Measuring Degrees of Incoherence

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Abstract

I discuss the construction of "measures of incoherence". These measures allow theorists to give a precise sense to the idea that agents can violate norms of probabilistic reasoning more or less severely. I will discuss previous attempts at providing such a measure, the reasons one might want such a measure, and how those objectives place different (and sometimes conflicting) constraints on the character of the measure one ought pick. I argue that degree of incoherence is best viewed as *purpose-dependent*. That is to say, there is no absolute measure of incoherence but, rather, how incoherent we ought view a person depends on the purpose for which we have decided to evaluate them.

Suppose there is some finite state space Ω. The finite number of states $\omega_i \in \Omega$ collectively form a partition of Ω. There is also an event space $\boldsymbol{E} \subseteq \mathcal{P}(\Omega)$. Each $\boldsymbol{A} \in \boldsymbol{E}$ is associated with an indicator function X_A. This is a function $\Omega \rightarrow \{0, 1\}$ which takes the values

$$X_A(\omega) = \begin{cases} 1 & \omega \in A \\ 0 & \omega \notin A \end{cases}$$

The gambler takes some of these indicator functions X_A and offers a number, denoted by p_A, which is their fair betting odds (or "prevision") on X_A. This means that for any stakes α (within their budget) the gambler is indifferent between accepting the gamble $\alpha_A(X_A - p_A)$ and the gamble $-\alpha_A(X_A - p_A)$. Call the full set of previsions the agent has offered their "credence function".

Suppose there is some finite collection of stakes $\alpha_1 ... \alpha_n$ such that for an agent's fair previsions $p_1 ... p_n$ on random variables $X_1 ... X_n$ a finite collection of gambles has the following property:

$\forall \omega \sum_1^n \alpha_i (X_i(\omega) - p_i) < 0$

In this scenario the agent in question is *incoherent*, and the collection of bets which witnesses this is called the *Dutch Book*.

Stated above are some of the definitions necessary for the famous Dutch book theorem (restricted to indicator functions on events) that philosophers are familiar with. It is noteworthy that this only provides the basis for a binary division of credence functions; there are credence functions that are incoherent, and those that are not incoherent. However, people are intuitively able to make more nuanced judgements than that. For instance, anything less than full belief in a tautology is incoherent. Suppose there are two credence functions, P_1 and P_2 such that $P_1(T) = 0.999999$ and $P_2(T) = 0.00001$. Both P_1 and P_2 are incoherent. However, one feels that P_2 is far *more* incoherent than P_1.

Attempts have been made to sharpen up the intuition which drives that judgement into a precise measure of degree of incoherence. There are two prominent strands of thought in the literature. There is the *cardinality of incoherency* strand, for which Lyle Zynda (Zynda 1996) may be taken as representative. The guiding idea here is to consider how large a subset of the credence function could be made coherent, or how many incoherent subsets it has. The larger the coherent subset the more coherent a credence function is. Or, alternately, the more incoherent subsets there are the more incoherent the credence function is. Then there is the *magnitude of incoherence* strand, for which the joint work of Mark Schervish, Teddy Seidenfeld and Joseph Kadane (Schervish *et al* 2002) is representative. The guiding idea here is to use the Dutch Book argument as a means of providing a measure. Roughly put - the greater the loss a cunning bookie could inflict upon the incoherent agent, the more incoherent they are. Although there are plenty of details (and attendant devils) to be worked out, these are the guiding intuitions behind each approach.

These two strands of thought can come apart. Consider $E = \{a, \neg a, b, \neg b\}$ with two credence functions on it. We write the credence functions as lists of pairs. For each pair, the first element is the indicator function for the event being bet upon, the second element is the prevision for that event. The credence functions are:

$C_1 = \{< X_a, 0.6 >, < X_{\neg a}, 0.6 >, < X_b, 0.6 >, < X_{\neg b}, 0.6 >\}$
$C_2 = \{< X_a, 0.6 >, < X_{\neg a}, 0.4 >, < X_b, 1 >, < X_{\neg b}, 1 >\}$

If one follows the guiding idea behind the cardinality measure of incoherence, C_1 will seem more incoherent than C_2. Whereas according to a magnitude measure of incoherence the opposite will seem to be the case. We are therefore faced with a theoretical puzzle - can the two strands be reconciled? If not, which (if any) should we prefer?

I will argue that what sort of measure we should prefer - and, consequently, the degree to which we should view somebody as incoherent - is *purpose dependent*. That is to say, the extent to which an agent violates the norms of probabilistic reasoning depends on the purpose for which we are evaluating that agent.

References

1. Schervish, M; Seidenfeld, T; & Kadane, J. 2002. 'Measuring Incoherence'. *Department of Statistics; Paper 29.*
   URL = http://repository.cmu.edu/statistics/29
2. Zynda, L. 1996. 'Coherence as an Ideal of Rationality' in *Synthese* vol.109 (2);175-216