

Open and Closed Mathematics: Student Experiences and Understandings

Jo Boaler, *King's College, London University*

This paper reports on 3-year case studies of 2 schools with alternative mathematical teaching approaches. One school used a traditional, textbook approach; the other used open-ended activities at all times. Using various forms of case study data, including observations, questionnaires, interviews, and quantitative assessments, I will show the ways in which the 2 approaches encouraged different forms of knowledge. Students who followed a traditional approach developed a procedural knowledge that was of limited use to them in unfamiliar situations. Students who learned mathematics in an open, project-based environment developed a conceptual understanding that provided them with advantages in a range of assessments and situations. The project students had been “apprenticed” into a system of thinking and using mathematics that helped them in both school and nonschool settings.

There is a growing concern among mathematics educators that many students are able to learn mathematics for 11 years or more but are then completely unable to use this mathematics in situations outside the classroom context. In various research projects adults and students have been presented with tasks in which they are required to make use of mathematics they have learned in school. These projects have shown that in real-world mathematical situations, adults and students do not use school-learned mathematical methods or procedures (Lave, Murtaugh, & de la Rocha, 1984; Masingila, 1993; Nunes, Schliemann, & Carraher, 1993). Lave (1988) compared adults' use of mathematics in shopping and test situations that presented similar mathematical demands. She found that the adults did not make use of their school-learned mathematics in shopping situations. More important, perhaps, she found that the adults did not regard the two situations as similar, and their choice of mathematical procedure depended more on their environment or context than on the actual mathematics within the tasks. As a result of this and other research, Lave used the term *situated learning* to describe the way in which learning is linked to the situation or context in which it takes place.

Lave (1988; Lave & Wenger, 1991) has provided a powerful critique of those theories of learning transfer that suggest that mathematics is simply learned in school and then lifted out of the classroom and applied to new situations. She and others in the field of situated cognition (Brown, Collins, & Duguid, 1989; Young, 1993)

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have been instrumental in raising awareness of the importance of the situation or context in which learning is encountered. One of the aims of this research study was to investigate Lave's notion of situated learning, particularly the factors that appear to influence school students when they encounter similar mathematical situations in various forms and contexts. I was particularly interested to discover whether different forms of teaching would create different forms of knowledge, which might then cause students to interact differently with the demands of new and unusual situations. To do this I contrasted two very different learning environments and monitored the effects of the environments on the understanding that students developed. My choice of mathematical environments was influenced by a number of factors that I describe below.

Various mathematics educators have suggested that students are unable to use school-learned methods and rules because they do not fully understand them. Educators relate this lack of understanding to the way that mathematics is taught. Schoenfeld (1988), for example, argued that teaching methods that focus on standard textbook questions encourage the development of procedural knowledge that is of limited use in nonschool situations. These and similar arguments have contributed to the growing support for open, or process-based, forms of mathematics. Supporters of process-based work argue that if students are given open-ended, practical, and investigative work that requires them to make their own decisions, plan their own routes through tasks, choose methods, and apply their mathematical knowledge, the students will benefit in a number of ways. The reported benefits generally relate to increased enjoyment and understanding (Perez, 1985/1986; Silver, 1994; Winograd, 1990/1991), to equality of opportunity (Burton, 1986), and even to enhanced transfer (Cognition and Technology Group at Vanderbilt, 1990). Research into the effectiveness of process-based teaching (see, e.g., Charles & Lester, 1984; Cobb et al., 1991; Cobb, Wood, Yackel, & Perlwitz, 1992) is, however, limited, partly because process-based mathematical learning environments are extremely rare in schools.

In recent years in the United Kingdom, there has been an official legitimization of open approaches to mathematics teaching through the government-sponsored Cockcroft report of 1982 (Cockcroft, 1982) and then through the National Curricula of 1989 and 1991 (Department of Education and Science, 1989, 1991), which made the teaching of process-based work statutory. In the United States, the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (1989) offered a similar legitimization of process-based approaches. More recently, we have experienced a Conservative backlash in the United Kingdom against all forms of progressive education and furtherance of a set of policies that politicians and the media have termed "back-to-basics." These policies include support for learning by rote, emphasis on arithmetic and numeracy, and the following of set methods and rules (Ball, 1994). These back-to-basics policies stand as a direct threat to the emergence of a new form of mathematics education that many believed was going to change school mathematics into a subject that students would find more enjoyable, understandable, and relevant (Burton, 1986, 1995).

These theoretical and historical developments formed the background to my research. In the face of opposing claims about the advantages of process-based work and of back-to-basics approaches, I chose to investigate, in detail, a process-based mathematical environment and to contrast this with a content-based, scheme-led¹ mathematical environment. A central part of this study included consideration of whether either approach served as an encouragement for students to use their mathematical knowledge in new and unusual situations.

RESEARCH METHODS

In order to contrast content and process-based mathematical environments, I conducted ethnographic, 3-year case studies (Eisenhart, 1988) of two schools. I chose to conduct in-depth, mainly qualitative case studies of the two schools' mathematical approaches because I wanted to monitor the relationships between the students' day-to-day experiences in classrooms and their developing understanding of mathematics. As part of the case studies, I performed a longitudinal cohort analysis of a "year group" of students in each school, while they moved from Year 9 (age 13) to Year 11 (age 16). The two case studies included a variety of qualitative and quantitative techniques. To understand the students' experiences of mathematics, I observed between 80 and 100 lessons in each school, usually taking the role of a participant observer (Eisenhart, 1988; Kluckhohn, 1940). I interviewed approximately 20 students and 4 teachers each year, analyzed comments elicited from students and teachers about classroom events (Beynon, 1985), gave questionnaires to all the students in my case study year groups ($n \approx 300$), and collected an assortment of background documentation. These methods, particularly the lesson observations and student interviews, enabled me to develop a comprehensive understanding of the students' experiences and to begin to view the world of school mathematics from the students' perspectives (Hammersley, 1992).

In addition to these methods, I gave the students various assessments during the 3-year period. Some of these I designed myself, but I also analyzed school and external examinations, such as the national mathematics examination (General Certificate of Secondary Education [GCSE]), taken by almost all 16-year-olds. All of these methods, both qualitative and quantitative, were used to inform each other in a continual process of interaction and reanalysis (Huberman & Crandall, 1982). To validate emerging perspectives, I made extensive use of triangulated data (Smith & Robbins, 1982), and the analyses developed in this paper were generally based on three or more different data sources. As the study developed, I used progressive focusing to form and shape new research ideas and perspectives, and my ongoing research design was continually influenced by events in the field.

¹In the United Kingdom a *mathematics scheme* refers to a series of textbooks, booklets, or cards that students work through, usually alone, without prior explanation from the teacher. Schemes are usually designed to cover all the necessary content in the curriculum, and many teachers use them as their sole teaching source.

In this paper I draw on lesson observations of classes in Years 9 to 11, interviews with students, questionnaire data, and the results of GCSE examinations, applied assessment activities, and short numerical questions.

THE TWO SCHOOLS

The two schools in the research study were chosen because their teaching methods were very different but their student bodies were very similar. I first chose a school that used a process-based mathematical approach, a practice that is rare in the United Kingdom; I refer to this school as Phoenix Park. I then selected a school that used a content-based mathematical approach and that had a student body that was almost identical to that of Phoenix Park. I refer to this school as Amber Hill.

The two schools both lie in the heart of mainly White, working-class communities located on the outskirts of large cities. Both schools are surrounded by council-owned houses (public housing) where the majority of the children live. Neither school is selective, and most parents choose the schools because of their proximity to their houses. In an analysis of socioeconomic status, derived from fathers' occupations, there were no significant differences between the cohorts in the two schools. Amber Hill is a secondary school that begins with Year 7, when students are 11 years of age. There were about 200 students in the year group I followed: 47% of these were females, 20% were from single-parent families, 68% were classified as working class, and 17% were from ethnic minorities. Phoenix Park is an "upper" school, which means that the students start in Year 9 when they are 13 years of age. There were approximately 110 students in the year group that I followed: 42% of these were females, 23% were from single-parent families, 79% were classified as working class, and 11% were from ethnic minorities. A comparison of the abilities of the students at the beginning of the research study was based on results of National Foundation for Educational Research (NFER) tests, which I standardized against national norms. These tests are similar to IQ tests and include questions assessing verbal reasoning, logic, and numeracy; 75% of Amber Hill students and 76% of Phoenix Park students scored below the national average on these tests, and there were no significant differences between the two sets of students. At the beginning of Year 9 (the start of my research), I gave the students a set of seven short questions assessing various aspects of number work. Some of these questions were contextualized, that is, they presented mathematical problems within described situations. There were no significant differences in the performance of the two sets of students on these tests.

I will now describe the distinguishing characteristics of the two schools and their mathematical approaches. Because there is not sufficient space to report my case study findings in detail here, I have referred to other sources when appropriate.

Amber Hill School

Amber Hill is a large, mixed, comprehensive school, run by an authoritarian (Ball, 1987) head teacher who tried to improve the school's academic record by

encouraging and, when possible, enforcing traditionalism. As a result, the school was unusually ordered and orderly, and any visitor walking the corridors would observe quiet and calm classrooms with students sitting in rows or small groups, usually watching the chalkboard or working through exercises. In mathematics lessons the teachers used the School Mathematics Project (SMP) 11–16 scheme, a content-based scheme that is used by the majority of mathematics departments in the United Kingdom. As part of this scheme students used individualized booklets in Years 7 and 8 and textbooks in Years 9, 10, and 11. The individualized booklets introduced students to mathematical procedures and techniques and then presented a range of questions for student practice. The different booklets each presented a different area of mathematics at a specific level of content. Work in the booklets was self-paced; when students finished one booklet, they collected another. Teachers interacted with students on an individual basis; there was no teaching from the front of the room.

In Year 9 the students moved to a more formal system of textbooks and instruction. The SMP textbooks are quite traditional in format. They generally explain a particular mathematical method and then present a series of exercises in which the students practice the method. Some of the questions are contextualized. The eight mathematics teachers all presented techniques and methods from the chalkboard for 15 to 20 minutes at the start of lessons; they then gave students questions to work through from the textbooks. At Amber Hill the students worked through these textbooks in every mathematics lesson in Years 9 to 11 apart from approximately 3 weeks of each of Years 10 and 11, when they were given an open-ended task. The eight teachers of mathematics at Amber Hill were all committed and experienced. The students were grouped into eight sets, based on their entry scores and teachers' beliefs about their abilities; Set 1 contained students of the highest attainment.

In my lesson observations at Amber Hill, I was repeatedly impressed by the motivation of the students, who would work through their exercises without complaint or disruption. In a small quantitative assessment of their time on task (Peterson & Swing, 1982), I recorded the numbers of students who were working 10 minutes into, halfway through, and 10 minutes before the end of each lesson. I observed eight lessons, each with approximately 30 students, and found that 100%, 99%, and 92% of the students appeared to be working at these three respective times. The first of these figures was particularly high because at this early point in lessons the students were always watching the teachers work through examples on the chalkboard.

Unfortunately, control and order in the mathematics classroom do not, on their own, ensure effective learning. My lesson observations, interviews, and questionnaires all showed that many students found mathematics lessons in Years 9, 10, and 11 extremely boring and tedious. In the lessons that I observed, students often demonstrated a marked degree of disinterest and a lack of involvement with their work. Thus, although the students worked hard and stayed on task throughout their lessons, they were extremely passive in their approach to their work and would dutifully complete exercises in lessons without any apparent desire to challenge or think about what they were doing (Boaler, 1997a). At the end of their Year 9 all my case

study cohorts ($n = 160$) completed a questionnaire that asked the students to write about aspects of lessons that they liked, disliked, or would like to be improved. This questionnaire prompted many students to write about the similarity of their mathematical experiences and the dominance of textbook work. In response to the question “What do you dislike about the way you do maths at school?” 49 students (31%) criticized the lack of variety in the school’s approach; 77 students (48%) also reported a lack of practical or activity-based experience. For example, one student commented, “Maths would be more interesting if we had some practical or group work. We barely ever do class activities; for example, we have done one all year.” The students believed that their mathematical experiences lacked variety, not only because they worked on textbooks for the vast majority of the time, but also because they regarded the questions within the books as very similar to each other: “I wish we had different questions, not three pages of sums on the same thing.”

In interviews conducted with students in Years 10 and 11, the students also referred to the monotony of their mathematical experience:

JB: Can you think of a maths lesson that you really enjoyed?

D: No.

P: They’re all the same.

D: I’m just not interested in it really; it’s just boring; they’re just all the same; you just go on. [Danielle and Paula, Year 10, Set 2]

The students’ comments noted in this paper represent a very small selection of those I received, but they were not unusual in any way. Indeed, comments were chosen because they typified the views presented by multiple students on several occasions. A large proportion of the students in the year group believed that mathematics lessons were too similar and monotonous. These beliefs were consistent across the eight mathematics sets, even though the teachers of these groups were quite different and varied in popularity and experience. The aspect that united the teachers was their common method of teaching: a 15–20 minute demonstration of method followed by the students working through questions in their textbooks, either alone or with their seating partners.

Rule-following behavior. As a result of approximately 100 lesson observations, I classified a variety of behaviors that seemed to characterize the Amber Hill students’ approaches to mathematics. One of these I termed *rule following* (Boaler, 1996, 1997a). Many of the Amber Hill students held a view that mathematics was all about memorizing a vast number of rules, formulas, and equations, and this view appeared to influence their mathematical behavior:

N: In maths there’s a certain formula to get to, say, from A to B, and there’s no other way to get to it, or maybe there is, but you’ve got to remember the formula; you’ve got to remember it. [Neil, Year 11, Set 7]

L: In maths you have to remember; in other subjects you can think about it. [Louise, Year 11, Set 1]

The students' views about the importance of remembering set rules, equations, and formulas seemed to have many negative implications. For example, in mathematical situations the students did not think it was appropriate to try to think about what to do; they thought they had to remember a rule or method they had used in a situation that was similar. However, because in mathematics lessons they were never encouraged to discuss different rules and methods or to think about why they may be useful in some situations and not others, the students did not know when situations were mathematically similar. Therefore, questions that did not require an obvious and simplistic use of a rule or formula caused students to become confused.

A second problem was provided for the students who thought that mathematics should be about understanding and sense making (Lampert, 1986). These students experienced a conflict at Amber Hill because they felt that their school's approach emphasized memory and rote learning:

- JB:* Is maths more about understanding work or remembering it?
J: More understanding; if you understand it, you're bound to remember it.
L: Yeah, but the way Mr. Langdon teaches, it's like he just wants us to remember it, when you don't really understand things.
JB: Do you find that it is presented to you as things you have got to remember, or is it presented to you as things you have got to work through and understand?
L: Got to be remembered.
J: Yeah, remember it—that's why we take it down in the back of our books, see; he wants us to remember it.

[Louise and Jackie, Year 10, Set 1]

The predominance of the students' belief in the importance of remembering rules was further demonstrated through a questionnaire devised in response to my fieldwork and given to the students in Year 10 ($n = 163$). One item of this questionnaire asked students what was more important in approaching a problem, remembering similar work done before or thinking hard about the work at hand. Sixty-four percent of students said that remembering similar work done before was more important. This view appeared to be consistent with the strategies they employed in class and, in many ways, was indicative of their whole approach to mathematics. The belief of many of the Amber Hill students that mathematics was all about learning set rules and equations seemed to have stopped them from trying to interpret situations mathematically. The Cognition and Technology Group at Vanderbilt (1990) has noted that when novices are introduced to concepts and theories, they often regard them as new "facts or mechanical procedures to be memorized" (1990, p. 3). The Amber Hill students rarely seemed to progress beyond this belief. This belief led also to a second, related aspect of their behavior that I have referred to as *cue based* (Boaler, 1996, 1997a).

Cue-based behavior. Often during lesson observations I witnessed students basing their mathematical thinking on what they thought was expected of them rather than on the mathematics within a question. Brousseau (1984) has talked about the *didactical contract* (p. 113), which causes pupils to base their mathematical thinking on whatever they think the teacher wants them to do. I was often aware that the

Amber Hill students used nonmathematical cues as indicators of the teacher's or the textbook's intentions. These sometimes related to the words of the teacher, but students would also use such cues as the expected difficulty of the question (what they thought should be demanded of them at a certain stage), the context of the question, or the teacher's intonation when talking to them. The following extract is taken from my field notes of a lesson at Amber Hill in Year 9, Set 1:

After a few minutes Nigel and Stephen start to complain because there is a question that "is a science question, not a maths question"; they decide they cannot do it, and I go over to help them. According to the problem, 53% of births are male babies and 47% female babies, but there are more females in the population. Students are asked to explain this. I ask Stephen if he has any idea, and he says, "Because men die quicker." I say that this is right and leave them. Soon most of the students are putting their hands up and asking for help on the same question. Carol, a high-attaining girl, has already completed all of the exercise but has left this question out and says that she cannot do it.

Later in the lesson, Helen has her hand up and I go over. The question says that "58.9 tonnes of iron ore has 6.7 tonnes of iron in it. What percentage of the ore is iron?" While I am reading this, Helen says, "I'm just a bit thick really." I ask Helen what she thinks she should do in the question, and she immediately tells me, correctly. When I tell her that she is right, she says, "But this is easier than the other questions we have been doing: in the others we have had to add things on and stuff first." A few minutes later two more girls ask me for help on the same question: both of these girls have already completed more difficult questions.

In this extract the girls gave up on the question on iron ore because the mathematical demand was different from what they had expected. The previous exercise had presented a series of abstract calculations in which the students were asked to work out percentages that required them to "add things on and stuff first." In the next exercise the questions were mathematically simpler, but they were contextualized. The writers of the textbook obviously regarded these as more difficult, but the girls were thrown by this, because they expected something more mathematically demanding. This expectation caused them to give up on the question. It is this sort of behavior that I have termed cue based, because the students were using irrelevant aspects of the tasks, rather than mathematical sense making or understanding, to cue them into the right method or procedure to use. Schoenfeld (1985) asserted that this sort of cue-based behavior is formed in response to conventional pedagogic practices that demonstrate set routines that should be learned in mathematics. This sort of behavior, which was common among the Amber Hill students, meant that if a question seemed inappropriately easy or difficult, if it required some non-mathematical thought, or if it required an operation other than the one they had just learned about, many students would stop working.

The Amber Hill teachers were aware that the students experienced these sorts of problems:

Students are generally good, unless a question is slightly different to what they are used to, or if they are asked to do something after a time lapse, if a question is written in words, or if they are expected to answer in words. If you look at the question and tell them that it's basically asking them to multiply 86 by 32, or something, they can do it, but otherwise they just look at the question and go blank. [Tim Langdon, Head of Mathematics]

However, the teachers believed that the students experienced difficulty because of the closed nature of their primary school experiences (ages 5 to 11), rather than because of their secondary school teaching.

To summarize, the students at Amber Hill were highly motivated and hard working, but many of them found mathematics lessons tedious and boring. A large number of the students also appeared to be influenced by an extremely set view of mathematics that they essentially regarded as a vast collection of exercises, rules, and equations that needed to be learned. These perceptions meant that when situations were slightly different from what they expected, or when they did not contain a cue suggesting the correct rule to use, many did not know what to do.

Phoenix Park School

Phoenix Park was different from Amber Hill in many respects; most differences derived from the school's commitment to progressive education. The students at the school were encouraged to take responsibility for their own actions and to be independent thinkers. There were few school rules, and lessons had a relaxed atmosphere. In mathematics lessons at Phoenix Park, the students worked on open-ended projects and in mixed-ability groups at all times, until January of their final year when they stopped their project work and started to practice examination techniques. At the beginning of their projects, the students were given a few different starting points among which they could choose, for example, "The volume of a shape is 216, what can it be?" or "What is the maximum sized fence that can be built out of 36 gates?" The students were then encouraged to develop their own ideas, formulate and extend problems, and use their mathematics. The approach was based on the philosophy that students should encounter a need to use mathematics in situations that were realistic and meaningful to them. If a student or a group of students needed to use some mathematics that they did not know about, the teacher would teach it to them. Each project lasted for 2 to 3 weeks, and at the end of the projects the students were required to turn in descriptions of their work and their mathematical activities. Prior to joining Phoenix Park, the students all attended schools that used the SMP scheme; therefore, until the end of Year 8 the students had experienced the same mathematical approach as the Amber Hill students. When, at the beginning of Year 9, the Amber Hill students moved from SMP booklets to textbooks, the Phoenix Park students moved from the same SMP booklets to a project-based approach.

A number of people have commented to me that Phoenix Park's approach must be heavily dependent on highly skilled and very rare teachers, but I am not convinced that this is true. The head of department who devised the approach left at the beginning of my research, and the new head of department was ambivalent toward the approach. A second teacher preferred textbooks, but tried to fit in. A third teacher believed in the approach but had a lot of problems controlling his classes and getting them to do the work. The fourth teacher was newly qualified at the start of the research. The four teachers that made up the mathematics department were all committed and hard working, but I did not regard them as exceptional.

In the 80 mathematics lessons I observed at Phoenix Park, there was very little

control or order and, in contrast to Amber Hill, no apparent structure to lessons. Students could take their work to another room and work unsupervised if they wanted to, because they were expected to be responsible for their own learning. In many of my lesson observations I was surprised by the number of students doing no work or choosing to work only for tiny segments of lessons. A study of the number of students working on task 10 minutes into, halfway through, and 10 minutes before the end of 11 lessons showed that 69%, 64%, and 58% of the students (approximately 30 in each lesson) were on task at the three respective times. What these figures do not show is that there were a few students in each class who appeared to do almost no work in any lesson. In Year 10 the students were given questionnaires in which they were asked to write a sentence describing their mathematics lessons. The three most popular descriptions from Phoenix Park students ($n = 75$) were “noisy” (23%), “a good atmosphere” (17%), and “interesting” (15%). These descriptions contrasted with the three most popular responses from Amber Hill students ($n = 163$), which were “difficult” (40%), something related to their teacher (36%), and “boring” (28%). When I asked students at Phoenix Park, in interviews, to describe their lessons to me, the single factor that was given the highest profile was the degree of choice they were given:

T: You get a choice.

JB: A choice between...?

T: A couple of things; you choose what you want to do and you carry on with that, and then you start another, different one.

JB: So you're not all doing the same thing at the same time?

Both: No.

JB: And can you do what you want in the activity, or is it all set out for you?

L: You can do what you want, really.

T: Sometimes it's set out, but you can take it further.

[Tanya and Laura, Year 10]

Students also talked about the relaxed atmosphere at Phoenix Park, the emphasis on understanding, and the need to explain methods.

I: It's an easier way to learn, because you're actually finding things out for yourself, not looking for things in the textbook.

JB: Was that the same in your last school, do you think?

I: No, like if we got an answer, they would say, “You got it right.” Here you have to explain how you got it.

JB: What do you think about that—explaining how you got it?

I: I think it helps you.

[Ian, Year 10]

During lesson observations at both schools, I frequently asked students to tell me what they were doing. At Amber Hill most students would tell me the textbook chapter title, and, if I inquired further, the exercise number. It was generally very difficult to obtain any further information. At Phoenix Park students would describe the problem they were trying to solve, what they had discovered so far, and what they were going to try next. In lessons at Phoenix Park the students discussed the

meaning of their work with each other and negotiated possible mathematical directions. In response to the questionnaire item concerning remembering or thinking, only 35% of the Phoenix Park students prioritized *remembering*, compared with 64% of the Amber Hill students. At its best the Phoenix Park approach seemed to develop the students' desires and abilities to think about mathematics in a way that the Amber Hill approach did not:

J: Solve the problems and think about other problems and solve them—problems that aren't connected with maths; think about them.

JB: You think that the way you do maths helps you to do that?

J: Yes.

JB: Things that aren't to do with maths?

J: Yes. It's more the thinking side to sort of look at everything you've got and think about how to solve it.

[Jackie, Phoenix Park, Year 10]

However, not all the feedback from Phoenix Park was positive. About one fifth of the students in each group liked neither the openness of the approach nor the freedom they were given. They did not work when left to their own devices, and they said that they preferred working from textbooks. Most of these students were boys, and often they were disruptive, not only in mathematics classes but throughout the school. Some of the girls described the problems these students experienced:

A: Well, I don't think they were stupid or anything. They just didn't want to do the work; they didn't want to find things out for themselves. They would have preferred it from the book—they needed to know straightaway sort of thing.

JB: Why were they like that, do you think?

A: Some people don't like to find things out for themselves. It depends on how they was [*sic*] taught in their middle school as well.

H: They just couldn't be bothered, really, to find anything out.

[Anna and Hilary, Phoenix Park, Year 11]

These girls offered an interesting, and probably quite accurate, perspective on some of the students. They did not want freedom or choice; they “couldn't be bothered to find anything out”; they “needed to know straightaway.” Certainly a major disadvantage of the school's approach seemed to relate to the small proportion of students who simply did not like it and chose to opt out of it.

The students' enjoyment of their mathematics teaching, at the two schools, appeared to be quite different. At Amber Hill there was a strong consensus about the shortcomings of the school's approach, but the Phoenix Park approach really seemed to divide the year group, most of whom liked it, although some hated it. The consensus at Amber Hill primarily related to the monotony of doing textbook work and the lack of freedom or choice the students were given. In Year 9, 263 students completed a questionnaire that asked them to describe what they disliked about mathematics at school; 44% of the Amber Hill students criticized the mathematics approach, and 64% of these students criticized the textbook system. Other common criticisms included the constant need to rush through work, the tendency for books to go “on and on,” the lack of freedom to work at their own level, and the lack of choice about

topics or order of topics. At Phoenix Park 14% of the students criticized the school's approach, and the most common response, from 23% of students, was to list nothing they disliked about mathematics at school. Table 1 presents all the responses given to three open questions (including the one discussed earlier); the responses have been combined to present an overview of the issues important to the students. The three items asked students to describe what they liked, disliked, and would like to change about mathematics lessons, and these prompted 382 comments from the 160 Amber Hill students and 202 comments from the 103 Phoenix Park students.

One of the most obvious differences demonstrated by these results is that when students were invited to give their own opinions about mathematics lessons, the Phoenix Park students chose to comment on the interest of their lessons and their enjoyment of open-ended work, whereas the Amber Hill students were more concerned about a lack of understanding and their dislike of textbooks.

Table 1
Percentages of Students' Responses to Items on Year 9 Open Questionnaire

Items	Amber Hill	Phoenix Park
Enjoy open-ended work	14	38
Dislike textbook work	22	0
Can't understand work	20	6
Can understand work	3	5
Work is interesting	4	21
Want more interesting work	15	19
Enjoy either working alone or with others	8	4
Pace is too fast	9	3
Pace is about right	0	3

A second important difference between the two schools related to gender preferences for ways of working (Boaler, 1997a, 1997b, 1997c). In questionnaires given to students in the two case study year groups in Years 9, 10, and 11, the boys were always significantly more positive and confident than the girls at Amber Hill, but there were never any significant differences between girls and boys at Phoenix Park. For example, in their Year 9 questionnaire 51% of the boys and 37% of the girls at Amber Hill reported enjoying mathematics all or most of the time ($\chi^2(2, N = 160) = 7.72, p < .05$); at Phoenix Park, 45% of the boys and 63% of the girls reported enjoying mathematics all or most of the time ($\chi^2(2, N = 103) = 3.18, p < .30$). When students were asked whether they were good, okay, or bad at the mathematics they did in school, 6% of the girls and 32% of the boys at Amber Hill regarded themselves as good ($\chi^2(2, N = 160) = 18.04, p < .001$); at Phoenix Park 23% of the girls and 22% of the boys regarded themselves as good ($\chi^2(2, N = 103) = 0.04, p < .98$).

I have attempted here to depict the primary distinguishing characteristics of the two schools' approaches and the students' responses to them. More detail on the schools can be found in Boaler (1996, 1997a), and an analysis of the gender-related implications of the different approaches is provided in Boaler (1997a, 1997b, 1997c). The effect of the ability grouping on students' performance is also considered

in Boaler (1997b). Ultimately, the success or otherwise of either of these two approaches must be ascertained through a consideration of the students' understanding of mathematics. In the next section, I present the results of applied assessment activities and formal, closed examinations.

STUDENT ASSESSMENTS

Lave (1988) has reported the different ways in which students perceive and think about the mathematics they encounter in school and in real-world situations. To gain more insight into this phenomenon, I decided to focus on applied activities situated within school. The ways in which students react to such tasks can never be used to predict the ways in which they will react to real-life mathematical situations. However, I believe that the combination of school settings and realistic constraints provided by applied tasks can give us important insight into the factors that influence a student's use of mathematical knowledge. Furthermore, if students are unable to make use or sense of their school mathematics in such tasks within school, it seems unlikely that they will make use of this mathematics when similar tasks are encountered in the real world with an even greater complexity of mathematical and nonmathematical variables.

The Architectural Activity

In the summer of Year 9, approximately half the students in the top four sets at Amber Hill ($n = 53$) and four of the mixed-ability groups at Phoenix Park ($n = 51$) were asked to consider a model and a plan of a proposed house and to solve two problems related to Local Authority design rules. Students were given a scale plan that showed different cross sections of a house and a scale model of the same house. To solve the problems, students needed to find information from different sources, choose their own methods, plan routes through the task, combine different areas of mathematical content, and communicate information. Because the students at Amber Hill were taken from the top half of the school's ability range and the students at Phoenix Park were not, there was a disparity in the attainment levels of the samples of students. The students in the Amber Hill sample had scored significantly higher on their mathematics NFER entry tests. However, my main aim was not to compare the overall performance of the students in the two schools, but rather to compare each individual's performance on the applied activity with his or her performance on a short written test. Approximately 2 weeks prior to the architectural task, the students took a pencil-and-paper test that assessed all the mathematical content they needed to use in the activity.

The architectural activity comprised two main sections. In the first section the students needed to decide whether the proposed house satisfied a council rule about proportion. The rule stated that the volume of the roof of a house must not exceed 70% of the volume of the main body of the house. The students therefore needed to find the volumes of the roof and of the house and to find the roof volume as a percentage of the house volume. To do so, students could use either the scale plan

or the model. The second council rule stated that roofs must not have an angle of less than 70° . The students therefore had to estimate the angle at the top of the roof (which was actually 45°) from either the plan or the model, a shorter and potentially easier task.

Grades for the two tasks were awarded as follows: A grade of 1 was given if the answer was correct or nearly correct, with one or two small errors; a grade of 2 was awarded if most or all of the answer was incorrect or if the problem was partially attempted. All the students made some attempt at the problems. The students' results for the two problems are shown with their test question results in Table 2. In the test the students were given three questions that assessed the mathematics involved in the proportion problem. These questions were decontextualized and asked students to find out the volume of a cuboid, the volume of a triangular prism (similar to the roof), and a percentage. Students were given a test grade of 1 if they answered all these questions correctly and a grade of 2 if they got one or more wrong (see Table 2).

Table 2
Volume and Angle Results from Activities and Tests

		Amber Hill ^a			Phoenix Park ^b				
		Volume							
		Activity grade			Activity grade				
		1	2		1	2			
Test grade	1	23	15	38	Test grade	1	23	8	31
	2	6	9	15		2	15	5	20
		29	24	53			38	13	51
		Angle							
		Activity grade			Activity grade				
		1	2		1	2			
Test grade	1	31	19	50	Test grade	1	40	8	48
	2	3	0	3		2	2	1	3
		34	19	53			42	9	51

^a $n = 53$. ^b $n = 51$.

From the roof-volume problem, Table 2 shows that at Amber Hill, 29 students (55%) attained a 1 in the activity, compared with 38 (75%) of the Phoenix Park students, despite the fact that the Amber Hill students were taken from the top half of the school's ability range. The table also shows that at Amber Hill, 15 students (28%) could do the mathematics when it was assessed in the test but could not use it in the activity, compared with 8 such students at Phoenix Park (16%). In addition, 15 students (29%) at Phoenix Park attained a 1 on the activity, despite getting one or more of the relevant test questions wrong, compared with 6 students (11%) at Amber Hill.

In the test on angle, the students were given a 45° angle (the same angle as the roof in the activity) and asked whether it was 20° , 45° , 90° or 120° . Of the Amber Hill students, 50 estimated this angle correctly in the test, but only 31 of these students estimated the 45° angle correctly in the applied activity. At Phoenix Park, 40 out of 48 students who recognized the angle in the test solved the angle problem. Paradoxically, the least successful students at Amber Hill were in Set 1, the highest

group. Eight of the 16 students did not solve the roof-volume problem, and 10 of the 16 students did not solve the angle problem. In both of these problems this failure emanated from an inappropriate choice of method. For example, in the angle problem, the 10 unsuccessful students attempted to use trigonometry to decide whether the angle of the roof, which was 45° , was more or less than 70° , but they failed to use the methods correctly. Successful students estimated the angle using their knowledge of the size of 90° angles. Unfortunately, the mere sight of the word *angle* seemed to prompt many of the Set 1 students at Amber Hill to think that trigonometry was required, even though this was clearly inappropriate in the context of the activity. The students seemed to take the word *angle* as a cue for the method to use.

In their Year 10, 100 students in each school completed another applied task and a short written test, and the same pattern of results emerged, but these were more marked. The Phoenix Park students gained significantly higher grades in all aspects of the applied task, and their performances on test and applied situations were very similar. In applied settings the Amber Hill students experienced difficulty using the mathematics that they could use in a test; this difficulty again related to their choice of methods.

Short Closed Questions

The fact that the Phoenix Park students gained higher grades in applied realistic situations may not be considered surprising, given the school's project-based approach. However, in traditional closed questions the Amber Hill students did not perform any better than the students at Phoenix Park. In a set of seven short written tests of numeracy that I devised, there were no significant differences in the results of the two schools, either at the beginning of Year 9 (the beginning of their new approaches) or at the end of Year 10 (2 years later). The GCSE examination is made up of short, fairly traditional, closed questions that assess content knowledge, apart from a few questions that are more applied. Students take these examinations at the end of Year 11. Entry into advanced courses of study, as well as many professional jobs, generally depends on gaining a grade of A, B, or C on this examination. A pass at GCSE is any grade from A to G. In their GCSE examinations, 11% of the Amber Hill cohort attained an A–C grade, and 71% passed the examination. At Phoenix Park 11% of the cohort attained an A–C grade, and 88% passed the examination. Significantly more of the Phoenix Park students than Amber Hill students attained an A–G pass ($\chi^2(1, N = 332) = 12.54, p < .001$), despite the fact that the GCSE examination was markedly different from anything the students were accustomed to at Phoenix Park. The A–C results from both of the schools and the A–G result from Amber Hill are considerably lower than the national averages for this examination, but the ability ranges of the cohorts at the two schools were also considerably lower than national averages.

At Amber Hill there were also significant differences in the attainment of girls and boys; 20% of the boys and 9% of the girls who entered the examination attained grades A–C ($\chi^2(1, N = 217) = 3.89, p < .05$). At Phoenix Park there were no significant differences in the achievement of girls and boys, with 13% of the boys

and 15% of the girls who entered the examination attaining grades A–C ($\chi^2(1, N = 115) = 0.12, p < .80$) (Boaler, 1997a).

DISCUSSION

The relative underachievement of the Amber Hill students in formal test situations may be considered surprising, both because the students worked hard in mathematics lessons and because the school's mathematical approach was extremely examination oriented. However, after many hours of observing and interviewing the students, I was not surprised by the relative performances of the two groups of students. The Amber Hill students had developed an *inert* (Whitehead, 1962) knowledge that they found difficult to use in anything other than textbook questions. In the examination, the students encountered difficulties because they found that the questions did not require merely a simplistic rehearsal of a rule or a procedure; the questions required students to understand what the question was asking and which procedure was appropriate. The questions further required the students to apply to new and different situations the methods they had learned. In interviews following their "mock" GCSE examinations,² Amber Hill students were clear about the reasons for their lack of success. The students agreed that they could not interpret the demands of the unfamiliar questions and that they could not see how to apply the procedures they had learned to the questions asked:

L: Some bits I did recognize, but I didn't understand how to do them; I didn't know how to apply the methods properly.

[Louise, Year 11, Set 3]

In their mathematics lessons the students had not experienced similar demands, for the textbook questions always followed from a demonstration of a procedure or method, and the students were never left to decide which method they should use. If the students were unsure of what to do in lessons, they would ask the teacher or try to read cues from the questions or from the contexts in which they were presented. In the examination the students tried to find similar cues, but they were generally unable to do so:

G: It's different, and like the way it's there like—not the same. It doesn't like tell you it, the story, the question; it's not the same as in the books, the way the teacher works it out. [Gary, Year 11, Set 4]

T: You can get a trigger, when she says like *simultaneous equations* and *graphs, graphically*. When they say like—and you know, it pushes that trigger, tells you what to do.

JB: What happens in the exam when you haven't got that?

T: You panic. [Trevor, Year 11, Set 3]

The students' responses to their examinations suggested that their textbook learning had encouraged them to develop an inert, procedural (Schoenfeld, 1985) knowledge that was of limited use to them. This development may be linked to

²In mock examinations, the students are given a previous year's GCSE examination, which they take under examination conditions.

the students' belief that mathematics was a rule-bound, memory-based subject:

L: In maths you have to remember; in other subjects you can think about it. But in exams the questions don't really give you clues on how to do them.

[Louise, Year 11, Set 1]

The students did not consider the examination questions holistically, nor did they try to interpret what to do because they believed that in mathematics "you have to remember." Unfortunately, their examination did not provide any cues that would help them access their memory.

Gibson (1986) is cited as providing the most extreme position on situated learning (Young, 1993) because he asserted that individuals develop meaning from situations by perceiving and acting and by creating meaning on the spot, rather than by using their memories of stored representations. Gibson's model seems important to consider alongside the reflections of the students at the two schools, because his description of the way in which people perceive and create meaning in new situations resonates with the way in which the Phoenix Park students described their examination experiences. The students at Phoenix Park believed that success in their examinations was enhanced by their desire and abilities to think about unfamiliar situations and determine what was required:

T: I think it allows—when you first come to the school and you do your projects, and it allows you to think more for yourself than when you were in middle school and you worked from the board or from books.

JB: And is that good for you, do you think?

T: Yes.

JB: In what way?

T: It helped with the exams where we had to...had to think for ourselves there and work things out.

[Tina, Year 11]

Indeed, it was this perceiving and interpreting of situations that seemed to characterize the real differences in the learning of the students at the two schools. When the students were presented with the angle problem in the architectural task, many of the Amber Hill students were unsuccessful, not because they were incapable of estimating an angle but because they could not interpret the situation correctly. In a second applied task given to students in Year 10, the Amber Hill students were again unable to determine what to do. In examinations they, similarly, could not interpret what the questions were asking. The Phoenix Park students were not as well versed in mathematical procedures, but they were able to interpret and develop meaning in the situations encountered:

JB: Did you feel in your exam that there were things you hadn't done before?

A: Well, sometimes, I suppose they put it in a way which throws you. But if there's stuff I actually haven't done before, I'll try and make as much sense of it as I can and try and understand it as best as I can. [Arran, Year 11]

The students at Phoenix Park seemed to have developed a predisposition to think about and to use mathematics in novel situations, and this tendency seemed to rest on two important principles: First, the students had the belief that mathematics involved active and flexible thought. Second, the students had developed an ability to adapt and change methods to fit new situations.

L: Well, if you find a rule or a method, you try and adapt it to other things. When we found this rule that worked with circles, we started to work out the percentages and then adapted it, so we just took it further and [used] different steps and tried to adapt it to new situations.
[Lindsey, Year 11]

This flexibility in approach, combined with the students' beliefs about the adaptable nature of mathematics and the need for reasoned thought, appeared to enhance the students' examination performance. The proportion of Phoenix Park students who passed the GCSE examination was higher than the national average, despite the initial attainment of the cohort and despite the fact that the students had not encountered all the areas of mathematics that were assessed in their examination. In previous years Phoenix Park had entered their students for a new form of examination that rewarded problem solving as well as procedural knowledge. The school achieved greater success on this new form of examination.³ Unfortunately, Conservative politicians in the United Kingdom caused the new examination, which held the potential for important improvements in mathematics education, to be abolished.

Another major difference in the learning of the students at the two schools related to their reported use of mathematics in real-world situations. The students at Amber Hill all spoke very strongly about their complete inability to make use of any school-learned methods in real situations, because they could not see any connection between what they had done in the classroom and the demands of their lives outside the classroom (Boaler, 1996, 1997a).

JB: When you use maths outside of school, does it feel like when you do maths in school or does it feel...?

K: No, it's different.

S: No way; it's *totally* different.

[Keith and Simon, Year 11, Set 6]

The students at Phoenix Park did not see a real difference between their school mathematics and the mathematics they needed outside of school.

JB: Do you think in the future, if you need to use maths in something, do you think you will be able to use what you're learning now or do you think you will just make up your own methods?

G: No, I think I'll remember. When I'm out of school now, I can connect back to what I done [*sic*] in class so I know what I'm doing.

[Gavin, Year 10]

In the following extract, Sue contrasted her project-based work with the few weeks of examination preparation the students received prior to their GCSE at Phoenix Park:

³When Phoenix Park first adopted a process-based approach, they were involved in a small-scale pilot of a new GCSE examination that assessed process as well as content. In 1994 the School Curriculum and Assessment Authority (SCAA) withdrew this examination, and the school was forced to enter students for a traditional, content-based examination. The proportion of students at the school attaining Grades A–C and A–G dropped from 32% and 97%, respectively, in 1993 to 12% and 84% in 1994. The school has now reintroduced textbook work in an attempt to raise examination performance.

- JB:* Do you think, when you use maths outside of school, it feels very different to using maths in school, or does it feel similar?
- S:* Very different from what we do now; if we do use maths outside of school, it's got the same atmosphere as how it used to be, but not now.
- JB:* What do you mean by "it's got the same atmosphere"?
- S:* Well, when we used to do projects, it was like that, looking at things and working them out, solving them—so it was similar to that, but it's not similar to this stuff now; it's—you don't know what this stuff is for really, except the exam.

[Sue, Year 11]

Sue's comments seem to capture the essence of the value of Phoenix Park's approach. When the students worked on projects, they needed to think for themselves, interpret situations, choose, combine, and adapt mathematical procedures, and this had "the same atmosphere" as the mathematical demands of the real world. The students at Phoenix Park had been enculturated into a system of working and thinking that appeared to be advantageous to them in new and unusual settings.

CONCLUSION

At Amber Hill, the mathematics teachers were not unusual. They were dedicated teachers who were effective at teaching textbook mathematical methods. The students they taught worked hard to learn these methods. Schoenfeld (1985) has described this type of textbook approach as being widespread, and it is certainly the predominant model adopted in the United Kingdom (Office for Standards in Education, 1993). The results of this research reveal some important limitations of this type of teaching. At Amber Hill, the students developed an inert, procedural knowledge that was of limited use to them in anything other than textbook situations.

A growing body of research has identified the effectiveness of apprenticeship learning (see, e.g., Lave, 1988; Chaiklin & Lave, 1993). A number of projects have managed to transport some of the positive features of cognitive apprenticeship into classroom settings (Cognition and Technology Group at Vanderbilt, 1990). The Phoenix Park approach shared some similarities with apprenticeship forms of learning, particularly because the students were introduced to new concepts and procedures only as part of authentic activities. The learning that the students developed in response to this approach appeared to be more usable than that developed from nonapprenticeship teaching. It seemed that the act of using mathematical procedures within authentic activities allowed the students to view the procedures as tools that they could use and adapt. The understandings and perceptions that resulted from these experiences seemed to lead to increased competence in transfer situations. But this competence supported Lave's rejection of theories of learning transfer (1988), because the students at Phoenix Park did not know *more* mathematics than the students at Amber Hill. Rather, the students were able to use mathematics because of three important characteristics: a willingness and ability to perceive and interpret different situations and develop meaning from them (Gibson, 1986) and in relation to them (Lave,

1993); a sufficient understanding of the procedures to allow appropriate procedures to be selected (Whitehead, 1962); and a mathematical confidence that enabled students to adapt and change procedures to fit new situations.

There were many indications from this study that the traditional back-to-basics mathematics approach of Amber Hill was ineffective in preparing students for the demands of the real world and was no more effective than a process-based approach for preparing students for traditional assessments of content knowledge. There were problems with the Phoenix Park approach, too, including the fact that some students spent much of their time not working. Nevertheless, the Phoenix Park students were able to achieve more in test and applied situations than the Amber Hill students; they also developed more positive views about the nature of mathematics, views I have not had space to report on in this paper. It would be easy to dismiss these results or to attribute them to some other factor, such as the quality of the teachers at Phoenix Park. But part of the value of ethnographic studies is the flexibility they allow researchers to investigate the influence of various factors, using the data that are most appropriate. After hundreds of hours spent in the classrooms at the two schools, after hearing the students' own accounts of their learning, after analyzing over 200 questionnaire responses for each year, and after consideration of the results of traditional and applied assessments, I have been able to isolate factors that have and have not been influential in the students' development of understanding. One important conclusion that I feel able to draw from this analysis is that a traditional textbook approach that emphasizes computation, rules, and procedures, at the expense of depth of understanding, is disadvantageous to students, primarily because it encourages learning that is inflexible, school-bound, and of limited use.

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Author

Jo Boaler, Lecturer, School of Education, King's College London, Cornwall House, Waterloo Road, London SE1 8WA, United Kingdom; joboaler@kcl.ac.uk