term deflection due to all the sustained loads and shrinkage, the short-term deflection due to the transitory live load, and any temperature-induced deflection. AS 3600 (1994) [1] limits the incremental deflection for members supporting masonry partitions to between span/500 and span/1000, depending on the provisions made to minimise the effect of movement.

Type (c) deflection problems include the perceptible springy vertical motion of floor systems and other vibration-related problems. Very little quantitative information for controlling vibration is available in codes of practice. ACI 318-99 [2] places a limit of span/360 on the short-term deflection of a floor due to live load. This limit provides a minimum requirement on the stiffness of members that may, in some cases, be sufficient to avoid problems of type (c).

Excessively wide cracks can be unsightly and spoil the appearance of an exposed concrete surface; they can allow the ingress of moisture accelerating corrosion of the reinforcement and durability failure; and, in exceptional cases, they can reduce the contribution of the concrete to the shear strength of a member. Excessively wide cracks in floor systems and walls may often be avoided by the inclusion of strategically placed contraction joints, thereby removing some of the restraint to shrinkage and reducing the internal tension. When cracking does occur, in order to ensure that crack widths remain acceptably small, adequate quantities of well distributed and well-anchored reinforcement must be included at every location where significant tension will exist.

The maximum crack width that may be considered to be acceptable in a given situation, depends on the type of structure, the environment and the consequences of excessive cracking. In corrosive and aggressive environments, crack widths should not exceed 0.1 - 0.2 mm. For members with one or more exposed surfaces, a maximum crack width of 0.3 mm should provide visual acceptability. For the sheltered interior of most buildings where the concrete is not exposed and aesthetic requirements are of secondary importance, larger crack widths may be acceptable (say 0.5 mm or larger).

3. Effects of Shrinkage

If concrete members were free to shrink, without restraint, shrinkage of concrete would not be a major concern to structural engineers. However, this is not the case. The contraction of a concrete member is often restrained by its supports or by the adjacent structure. Bonded reinforcement also restrains shrinkage. Each of these forms of restraint involve the imposition of a gradually increasing tensile force on the concrete which may lead to time-dependent cracking (in previously uncracked regions), increases in deflection and a widening of existing cracks. Restraint to shrinkage is probably the most common cause of unsightly cracking in concrete structures. In many cases, these problems arise because shrinkage has not been adequately considered by the structural designer and the effects of shrinkage are not adequately modelled in the design procedures specified in codes of practice for crack control and deflection calculation.

The advent of shrinkage cracking depends on the degree of restraint to shrinkage, the extensibility and strength of the concrete in tension, tensile creep and the load induced tension existing in the member. Cracking can only be avoided if the gradually increasing tensile stress induced by shrinkage, and reduced by creep, is at all times less than the tensile strength of the concrete. Although the tensile

strength of concrete increases with time, so too does the elastic modulus and, therefore, so too does the tensile stress induced by shrinkage. Furthermore, the relief offered by creep decreases with age. The existence of load induced tension in uncracked regions accelerates the formation of time-dependent cracking. In many cases, therefore, shrinkage cracking is inevitable. The control of such cracking requires two important steps. First, the shrinkage-induced tension and the regions where shrinkage cracks are likely to develop must be recognised by the structural designer. Second, an adequate quantity and distribution of anchored reinforcement must be included in these regions to ensure that the cracks remain fine and the structure remains serviceable.

3.1 What is Shrinkage?

Shrinkage of concrete is the time-dependent strain measured in an unloaded and unrestrained specimen at constant temperature. It is important from the outset to distinguish between plastic shrinkage, chemical shrinkage and drying shrinkage. Some high strength concretes are prone to plastic shrinkage, which occurs in the wet concrete, and may result in significant cracking during the setting process. This cracking occurs due to capillary tension in the pore water. Since the bond between the plastic concrete and the reinforcement has not yet developed, the steel is ineffective in controlling such cracks. This problem may be severe in the case of low water content, silica fume concrete and the use of such concrete in elements such as slabs with large exposed surfaces is not recommended.

Drying shrinkage is the reduction in volume caused principally by the loss of water during the drying process. Chemical (or endogenous) shrinkage results from various chemical reactions within the cement paste and includes hydration shrinkage, which is related to the degree of hydration of the binder in a sealed specimen. Concrete shrinkage strain, which is usually considered to be the sum of the drying and chemical shrinkage components, continues to increase with time at a decreasing rate. Shrinkage is assumed to approach a final value, ϵ_{sc}^* , as time approaches infinity and is dependent on all the factors which affect the drying of concrete, including the relative humidity and temperature, the mix characteristics (in particular, the type and quantity of the binder, the water content and water-to-cement ratio, the ratio of fine to coarse aggregate, and the type of aggregate), and the size and shape of the member.

Drying shrinkage in high strength concrete is smaller than in normal strength concrete due to the smaller quantities of free water after hydration. However, endogenous shrinkage is significantly higher.

For normal strength concrete ($f'_c \leq 50$ MPa), AS3600 suggests that the design shrinkage (which includes both drying and endogenous shrinkage) at any time after the commencement of drying may be estimated from

$$\epsilon_{cs} = k_1 \epsilon_{cs,b} \quad (1)$$

where $\epsilon_{cs,b}$ is a basic shrinkage strain which, in the absence of measurements, may be taken to be 850×10^{-6} (note that this value was increased from 700×10^{-6} in the recent Amendment 2 of the Standard); k_1 is obtained by interpolation from Figure 6.1.7.2 in the Standard and depends on the time since the commencement of drying, the environment and the concrete surface area to volume ratio. A hypothetical thickness, $t_h = 2A/u_e$, is used to take this into account, where A is the cross-sectional area of the member and u_e is that portion of the section perimeter exposed to the atmosphere plus half the total perimeter of any voids contained within the section.

AS3600 states that the actual shrinkage strain may be within a range of plus or minus 40% of the value predicted (increased from $\pm 30\%$ in Amendment 2 to AS3600-1994). In the writer's opinion,

this range is still optimistically narrow, particularly when one considers the size of the country and the wide variation in shrinkage measured in concretes from the various geographical locations. Equation 1 does not include any of the effects related to the composition and quality of the concrete. The same value of ϵ_{cs} is predicted irrespective of the concrete strength, the water-cement ratio, the aggregate type and quantity, the type of admixtures, etc. In addition, the factor k_1 tends to overestimate the effect of member size and significantly underestimate the rate of shrinkage development at early ages.

The method should be used only as a guide for concrete with a low water-cement ratio (<0.4) and with a well graded, good quality aggregate. Where a higher water-cement ratio is expected or when doubts exist concerning the type of aggregate to be used, the value of ϵ_{cs} predicted by AS3600 should be increased by at least 50%. The method in the Standard for the prediction of shrinkage strain is currently under revision and it is quite likely that significant changes will be proposed with the inclusion of high strength concretes.

A proposal currently being considered by Standards Australia, and proposed by Gilbert (1998) [9], involves the total shrinkage strain, ϵ_{cs} , being divided into two components, endogenous shrinkage, ϵ_{cse} , (which is assumed to develop relatively rapidly and increases with concrete strength) and drying shrinkage, ϵ_{csd} (which develops more slowly, but decreases with concrete strength). At any time t (in days) after pouring, the endogenous shrinkage is given by

$$\epsilon_{cse} = \epsilon_{cse}^* (1.0 - e^{-0.1t}) \quad (2)$$

where ϵ_{cse}^* is the final endogenous shrinkage and may be taken as $\epsilon_{cse}^* = (3f'_c - 50) \times 10^{-6}$, where f'_c is in MPa. The basic drying shrinkage ϵ_{csd}^* is given by

$$\epsilon_{csd}^* = (1100 - 8f'_c) \times 10^{-6} \geq 250 \times 10^{-6} \quad (3)$$

and at any time t (in days) after the commencement of drying, the drying shrinkage may be taken as

$$\epsilon_{csd} = k_1 \epsilon_{csd}^* \quad (4)$$

The variable k_1 is given by
$$k_1 = \frac{k_4 k_5 t^{0.8}}{t^{0.8} + (t_h / 7)} \quad (5)$$

where $k_4 = 0.8 + 1.2e^{-0.005t_h}$ and k_5 is equal to 0.7 for an arid environment, 0.6 for a temperate environment and 0.5 for a tropical/coastal environment. For an interior environment, k_5 may be taken as 0.65. The value of k_1 given by Equation 5 has the same general shape as that given in Figure 6.1.7.2 in AS3600, except that shrinkage develops more rapidly at early ages and the reduction in drying shrinkage with increasing values of t_h is not as great.

The final shrinkage at any time is therefore the sum of the endogenous shrinkage (Equation 2) and the drying shrinkage (Equation 4). For example, for specimens in an interior environment with hypothetical thicknesses $t_h = 100$ mm and $t_h = 400$ mm, the shrinkage strains predicted by the above model are given in Table 1.

Table 1 Design shrinkage strains predicted by proposed model for an interior environment.

t_h	f'_c	ϵ_{cse}^* ($\times 10^{-6}$)	ϵ_{csd}^* ($\times 10^{-6}$)	Strain at 28 days ($\times 10^{-6}$)			Strain at 10000 days ($\times 10^{-6}$)		
				ϵ_{cse}	ϵ_{csd}	ϵ_{cs}	ϵ_{cse}	ϵ_{csd}	ϵ_{cs}
100	25	25	900	23	449	472	25	885	910
	50	100	700	94	349	443	100	690	790
	75	175	500	164	249	413	175	493	668
	100	250	300	235	150	385	250	296	546
400	25	25	900	23	114	137	25	543	568
	50	100	700	94	88	182	100	422	522
	75	175	500	164	63	227	175	303	478
	100	250	300	235	38	273	250	182	432

3.2 Shrinkage in Unrestrained and Unreinforced Concrete (Gilbert, 1988) [7]

Drying shrinkage is greatest at the surfaces exposed to drying and decreases towards the interior of a concrete member. In Fig. 1a, the shrinkage strains through the thickness of a plain concrete slab, drying on both the top and bottom surfaces, are shown. The slab is unloaded and unrestrained.

The mean shrinkage strain, ϵ_{cs} in Fig. 1, is the average contraction. The non-linear strain labelled $\Delta\epsilon_{cs}$ is that portion of the shrinkage strain that causes internal stresses to develop. These self-equilibrating stresses (called eigenstresses) produce the elastic and creep strains required to restore compatibility (ie. to ensure that plane sections remain plane). These stresses occur in all concrete structures and are tensile near the drying surfaces and compressive in the interior of the member. Because the shrinkage-induced stresses develop gradually with time, they are relieved by creep. Nevertheless, the tensile stresses near the drying surfaces often overcome the tensile strength of the immature concrete and result in surface cracking, soon after the commencement of drying. Moist curing delays the commencement of drying and may provide the concrete time to develop sufficient tensile strength to avoid unsightly surface cracking.

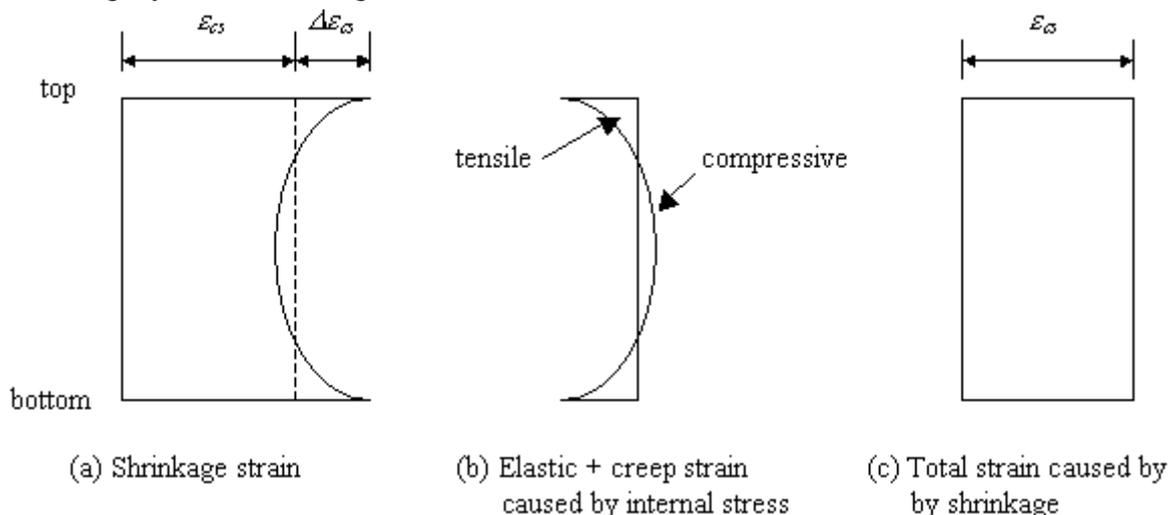


Fig. 1 - Strain components caused by shrinkage in a plain concrete slab.

The elastic plus creep strains caused by the eigenstresses are equal and opposite to $\Delta\epsilon_{cs}$ and are shown in Fig. 1b. The total strain distribution, obtained by summing the elastic, creep and shrinkage strain components, is linear (Fig. 1c) thus satisfying compatibility. If the drying conditions are the

same at both the top and bottom surfaces, the total strain is uniform over the depth of the slab and equal to the mean shrinkage strain, ϵ_{cs} . It is this quantity that is usually of significance in the analysis of concrete structures. If drying occurs at a different rate from the top and bottom surfaces, the total strain distribution becomes inclined and a warping of the member results.

3.3 Shrinkage in an unrestrained reinforced concrete member (Gilbert, 1986)

In concrete structures, unrestrained contraction and unrestrained warping are unusual. Reinforcement embedded in the concrete provides restraint to shrinkage. As the concrete shrinks, the reinforcement is compressed and imposes an equal and opposite tensile force on the concrete at the level of the reinforcement. If the reinforcement is not symmetrically placed on a section, a shrinkage-induced curvature develops with time. Shrinkage in an unsymmetrically reinforced concrete beam or slab can produce deflections of significant magnitude, even if the beam is unloaded.

Consider the unrestrained, singly reinforced, simply-supported concrete beam shown in Figure 2a and the small beam segment of length Δx . The shrinkage induced stresses and strains on an uncracked and on a cracked cross-section are shown in Figures 2b and 2c, respectively.

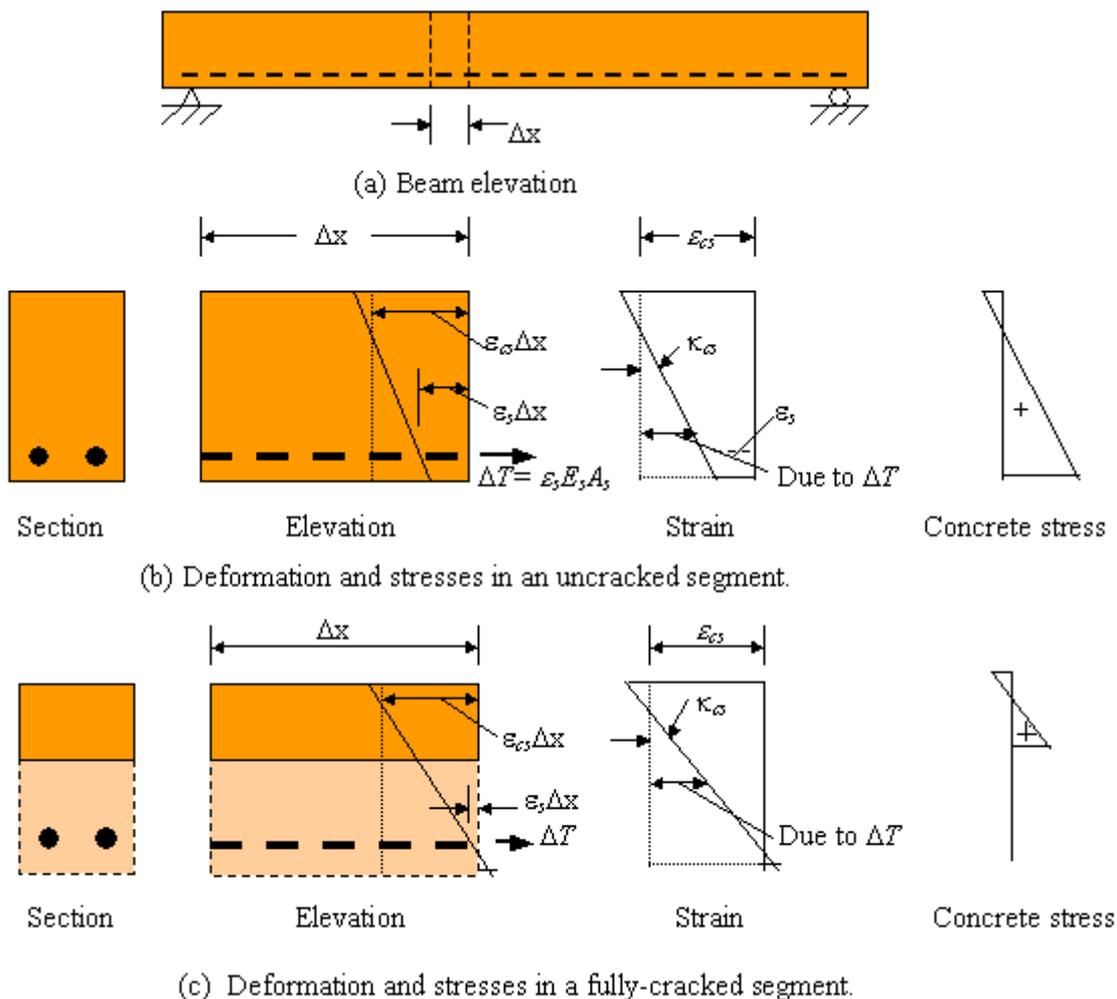


Fig. 2 - Shrinkage warping in a singly reinforced beam.

As the concrete shrinks, the bonded reinforcement imposes a tensile restraining force, ΔT , on the concrete at the level of the steel. This gradually increasing tensile force, acting at some eccentricity to the centroid of the concrete cross-section, produces curvature (elastic plus creep) and a gradual

warping of the beam. It also may cause cracking on an uncracked section or an increase in the width of existing cracks in a cracked member. For a particular shrinkage strain, the magnitude of ΔT depends on the quantity of reinforcement and on whether or not the cross-section has cracked.

Shrinkage strain is independent of stress, but shrinkage warping is not independent of the load and is significantly greater in a cracked beam than in an uncracked beam, as indicated in Fig. 2. The ability of the concrete section to carry tensile stress depends on whether or not the section has cracked, ie. on the magnitude of the applied moment, among other things. ΔT is much larger on the uncracked section of Fig. 2b than on the cracked section of Fig. 2c. Existing design procedures for the calculation of long-term deflection fail to adequately model the additional cracking that occurs with time due to ΔT and the gradual breakdown of tension stiffening with time (also due to ΔT), and consequently often greatly underestimate final deformations.

Compressive reinforcement reduces shrinkage curvature. By providing restraint at the top of the section, in addition to the restraint at the bottom, the eccentricity of the resultant tension in the concrete is reduced and, consequently, so is the shrinkage curvature. An uncracked, symmetrically reinforced section will suffer no shrinkage curvature. Shrinkage will however induce a uniform tensile stress which when added to the tension caused by external loading may cause time-dependent cracking.

3.4 Shrinkage in a restrained reinforced concrete member (Gilbert, 1992) [8]

Structural interest in shrinkage goes beyond its tendency to increase deflections due to shrinkage warping. External restraint to shrinkage is often provided by the supports of a structural member and by the adjacent structure. When flexural members are also restrained at the supports, shrinkage causes a build-up of axial tension in the member, in addition to the bending caused by the external loads. Shrinkage is usually accommodated in flexural members by an increase in the widths of the numerous flexural cracks. However, for members not subjected to significant bending and where restraint is provided to the longitudinal movements caused by shrinkage and temperature changes, cracks tend to propagate over the full depth of the cross-section. Excessively wide cracks are not uncommon. Such cracks are often called *direct tension cracks*, since they are caused by direct tension rather than by flexural tension. In fully restrained direct tension members, relatively large amounts of reinforcement are required to control the load independent cracking.

Consider the fully-restrained member shown in Fig. 3a. As the concrete shrinks, the restraining force $N(t)$ gradually increases until the first crack occurs when $N(t) = A_c f_t$, usually within two weeks from the commencement of drying, where A_c is the cross-sectional area of the member and f_t is the tensile strength of the concrete. Immediately after first cracking, the restraining force reduces to N_{cr} , and the concrete stress away from the crack is less than the tensile strength of the concrete. The concrete on either side of the crack shortens elastically and the crack opens to a width w , as shown in Fig. 3b. At the crack, the steel carries the entire force N_{cr} and the stress in the concrete is obviously zero. In the region immediately adjacent to the crack, the concrete and steel stresses vary considerably and there exists a region of partial bond breakdown. At some distance s_o on each side of the crack, the concrete and steel stresses are no longer influenced directly by the presence of the crack, as shown in Figs 3c and 3d.

In Region 1, where the distance from the crack is greater than or equal to s_o , the concrete and steel stresses are σ_{c1} and σ_{s1} , respectively. Since the steel stress (and hence strain) at the crack is tensile and the overall elongation of the steel is zero (full restraint), σ_{s1} must be compressive. Equilibrium requires that the sum of the forces carried by the concrete and the steel on any cross-section is equal to the restraining force. Therefore, with the force in the steel in Region 1 being compressive, the

force carried by the concrete ($A_c \sigma_{c1}$) must be tensile and somewhat greater than the restraining force (N_{cr}). In Region 2, where the distance from the crack is less than s_o , the concrete stress varies from zero at the crack to σ_{c1} at s_o from the crack. The steel stress varies from σ_{s2} (tensile) at the crack to σ_{s1} (compressive) at s_o from the crack, as shown.

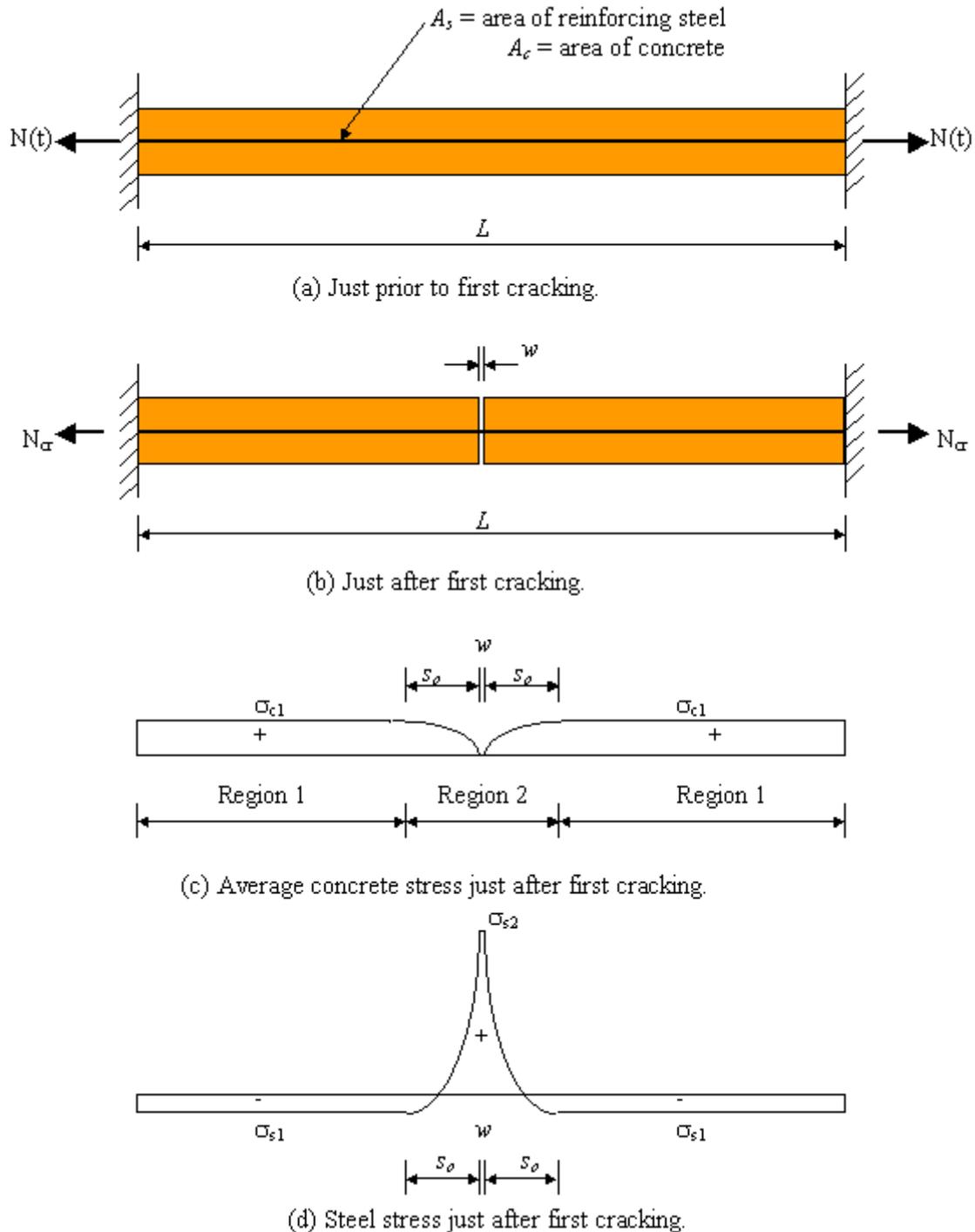


Fig. 3 - First cracking in a restrained direct tension member.

To determine the crack width w and the concrete and steel stresses in Fig. 3, the distance s_o over which the concrete and steel stresses vary, needs to be known and the restraining force N_{cr} needs to

be calculated. An approximation for s_o maybe obtained using the following equation, which was proposed by Favre et al. (1983) [6] for a member containing deformed bars or welded wire mesh:

$$s_o = d_b / 10 \rho \tag{6}$$

where d_b is the bar diameter, and ρ is the reinforcement ratio A_s / A_c . Base and Murray (1982) used a similar expression.

Gilbert (1992) showed that the concrete and steel stresses immediately after first cracking are

$$\sigma_{c1} = \frac{N_{cr} - \sigma_{s1} A_s}{A_c} = \frac{N_{cr}(1 + C_1)}{A_c}; \quad \sigma_{s1} = \frac{2 s_o}{3 L - 2 s_o} \frac{N_{cr}}{A_s} = -C_1 \frac{N_{cr}}{A_s}; \quad \text{and} \quad \sigma_{s2} = \frac{N_{cr}}{A_s} \tag{7}$$

where $C_1 = 2 s_o / (3L - 2 s_o)$. If n is the modular ratio, E_s / E_c , the restraining force immediately after first cracking is

$$N_{cr} = \frac{n \rho f_t A_c}{C_1 + n \rho (1 + C_1)} \tag{8}$$

With the stresses and deformations determined immediately after first cracking, the subsequent long-term behaviour as shrinkage continues must next be determined. After first cracking, the concrete is no longer fully restrained since the crack width can increase with time as shrinkage continues. A state of partial restraint therefore exists after first cracking. Subsequent shrinkage will cause further gradual increases in the restraining force $N(t)$ and in the concrete stress away from the crack, and a second crack may develop. Additional cracks may occur as the shrinkage strain continues to increase with time. However, as each new crack forms, the member becomes less stiff and the amount of shrinkage required to produce each new crack increases. The process continues until the crack pattern is established, usually in the first few months after the commencement of drying. The concrete stress history in an uncracked region is shown diagrammatically in Fig. 4. The final average crack spacing, s , and the final average crack width, w , depend on the quantity and distribution of reinforcement, the quality of bond between the concrete and steel, the amount of shrinkage, and the concrete strength. Let the final shrinkage-induced restraining force be $N(\infty)$.

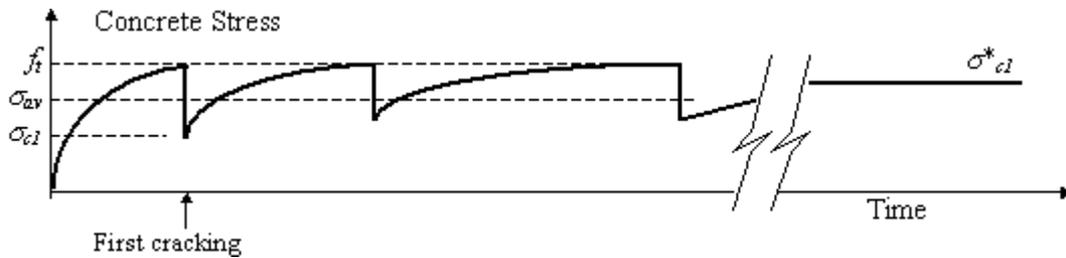


Fig. 4 - Concrete stress history in uncracked Region 1 (Gilbert, 1992) [8]

After all shrinkage has taken place and the final crack pattern is established, the average concrete stress at a distance greater than s_o from the nearest crack is σ^*_{c1} and the steel stresses at a crack and at a distance greater than s_o from a crack are σ^*_{s2} and σ^*_{s1} , respectively. Gilbert (1992) [8] developed the following expressions for the final restraining force $N(\infty)$ and the final average crack width w : Provided the steel quantity is sufficiently large, so that yielding does not occur at first cracking or subsequently, the final restraining force is given by

$$N(\infty) = - \frac{n^* A_s}{C_2} (\sigma_{av} + \epsilon_{cs}^* E_e^*) \tag{9}$$

ϵ_{cs}^* is the final shrinkage strain; E_e^* is the final effective modulus of the concrete and is given by $E_e^* = E_c / (1 + \phi^*)$; ϕ^* is the final creep coefficient; n^* is the effective modular ratio (E_s / E_e^*); $C_2 =$

$2s_o/(3s - 2s_o)$; and σ_{av} is the average stress in the uncracked concrete (see Fig. 4) and may be assumed to be $(\sigma_{c1} + f_t)/2$. The maximum crack spacing is

$$s = \frac{2 s_o (1 + \xi)}{3\xi} \quad (10)$$

and ξ is given by

$$\xi = \frac{-n^* \rho (\sigma_{av} + \varepsilon_{cs}^* E_e^*)}{n^* \rho (\sigma_{av} + \varepsilon_{cs}^* E_e^*) + f_t} \quad (11)$$

The final steel stress at each crack and the final concrete stress in Regions 1 (further than s_o from a crack) are, respectively,

$$\sigma_{s2}^* = N(\infty)/A_s \quad \text{and} \quad \sigma_{c1}^* = N(\infty)(1 + C_2)/A_c < f_t \quad (12)$$

Provided the steel at the crack has not yielded, the final crack width is given by

$$w = - \left[\frac{\sigma_{c1}^*}{E_e^*} \left(s - \frac{2}{3} s_o \right) + \varepsilon_{cs}^* s \right] \quad (13)$$

When the quantity of steel is small, such that yielding occurs at first cracking, uncontrolled and unserviceable cracking will result and the final crack width is wide. In this case,

$$\sigma_{s1}^* = \frac{n^* \rho f_y + \varepsilon_{cs}^* E_s}{1 + n^* \rho}; \quad \sigma_{s2}^* = f_y; \quad \text{and} \quad \sigma_{c1}^* = \frac{f_y A_s - \sigma_{s1}^* A_s}{A_c} \quad (14)$$

and the final crack width is

$$w = - \frac{\sigma_{s1}^* (3L - 2 s_o) + 2 s_o f_y}{3 E_s} \quad (15)$$

where L is the length of the restrained member.

Numerical Example:

Consider a 5 m long and 150 mm thick reinforced concrete slab, fully restrained at each end. The slab contains 12-mm diameter deformed longitudinal bars at 300 mm centres in both the top and bottom of the slab ($A_s = 750 \text{ mm}^2/\text{m}$). The concrete cover to the reinforcement is 30 mm. Estimate the spacing, s , and final average width, w , of the restrained shrinkage cracks.

Take $\phi^* = 2.5$, $\varepsilon_{cs}^* = -600 \times 10^{-6}$, $f_t = 2.0 \text{ MPa}$, $E_c = 25000 \text{ MPa}$, $E_s = 200000 \text{ MPa}$, $n = 8$ and $f_y = 400 \text{ MPa}$. The reinforcement ratio is $\rho = 0.005$ and from Equation 6,

$$s_o = \frac{12}{10 \times 0.005} = 240 \text{ mm.}$$

The final effective modulus is $E_e^* = \frac{25,000}{1 + 2.5} = 7143 \text{ MPa}$ and the corresponding effective modular ratio is $n^* = 28$. The constant $C_1 = 2 s_o / (3L - 2 s_o) = 2 \times 240 / (3 \times 5000 - 2 \times 240) = 0.0331$ and from Equation 8, the restraining force immediately after first cracking is

$$N_{cr} = \frac{8 \times 0.005 \times 2.0 \times 150,000}{0.0331 + 8 \times 0.005 (1 + 0.0331)} = 161,300 \text{ N/m}$$

The steel stress at the crack $\sigma_{s2} = 161300/750 = 215 \text{ MPa}$ and the concrete stress σ_{c1} is obtained from Equation 7: $\sigma_{c1} = 161300 (1 + 0.0331)/150000 = 1.11 \text{ MPa}$.

The average concrete stress may be approximated by $\sigma_{av} = (1.11 + 2.0)/2 = 1.56 \text{ MPa}$ and from Equation 11:

$$\xi = \frac{-28 \times 0.005 (1.56 - 0.0006 \times 7143)}{28 \times 0.005 (1.56 - 0.0006 \times 7143) + 2.0} = 0.236$$

The maximum crack spacing is determined using Equation 10:

$$s = \frac{2 \times 240 (1 + 0.236)}{3 \times 0.236} = 837 \text{ mm}$$

The constant C_2 is obtained from $C_2 = (2 \times 240)/(3 \times 839 - 2 \times 240) = 0.236$ and the final restraining force is calculated using Equation 9:

$$N(\infty) = -\frac{28 \times 750}{0.236} (1.56 - 0.0006 \times 7143) = 242,670 \text{ N/m}$$

From Equation 12, $\sigma_{s2}^* = 323 \text{ MPa}$, $\sigma_{c1}^* = 1.99 \text{ MPa}$ and, consequently, $\sigma_{s1}^* = -76.4 \text{ MPa}$. The final crack width is determined using Equation 13:

$$w = -\left[\frac{1.99}{7143} \left(839 - \frac{2}{3} \times 240 \right) - 0.0006 \times 839 \right] = 0.313 \text{ mm.}$$

Tables 2 and 3 contain results of a limited parametric study showing the effect of varying steel area, bar size, shrinkage strain and concrete tensile strength on the final restraining force, crack width, crack spacing and steel stress in a 150 mm thick slab, fully-restrained over a length of 5 m.

Table 2 Effect of steel area and shrinkage strain on direct tension cracking. ($\phi^* = 2.5, f_t = 2.0 \text{ MPa}$ and $d_b = 12 \text{ mm}$)

A_s mm ²	ρ	$\epsilon_{cs}^* = -0.0006$				$\epsilon_{cs}^* = -0.00075$				$\epsilon_{cs}^* = -0.0009$			
		$N(\infty)$ kN	σ_{s2}^* MPa	s mm	w mm	$N(\infty)$ kN	σ_{s2}^* MPa	s mm	w mm	$N(\infty)$ kN	σ_{s2}^* MPa	s mm	w mm
375	.0025	150	400	-	1.37	150	400	-	2.03	150	400	-	2.68
450	.003	180	400	-	1.35	180	400	-	2.01	180	400	-	2.66
600	.004	240	400	-	1.22	234	390	913	0.49	216	360	717	0.50
750	.005	243	324	837	0.31	220	294	601	0.33	197	264	469	0.34
900	.006	233	259	601	0.23	206	229	427	0.24	179	199	332	0.24
1050	.007	224	214	453	0.18	193	184	320	0.18	161	154	247	0.19
1200	.008	215	170	354	0.14	179	149	248	0.15	143	119	191	0.15

Table 3 Effect of bar diameter and concrete tensile strength on direct tension cracking. ($\phi^* = 2.5, \epsilon_{cs}^* = -0.0006, A_s = 900 \text{ mm}^2$ and $\rho = 0.006$)

d_b (mm)	$f_t = 2.0 \text{ MPa}$				$f_t = 2.5 \text{ MPa}$			
	$N(\infty)$ kN	σ_{s2}^* MPa	s mm	w mm	$N(\infty)$ kN	σ_{s2}^* MPa	s mm	w mm
6	237	263	317	0.12	323	359	482	0.14
10	234	260	508	0.19	320	356	758	0.23
12	233	259	601	0.23	319	354	889	0.27
16	232	257	781	0.30	317	352	1141	0.35
20	230	256	956	0.37	315	350	1385	0.42

4. Control of deflection

The control of deflections may be achieved by limiting the calculated deflection to an acceptably small value. Two alternative general approaches for deflection calculation are specified in AS3600 (1), namely ‘deflection by refined calculation’ (Clause 9.5.2 for beams and Clause 9.3.2 for slabs) and ‘deflection by simplified calculation’ (Clause 9.5.3 for beams and Clause 9.3.3 for slabs). The former is not specified in detail but allowance should be made for cracking and tension stiffening, the shrinkage and creep properties of the concrete, the expected load history and, for slabs, the two-way action of the slab.

The long-term or time-dependent behaviour of a beam or slab under sustained service loads can be determined using a variety of analytical procedures (Gilbert, 1988) [7], including the Age-Adjusted Effective Modulus Method (AEMM), described in detail by Gilbert and Mickleborough (1997) [12]. The use of the AEMM to determine the instantaneous and time-dependent deformation of the critical cross-sections in a beam or slab and then integrating the curvatures to obtain deflection, is a *refined calculation method* and is recommended.

Using the AEMM, the strain and curvature on individual cross-sections at any time can be calculated, as can the stress in the concrete and bonded reinforcement or tendons. The routine use of the AEMM in the design of concrete structures for the serviceability limit states is strongly encouraged.

However, in most design situations, the latter approach (*deflection by simplified calculation*) is generally used and its limitations are discussed in detail below.

4.1 Deflection by Simplified Calculation - AS3600:

The instantaneous or short-term deflection of a beam may be calculated using the mean value of the elastic modulus of concrete at the time of first loading, E_{cj} , together with the effective second moment of area of the member, I_{ef} . The effective second moment of area involves an empirical adjustment of the second moment of area of a cracked member to account for tension stiffening (the stiffening effect of the intact tensile concrete between the cracks). For a given cross-section, I_{ef} is calculated using Branson's formula (Branson, 1963):

$$I_{ef} = I_{cr} + (I - I_{cr})(M_{cr}/M_s)^3 \leq I_{e,max} \quad (16)$$

where I_{cr} is the second moment of area of the fully-cracked section (calculated using modular ratio theory); I is the second moment of area of the gross concrete section about its centroidal axis; M_s is the maximum bending moment at the section, based on the short-term serviceability design load or the construction load; and M_{cr} is the cracking moment given by

$$M_{cr} = Z(f'_{cf} - f_{cs} + P/A_g) + Pe \geq 0.0 \quad (17)$$

Z is the section modulus of the uncracked section, referred to the extreme fibre at which cracking occurs; f'_{cf} is the characteristic flexural tensile strength of concrete; f_{cs} is the maximum shrinkage induced tensile stress on the uncracked section at the extreme fibre at which cracking occurs and may be taken as

$$f_{cs} = \left(\frac{1.5p}{1 + 50p} E_s \varepsilon_{cs} \right) \quad (18)$$

where p is the reinforcement ratio (A_{st}/bd) and ε_{cs} is the final design shrinkage strain.

The maximum value of I_{ef} at any cross-section, $I_{e,max}$ in Equation 16, is I when $p = A_{st}/bd \geq 0.005$ and $0.6I$ when $p < 0.005$.

Alternatively, as a further simplification but only for reinforced concrete members, I_{ef} at each nominated cross-section for rectangular sections may be taken as equal to

$$(0.02 + 2.5p)bd^3 \quad \text{when } p \geq 0.005 \quad \text{and} \quad (0.1 - 13.5p)bd^3 \quad \text{when } p < 0.005.$$

The value of I_{ef} for the member is determined from the value of I_{ef} at midspan for a simple-supported beam. For interior spans of continuous beams, I_{ef} is half the midspan value plus one quarter of the value at each support, and for end spans of continuous strips, I_{ef} is half the midspan value plus half the value at the continuous support. For a cantilever, I_{ef} is the value at the support.

The term f_{cs} in Equation 17 was introduced in Amendment 2 to AS3600 to allow for the tension that inevitably develops due to the restraint to shrinkage provided by the bonded tensile reinforcement.

Equation 18 is based on the expression proposed in Gilbert (1998) [9], by assuming conservative values for the elastic modulus and the creep coefficient of concrete and assuming about 40% of the final shrinkage has occurred at the time of cracking. In the calculation of f_{cs} using Equation 18, the final or long-term value of ε_{cs} in the concrete should be used.

This allowance for shrinkage induced tension is particularly important in the case of lightly reinforced members (including slabs) where the tension induced by the full service moment alone might not be enough to cause cracking. In such cases, failure to account for shrinkage may lead to deflection calculations based on the uncracked section properties. This usually grossly underestimates the actual deflection. For heavily reinforced sections, the problem is not so significant, as the service loads are usually well in excess of the cracking load and the ratio of cracked to uncracked stiffness is larger.

For the calculation of long-term deflection, one of two approaches may be used. For reinforced or prestressed beams, the creep and shrinkage deflections can be calculated separately (using the material data specified in the Standard and the principles of mechanics). Alternatively, for reinforced concrete beam, long-term deflection can be crudely approximated by multiplying the immediate deflection caused by the sustained load by a multiplier k_{cs} given by

$$k_{cs} = [2 - 1.2(A_{sc}/A_{st})] \geq 0.8 \quad (19)$$

where the ratio A_{sc}/A_{st} is taken at midspan for a simple or continuous span and at the support for a cantilever.

4.2 What is Wrong with the AS3600 Simplified Procedure and How to Improve it:

The current simplified approach for the calculation of final deflection fails to adequately predict the long-term or time-dependent deflection (by far the largest portion of the total deflection in most reinforced and prestressed concrete members). Shrinkage induced curvature and the resulting deflection is not adequately accounted for when using k_{cs} and no account is taken of the actual creep and shrinkage properties of the concrete. The introduction of f_{cs} in the estimation of the cracking moment is a positive step in improving the procedure, by recognising that early shrinkage can induce tension that significantly reduces the cracking moment and significantly reduces the instantaneous stiffness with time. However, the gradual reduction in I_{ef} with time due to shrinkage and cyclic loading is still not fully accounted for. To better model the breakdown of tension stiffening with time, Equation 18 should be replaced by Equation 20, which was originally proposed by Gilbert (1999a) [10] but was modified by Standards Australia (for political, rather than technical, reasons).

$$f_{cs} = \left(\frac{2.5p}{1+50p} E_s \varepsilon_{sh} \right) \quad (20)$$

A further criticism of the simplified approach is the use of the second moment of area of the gross concrete section I in Equation 16. It is unnecessarily conservative to ignore the stiffening effect of the bonded reinforcement in the calculation of the properties of the uncracked cross-section.

The use of the deflection multiplier k_{cs} to calculate time-dependent deflections is simple and convenient and, provided the section is initially cracked under short term loads, it sometimes provides a 'ball-park' estimate of final deflection. However, to calculate the shrinkage induced deflection by multiplying the load induced short-term deflection by a long-term deflection multiplier is fundamentally wrong. Shrinkage can cause significant deflection even in unloaded members (where the short-term deflection is zero). The approach ignores the creep and shrinkage characteristics of the concrete, the environment, the age at first loading and so on. At best, it provides a very approximate estimate. At worst, it is not worth the time involved in making the calculation.

It is, however, not too much more complicated to calculate long-term creep and shrinkage deflection separately. As mentioned previously, well established and reliable methods are available for calculating the time-dependent behaviour of reinforced and prestressed concrete cross-sections (Gilbert, 1988) [7]. A simple method suitable for routine use in design is outlined below.

The load induced curvature, $\kappa(t)$, (instantaneous plus creep) at any time t due to sustained service actions may be expressed as

$$\kappa(t) = \kappa_i(t)(1 + \phi/\alpha) \quad (21)$$

where $\kappa_i(t)$ is the instantaneous curvature due to the sustained service moment M_s ($\kappa_i(t) = M_s/E_c I_{ef}$); for an uncracked cross-section I_{ef} should be taken as the second moment of area of the uncracked transformed section, while for a cracked section, I_{ef} should be calculated from Equation 16 (with f_{cs} calculated using Equation 20 when estimating the final long-term curvature); ϕ is the creep coefficient at time t ; and α is a term that accounts for the effects of cracking and the 'braking' action of the reinforcement on creep and may be estimated from Equations 22a, 22b or 22c.

For a cracked reinforced concrete section in pure bending, $\alpha = \alpha_1$, where

$$\alpha_1 = [0.48 p^{-0.5}] [1 + (125 p + 0.1)(A_{sc}/A_{st})^{1.2}] \quad (22a)$$

For an uncracked reinforced or prestressed concrete section, $\alpha = \alpha_2$, where

$$\alpha_2 = [1.0 - 15.0 p] [1 + (140 p - 0.1)(A_{sc}/A_{st})^{1.2}] \quad (22b)$$

where $p = A_{st}/b d_o$ and A_{st} is the equivalent area of bonded reinforcement in the tensile zone at depth d_o (the depth from the extreme compressive fibre to the centroid of the outermost layer of tensile reinforcement). The area of any bonded reinforcement in the tensile zone (including bonded tendons) not contained in the outermost layer of tensile reinforcement (ie. located at a depth d_1 less than d_o) should be included in the calculation of A_{st} by multiplying that area by d_1/d_o . For the purpose of the calculation of A_{st} , the tensile zone is that zone that would be in tension due to the applied moment acting in isolation. A_{sc} is the area of the bonded reinforcement in the compressive zone.

For a cracked, partially prestressed section or for a cracked reinforced concrete section subjected to bending and axial compression, α may be taken as

$$\alpha = \alpha_2 + (\alpha_1 - \alpha_2)(d_n/d_n)^{2.4} \quad (22c)$$

where d_n is the depth of the intact compressive concrete on the cracked section and d_{n1} is the depth of the intact compressive concrete on the cracked section ignoring the axial compression and/or the prestressing force (ie. the value of d_n for an equivalent cracked reinforced concrete section containing the same quantity of bonded reinforcement).

The shrinkage induced curvature on a reinforced or prestressed concrete section can be approximated by

$$\kappa_{sh} = \left[\frac{k_r \epsilon_{sh}}{D} \right] \quad (23)$$

where D is the overall depth of the section, A_{st} and A_{sc} are as defined under Equation 22b above, and the factor k_r depends on the quantity and location of the bonded reinforcement and may be estimated from Equations 24a, 24b, 24c or 24d.

For an uncracked cross-section, $k_r = k_{r1}$, where

$$k_{r1} = (100 p - 2500 p^2) \left(\frac{d_o}{0.5D} - 1 \right) \left(1 - \frac{A_{sc}}{A_{st}} \right)^{1.3} \quad \text{when } p = A_{st}/b d_o \leq 0.01 \quad (24a)$$

$$k_{r1} = (40p + 0.35) \left(\frac{d_o}{0.5D} - 1 \right) \left(1 - \frac{A_{sc}}{A_{st}} \right)^{1.3} \quad \text{when } p = A_{st}/b d_o > 0.01 \quad (24b)$$

For a cracked reinforced concrete section in pure bending, $k_r = k_{r2}$, where

$$k_{r2} = 1.2 \left(1 - 0.5 \frac{A_{sc}}{A_{st}} \right) \left(\frac{D}{d_o} \right) \quad (24c)$$

For a cracked, partially prestressed section or for a cracked reinforced concrete section subjected to bending and axial compression, k_r may be taken as

$$k_r = k_{r1} + (k_{r2} - k_{r1})(d_{n1}/d_n) \quad (24d)$$

where d_n is the depth of the intact compressive concrete on the cracked section and d_{n1} is the depth of the intact compressive concrete on the cracked section ignoring the axial compression and/or the prestressing force (ie. the value of d_n after cracking for an equivalent cracked reinforced concrete section containing the same quantity of bonded reinforcement).

Equations 22, 23 and 24 have been developed from parametric studies of a wide range of cross-sections analysed using the Age-Adjusted Effective Modulus Method of analysis (with typical results of such analyses presented and illustrated by Gilbert, 2000).

When the load induced and shrinkage induced curvatures are calculated at selected sections along a beam or slab, the deflection may be obtained by double integration. For a reinforced or prestressed concrete continuous span with the degree of cracking varying along the member, the curvature at the left and right supports, κ_1 and κ_r and the curvature at midspan κ_m may be calculated at any time after loading and the deflection at midspan Δ may be approximated by assuming a parabolic curvature diagram along the span, ℓ :

$$\Delta = \frac{\ell^2}{96} (\kappa_1 + 10 \kappa_m + \kappa_r) \quad (25)$$

The above equation will give a reasonable estimate of deflection even when the curvature diagram is not parabolic and is a useful expression for use in deflection calculations.

4.3 Deflection Calculations - Worked Examples:

Example 1

A reinforced concrete beam of rectangular section (800 mm deep and 400 mm wide) is simply-supported over a 12 m span and is subjected to a uniformly distributed sustained service load of 22.22 kN/m. The longitudinal reinforcement is uniform over the entire span and consists of 4 Y32 bars located in the bottom at an effective depth of 750 mm ($A_{st} = 3200 \text{ mm}^2$) and 2 Y32 bars in the top at a depth of 50 mm below the top surface ($A_{sc} = 1600 \text{ mm}^2$). Calculate the instantaneous and long-term deflection at midspan, assuming the following material properties:

$f'_c = 32 \text{ MPa}$; $f'_{cf} = 3.39 \text{ MPa}$; $E_c = 28,570 \text{ MPa}$; $E_s = 2 \times 10^5 \text{ MPa}$; $\phi = 2.5$; and $\epsilon_{cs} = 0.0006$.
For each cross-section, $p = A_{st}/bd = 0.0107$.

The section at midspan:

The sustained bending moment is $M_s = 400 \text{ kNm}$. The second moments of area of the uncracked transformed cross-section, I , and the full-cracked transformed section, I_{cr} , are $I = 20,560 \times 10^6 \text{ mm}^4$ and $I_{cr} = 7,990 \times 10^6 \text{ mm}^4$. The bottom fibre section modulus of the uncracked section is $Z = I/y_b = 52.7 \times 10^6 \text{ mm}^3$. From Equation 20,

$$f_{cs} = \left(\frac{2.5 \times 0.0107}{1 + 50 \times 0.0107} \times 2 \times 10^5 \times 0.0006 \right) = 2.09 \text{ MPa}$$

and the time-dependent cracking moment is obtained from Equation 17:

$$M_{cr} = 52.7 \times 10^6 (3.39 - 2.09) = 68.5 \text{ kNm.}$$

From Equation 16, the effective second moment of area is

$$I_{ef} = [7990 + (20560 - 7990)(68.5/400)^3] \times 10^6 = 8050 \times 10^6 \text{ mm}^4$$

The instantaneous curvature due to the sustained service moment is therefore

$$\kappa_i(t) = \frac{M_s}{E_c I_{ef}} = \frac{400 \times 10^6}{28570 \times 8050 \times 10^6} = 1.74 \times 10^{-6} \text{ mm}^{-1}.$$

From Equation 22a: $\alpha_1 = [0.48 \times 0.0107^{-0.5}][1 + (125 \times 0.0107 + 0.1)(1600/3200)^{1.2}] = 7.55$
and the load induced curvature (instantaneous plus creep) is obtained from Equation 21:

$$\kappa(t) = 1.74 \times 10^{-6} (1 + 2.5/7.55) = 2.32 \times 10^{-6} \text{ mm}^{-1}.$$

From Equation 24c: $k_r = k_{r2} = 1.2 \times (1 - 0.5 \times \frac{1600}{3200}) \frac{800}{750} = 0.96$

and the shrinkage induced curvature is obtained from Equation 23:

$$\kappa_{cs} = \left[\frac{0.96 \times 600 \times 10^{-6}}{800} \right] = 0.72 \times 10^{-6} \text{ mm}^{-1}$$

The instantaneous and final time-dependent curvatures at midspan are therefore

$$\kappa_i = 1.74 \times 10^{-6} \text{ mm}^{-1} \quad \text{and} \quad \kappa = \kappa(t) + \kappa_{cs} = 3.04 \times 10^{-6} \text{ mm}^{-1}.$$

The section at each support:

The sustained bending moment is zero and the section remains uncracked. The load-dependent curvature is therefore zero. However, shrinkage curvature develops with time. From Equation 24b:

$$k_r = k_{r1} = (40 \times 0.0107 + 0.35) \left(\frac{750}{0.5 \times 800} - 1 \right) \left(1 - \frac{1600}{3200} \right)^{1.3} = 0.276$$

and the shrinkage induced curvature is estimated from Equation 23:

$$\kappa_{cs} = \left[\frac{0.276 \times 600 \times 10^{-6}}{800} \right] = 0.21 \times 10^{-6} \text{ mm}^{-1}$$

Deflections:

The instantaneous and final long-term deflections at midspan, Δ_i and Δ_{LT} , respectively, are obtained from Equation 25:

$$\Delta_i = \frac{12000^2}{96} (0 + 10 \times 1.74 + 0) \times 10^{-6} = 26.1 \text{ mm}$$

$$\Delta_{LT} = \frac{12000^2}{96} (0.21 + 10 \times 3.04 + 0.21) \times 10^{-6} = 46.0 \text{ mm} (= \text{span}/260)$$

It is of interest to note that using the current approach in AS3600, with $k_{cs} = 1.4$ (from Equation 19), the calculated final deflection is 60.9 mm.

Example 2

A post-tensioned concrete beam of rectangular section (800 mm deep and 400 mm wide) is simply-supported over a 12 m span and is subjected to a uniformly distributed sustained service load of 38.89 kN/m. The beam is prestressed with a single parabolic cable consisting of 15/12.7mm diameter strands ($A_p = 1500 \text{ mm}^2$) with $d_p = 650 \text{ mm}$ at midspan and $d_p = 400 \text{ mm}$ at each support. The duct containing the tendons is filled with grouted soon after transfer. The longitudinal reinforcement is uniform over the entire span and consists of 4 Y32 bars located in the bottom at an effective depth of 750 mm ($A_s = 3200 \text{ mm}^2$) and 2 Y32 bars in the top at a depth of 50 mm below the top surface ($A_{sc} = 1600 \text{ mm}^2$). For the purpose of this exercise, the initial prestressing force in the tendon is assumed to be 2025 kN throughout the member and the relaxation loss is 50 kN. Calculate the instantaneous and long-term deflection at midspan, assuming the following material properties:

$$f'_c = 32 \text{ MPa}; f'_{cf} = 3.39 \text{ MPa}; E_c = 28,570 \text{ MPa}; E_s = 2 \times 10^5 \text{ MPa}; \phi = 2.5; \text{ and } \epsilon_{cs} = 0.0006.$$

The section at midspan:

The sustained bending moment is $M_s = 700$ kNm. The centroidal axis of the uncracked transformed cross-section is located at a depth of 415.7 mm below the top fibre and the second moment of area is $I = 21,070 \times 10^6 \text{ mm}^4$. The top and bottom fibre concrete stresses immediately after first loading (due the applied moment and prestress) are -10.11 MPa and -1.55 MPa, respectively (both compressive). The section remains uncracked throughout.

The instantaneous curvature due to the sustained service moment is

$$\kappa_i(t) = \frac{M_s - Pe}{E_c I} = \frac{700 \times 10^6 - 2025 \times 10^3 \times 234.3}{28570 \times 21070 \times 10^6} = 0.375 \times 10^{-6} \text{ mm}^{-1}.$$

From Equation 22a, with $A_{sc} = 1600 \text{ mm}^2$, $A_{st} = A_s + A_p d_p/d_o = 3200 + 1500 \times 650/750 = 4500 \text{ mm}^2$ and, therefore $p = A_{st}/b d_o = 4500/(400 \times 750) = 0.015$:

$$\alpha_2 = [1.0 - 15.0 \times 0.015][1 + (140 \times 0.015 - 0.1)(1600/4500)^{1.2}] = 1.22$$

and the load induced curvature (instantaneous plus creep) is obtained from Equation 21:

$$\kappa(t) = 0.375 \times 10^{-6} (1 + 2.5/1.22) = 1.14 \times 10^{-6} \text{ mm}^{-1}.$$

$$\text{From Equation 24b: } k_r = k_{rI} = (40 \times 0.015 + 0.35) \left(\frac{750}{0.5 \times 800} - 1 \right) \left(1 - \frac{1600}{4500} \right) = 0.536$$

and the shrinkage induced curvature is obtained from Equation 23:

$$\kappa_{cs} = \left[\frac{0.54 \times 600 \times 10^{-6}}{800} \right] = 0.40 \times 10^{-6} \text{ mm}^{-1}$$

The instantaneous and final time-dependent curvatures at midspan are therefore

$$\kappa_i = 0.375 \times 10^{-6} \text{ mm}^{-1} \quad \text{and} \quad \kappa = \kappa(t) + \kappa_{cs} = 1.54 \times 10^{-6} \text{ mm}^{-1}.$$

The section at each support:

The sustained bending moment is zero and the section remains uncracked. The centroidal axis of the uncracked transformed cross-section (with A_p located at a depth of 400 mm) is located at a depth of 409.4 mm below the top fibre and the second moment of area is $I = 20,560 \times 10^6 \text{ mm}^4$. The prestressing steel is located 9.4 mm above the centroidal axis of the transformed section, so that the prestressing force induces a small instantaneous positive curvature. Shrinkage (and creep) curvature develops with time.

The instantaneous curvature is

$$\kappa_i(t) = \frac{M_s - Pe}{E_c I} = \frac{0 - 2025 \times 10^3 \times -9.4}{28570 \times 20560 \times 10^6} = 0.032 \times 10^{-6} \text{ mm}^{-1}.$$

From Equation 22a, with $A_{sc} = 1600 \text{ mm}^2$, $A_{st} = A_{sI} + A_p d_p/d_o = 3200 + 1500 \times 400/750 = 4000 \text{ mm}^2$ and, therefore $p = A_{st}/b d_o = 4000/(400 \times 750) = 0.0133$:

$$\alpha_2 = [1.0 - 15.0 \times 0.0133][1 + (140 \times 0.0133 - 0.1)(1600/4000)^{1.2}] = 1.27$$

and the load induced curvature (instantaneous plus creep) is obtained from Equation 21:

$$\kappa(t) = 0.032 \times 10^{-6} (1 + 2.5/1.27) = 0.09 \times 10^{-6} \text{ mm}^{-1}.$$

From Equation 24b:

$$k_r = k_{rI} = (40 \times 0.0133 + 0.35) \left(\frac{750}{0.5 \times 800} - 1 \right) \left(1 - \frac{1600}{4000} \right)^{1.3} = 0.398$$

and the shrinkage induced curvature is estimated from Equation 23:

$$\kappa_{cs} = \left[\frac{0.398 \times 600 \times 10^{-6}}{800} \right] = 0.30 \times 10^{-6} \text{ mm}^{-1}$$

The instantaneous and final time-dependent curvatures at the supports are therefore

$$\kappa_i = 0.032 \times 10^{-6} \text{ mm}^{-1} \quad \text{and} \quad \kappa = \kappa(t) + \kappa_{cs} = 0.39 \times 10^{-6} \text{ mm}^{-1}.$$

Deflections:

The instantaneous and final long-term deflections at midspan, Δ_i and Δ_{LT} , respectively, are obtained from Equation 25:

$$\Delta_i = \frac{12000^2}{96} (0.032 + 10 \times 0.375 + 0.032) \times 10^{-6} = 5.7 \text{ mm}$$

$$\Delta_{LT} = \frac{12000^2}{96} (0.39 + 10 \times 1.54 + 0.39) \times 10^{-6} = 24.3 \text{ mm}$$

In this example, the ratio of final to instantaneous deflection is 4.3.

5. Control of flexural cracking

5.1 The requirements of AS3600

In AS3600-1994, the control of flexural cracking is deemed to be satisfactory, providing the designer satisfies certain detailing requirements. These involve maximum limits on the centre-to-centre spacing of bars and on the distance from the side or soffit of the beam to the nearest longitudinal bar. These limits do not depend on the stress in the tensile steel under service loads and have been found to be unreliable when the steel stress exceeds about 240 MPa. The provisions of AS3600-1994 oversimplify the problem and do not always ensure adequate control of cracking.

With the current move to higher strength reinforcing steels (characteristic strengths of 500 MPa and above), there is an urgent need to review the crack-control design rules in AS3600 for reinforced concrete beams and slabs. The existing design rules for reinforced concrete flexural elements are intended for use in the design of elements containing 400 MPa bars and are sometimes unconservative. They are unlikely to be satisfactory for members in which higher strength steels are used, where steel stresses at service loads are likely to be higher due to the reduced steel area required for strength.

Standards Australia has established a Working Group to investigate and revise the crack control provisions of the current Australian Standard to incorporate recent developments and to accommodate the use of high of high strength reinforcing steels. A theoretical and experimental investigation of the critical factors that affect the control of cracking due to restrained deformation and external loading is currently underway at the University of New South Wales. The main objectives of the investigation are to gain a better understanding of the factors that affect the spacing and width of cracks in reinforced concrete elements and to develop rational and reliable design-oriented procedures for the control of cracking and the calculation of crack widths.

As an interim measure, to allow the immediate introduction of 500 MPa steel reinforcement, the deemed to comply crack control provisions of Eurocode 2 (with minor modifications) have been included in the recent Amendment 2 of the Standard. In Gilbert (1999b) [11] and Gilbert et al. (1999) [13], the current crack control provisions of AS 3600 were presented and compared with the corresponding provisions in several of the major international concrete codes, including BS 8110, ACI 318 and Eurocode 2. A parametric evaluation of the various code approaches was also undertaken to determine the relative importance in each model of such factors as steel area, steel stress, bar diameter, bar spacing, concrete cover and concrete strength on the final crack spacing and crack width. The applicability of each model was assessed by comparison with some local crack width measurements and problems were identified with each of the code models. Gilbert et al (1999) conclude that the provisions of Eurocode 2 appear to provide a more reliable means for ensuring adequate crack control than either BS 8110 or ACI 318, but that all approaches fail to adequately account the increase in crack widths that occurs with time.

In Amendment 2, Clause 8.6.1 *Crack control for flexure in reinforced beams* has been replaced with the following:

8.6.1 Crack control for flexure and tension in reinforced beams Cracking in reinforced beams subjected to flexure or tension shall be deemed to be controlled if the appropriate requirements in (a) and (b), and either (c) or (d) are satisfied. For the purpose of this Clause, the resultant action is considered to be *flexure* when the tensile stress distribution within the section prior to cracking is triangular with some part of the section in compression, or *tension* when the whole of the section is in tension.

- (a) The minimum area of reinforcement required in the tensile zone ($A_{st,min}$) in regions where cracking shall be taken as

$$A_{gt} = 3 k_s A_{ct} / f_s$$

where

k_s = a coefficient which takes into account the shape of the stress distribution within the section immediately prior to cracking, and equals 0.6 for flexure and 0.8 for tension.

A_{ct} = the area of concrete in the tensile zone, being that part of the section in tension assuming the section is uncracked; and

f_s = the maximum tensile stress permitted in the reinforcement after formation of a crack, which shall be the lesser of the yield strength of the reinforcement (f_{sy}) and the maximum steel stress in Table 8.6.1(A) for the largest nominal bar diameter (d_b) of the bars in the section.

- (b) The distance from the side or soffit of a beam to the centre of the nearest longitudinal bar shall not be greater than 100mm. Bars with a diameter less than half the diameter of the largest bar in the cross-section shall be ignored. The centre-to-centre spacing of bars near a tension face of the beam shall not exceed 300 mm.
- (c) For beams subjected to tension, the steel stress (f_{scr}), calculated for the load combination for the short-term serviceability limit states assuming the section is cracked, does not exceed the maximum steel stress given in Table 8.6.1(A) for the largest nominal diameter (d_b) of the bars in the section.
- (d) For beams subjected to flexure, the steel stress (f_{scr}), calculated for the load combination for the short-term serviceability limit states assuming the section is cracked, does not exceed the maximum steel stress given in Table 8.6.1(A) for the largest nominal diameter (d_b) of the bars in the tensile zone under the action of the design bending moment. Alternatively, the steel stress does not exceed the maximum stress determined from Table 8.6.1(B) for the largest centre-to-centre spacing of adjacent parallel bars in the tensile zone. Bars with a diameter less than half the diameter of the largest bar in the cross-section shall be ignored when determining spacing.

TABLE 8.6.1(A)
MAXIMUM STEEL STRESS
FOR TENSION OR FLEXURE IN BEAMS

Maximum steel stress (MPa)	Nominal bar diameter, d_b , (mm)
160	32
200	25
240	20
280	16
320	12
360	10
400	8
450	6

TABLE 8.6.1(B)
MAXIMUM STEEL STRESS
FOR FLEXURE IN BEAMS

Maximum steel stress (MPa)	Centre-to-centre spacing (mm)
160	300
200	250
240	200
280	150
320	100
360	50
Note: Linear interpolation may be used.	

The amendment is similar to the crack control provisions in Eurocode 2. In essence, the amendment requires the quantity of steel in the tensile region to exceed a minimum area, $A_{st,min}$, and places a maximum limit on the steel stress depending on either the bar diameter or the centre-to-centre spacing of bars. As in the existing clause, a maximum limit of 100 mm is also placed on the distance from the side or soffit of a beam to the nearest longitudinal bar.

5.2 Calculation of Flexural Crack Widths

An alternative approach to flexural crack control is to calculate the *design crack width* and to limit this to an acceptably small value. The writer has proposed an approach for calculating the design crack width (Gilbert 1999b). This approach is similar to that proposed in Eurocode 2, but modified to include shrinkage shortening of the intact concrete between the cracks in the tensile zone and to more realistically represent the increase in crack width if the cover is increased.

The design crack width, w , may be calculated from

$$w = \beta_m s_{rm} (\varepsilon_{sm} + \varepsilon_{cs,t}) \quad (26)$$

where s_{rm} is the average final crack spacing; β_m is a coefficient relating the average crack width to the design value and may be taken as $\beta_m = 1.0 + 0.025c \geq 1.7$; c is the distance from the concrete surface to the nearest longitudinal reinforcing bar; and ε_{sm} is the mean strain allowing for the effects of tension stiffening and may be taken as

$$\varepsilon_{sm} = (\sigma_s/E_s)[1 - \beta_1\beta_2(\sigma_{sr}/\sigma_s)^2] \quad (27)$$

where σ_s is the stress in the tension steel calculated on the basis of a cracked section; σ_{sr} is the stress in the tension steel calculated on the basis of a cracked section under the loading conditions causing first cracking; β_1 depends on the bond properties of the bars and equals 1.0 for high bond bars and 0.5 for plain bars; and β_2 accounts for the duration of loading and equals 1.0 for a single, short-term loading and 0.5 for a sustained load or for many cycles of loading.

The average final spacing of flexural cracks, s_{rm} (in mm), can be calculated from

$$s_{rm} = 50 + 0.25 k_1 k_2 d_b / \rho \quad (28)$$

where d_b is the bar size (or average bar size in the section) in mm; k_1 accounts for the bond properties of the bar and, for flexural cracking, $k_1 = 0.8$ for high bond bars and $k_1 = 1.6$ for plain bars; k_2 depends on the strain distribution and equals 0.5 for bending; and ρ_r is the effective reinforcement ratio, $A_s/A_{c,eff}$ where A_s is the area of reinforcement contained within the effective tension area, $A_{c,eff}$. The effective tension area is the area of concrete surrounding the tension steel of depth equal to 2.5 times the distance from the tension face of the section to the centroid of the reinforcement, but not greater than $1/3$ of the depth of the tensile zone of the cracked section, in the case of slabs.

$\varepsilon_{cs,t}$ is the shrinkage induced shortening of the intact concrete at the tensile steel level between the cracks. For short-term crack width calculations, $\varepsilon_{cs,t}$ is zero. Using the age-adjusted effective modulus method and a shrinkage analysis of a singly reinforced concrete section, see Gilbert (1988), it can be shown that

$$\varepsilon_{cs,t} = \varepsilon_{cs} / (1 + 3 p \bar{n}) \quad (13)$$

where p is the tensile reinforcement ratio for the section (A_{st}/bd); \bar{n} is the age-adjusted modular ratio (E_s/E_{ef}); E_{ef} is the age-adjusted effective modulus for concrete ($E_{ef} = E_c / (1 + 0.8\phi)$); and ε_{cs} and ϕ are final long-term values of shrinkage strain and creep coefficient, respectively.

The above procedure overcomes the major deficiencies in current code procedures and more accurately agrees with laboratory and field measurements of crack widths. In Tables 4 and 5, crack widths calculated using the proposed procedure are presented for rectangular slab and beam sections. In each case, $\varepsilon_{cs} = -0.0006$ and $\phi = 3.0$. In general, the calculated crack widths are larger than those predicted by either ACI or EC2, but unlike these codes, the proposed model will signal serviceability problems to the structural designer in most situations where excessive crack widths are likely.

It should be pointed out that the steel stress under *sustained* service loads is usually less than 200 MPa for beams and slabs designed using 400 MPa steel. The range of steel stresses in Tables 4 and 5 are more typical of situations in which 500 MPa steel is used.

Table 4 Calculated final flexural crack widths in a 200 mm thick slab

Effective depth, d (mm)	Bar diam d_b (mm)	Area of tensile steel, A_{st} (mm ² /m)	Bar spacing, s (mm)	Crack width (mm)		
				Steel stress, σ_s (MPa)		
				200	250	300
174	12	1044	108	0.226	0.279	0.330
172	16	1032	195	0.267	0.331	0.392
170	20	1020	308	0.309	0.384	0.455
168	24	1008	449	0.352	0.438	0.519
166	28	996	618	0.394	0.492	0.585

Table 5 Calculated final flexural crack widths for beam ($b = 400$ mm and $d = 400$ mm)

Bar diam d_b (mm)	No. of bars	A_{st} (mm ²)	$p = A_{st}/bd$	Crack width (mm)					
				Cover = 25 mm			Cover = 50 mm		
				Steel stress, σ_s (MPa)			Steel stress, σ_s (MPa)		
				200	250	300	200	250	300
20	2	620	.0039	.309	.397	.479	.488	.646	.791
20	3	930	.0058	.267	.326	.384	.414	.513	.607
20	4	1240	.0078	.231	.280	.327	.349	.425	.498
24	2	900	.0056	.314	.386	.455	.480	.596	.707
24	3	1350	.0084	.251	.304	.355	.369	.449	.526
24	4	1800	.0113	.214	.258	.301	.304	.367	.430
28	2	1240	.0078	.299	.362	.424	.434	.529	.621
28	3	1860	.0116	.234	.281	.329	.325	.393	.459
32	2	1600	.0100	.285	.344	.402	.394	.477	.558

6. Conclusions

The effects of shrinkage on the behaviour of reinforced and prestressed concrete members under sustained service loads has been discussed. In particular, the mechanisms of shrinkage warping in unsymmetrically reinforced elements and shrinkage cracking in restrained direct tension members has been described. Recent amendments to the serviceability provisions of AS3600 have been outlined and techniques for the control of deflection and cracking are presented. Reliable procedures for the prediction of long-term deflections and final crack widths in flexural members have also been proposed and illustrated by examples.

Acknowledgment

This paper stems from a continuing study of the serviceability of concrete structures at the University of New South Wales. The work is currently funded by the Australian Research Council through two ARC Large Grants, one on deflection control of reinforced concrete slabs and one on crack control in concrete structures. The support of the ARC and UNSW is gratefully acknowledged.

REFERENCES

1. AS3600-1994, Australian Standard for Concrete Structures, *Standards Australia*, Sydney, (1994).
2. ACI318-95, Building code requirements for reinforced concrete, *American Concrete Institute*, Committee 318, Detroit, 1995.
3. Base, G.D. and Murray, M.H., "New Look at Shrinkage Cracking", *Civil Engineering Transactions*, IEAust, V.CE24, No.2, May 1982, 171pp.
4. Branson, D.E., "Instantaneous and Time-Dependent Deflection of Simple and Continuous RC Beams", *Alabama Highway Research Report*, No.7, Bureau of Public Roads, 1963.
5. DD ENV-1992-1-1 Eurocode 2, Design of Concrete Structures, British Standards Institute, 1992.
6. Favre, R., et al., "Fissuration et Deformations", *Manual du Comite Ewo-International du Beton (CEB)*, Ecole Polytechnique Federale de Lausanne, Switzerland, 1983, 249 p.
7. Gilbert, R.I., "Time Effects in Concrete Structures", *Elsevier Science Publishers*, Amsterdam, 1988, 321p.
8. Gilbert, R.I., "Shrinkage Cracking in Fully Restrained Concrete Members", *ACI Structural Journal*, Vol. 89, No. 2, March-April 1992, pp 141-149
9. Gilbert, R.I., "Serviceability Considerations and Requirements for High Performance Reinforced Concrete Slabs", *Proceedings International Conference On High Performance High Strength Concrete*, Curtin University of Technology, Perth, Western Australia, August 1998, pp 425-439.
10. Gilbert, R.I., "Deflection Calculations for Reinforced Concrete Structures - Why we sometimes get it Wrong", *ACI Structural Journal*, Vol. 96, No. 6, November-December 1999(a), pp 1027 - 1032.
11. Gilbert, R.I., "Flexural Crack Control for Reinforced Concrete Beams and Slabs: An Evaluation of Design Procedures", ACMSM 16, Proceedings of the 16th Conference on the Mechanics of Structures and Materials, Sydney, Balkema, Rotterdam, 1999(b), pp 175-180.
12. Gilbert, R.I. and Mickleborough, "Design of Prestressed Concrete", E & FN Spon, London, 2nd Printing, 1997, 504p.
13. Gilbert, R.I., Patrick, M. and Adams, J.C., "Evaluation of Crack Control Design Rules for Reinforced Concrete Beams and Slabs", *Concrete 99, Biennial Conference of the Concrete Institute of Australia*, Sydney, 1999, pp 21-29.



Ian Gilbert is Professor of Civil Engineering and Head of the School of Civil and Environmental Engineering at the University of New South Wales. His main research interests have been in the area of serviceability and the time-dependent behaviour of concrete structures. His publications include three books and over one hundred refereed papers in the area of reinforced and prestressed concrete structures. He has served on Standards Australia's Concrete Structures Code Committee BD/2 since 1981 and was actively involved in the development of AS3600. He is currently chairing two of the Working Groups (WG2 – Anchorage and WG7 – Serviceability) established to review AS3600. Professor Gilbert was awarded the Chapman Medal by the IEAust in 2000 and is the 2001 Eminent Speaker for the Structural College, IEAust.