Why Are Those Options Smiling?

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ABSTRACT

This paper tests whether the true smile in implied volatilities is flat. The smile in observed Black-Scholes implied volatilities has often been attributed to deficiencies in the B-S model, such as the assumption of constant volatility, which cause the implied volatilities calculated using the B-S formula to differ from the true volatilities. If such deficiencies are the sole cause, then if the implied volatilities were calculated correctly (i.e., using the true though possibly unknown model), the smile should disappear or become flat. Using stock index options data, we test and reject the hypothesis that the true smile in stock index option prices is flat. If the true smile is flat, then a trading strategy in which one buys options at the bottom of the incorrect Black-Scholes smile and sells options at the top(s) should not be profitable *even on a pre-transaction-cost basis*. However, we find that such a delta-gamma neutral strategy yields substantial pre-transaction-cost profits. Moreover, the profits are large when the B-S model predicts large profits and small when small profits are predicted. Our results indicate that while part of the observed Black-Scholes smile appears due to deficiencies in the Black-Scholes model, a substantial part reflects a smile in the true implied volatilities. We argue that the true smile persists despite these substantial pre-transaction-cost profits, because maintaining the trading portfolio's original low risk profile requires frequent re-balancing which quickly eats away the profits.

Why Are Those Options Smiling?

One of the most persistent and well-documented financial market anomalies is the cross-sectional implied volatility "smile" or "sneer" reflected in option prices. Since they supposedly represent the market's expectation of likely volatility over the remaining lives of the options, implied volatilities should be the same for all options with the same maturity date observed at the same time. However, it is well-known that in many markets Black-Scholes (B-S) implied volatilities for options with the same expiration date but different strike prices consistently differ cross-sectionally often displaying a persistent "smile" or "sneer" pattern.

One explanation of the smile, and the most popular to date, is that it reflects erroneous assumptions by the Black-Scholes model about the distribution of the underlying asset's returns - specifically either the B-S assumption of constant volatility over the life of the option or the assumption that the underlying asset price follows a geometric Brownian motion. Numerous authors have shown that if returns follow a jump process or if volatility varies stochastically (or even deterministically) over time, implied volatilities calculated using the Black-Scholes formula will (or may) display a smile pattern even though the true smile is flat.[1] However, other studies, such as Das and Sundaram (1999) and Tompkins (2001), have concluded that these "corrected" models cannot fully explain the smile arguing that either the implied smile is too shallow or that the models cannot explain the smile term structure.[2]

An alternative, though not mutually exclusive, explanation of the smile is that it is due to a failure of the no-arbitrage assumption. In other words, it is possible that even correctly calculated implied volatilities display a smile pattern and that for some reason arbitrage fails to flatten it.

In this paper, we test whether the true smile or sneer in equity index options (the most studied smile in the literature) is flat. Put another way, we test whether the observed smile in B-S volatilities can be wholly attributed to errors in the Black-Scholes formula. While several recent studies have tested whether specific alternative option pricing models can explain the B-S smile and conclude they cannot, this leaves open the question of whether there is a true but undiscovered model which could. For instance, while Das and Sundaram (1999) show that jump-diffusion models cannot explain the smile at long maturities and stochastic volatility models cannot explain the smile at short maturities, a model which combined both a jump process and stochastic volatility conceivably could. An important advantage of our test is that it does not require specifying and testing an alternative model to B-S. Whatever the true underlying option pricing model, if the observed B-S smile is wholly due to erroneous distributional assumptions in the B-S model, i.e., if the true (though unobserved) smile is flat, then a strategy of buying options at or near the bottom of the B-S smile and selling options at or near the top of the smile in delta neutral proportions should not yield excess profits *even on a pre-transaction-cost basis*. However, we find that such trading strategy based on the B-S model yields substantial pre-transaction-cost profits. Moreover, these profits vary roughly in line with the B-S model's predictions while they should not if the true smile is flat. On the other hand, the trading profits are not as large as the B-S formula predicts, suggesting that the true smile is indeed flatter than the observed B-S smile, i.e., that part of the smile may be due to deficiencies in the B-S model, such as the assumptions that volatility is constant and that returns are log-normally distributed. While an advantage of our approach is that it is not conditional on a particular alternative to B-S, this also means that it does not predict a precise shape for the smile as alternative models can.

Our finding that a trading strategy based on the smile yields large profits raises two questions. First, if the smile is not wholly due to erroneous distributional assumptions in the B-S formula, what does cause it? Second, why isn't the smile eliminated by traders employing the profitable strategy we document? Regarding the former, we hypothesize that the equity market smile or smirk is partially caused by hedging pressures - in particular, hedgers buying out-of-the-money puts to protect their portfolios against a possible stock market crash. We posit that these hedge-driven purchases drive up the prices, and therefore the implied volatilities (IV), of low-strike-price options.[3] This interpretation is supported by trading volume patterns, and by the fact that IV's calculated from low strike prices tend to grossly exceed actual volatilities. Also, our calculations suggest that part of the smile is probably due to errors in the B-S formula, i,e, that B-S implied volatilities do smile more than the true volatilities.

The second question raised by our results is why doesn't trading based on the documented profits to our trading strategy flatten the smile? In other words, do our findings imply that the markets are inefficient? Indeed, this objection was raised even before we began our investigation by Rubinstein in his 1994 AFA Presidential Address (Rubinstein, 1994) in which he developed his own model capable of explaining the smile. Rubinstein wrote,

> This [Rubinstein's] discussion overlooks one possibility: the Black-Scholes formula is true but the market for options is inefficient. This would imply that investors using the Black-Scholes formula and simply following a strategy of selling low striking price index options and buying high striking price index options during the 1988-1992 period should have made considerable profits. I have not tested for this possibility, given my priors concerning market inefficiency and in the face of the large profits that would have been possible under this hypothesis, I will suppose that it would be soundly rejected and not pursue the matter further, or leave it to skeptics whose priors would justify a different research strategy.

We find that traders following such a strategy could make large profits but show that this does not mean the markets are inefficient. While our trading portfolios are initially hedged against changes in the price of the underlying asset, the S&P 500 index, we show that maintaining this delta neutrality requires quite frequent re-balancing entailing substantial transaction costs which quickly eat away the profits. Sizable profits net of transaction costs are only possible without frequent re-balancing requiring acceptance of substantial risk. Thus, our finding that the smile is not entirely due to errors in Black-Scholes does not mean that the market is inefficient.

It should be emphasized that the finding that profits net of transactions costs are small if the portfolio is re-balanced frequently does not rescue the view that the smile is wholly due to the B-S model's assumption of log-normal returns with constant volatility. If the true smile is flat, a trading strategy based on the incorrect smile should not be profitable *even on a pre-transaction-cost basis*.

The paper is organized as follows. In the next section, we describe our data and document the volatility smile or sneer. In section II, we explain how we construct delta and gamma neutral trading portfolios to test whether the true smile is flat. Results are presented in III. In IV, we explore why trading based on the smile fails to fully flatten it and present our conjecture on the cause of the smile. Section V concludes the paper.

I. Implied Volatility Patterns in the Equity Index Market

A. Data

Our data set consists of daily closing prices of options on S&P 500 futures traded on the Chicago Mercantile Exchange from January 1, 1988 through April 30, 1998 and closing prices of the S&P 500 futures themselves. We chose the S&P 500 Index because its smile is the most studied in the literature. We chose to study options on the futures rather than the cash index for several reasons. While options on the cash index and options on the nearby futures are virtually indistinguishable, the latter have several advantages: (1) the implied volatility calculations are not dependent on unknown future dividends, (2) since the options and futures markets close at the same time, non-synchronous prices are not a serious problem, and (3) arbitrage between options and the futures contract is easier than between options and all 500 stocks. Since it is well-documented that a shift in the smile occurred following the 1987 crash, our data set begins after that date.[4]

In order to examine separately both short and long-term options, each day we observe the prices of two types of options: (1) options maturing in 2 to 6 weeks, and (2) options maturing in 13 to 26 weeks. The CME trades options maturing in March, June, September, and December (the calendar options) and in each of the next three months. For the short-term (two to six week) options, we switch from the nearby option to the next contract on the ninth trading day before expiration. We restrict our data to options with at least two weeks to maturity since (1) the time value of very short-lived options relative to their bid-ask spreads is small and (2) the set of traded strike prices shrinks as expiration approaches. For the longer-term options data set, we collect prices for the second calendar option. For example, we observe prices of the June options in January, February, and through the first of March. When the March option matures, we switch to the September option, and when the June option matures, we switch to the December option.

Each day we observe both calls and puts at a variety of strike prices for options with the same expiration date. In the S&P 500 futures options market, these strikes are in increments of 5 points, e.g., 825, 830, 835 etc. Since trading is light in some far-from-the-money contracts, we restrict our sample to the first eight out-of-the-money and first eight in-the-money contracts relative to the underlying S&P 500 futures price. When, for instance, the S&P 500 futures index is 1001, we collect prices for options with strike prices from 965 to 1040. In summary, each day we observe and calculate implied volatilities for up to sixteen calls and up to sixteen puts for each of our two data sets. However, since all 32 options do not trade each day, we do not always have 32 observations. In Figure 1 we report average daily trading volumes at each relative strike price. Note that trading is higher in out-of-the-money than in in-the-money-options and for the shorter term options is generally higher the closer the strike is to the underlying index future price.

Using Black's (1976) model for options on futures,[5] day t closing prices for both the option and S&P 500 futures, and 3-month T-bill rates, we solve for the implied standard deviation, $ISD_{j,t}$, on each of the (32 or less) options j observed on day t. For easier interpretation, these are annualized by multiplying by the square root of 252, the approximate number of trading days each year.

B. The Smile

The smile in our data sets is reported in Table 1 and in Figure 2. For each option j on every day t, we calculate both the implied standard deviation, $ISD_{j,t}$, and the relative percentage "moneyness" of option j's strike price measured as $(K_{j,t}/F_t)$-1 where $K_{j,t}$ is option j's strike price and F_t is the underlying futures price on day t. Time series means of both ISD and (K/F)-1 are reported in Table 1 and the former is graphed against the latter in Figure 2. The following nomenclature is used in Table 1 to identify calls and puts and strike price groups j. The first letter, "C" or "P," indicates **c**all or **p**ut, the second, "I" or "O", indicates whether the option is **i**n or **o**ut of the money, and the last digit, "1" through "8", reports the strike price position relative to the underlying futures price where "1" is the closest to the money and "8" is the furthest in- or out-of-the-money. For example, CI3 indicates an in-the-money call option whose strike price is the third strike below the futures price.

As shown in Figure 2A, for the shorter-term options, there is a strong "smile," or "smirk" pattern in this market with the implied standard deviation consistently falling as the strike price increases until moderately high strike prices are reached at which point the ISDs start to rise. The cross-sectional differences in implied volatilities are striking and sizable in that implied volatilities at the peak of the smile are over 50% higher than those at the trough. Clearly, the null hypothesis that the ISDs do not differ cross-sectionally is rejected. As shown in Figure2B, the smile in the longer term options is not as pronounced and never turns up.

In Table 1 and Figure 2, we also include a measure of actual or "realized" volatility. For each day t, RLZ_t, is calculated as the standard deviation of returns over the period from day t through the option's expiration date:

$$RLZ_t = \sqrt{252 \times [\frac{1}{N-t-1} \sum_{s=t+1}^{N} (R_s - \bar{R})^2]} \quad (1)$$

where $R_s = \ln(F_s / F_{s-1})$ and F_s is the final futures price on day s. Since all options j observed on day t expire on the same day, RLZ only has a t subscript.

The mean of RLZ_t, is shown in both Table 1 and Figure 2. As shown there, in both data sets both put and call ISDs at all sixteen strike prices tend to overestimate actual realized volatility over the life of the options and this over-estimation is substantial for many options. For all 32 options at both maturities, the null that $ISD_{j,t}$ is an unbiased predictor of RLZ_t is rejected at the .01 level.[6] As further evidence, in columns 6 and 13 of Table 1, we report the frequency with which the implied standard deviation overestimates actual volatility over the life of the option. As shown there, for both puts and calls, the ISDs calculated from the five lowest strike prices overestimated RLZ_t more than 85% of the time in both data sets. Even at-the-money ISDs overestimated RLZ_t more than 75% of the time. Clearly, the tendency for implied volatilities to over-estimate actual subsequent volatility is strong. These findings tend to support the position of Canina and Figlewski (1993), in their argument with Christensen and Prabhala (1998), that implied volatility does not predict actual volatility very well in this market. One caveat is in order however. Since our data set begins in 1988, it is possible that the ISDs tended to exceed realized volatility partially because the former reflected the possibility of another 1987 style market crash which did not in fact reoccur . If the data set is extended backward through the 1987 crash, then the mean ISDs at the bottom of the smile are insignificantly different from the mean RLZ.

As previously noted, we do not observe option prices at all sixteen strike prices every day. If far-from-the-money option prices tend to be observed more when expected volatility is high or low, then the smile in Figure 2 could be a biased representative of the true cross-sectional smile. Accordingly, we recalculate the smile as follows. First, on each day t, we calculate the average

ISD of the four nearest-the-money options: $CI1_t$, $CO1_t$, $PO1_t$, and $PI1_t$ which we label A_t. For each of options observed on day t, we then calculate the ratio $R_{j,t} = ISD_{j,t}/A_t$. Group j means of this ratio are reported in columns 5 and 12 of Table 1 and graphed in Figure 3. As shown there, the smile calculated from the short term options now resembles a sneer which levels off rather than increasing at high strike prices and most of the differences between call and put ISDs disappear.[7] The smile calculated from longer term options is now a virtually linear downward sloping line.

II. Testing Whether the True Smile is Flat

A. The Testing Procedure.

Presuming that the smile is due to errors in the B-S formula, researchers have focused on developing pricing models based on alternative assumptions which could explain it, such as models based on stochastic volatility, deterministic models in which volatility rises if prices fall, or models based on alternative return distributions (e.g. jump-diffusion processes). The presumption is that, if measured correctly, there would be no smile, i.e., that the true implied volatilities are identical for all options with the same expiry. Consider a trading strategy in which one buys options which the B-S formula identifies as underpriced (that is options near the bottom of the smile) and sells options which are overpriced according to B-S (those near the top of the smile). If indeed the true smile is flat, that is if the true cross-sectional ISD's are all equal and a smile is only observed because of inappropriate distributional assumptions in the B-S formula, then this strategy should not return abnormal profits even on a pre-transaction-cost basis since the options which B-S identifies as overvalued are not in fact overpriced and those B-S identifies as undervalued are not truly undervalued. However, if B-S is substantially correct, this strategy should be profitable.

To test whether the true smile is flat, we implement a trading strategy in which we buy puts and calls near the bottom of the B-S smile and simultaneously sell puts and calls near the top of the B-S smile in proportions such that net investment is zero and the portfolio is hedged against changes in the underlying S&P 500 index. After one, five, or ten days, we close our position. To implement this trading strategy we proceed as follows:

1. For each day t in our data set, we restrict the set of tradable options to those whose time value is at least $1.00 and whose strike price is within five percent of the underlying S&P 500 index futures price.[8] In order to maximize potential profits, we should normally sell the options with the highest ISDs, which according to Table 1 and Figures 3 and 4 would normally be CI8 and PO8. However, as illustrated in Figure 1, trading in deep-in-the-money options, like CI8, is relatively light so it is possible that traders would have difficulty unwinding positions involving these more extreme options. Also, since S&P 500 options are quoted in increments of $.05, ISDs calculated from very low price options may not be reliable particularly at short maturities. Hence, we place these restrictions on our choices.[9]

2. From among the options meeting these restrictions on day t, we choose the call, CH, and put, PH, with the highest implied volatilities subject to the condition that the strike price, $K_{j,t}$ is less than the futures price, F_t. Likewise, we find the call, CL, and put, PL, with the lowest ISDs subject to the condition $K_{j,t}>F_t$. We form a trading portfolio from these four options on day t if $ISD_{CH,t}-ISD_{CL,t}>.03$ and $ISD_{PH,t}-ISD_{PL,t}>.03$. Since the intention of the trading strategy is to exploit ISD differences, we require that a difference of at least .03 exist. These requirements are not met on 618 of 2611 trading days in the short-term sample and on 889 of 2582 trading days for the longer-term sample. Descriptive statistics on these options: their implied standard deviations, their relative strike prices, and their Greeks: delta, gamma, theta, and vega are reported in Table 2. As shown there, for the short-term options, the difference between $ISD_{CH,t}$ and $ISD_{CL,t}$

averages .051 while the difference between $ISD_{PH,t}$ and $ISD_{PL,t}$ averages .052. For the longer term options, the figures are .044 and .051.

3. We form trading portfolios which are hedged against changes in the S&P 500 index according to the Black-Scholes model. Let $P_{CL,t}$, $\Delta_{CL,t}$, and $\gamma_{CL,t}$ represent the price, delta, and gamma[10] respectively of the low ISD call on day t and similarly for CH, PH, and PL. Let $N_{CL,t}$ and $N_{PL,t}$ represent the number of contracts in options CL and PL which are purchased on day t and let $N_{CH,t}$ and $N_{PH,t}$ represent the number of contracts in CH and PH which are sold or written on day t. $N_{CL,t}$, $N_{PL,t}$, $N_{CH,t}$, and $N_{PH,t}$ are chosen to satisfy the following requirements:

$$N_{CL,t}\,P_{CL,t} + N_{PL,t}\,P_{PL,t} - N_{CH,t}\,P_{CH,t} - N_{PH,t}\,P_{PH,t} = 0 \qquad (2)$$

$$N_{CL,t}\,P_{CL,t} + N_{PL,t}\,P_{PL,t} = N_{CH,t}\,P_{CH,t} + N_{PH,t}\,P_{PH,t} = \$100 \qquad (3)$$

$$N_{CL,t}\,\Delta_{CL,t} + N_{PL,t}\,\Delta_{PL,t} - N_{CH,t}\,\Delta_{CH,t} - N_{PH,t}\,\Delta_{PH,t} = 0 \qquad (4)$$

$$N_{CL,t}\,\gamma_{CL,t} + N_{PL,t}\,\gamma_{PL,t} - N_{CH,t}\,\gamma_{CH,t} - N_{PH,t}\,\gamma_{PH,t} = 0 \qquad (5)$$

Equation 2 imposes the condition that the trading portfolio is costless (ignoring transaction costs), i.e., that sale of the high ISD options finances the purchase of the low ISD options. To make the portfolios comparable, equation 3 standardizes all portfolios to a nominal gross value of $100. In other words, $100 is raised by selling the high ISD options and these funds are used to purchase $100 of low ISD options.[11]

Equation 4 ensures that the trading portfolio is delta neutral so that the value of the portfolio is unaffected by small changes in the underlying S&P 500 index (assuming the B-S pricing formula is correct). Since option prices are convex, an option’s delta changes as the underlying asset’s price changes so a portfolio which was originally delta neutral may not be after the underlying index futures price changes. Equation 5 imposes the condition that the portfolio also be gamma neutral, i.e., that small changes in the underlying index leave the portfolio delta unchanged at zero. If the B-S formula is correct, equation 5 should increase the range over which the portfolio is immunized against changes in the underlying asset price.[12] If the B-S formula is incorrect, then equations 4 and 5 may not immunize the portfolio against changes in the underlying S&P 500 index - a question to which we shall return below.

Each day we form the trading portfolios by solving equations 2-5 for the four N's. Descriptive statistics on the composition and characteristics of the resulting portfolios are provided in Table 3. As reported there, on an average day in the short-term sample, we write 2.39 high-ISD call contracts raising $48.83 and 24.57 high-ISD put contracts raising $51.17. We use this $100 to buy an average of 5.73 low-ISD put contracts at an average cost of $81.12 and 11.06 low-ISD call contracts at an average cost of $18.88.

B. Portfolio Characteristics.

In Table 3, we also report the portfolios' time t "Greeks". Of course delta and gamma are zero by construction. In <u>all</u> of our portfolios in both samples, theta is positive meaning that ceteris paribus the value of the trading portfolio should increase as time passes.[13] This is by design and the source of the positive expected trading profits of our strategy according to B-S. All other things equal, all options tend to lose value as time passes and the time-to-expiration becomes shorter. Consequently, a given ISD difference between two options translates into a smaller price difference as time passes. The greater an option's time value, the further it falls as expiration approaches so, as reported in Table 2, all other things equal, the high ISD options (which we sell) have larger negative thetas than the low ISD options (which we buy). Moreover, as reported in Table 3 we generally sell more options than we buy. Consequently, the trading portfolios always have positive thetas implying that the portfolio's value increases as time passes. As reported in Table 2, longer term options have much smaller thetas than short-term options, i.e., the passage of one day matters much more for an option with only 20 days to maturity than it does for one with 120 days to maturity. Consequently, the theta for the longer term trading portfolios is much smaller: a mean of only 128.5 versus 506.7 for the shorter term trading portfolios.

All other things equal, the expected trading profits according to the B-S formula are equal to the portfolio's theta times the length of the holding period. As shown in Table 3, the mean theta for the short-term trading portfolios is 506.7315 so, ceteris paribus, if the average year consists of 252 trading days, the mean predicted one-day holding period profit according to the Black-Scholes formula is 506.7315/252=$2.011 - a substantial one-day profit on a $100 nominal position with $0 net investment.[14] For the longer term options, the predicted one-day holding period profit is a much smaller 128.4951/252=$0.510. In summary, predicted profits are much higher for our short-term option portfolios than for our portfolios of longer-term options.

Since our portfolios are both delta and gamma neutral, the expected determinants of the holding period returns besides theta are the shape and height of the smile. In every trading portfolio in both samples, vega is negative implying that an equal increase (decrease) in implied volatility across all four options in the portfolio should lower (raise) the portfolio's value. Measuring the height of the smile as the average ISD on the four at-the-money options, CI1, CO1, PI1, and PO1, we find that increases and decreases in the height of the smile approximately average out in our samples but that the average change is a small decrease implying a small profit on average.[15] We would also expect to realize trading profits (losses) if after the portfolio is formed, the smile flattens (steepens). A flattening of the smile would imply a rise in the price of the low ISD options, CL_t and PL_t, which we buy, and a fall in the prices of the high ISD options, CH_t and PH_t, which we sell, so profits should be observed. While we observe the smile both steepening and flattening in our samples, the former is more common implying small losses on average.[16] We have already observed that the smile tends to be flatter in our long-term sample than in our short-term sample and this flattening pattern tends to be observed within the two samples as well.

III. Results

A. Trading Profits

In Panels A and B of Table 4, we report mean profits and losses without transaction costs for 1, 5, and 10 (market) day holding periods for our short-term and long-term samples respectively. For the short-term sample, we also report profits if the portfolio is held to maturity. For the 5 and 10 day holding periods, we restrict the data set to options which will have at least 10 days remaining to expiration at the end of the holding period since low trading volume in far-from-the-money-close-to-expiration options may make closing these positions close to expiration difficult.[17]

As reported in Panel A of Table 4, average profits in the short-term sample are $0.86, $4.35, and $ 8.11, and for the 1, 5, and 10 day holding periods respectively and $10.46 if the positions are held until maturity. These are substantial profits for costless positions with a nominal value of $100; all except the profits on positions held unchanged to maturity are significant at the .0001 level.[18] Over a one-day holding period, 63.1% of the trades are profitable and 65.1% are profitable over a five day horizon. Clearly these results reject the hypothesis that the smile is totally due to erroneous distributional assumptions by the Black-Scholes model which would disappear if the correct pricing formula were used. Options which the B-S model predicts are under and over-priced appear on average to be so.

As predicted by the B-S thetas in Table 3, the mean holding period profits for the longer term options are much smaller: $0.19, $1.08, and $2.50 for the 1, 5, and 10 day holding periods respectively but these too are significant at the .0001 level. Over a one-day holding period 57.7% of the trades are profitable and 62.5% are profitable over a five day horizon.

While large, positive, and highly significant, the average profits in both panels A and B of Table 4 are considerably less than the B-S model predicts. As noted above, based on the thetas in Table 3, the B-S model predictgs an average 1-day holding period profit of $2.01 in the short-term sample and an average 1-day profit of $0.51 in the long-term sample. Actual profit averages in Table 4 are $0.86 and $0.19 respectively suggesting that *part* of the smile may indeed be due to errors in the B-S model.

In Table 4, we also report the standard deviation of profits and measures of skewness and kurtosis. Despite the fact that the portfolios are delta neutral, profits are fairly variable even for short holding periods. We examine the reasons for this in section B below. As expected, the standard deviation increases sharply as longer holding periods are considered. Profits for the 1-day holding periods display extreme kurtosis. This was expected. Since the portfolios are delta neutral, small changes in the underlying futures (which dominate in the 1-day sample) should have no impact on profits, so profits tend to be peaked around the small positive profits predicted by theta. Over the longer holding periods, as larger changes in the underlying futures become more common and impact profits more, this peakedness is reduced.

As reported in Table 4, profits over a one-day holding period are negatively skewed while returns over longer periods are positively skewed. Recall that returns on bought options are positively skewed while returns on sold options are negatively skewed. Since we sell more options than we buy (Table 3), we expect the portfolio returns to be negatively skewed which is the case over the shorter holding periods. The positive skewness over the longer holding periods is apparently due to the fact that the observed period was generally a bull market period with few large falls in the S&P 500 index. Consequently, while the out-of-the-money call, CL, (positive skew) finished in-the-money 25% of the time at expiration, the out-of-the-money put, PH, (negative skew) finished in the money only 4.3% of the time.

The profits in Panels A and B of Table 4 are ex-post in the sense that it is assumed that the trader can trade at the same time t prices used to choose which options will be bought or sold and in what quantities. In reality, once the trader has observed the prices, determined the composition of his portfolio, and placed his order, the prices may have changed. Consequently, in Panels C and D we present ex-ante profits. Again the options to be traded are determined on day t and the proportions, $N_{CL,t}$, $N_{CH,t}$, $N_{PL,t}$, and $N_{PH,t}$ are determined using the prices, deltas, and gammas on day t. However, the portfolio's are formed on day t+1 based on the prices at that time and unwound on day t+2 (for the 1-day holding period), day t+6 (for the 5-day holding period) or day t+11 (for the 10-day holding period). Again, we observe significant positive holding period returns for all holding periods. Indeed, the profits in Panel C actually slightly exceed those in Panel A for the one, five and ten-day holding periods and those in panel D exceed the ex-post results in panel B for the five and ten-day holding periods. Clearly, there is ample time to run the calculations, determine the composition of the trading portfolio, place one's order, and still make money. Since it makes little difference and one day is much longer than necessary to determine the portfolio's composition and execute the trades, we henceforth work with the ex-post profits in Panels A and B.

In summary, our results decidedly reject the null that the true smile is flat. If the observed B-S smile is totally a result of deficiencies in the B-S formula, then options which the B-S formula identifies as over- or under-valued are not in fact over- and under-valued. Consequently, a strategy of buying options at the bottom of the B-S smile and selling those at the top should not be profitable even before deducting transaction costs. We find that it clearly is. On the other hand, observed profits are not as high as the B-S model predicts implying that the B-S model does not hold exactly.

B. Determinants of Trading Profits.

Having shown that this trading strategy is profitable on average (particularly in the short-term sample), we now examine the profits, their variability, and their determinants more closely. We are primarily interested in two questions. First, do the profits behave as the B-S model predicts? Specifically, are profits large when the model predicts large profits and small when it predicts small profits? If so, then we have further evidence that the smile is not wholly due to deficiencies in the B-S formula. If not, then either the true smile is not as steep as the B-S smile and/or the B-S model doesn't hold? Second, since the portfolios are both delta and gamma neutral, why are the strategies unprofitable about one third of the time and why are the profits so variable?

According to a Taylor series expansion of the B-S option pricing formula where the price term is taken to the second order and all other determinants to the first order, the change in the price of an individual option (ΔP) can be expressed as:

$$\Delta P = \text{Theta}\,(\Delta T) + \text{Delta}(\Delta F) - (1/2)\,\text{Gamma}\,(\Delta F)^2 + \text{Vega}(\Delta\sigma) + \text{Rho}\,(\Delta r) + U \qquad (6a)$$

where F is the underlying asset price, T is the time to maturity of the option, σ is the volatility (ISD), and r is the interest rate. In our sample as in most, the impact of changes in the interest rate, r, is nil so this term is dropped hereafter.

Since ΔT and ΔF are identical for each of the four options in our portfolios, the resulting Taylor series approximation for one of our portfolios is:

$$\Delta P = \text{Theta}'\,(\Delta T) + \text{Delta}'\,(\Delta F) - (1/2)\,\text{Gamma}'\,(\Delta F)^2 + \sum_{j=1}^{4} N_j \text{Vega}_j(\Delta\sigma_j) + U \qquad (6b)$$

In equation 6b $Theta^{/}$ represents the portfolio's Theta, i.e., $Theta^{/} = \sum_{j=1}^{4} N_j Theta_j$ where N_j is the number of contracts of option j in the portfolio and $Theta_j$ is option j's theta. $Delta^{/}$ and $Gamma^{/}$ are portfolio measures defined similarly.

However, since $\Delta\sigma$ differs for each of the four options in the portfolio, we cannot replace the Vega term in equation 6a by a single portfolio equivalent. We hypothesize that the effect of changes in the four implied volatilities on the portfolio's value can be effectively summarized by two measures: changes in the height of the smile and changes in its slope. First, we hypothesize that the impact of overall changes in the height of the smile can be expressed as $Vega^{/}(\Delta\bar{\sigma})$ where $Vega^{/} = \sum_{j=1}^{4} N_j Vega_j$ represents the portfolio's Vega as defined above and as reported in Table 3 and $\Delta\bar{\sigma}$ is the change in the overall height of the smile measured as the average ISD of the four at-the-money options: CI1, CO1, PI1, and PO1. Second, since a flattening (steepening) of the smile means a rise (fall) in the value of the low volatility options, which we buy, relative to the value of the high volatility options, which we sell, we expect a flattening to result in profits and a steepening to result in losses. We posit that this effect can be approximated as a linear function of the change in the slope, ΔSL, where the slope, SL, is measured as the weighted average of the ISDs of the two high ISD options (weighted by their relative weights in the portfolio) minus the weighted average of the two low ISD options. Consequently, our modified Taylor series type expression for the profits, ΔP_i, on trading portfolio i is:[19]

$$\Delta P_i = Theta^{/}_i(\Delta T) + Delta^{/}_i(\Delta F_i) - .5\,Gamma^{/}_i(\Delta F_i)^2 + Vega^{/}_i(\Delta\bar{\sigma}_i) + e(\Delta SL_i) + U_i \quad (7)$$

Suppose now that we estimate the equation:

$$\Delta P_i = a + b(\Delta F_i) + c(\Delta F_i)^2 + d(\Delta\bar{\sigma}_i) + e(\Delta SL_i) + U_i \quad (8)$$

across all our portfolios. For all our portfolios, $Delta^{/}_i$ and $Gamma^{/}_i$ are zero by construction for all portfolios so B-S implies b=0 and c=0. For an x day holding period, $\Delta T = x/252$ for all i, so the a parameter should equal (x/252) times the average theta. Given the negative vegas in Table 3, we expect d<0. Finally, we expect e<0, that is that a steeping (flattening) of the smile should lead to losses (profits).

Results of estimations of equation 8 for 1, 5, and 10 day holding periods are reported in Table 5 with t values in parentheses. The results are mixed. The intercepts, which measure average profits if there is no change in the smile and no change in the S&P 500 futures, are positive and significant as expected. They are higher than the average profits in Table 4 but still somewhat lower than the average profits implied by the thetas in Table 3. For instance, for the shorter-term options, the intercept in the 1-day holding period equation, which measures average profits if the underlying futures price doesn't change and if there is no change in the position or slope of the smile, is $1.39 versus average profits of $0.86 in Table 4. This is still less than the profits predicted by the theta in Table 3, i.e., 506.735/252=$2.01. Again, the implication is that the true smile may be flatter than the B-S smile. On the other hand, the intercepts are well behaved in that the intercepts for the 5 and 10 day holding periods are approximately 5 and 10 times the intercept for the 1-day holding period.

Interestingly, although the portfolios were constructed so that their B-S deltas and gammas were zero by construction, the hypothesis that the true deltas and gammas are zero, i.e., that b=c=0, is clearly rejected. For instance, for a 1-day holding period for shorter-term options, the result in Table 5 implies an average delta of 0.067. The implication is that either the delta and gamma neutral properties only hold for extremely small changes in F or the B-S formula expression for delta is incorrect.[20]

As expected, e<0 implying that a steepening of the smile over the holding period leads to losses while a flattening leads to profits. This term is highly significant in all six regressions. Consistent with the earlier observation that the smile tends to steepen as maturity approaches, in all six data sets, ΔSL is slightly positive on average implying losses on average due to this variable.

On the other hand, changes in the height of the smile do not consistently have the expected impact on profits. Given the negative vegas in Table 3, we expected a negative coefficient d. However, in two of the regressions for shorter-term options d>0.

While they provide insight into the behavior of the profits and losses on our trading portfolios, the estimations of equation 8 in Table 5 are not completely satisfactory. Specifically, the estimations in Table 5 ignore the fact that $\text{Theta}^{/}_{i}$ and $\text{Vega}^{/}_{i}$ differ for each portfolio i. According to the B-S formula, portfolios with high thetas should be more profitable than those with lower thetas. This is in fact the case in our data. When we split our sample of trading shorter maturity portfolios into two subsamples: (1) portfolios whose estimated theta exceeds the median theta value of 464.3 and (2) portfolios with thetas below 464.3, 1-day holding period profits average \$1.28 for the first subsample versus \$.44 for the second - a difference which is significant at the .0001 level. Vega also differs across portfolios but equation 8 ignores this.

Although the portfolios were designed to be delta and gamma neutral initially, the results in Table 5 indicate that either delta and gamma change over the holding period or the initial values were incorrect. Checking whether the parameters change, we find that at the end of the next trading day, the average absolute B-S delta is .378 and the average absolute gamma is .0852. Clearly, the initial zero values for delta and gamma are not fully representative of their levels over the entire holding period.

Since the Greeks change over time, an average of their values at the beginning and end of the holding period should be a better measure of the portfolio's sensitivity to that factor over the holding period than the initial value alone. Consequently, we define $\overline{\text{Delta}}^{/}_{i}$ as the average of portfolio i's deltas at the beginning (zero by construction) and end of the holding period. We then calculate the term $\overline{\text{Delta}'}_i(\Delta F_i)$. Repeating this averaging procedure for the other Greek terms, we calculate $\overline{\text{Gamma}'}_i(\Delta F_i)^2$ $\overline{\text{Vega}'}_i(\Delta\overline{\sigma}_i)$, and $\overline{\text{Theta}'}_i(\Delta T)$. We then estimate the equation:

$$\Delta P_i = w + a[\overline{\text{Theta}'}_i(\Delta T)] + b[\overline{\text{Delta}'}_i(\Delta F_i)] + c[-.5\overline{\text{Gamma}'}_i(\Delta F_i)^2] + d[\overline{\text{Vega}'}_i(\Delta\overline{\sigma}_i)] + e(\Delta SL_i) + U_i \quad (9)$$

If (1) the first (and second) order terms in equation 9 are independent of the higher order terms which are represented by U_i, (2) the Greeks are constant over the holding period, and (3) there is no measurement error, the B-S formula implies w=0 and a=b=c=d=1.0. However, since $\overline{\text{Delta}'}_i$, $\overline{\text{Gamma}'}_i$, and $\overline{\text{Vega}'}_i$, are midpoints of changing Greeks and $\Delta\sigma_i$ is an approximation, this will not necessarily be the case in our estimations.

Results are reported in Table 6. All coefficients have the predicted sign and all are significant at the .0001 level in all six regressions. As hypothesized, portfolios with high thetas tend to have higher profits than portfolios with lower thetas. The coefficients of the Delta and Gamma terms in Table 6 confirm that the portfolios do not remain hedged against changes in the underlying S&P 500 index and that part of the profit variation is attributable to changes in this index. Clearly, profits also vary with changes in the height and slope of the smile. These five variables also explain most of the variation in profits across our portfolios; adjusted R^2s vary from .822 to .955.

While all coefficients have the expected sign and are highly significant, in most regressions a, b, c, and d are also significantly from the 1.0 values implied by the B-S model. What is unclear is whether this is due to correlation with higher order terms excluded from the expression, to the approximate nature of some of our variables, or to deficiencies in the B-S model.

To measure the relative importance of each of the terms in question 9, we calculate the impact on profits of a one standard deviation change in each variable. These results are shown in brackets below the coefficient and t-value estimates in Table 6. Although the portfolios were designed to be delta neutral, changes in the underlying S&P 500 index clearly have a major impact on profits and losses - particularly in the short-term options data set. In all three shorter-term portfolio regressions, a one standard deviation change in $\overline{\text{Delta}}_i\,(\Delta F_i)$ has the greatest impact of the five variables. The gamma term ranks second in terms of this measure of importance in two of these three regressions.

Reflecting the fact that long-term options are more sensitive to volatility changes than short-term options, changes in the height and slope of the smile are relatively more important in the long-term options data set - though in both data sets changes in the slope of the smile prove considerably more important than changes in its height. In two of the three longer-term portfolio regressions, a one standard deviation change in the slope of the smile has the greatest estimated impact of the five variables.

Since it is the source of the positive expected profits to our trading strategy, we are particularly interested in the results for the theta term in the regressions. Since this variable is significant, it is clear that profits do vary positively with a portfolio's theta. However, this variable proves less important in explaining the profit variations than changes in the underlying S&P 500 index and changes in the slope of the smile. Consistent with our earlier findings but inconsistent with the B-S model's prediction, the coefficient of $\overline{\text{Theta}'}_i\,(\Delta T)$ is significantly less that 1.0 indicating that while profits vary with theta, they do not vary as strongly as the B-S model predicts.

Unlike ΔF_i, $\Delta\sigma_i$, and ΔSL_i, ΔT is known at the beginning of the period. Thus, $\text{Theta}'_i(\Delta T)$ provides a measure of the *predicted* variation in profits across portfolios while $\overline{\text{Theta}'}_i\,(\Delta T)$ does not since it is based on the theta value at the end as well as the beginning of the holding period. Consequently, we also estimate equation 9 using $\text{Theta}'_i(\Delta T)$ instead of $\overline{\text{Theta}'}_i\,(\Delta T)$ with the results shown in Table 7. The results in Table 7 do not differ much from those in Table 6 and both tables make clear that this variable explains relatively little of the profit variation.

In summary, while our trading strategy is profitable on average, profits vary considerably from portfolio to portfolio. Although part of this variation across portfolios is predictable due to the known differences in theta across portfolios, most is due to the changes in the underlying S&P 500 index and to changes in the height and slope of the smile which are not predictable. Consequently, the strategy entails substantial risk particularly over the longer holding periods despite the fact that the portfolios were designed to be delta and gamma neutral. The profit variation across portfolios is roughly but not completely consistent with the B-S model's prediction. Profits appear more sensitive to changes in the S&P 500 index than the B-S model predicts and less sensitive to changes in the height of the smile and to variations in theta. Clearly B-S correctly identifies over- and under-priced options. However, the profits based on these mispricings do not perfectly match the B-S model's predictions.

IV. Why Does the Smile Exist and Persist?

A. Risk, Re-Balancing, and Transaction Costs.

Our results in Table 4 raise an obvious question: "If such large trading profits are possible, why is the smile not flattened by traders following this profitable strategy"? As traders buy options at the bottom of the smile and sell options at the top of the smile, ISDs on the former should rise and ISDs on the latter should fall flattening the smile. Since the smile persists, this obviously does not happen enough to eliminate the smile. Why? We think the answer involves a combination of risk and transaction costs.

Although our trading portfolios are both delta and gamma neutral initially, we have seen that they quickly lose this immunization against changes in the S&P 500 index. The portfolios are also exposed to changes in the slope of the smile (and to a lesser extent its height) which impact profits and this is particularly true of the longer term portfolios. Consequently, the strategy is fairly risky especially over the five- and ten-day holding periods. While mean 5-day profits in the short-term options data set are $4.35 the standard deviation is $10.97. In 10% of the trades our trader would have lost $6.15 or more in five days following our strategy.

Since, as shown in Table 6, most of the risk in the short-term data set is due to changes in the underlying S&P 500 index, this risk could be reduced substantially by re-balancing the portfolios frequently to make them delta neutral but this would entail transaction costs. In the options on futures market these transaction costs consist of brokerage commissions and bid-ask spreads. The impact of brokerage commissions on profits is shown in panel A of Table 8 where we subtract estimated commissions from the profits reported in Table 4. Commissions at on-line and discount brokerages range from $1.50 to $2.25 per contract and are higher at full-service brokerages. We assume a one-way commission of $2.00 per contract. Prior to Nov 1, 1997, one S&P 500 futures contract called for payment of 500 times the index. On 11/1/97, this was changed to 250 times the index. Consequently, a $2.00 commission translates into a transaction cost of $.004 per index unit prior to 11/1/97 and $.008 thereafter. As shown in Panel A of Table 8, these commission charges reduce the average profit figures for the portfolios of short-term options about $.35 from those in Table 4 and reduce the average profits on the longer-term option portfolios about $.20. Except for the 1-day profits on the longer-term portfolios, all profit means remain positive and significant at the .0001 level.

Except for floor traders, traders following our trading strategy would also face bid-ask spreads. Since there are no specialists, bid-ask spreads cannot be directly observed in this market but it is commonly accepted that the bid-ask spread in the S&P 500 futures options market is generally one tick or $.05. Since some of our options are fairly low in price (C_L and P_H average $2.23 and $3.22 respectively in the short-term data set), this translates into a fairly major expense in percentage terms particularly in the short-term data set. In Panel B we report profits assuming both a bid-ask spread of $.05 and a commission of $2.00. With these costs added, the portfolios are no longer profitable on average over a 1-day period in either market and are not profitable over a 5-day period in the case of the longer-term options. Average profits remain positive over longer holding periods in the case of the short-term options but these are quite risky as seen from the high standard deviations.

In summary, it is possible to make profits by trading on the smile but only by accepting substantial risk. Over a 1-day holding period, transaction costs completely eliminate all profits. In the case of shorter-term options, positive profits can be earned on average after transactions costs over 5 and 10 day holding periods, but only without re-balancing which means accepting substantial risk. Frequent re-balancing to reduce this risk would entail additional transaction costs which would quickly eliminate the positive expected profits.[21] Consequently, while we reject the hypothesis that the true (or correctly calculated) smile is flat, it does not follow that the market is inefficient. Since the B-S model correctly identifies over- and under-priced options, it is clear that the smile cannot be wholly attributed to errors in the B-S model. However, it is impossible to profit from these mis-pricings without accepting substantial risk.

B. An Alternative Explanation of the Smile

If the smile is not wholly caused by errors in the B-S formula, what is responsible? Although we can offer only circumstantial evidence, we conjecture that the smile is partially due to hedging pressures. Holders of common stock portfolios can hedge against declines in the market value of their portfolios by buying out-of-the-money S&P 500 puts. This is a commonly cited use of the market and as shown in Figure 1, trading is highest in out-of-the-money puts. Such purchases for hedging purposes would tend to drive up the price of these puts and therefore their implied standard deviations. As the ISD's on out-of-the-money puts rise relative to calls with the same strike prices, this would set off put-call-parity arbitrage, which (unlike the trading strategy documented here) is relatively riskless since the two strike prices are the same. Such arbitrage would equalize ISDs on puts and calls with the same strike prices (and as shown in

Figure 3, they are virtually identical) while moderating the rise in the put ISDs. This explanation is consistent with both the high trading volume in out-of-the-money puts and the fact that generally ISD's on low strike price options far exceed actual ex-post volatility in the S&P 500 index.

V. Conclusions

We have shown that the smile in equity index options cannot be wholly attributed to deficiencies in the Black-Scholes formula, such as the presumption of constant volatility or the presumption of a log-normal return distribution. If one buys options at the bottom of the B-S smile and sells options at the top of the smile in a delta-neutral ratio, the strategy yields substantial pre-transaction-cost profits. This implies that, although the true (or correctly calculated) smile may be flatter than that calculated using the B-S model, it is clearly far from flat. Despite its supposed deficiencies, the Black-Scholes formula performs reasonably well in correctly identifying mis-priced options.

We have also shown that the existence of a true smile does not mean that the markets are inefficient. While trading on the smile is quite profitable, it is also highly risky because the slope of the smile changes and because the portfolio's initial delta-gamma neutral property can only be maintained with quite frequent re-balancing which would eat up the profits. Consequently, the existence of a smile is not incompatible with market efficiency.

Table 1A

The Implied Volatility Smile: Implied Volatilities and Moneyness by Strike Price for Short Maturity Options

Calls							Puts						
Strike Price	Implied Standard Deviation		Mean K/F -1	Mean ISD Ratio	% of obs > RLZ	Obs	Strike Price	Implied Standard Deviation		Mean K/F -1	Mean ISD Ratio	% of obs > RLZ	Obs
	Mean	Std. Dev.						Mean	Std. Dev.				
CI8	0.2200	0.0507	-0.0740	1.4840	0.9403	1,306	PO8	0.2269	0.0568	-0.0853	1.5390	0.9602	2,487
CI7	0.2110	0.0498	-0.0666	1.4186	0.9424	1,475	PO7	0.2184	0.0566	-0.0754	1.4598	0.9557	2,574
CI6	0.2008	0.0473	-0.0592	1.3542	0.9370	1,729	PO6	0.2071	0.0539	-0.0640	1.3802	0.9464	2,595
CI5	0.1913	0.0467	-0.0496	1.2857	0.9203	1,956	PO5	0.1964	0.0530	-0.0525	1.3007	0.9301	2,603
CI4	0.1816	0.0471	-0.0397	1.2199	0.9084	2,248	PO4	0.1859	0.0515	-0.0409	1.2245	0.9115	2,609
CI3	0.1723	0.0468	-0.0289	1.1499	0.8799	2,447	PO3	0.1755	0.0502	-0.0293	1.1516	0.8834	2,608
CI2	0.1645	0.0473	-0.0176	1.0838	0.8452	2,564	PO2	0.1657	0.0489	-0.0177	1.0841	0.8461	2,605
CI1	0.1568	0.0469	-0.0060	1.0247	0.7989	2,596	PO1	0.1568	0.0474	-0.0060	1.0242	0.8010	2,598
CO1	0.1497	0.0460	0.0056	0.9752	0.7509	2,605	PI1	0.1493	0.0460	0.0056	0.9756	0.7538	2,547
CO2	0.1443	0.0448	0.0173	0.9386	0.7083	2,605	PI2	0.1433	0.0449	0.0170	0.9369	0.7089	2,442
CO3	0.1404	0.0435	0.0289	0.9139	0.6746	2,603	PI3	0.1405	0.0432	0.0281	0.9081	0.6695	2,085
CO4	0.1382	0.0419	0.0403	0.9022	0.6579	2,575	PI4	0.1410	0.0413	0.0380	0.8908	0.6511	1,625
CO5	0.1380	0.0404	0.0507	0.9038	0.6466	2,414	PI5	0.1448	0.0404	0.0474	0.8909	0.6545	1,230
CO6	0.1411	0.0402	0.0596	0.9082	0.6637	2,052	PI6	0.1519	0.0403	0.0584	0.9131	0.7119	972
CO7	0.1459	0.0409	0.0671	0.9092	0.6696	1,592	PI7	0.1649	0.0413	0.0679	0.9509	0.7176	694
CO8	0.1519	0.0409	0.0720	0.9052	0.6593	1,168	PI8	0.1764	0.0429	0.0749	1.0002	0.7338	556
RLZ	0.1299	0.0564	--------	--------	--------	2,611							

Based on daily observations from Jan. 1, 1988 through April 30, 1998 of options on S&P 500 futures maturing in 2 to 6 weeks.

In the "Strike Price" column, the first letter © or P) stands for a **C**all or a **P**ut; the second letter (I or O) refers to **I**n-the-money or **O**ut-of-the-money; and the last digit indicates the relative position of an option from the money where 1 indicates that the option is the nearest-to-the-money and 2 indicates that the option is the second nearest-to-the-money etc.

RLZ stands for the realized volatility over the remaining life of the options.

(K/F -1) is a measure of how far in or out of the money an option is. K = strike price. F=underlying S&P 500 index futures price.

The " ISD Ratio" in columns 5 and 12 measures the ratio of the implied standard deviation at that strike price to the average ISD of the four at-the-money options: CI1, CO1, PI1, and PO1.

Table 1B

The Implied Volatility Smile: Implied Volatilities and Moneyness by Strike Price for Long Maturity Options

Calls							Puts						
Strike Price	Implied Standard Deviation		Mean K/F -1	Mean ISD Ratio	% of obs > RLZ	Obs	Strike Price	Implied Standard Deviation		Mean K/F -1	Mean ISD Ratio	% of obs > RLZ	Obs
	Mean	Std. Dev.						Mean	Std. Dev.				
CI8	0.1857	0.0365	-0.0787	1.2219	0.9082	1,307	PO8	0.1930	0.0450	-0.0848	1.2328	0.9178	2,140
CI7	0.1815	0.0359	-0.0693	1.1888	0.8921	1,473	PO7	0.1907	0.0450	-0.0752	1.2002	0.9116	2,263
CI6	0.1784	0.0367	-0.0596	1.1635	0.8824	1,556	PO6	0.1875	0.0451	-0.0647	1.1707	0.9004	2,359
CI5	0.1759	0.0386	-0.0499	1.1340	0.8854	1,727	PO5	0.1826	0.0448	-0.0530	1.1372	0.8944	2,395
CI4	0.1729	0.0399	-0.0396	1.1063	0.8709	1,928	PO4	0.1775	0.0440	-0.0414	1.1064	0.8821	2,418
CI3	0.1697	0.0407	-0.0289	1.0736	0.8694	2,198	PO3	0.1726	0.0431	-0.0295	1.0739	0.8724	2,461
CI2	0.1662	0.0407	-0.0177	1.0441	0.8532	2,269	PO2	0.1682	0.0429	-0.0179	1.0443	0.8608	2,413
CI1	0.1625	0.0409	-0.0060	1.0132	0.8389	2,390	PO1	0.1631	0.0420	-0.0062	1.0134	0.8480	2,402
CO1	0.1591	0.0414	0.0057	0.9863	0.8188	2,395	PI1	0.1590	0.0405	0.0056	0.9862	0.8248	2,260
CO2	0.1546	0.0412	0.0174	0.9589	0.8046	2,451	PI2	0.1549	0.0404	0.0173	0.9571	0.8311	2,042
CO3	0.1518	0.0412	0.0292	0.9351	0.7708	2,387	PI3	0.1524	0.0394	0.0292	0.9320	0.8090	1,681
CO4	0.1481	0.0409	0.0407	0.9116	0.7383	2,312	PI4	0.1512	0.0385	0.0404	0.9059	0.7853	1,332
CO5	0.1457	0.0416	0.0519	0.8907	0.7061	2,133	PI5	0.1504	0.0381	0.0512	0.8873	0.7598	1,095
CO6	0.1431	0.0427	0.0617	0.8718	0.6648	1,954	PI6	0.1483	0.0381	0.0611	0.8666	0.7243	885
CO7	0.1414	0.0420	0.0711	0.8582	0.6043	1,683	PI7	0.1476	0.0407	0.0710	0.8526	0.6823	702
CO8	0.1408	0.0435	0.0790	0.8469	0.5785	1,497	PI8	0.1467	0.0391	0.0794	0.8427	0.6197	610
RLZ	0.1320	0.0414	--------	--------	--------	2,582							

Based on daily observations from Jan. 1, 1988 through April 30, 1998 of options on S&P 500 futures maturing in 13 to 26 weeks.

In the "Strike Price" column, the first letter © or P) stands for a **C**all or a **P**ut; the second letter (I or O) refers to **I**n-the-money or **O**ut-of-the-money; and the last digit indicates the relative position of an option from the money where 1 indicates that the option is the nearest-to-the-money and 2 indicates that the option is the second nearest-to-the-money etc.

RLZ stands for the realized volatility over the remaining life of the options.

(K/F -1) is a measure of how far in or out of the money an option is. K = strike price. F=underlying S&P 500 index futures price.

The " ISD Ratio" in columns 5 and 12 measures the ratio of the implied standard deviation at that strike price to the average ISD of the four at-the-money options: CI1, CO1, PI1, and PO1.

Table 2
Characteristics of Options Included in the Trading Portfolios

Characteristics of the four options used to form costless delta-gamma neutral trading portfolios are reported. The options are: the call, CH, with the highest implied standard deviation (ISD) that day, the call, CL, with the lowest ISD that day and the puts, PH and PL, with the highest and lowest ISDs respectively- all subject to the restrictions outlined in the text designed to restrict the choice set to actively traded options. The portfolios are formed each day between 1/1/88 and 4/30/98 that the restrictions are met. There are 1993 portfolios of short-term options, which expire in 2 to 6 weeks, and 1693 portfolios of long-term options expiring in 13 to 26 weeks.

	Short Term Options		**Long Term Options**	
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**
Price				
CL	$2.2299	$1.9137	$3.7180	$4.7760
CH	$24.1104	$10.8425	$27.7118	$9.2354
PL	$18.1288	$10.5401	$23.9274	$8.6318
PH	$3.2182	$3.0568	$6.5470	$5.3559
Implied Standard Deviations (ISD)				
CL	0.1382	0.0425	0.1307	0.0372
CH	0.1887	0.0490	0.1746	0.0403
PL	0.1391	0.0427	0.1426	0.0390
PH	0.1910	0.0492	0.1930	0.0427
ISD Difference				
CH - CL	0.0506	0.0135	0.0440	0.0100
PH - PL	0.0520	0.0138	0.0505	0.0121
Delta				
CL	0.2022	0.0550	0.1750	0.0644
CH	0.7987	0.0640	0.6701	0.0455
PL	-0.7684	0.0687	-0.6436	0.0687
PH	-0.1871	0.0556	-0.2017	0.0463
Gamma				
CL	0.0176	0.0077	0.0078	0.0015
CH	0.0120	0.0044	0.0083	0.0017
PL	0.0185	0.0077	0.0105	0.0022
PH	0.0114	0.0041	0.0058	0.0009
Theta				
CL	-36.4379	24.0552	-12.4763	10.1061
CH	-49.0406	35.2810	-20.9551	9.9192
PL	-38.2057	25.1293	-17.4284	8.9932
PH	-49.5724	35.7168	-19.5517	10.6530
Vega				
CL	38.5806	17.8024	66.1941	31.0741
CH	38.7303	20.5433	90.8758	26.7389
PL	41.5132	20.0251	93.1870	28.7095
PH	37.7728	20.4315	73.2295	29.2240

Table 3

Composition and Characteristics of the Trading Portfolios

Characteristics of costless delta-gamma-neutral trading portfolios are reported. The portfolios are formed by shorting N_{CH} option contracts in the high ISD call, CH, and shorting N_{PH} units of the high ISD put, PH. We use these funds to buy N_{CL} contracts in the low ISD call, CL, and N_{PL} contracts in the low ISD put. The portfolios are constructed to be both costless and delta-gamma neutral. The portfolios are formed each day between 1/1/88 and 4/30/98 that the restrictions are met. There are 1993 portfolios of short-term options, which expire in 2 to 6 weeks, and 1693 portfolios of long-term options expiring in 13 to 26 weeks.

	Short Term Options		**Long Term Options**	
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**
Number of options (N) bought or sold				
CL	11.0570	4.0904	5.8953	2.8229
CH	2.3898	0.9643	1.2771	0.4197
PL	5.7298	2.4796	3.8738	0.9788
PH	24.5722	12.5072	13.1507	5.0991
Dollar Amount (N*Price)				
CL	$18.8827	$3.6389	$14.5662	$4.2157
CH	$48.8280	$7.5501	$33.1391	$7.4605
PL	$81.1174	$3.6389	$85.4338	$4.2157
PH	$51.1720	$7.5501	$66.8609	$7.4605
Portfolio Greeks				
Delta	0.0000	0.0000	0.0000	0.0000
Gamma	0.0000	0.0000	0.0000	0.0000
Theta	506.7315	238.2896	128.4951	50.0364
Vega	-242.2520	115.3106	-291.6542	119.6553

Table 4

Trading Profits

We report pre-transaction-cost trading profits and losses for the zero cost trading portfolios described in Table 3 in which we long puts and calls with low Black-Scholes implied standard deviations and short puts and calls with high B-S ISDs. The portfolios are initially both delta and gamma neutral. The trading portfolios are held one, five, or ten market days without re-balancing. The one, five, and ten day holding periods end at least ten days before the options expire. In Panels A and B we report ex post profits in which the trading portfolios are formed at the same day t prices used to determine the composition of the portfolios. In Panels C and D, we report ex ante profits for portfolios established using the prices on day t+1. The short-term option portfolios mature in 2 to 6 weeks and the long-term in 13 to 26 weeks.

Holding Period	Profits				
	Mean	**t**	**Std. Dev.**	**Skewness**	**Kurtosis**
Panel A: Ex - Post Results for Short Term Options					
1 Day	$0.859	10.17	$3.769	-2.229	38.813
5 Days	$4.354	9.96	$10.972	0.483	10.408
10 Days	$8.110	6.19	$19.292	1.611	11.648
Expiration	$10.461	2.17	$67.590	0.226	0.247
Panel B: Ex - Post Results for Long Term Options					
1 Day	$0.192	4.24	$1.862	-0.450	5.205
5 Days	$1.083	7.13	$3.952	0.056	2.037
10 Days	$2.503	6.86	$6.717	1.674	16.846
Panel C: Ex-Ante Results for Short Term Options					
1 Day	$1.031	8.08	$5.689	-1.442	48.396
5 Days	$4.739	8.14	$14.590	0.057	33.545
10 Days	$8.321	5.52	$22.141	0.376	15.702
Expiration	$9.598	1.98	$67.701	0.235	0.280
Panel D: Ex-Ante Results for Long Term Options					
1 Day	$0.169	3.62	$1.923	-0.579	7.823
5 Days	$1.174	6.99	$4.366	0.835	22.561
10 Days	$2.590	6.70	$7.111	1.541	17.407

Table 5 - Trading Profit Regressions

This table reports regressions estimations of the equation: $\Delta P_i = a + b(\Delta F_i) + c(\Delta F_i)^2 + d(\Delta\bar{\sigma}_i) + e(\Delta SL_i) + U_i$ where ΔP_i is the profit or loss on trading portfolio i, ΔF_i is the change in the S&P 500 futures over the holding period, $\Delta\bar{\sigma}_i$ is the change in the implied volatility on at the money options, and ΔSL_i is the change in the slope of the smile over the holding period. The portfolios, which are formed daily over the period 1/1/1988 - 4/30/1998, are described in the text and in Table 3. T-values are shown in parentheses. The short-term option portfolios mature in 2 to 6 weeks and the long-term in 13 to 26 weeks.

	Short Term Options			**Long Term Options**		
Variable	**1 day holding period**	**5 day holding period**	**10 day holding period**	**1 day holding period**	**5 day holding period**	**10 day holding period**
Intercept	1.3957 (20.99)	7.5926 (28.22)	14.905 (25.57)	0.4360 (20.07)	2.1819 (38.66)	4.1403 (38.49)
ΔF - change in S&P 500 futures	0.0671 (3.68)	0.1783 (5.07)	0.2316 (3.90)	-0.0383 (-4.98)	-0.0590 (-6.44)	-0.0479 (-3.64)
ΔF^2	-0.0079 (-17.66)	-0.0072 (-10.50)	-0.0061 (-5.71)	-0.0045 (-7.54)	-0.0033 (-8.79)	-0.0024 (-5.42)
$\Delta\bar{\sigma}$ - change in height of smile	-80.647 (-10.32)	13.991 (0.92)	111.26 (4.50)	-98.212 (-20.84)	-115.98 (-20.24)	-87.378 (-11.63)
ΔSL - change in slope of smile	-474.30 (-26.48)	-549.69 (-21.26)	-623.31 (-18.25)	-687.30 (-79.30)	-701.44 (-62.46)	-729.77 (-47.51)
adjusted R^2	.391	.289	.320	.800	.743	.652

Table 6 – Regression Estimates of Determinants of Trading Profits

This table reports regression estimates of the equation:

$$\Delta P_i = w + a[\overline{Theta'}_i(\Delta T)] + b[\overline{Delta'}_i(\Delta F_i)] + c[-.5\overline{Gamma'}_i(\Delta F_i)^2] + d[\overline{Vega'}_i(\Delta\overline{\sigma}_i)] + e(\Delta SL_i) + U_i$$

where ΔP_i is the profit or loss on portfolio i over the holding period, ΔF_i is the change in the S&P 500 futures over the holding period, $\Delta\overline{\sigma}_i$ is the change in the implied volatility on at the money options, and ΔSL_i is the change in the slope of the smile over the holding period. The Greek terms, e.g., $\overline{Theta'}_i$, are averages of the portfolio's B-S Greeks at the beginning and end of the holding period. The portfolios, which are formed daily over the period 1/1/1988 - 4/30/1998, are described in the text and in Table 3. T-values are shown in parentheses. The estimated impact on profits of a one standard deviation change in each variable is shown in brackets. The short-term option portfolios mature in 2 to 6 weeks and the long-term in 13 to 26 weeks.

	Short Term Options			**Long Term Options**		
Variable	**1 day holding period**	**5 day holding period**	**10 day holding period**	**1 day holding period**	**5 day holding period**	**10 day holding period**
Intercept	0.3415 (4.22)	2.5907 (13.78)	6.1432 (20.64)	0.1372 (2.70)	0.4713 (4.74)	1.0410 (7.41)
$\overline{Theta'}_i(\Delta T)$	0.7297 (19.29) [0.823]	0.6352 (30.75) [3.580]	0.5315 (29.46) [5.855]	0.6611 (7.05) [0.134]	0.7909 (20.51) [0.893]	0.7644 (26.64) [1.946]
$\overline{Delta'}_i(\Delta F_i)$	1.4378 (40.70) [3.809]	1.3702 (70.60) [14.579]	1.2925 (84.96) [21.768]	1.4234 (13.29) [0.422]	1.3484 (26.40) [1.917]	1.3451 (37.10) [5.561]
$-.5\overline{Gamma'}_i(\Delta F_i)^2$	0.7349 (13.92) [1.386]	0.8301 (29.18) [7.663]	0.8003 (31.24) [11.565]	1.2988 (6.16) [0.211]	1.3592 (17.96) [1.564]	1.1204 (21.85) [3.398]
$\overline{Vega'}_i(\Delta\overline{\sigma}_i)$	0.3282 (22.25) [1.013]	0.4262 (23.92) [2.534]	0.5128 (19.53) [3.091]	0.3273 (22.65) [0.508]	0.5099 (34.51) [1.560]	0.5342 (36.74) [2.231]
ΔSL_i	-580.15 (-61.91) [-2.557]	-545.65 (-61.98) [-6.298]	-496.98 (-51.60) [-9.399]	-683.00 (-86.68) [-1.626]	-677.52 (-84.85) [-3.103]	-668.02 (-77.73) [-4.172]
adjusted R^2	.822	.924	.955	.834	.870	.896

Table 7 – Profit Regressions with Ex Ante Thetas

This table reports regression estimates of the equation:

$$\Delta P_i = w + a[\mathrm{Theta}'_i(\Delta T)] + b[\overline{\mathrm{Delta}'_i}(\Delta F_i)] + c[-.5\overline{\mathrm{Gamma}'_i}(\Delta F_i)^2] + d[\overline{\mathrm{Vega}'_i}(\Delta\overline{\sigma}_i)] + e(\Delta SL_i) + U_i$$

where ΔP_i is the profit or loss on portfolio i over the holding period, ΔF_i is the change in the S&P 500 futures over the holding period, $\Delta\overline{\sigma}_i$ is the change in the implied volatility on at the money options, and ΔSL_i is the change in the slope of the smile over the holding period. The bars over the Greek terms, e.g., $\overline{\mathrm{Delta}'_i}$, indicates an average of the portfolio's B-S Greeks at the beginning and end of the holding period. However Theta'_i is the theta at the beginning of the holding period. The portfolios, which are formed daily over the period 1/1/1988 - 4/30/1998, are described in the text and in Table 3. T-values are shown in parentheses. The estimated impact on profits of a one standard deviation change in each variable is shown in brackets. The short-term portfolios mature in 2 to 6 weeks and the long-term in 13 to 26 weeks.

	Short Term Options			**Long Term Options**		
Variable	**1 day holding period**	**5 day holding period**	**10 day holding period**	**1 day holding period**	**5 day holding period**	**10 day holding period**
Intercept	0.4305 (5.04)	2.4420 (10.58)	6.0056 (13.35)	0.1593 (3.08)	0.6994 (6.44)	1.2430 (7.77)
$\mathrm{Theta}_i(\Delta T)$	0.6833 (16.98) [0.646]	0.6440 (24.93) [2.098]	0.5432 (18.66) [2.728]	0.6152 (6.45) [0.122]	0.6853 (16.38) [0.680]	0.7086 (21.78) [1.407]
$\overline{\mathrm{Delta}'_i}(\Delta F_i)$	1.5997 (45.85) [4.237]	1.5952 (85.06) [16.973]	1.4749 (93.48) [24.840]	1.4491 (13.49) 0.429]	1.4880 (28.32) [2.115]	1.5501 (41.26) [6.409]
$-.5\overline{\mathrm{Gamma}'_i}(\Delta F_i)^2$	1.0376 (20.12) [1.957]	1.2361 (46.96) [11.360]	1.1870 (47.19) [17.153]	1.4091 (6.67) [0.229]	1.6881 (22.00) [1.943]	1.6071 (31.45) [4.875]
$\overline{\mathrm{Vega}'_i}(\Delta\overline{\sigma}_i)$	0.2789 (19.15) [0.860]	0.3129 (17.21) [1.861]	0.3275 (11.19) [1.974]	0.3188 (22.08) [0.495]	0.4717 (30.86) [1.443]	0.4827 (31.68) [2.016]
ΔSL_i	-588.89 (-61.64) [-2.596]	-568.11 (-59.75) [-6.557]	-518.38 (-45.87) [-9.804]	-681.76 (-86.34) [-1.623]	-670.87 (-80.76) [-3.072]	-652.16 (-71.69) [-4.073]
adjusted R^2	.816	.912	.939	.833	.859	.883

Table 8
Post-transaction-cost Trading Profits

We report trading profits and losses after deducting measures of transaction costs for the zero net cost trading portfolios described in Table 3 in which we long puts and calls with low Black-Scholes implied standard deviations and short puts and calls with high B-S ISDs. The portfolios are initially both delta and gamma neutral and the portfolios are formed at the same prices used to determine the composition of the portfolio. The trading portfolios are held one, five, or ten market days without re-balancing. The one, five, and ten day holding periods end at least ten days before the options expire. In Panel A, we assume a brokerage commission of $2.00 per contract. In Panel B we assume a $2.00 commission and a bid/ask spread of $.05. The short-term portfolios mature in 2 to 6 weeks and the long-term in 13 to 26 weeks.

Holding Period	Short Term Options				Long Term Options			
	Mean	**t**	**Std. Dev.**	**Obs**	**Mean**	**t**	**Std. Dev.**	**Obs**
Panel A: With Brokerage Commission Costs								
1 Day	$0.503	5.96	$3.763	1,992	-$0.002	-0.05	$1.863	1,693
5 Days	$4.017	9.19	$10.964	1,574	$0.889	4.14	$3.949	1,690
10 Days	$7.794	5.95	$19.290	1,084	$2.309	6.33	$6.712	1,693
Expiration	$10.105	2.09	$67.707	1,992	---------------	---------------	---------------	---------------
Panel B: With Commission Costs and a Bid-Ask Spread								
1 Day	-$1.685	-19.56	$3.844	1,992	-$1.212	-25.93	$1.923	1,693
5 Days	$1.942	4.45	$10.948	1,574	-$0.321	-1.49	$3.956	1,690
10 Days	$5.841	4.46	$19.285	1,084	$1.099	3.02	$6.693	1,693
Expiration	$7.917	1.64	$67.654	1992	---------------	---------------	---------------	---------------

Figure 1A
Average Daily Trading Volumes
Options on S&P 500 Futures
2 to 6 Weeks to Expiration Arranged by Strike Price

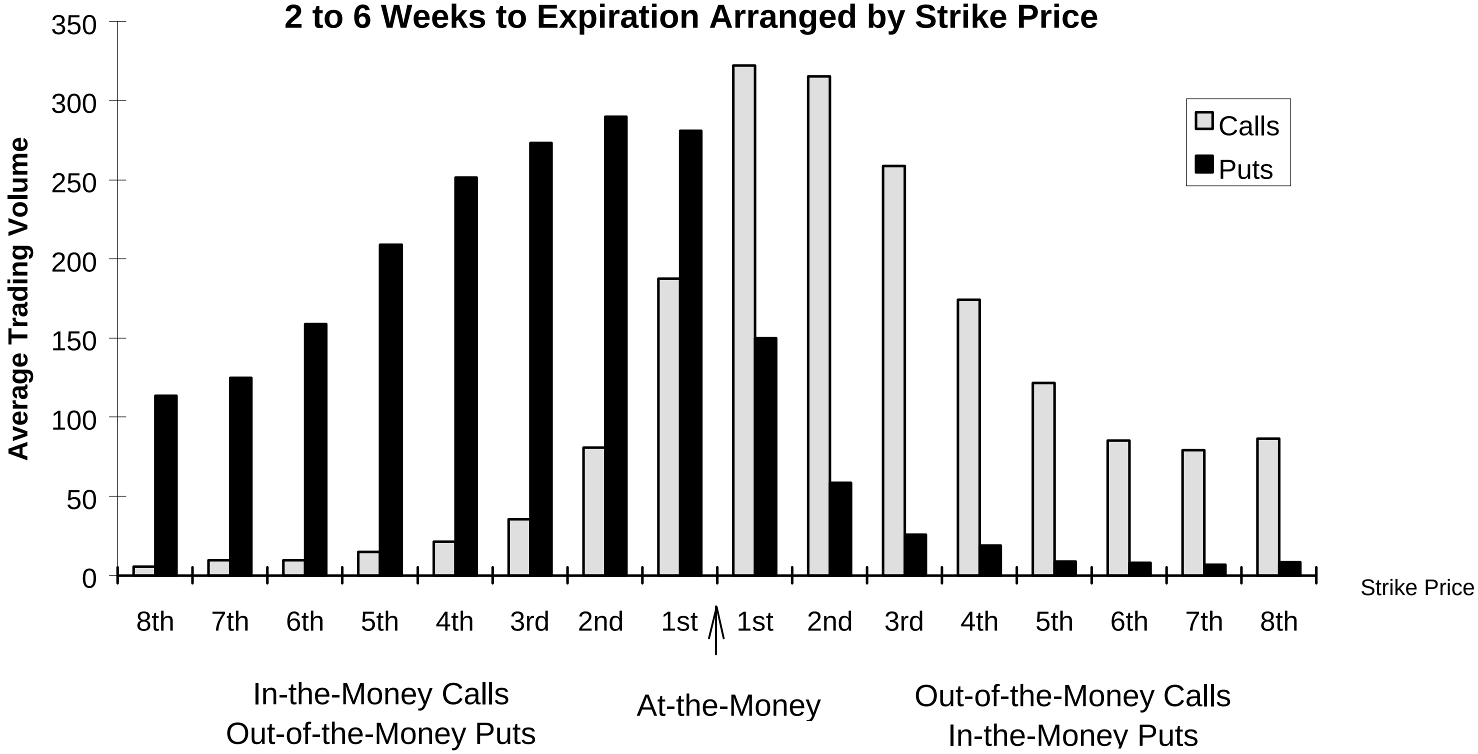


Figure 1B
Average Daily Trading Volumes
Options on S&P 500 Futures
13 to 26 Weeks to Expiration Arranged by Strike Price

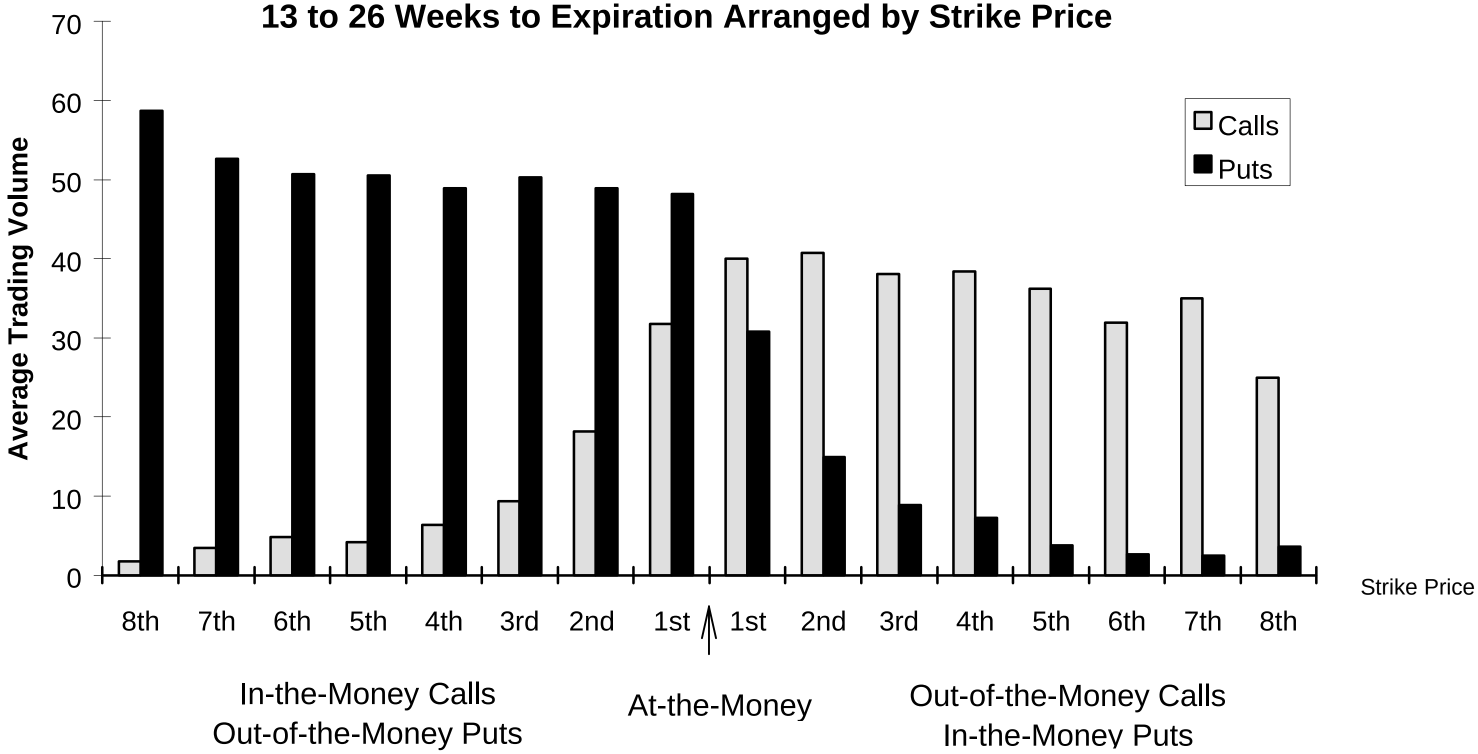
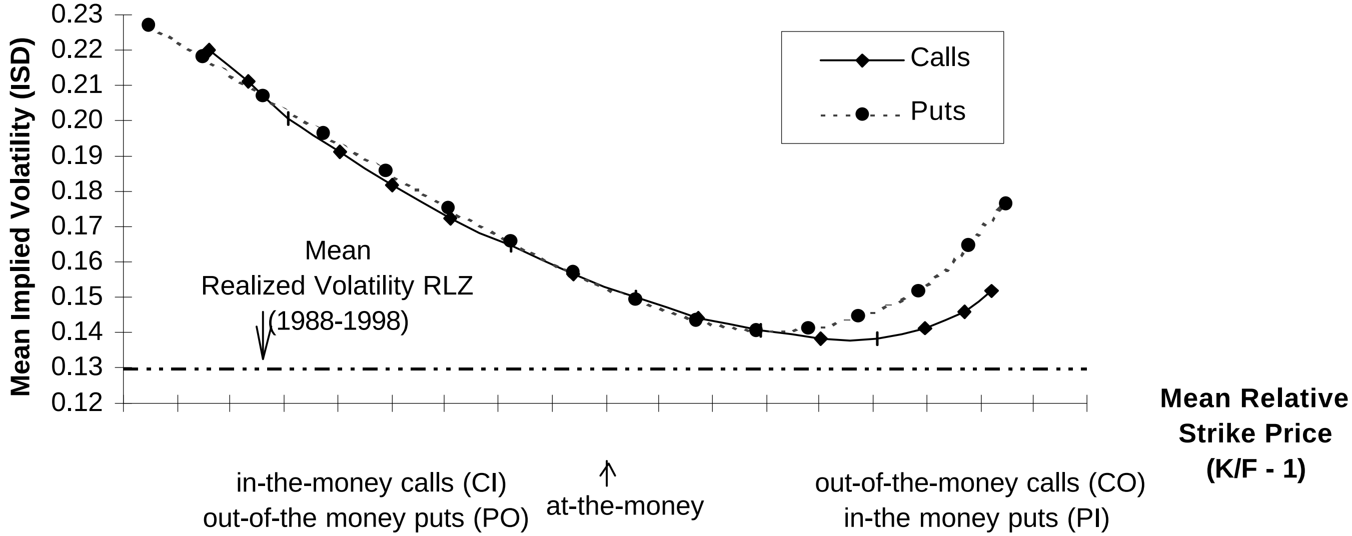
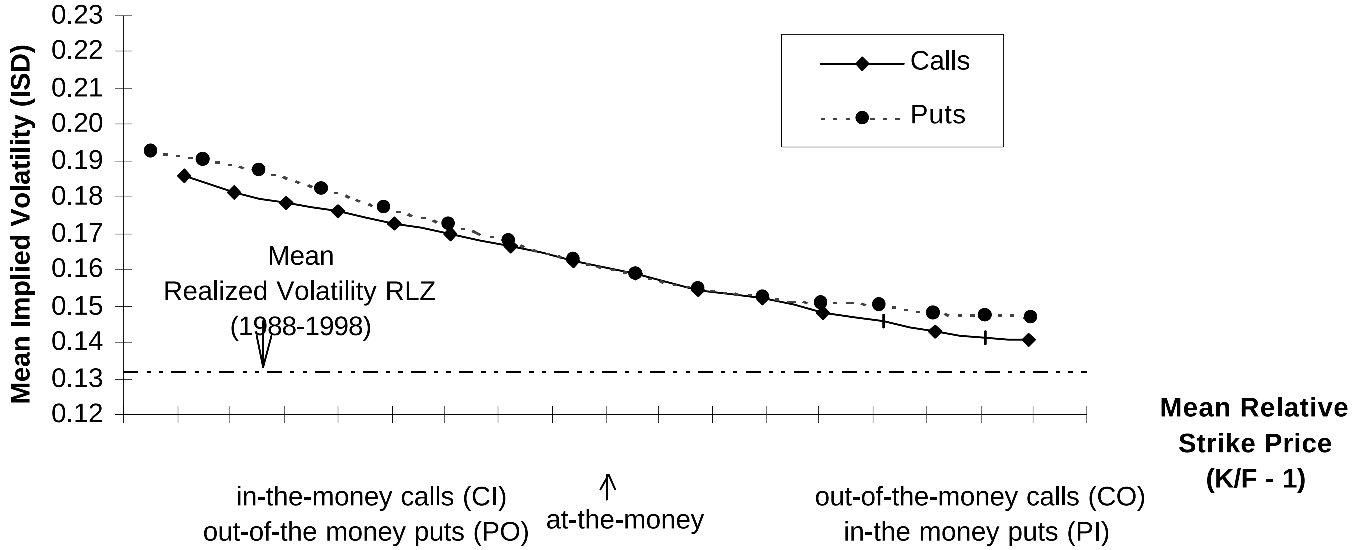


Figure 2A
The Implied Volatility Smile
S&P 500 Futures Options - 2 to 6 Weeks to Expiration



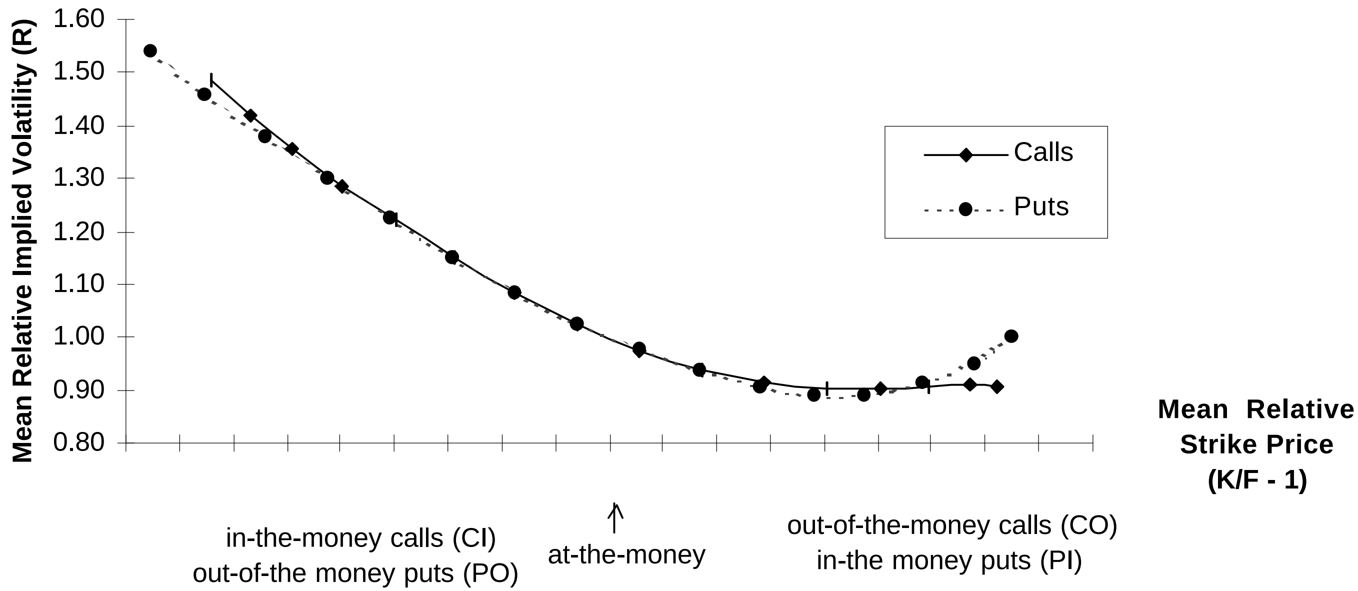
Note: The mean implied standard deviation (ISD) over the period 1/1/1988-4/30/1998 is graphed against the mean rela strike price for 16 call option groupings and 16 put option groupings. These groupings differ by strike price. Each day observe the first eight in-the-money calls and first eight out-of-the-money calls and the same for puts. For example, th farthest right diamond shows the mean relative strike price and the mean implied volatility for the eighth from-the-mone the-money call. The relative strike price is defined as (K/F - 1) where K is the strike price and F is the underlying futu price. For comparison, the mean realized volatility, calculated as the actual standard deviation of returns over the lif options in the data set is also shown.

Figure 2B
The Implied Volatility Smile
S&P 500 Futures Options - 13 to 26 Weeks to Expiration



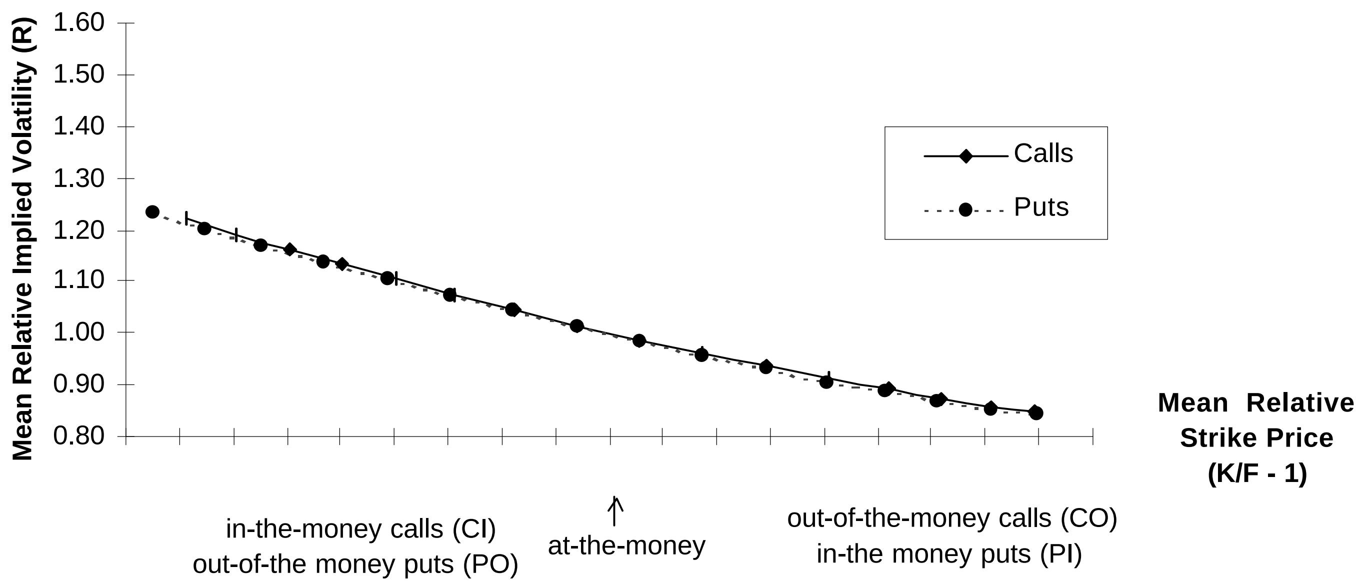
Note: The mean implied standard deviation (ISD) over the period 1/1/1988-4/30/1998 is graphed against the mean rela strike price for 16 call option groupings and 16 put option groupings. These groupings differ by strike price. Each day observe the first eight in-the-money calls and first eight out-of-the-money calls and the same for puts. For example, th farthest right diamond shows the mean relative strike price and the mean implied volatility for the eighth from-the-mone the-money call. The relative strike price is defined as (K/F - 1) where K is the strike price and F is the underlying futu price. For comparison, the mean realized volatility, calculated as the actual standard deviation of returns over the lif options in the data set is also shown.

Figure 3A
The Implied Volatility Smile in Relative Terms
Options Expiring in 2 to 6 Weeks



Note: The mean relative implied standard deviation (R) over the period 1/1/1988-4/30/1998 is graphed against the mear strike price for 16 call option groupings and 16 put option groupings which differ by strike price. Each day we observe eight in-the-money calls and first eight out-of-the-money calls and the same for puts. For example, the farthest right di shows the mean relative strike price and the mean relative implied volatility for the eighth from-the-money, out-of-the-m call. The relative strike price is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. axis measures the mean ratio of the implied standard deviation for that strike price relative to the average of the two ne the-money calls and the two nearest-the-money puts. The diamonds represent calls and the dots represent puts.

Figure 3b
The Implied Volatility Smile in Relative Terms
Options Expiring in 13 to 26 Weeks



Note: The mean relative implied standard deviation (R) over the period 1/1/1988-4/30/1998 is graphed against the mear strike price for 16 call option groupings and 16 put option groupings which differ by strike price. Each day we observe eight in-the-money calls and first eight out-of-the-money calls and the same for puts. For example, the farthest right di shows the mean relative strike price and the mean relative implied volatility for the eighth from-the-money, out-of-the-m call. The relative strike price is defined as (K/F - 1) where K is the strike price and F is the underlying futures price. axis measures the mean ratio of the implied standard deviation for that strike price relative to the average of the two ne the-money calls and the two nearest-the-money puts. The diamonds represent calls and the dots represent puts.

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ENDNOTES

1. Examples of smile-consistent jump-diffusion models are: Merton (1976), Ball and Torous (1985), Jarrow and Rosenfeld (1984), Amin (1993), and Bates (1996), which augment the B-S return distribution with a Poisson-driven jump process. Examples of smile-consistent stochastic volatility models include: Hull and White (1987a), Wiggins (1987), Amin and Ng (1993), and Heston (1993). Smile consistent deterministic volatility models include: Derman and Kani (1994), Rubinstein (1985, 1994), and Jackwerth (1997).

2. For instance, Heynen (1994) finds that the observed smile pattern is inconsistent with various stochastic volatility models. Jorion (1988) concludes that jump processes cannot explain the smile while Bates (1996) concludes the same for stochastic volatility models. Das and Sundaram (1999) find that the implied volatility smiles implied by stochastic volatility models are too shallow and that jump-diffusion models imply a smile only at short maturities. Dumas, Fleming, and Whaley (1998) conclude that Rubinstein's deterministic tree model forecasts less well than the naive B-S model.

3. As explained below, while most hedger interest in this market is concentrated in out-of-the-money puts, we posit that put-call parity arbitrage pulls up implied volatilities on calls at the same strike prices as well.

4. Beginning in 1988 has other advantages as well. Trading was light in the first years (1983-1987) of the market so it may not have been as efficient. Also, until serial options were introduced in August 1987, only options maturing in March, June, September, and December were traded. After that date we have a continuous monthly series. In a separate paper, one of the authors has examined the 1983-1987 period. Our results are not sensitive to this choice.

5. Black's model is a simplified version of B-S adjusted for the facts that (1) futures pay no dividends, and (2) futures entail no investment at time t. While Black's model is for European options, S&P 500 futures options are American. While use of a European option model introduces a small upward bias in implied volatility, Jorion (1995) shows that this bias is small, e.g., a 12% volatility is measured as 12.02%. In any case, our tests test whether the smile is due to inappropriate assumptions in the B-S model including the European options assumption.

6. Note that the observations of RLZ_t are not independent. For example, the realized volatilities calculated for an option with 20 days to expiration and the same option one day latter with 19 days to expiration only differ in that one day is dropped from the sample over which RLZ is calculated for the option with 19 days to expiration. The differences are significant even after adjusting for this dependence.

7. The small differences between the implied volatilities for puts and calls observed at relatively high strike prices could represent the possibility of early exercise on deep-in-the-money puts.

8. For the longer-term options, we relax the 5% restriction to 10% for out-of-the-money options.

9. With these restrictions, there were still one to three (depending on the holding period) observations in the shorter-term data set (out of 1992 to 1084 total observations) in which at least one of the four options was not traded on the day the positions were unwound. As explained below, since the time value of these options is nil, their impact on the results is nil.

10. Delta and gamma are the option price's first and second derivatives which respect to the underlying S&P 500 index according to Black's options on futures model.

11.   As is conventional, all prices are quoted in terms of one S&P 500 index unit. Until 11/1/97 when it was changed to 250, each S&P 500 futures and option contract called for paying 500 times the quoted price so this trading portfolio would actually represent a $50,000 gross value before 11/1/97 and $25,000 afterwards.

12. Comparing the relative performance of various hedging schemes, Hull and White (1987b) find that delta-gamma neutral hedging performs well when the traded option has a short maturity as here and a relatively constant implied volatility. We have also examined portfolios in which equation 5 is replaced by a requirement that the portfolios be vega neutral, i.e., that an equal across-the-board change in implied volatility leave the portfolio value unchanged. The overall profitability and risk of these delta-vega neutral portfolios are similar to the results presented for the delta-gamma neutral portfolios - although (as expected) they are more sensitive to changes in the underlying asset price and less to overall changes in implied volatility.

13. Theta is not measured consistently in the literature. Some express theta as the partial derivative with respect to time so that it is negative for all options. Others express it as the partial with respect to the time to expiration so that it is positive. We use the former convention.

14. This assumes that there are 252 trading days in an average year.

15. For the shorter term options the mean vega is -242.25 and the average change in the mean ISD is -.000502 implying an average profit of $0.122. For the longer term options, the mean vega is -291.6542 and the average change in the mean ISD is -.000267 implyin an average profit of $0.078.

16. Estimates are presented later.

17. This means that the number of observations for short term sample underlying the 5 and 10 day results is smaller (1574 and 1084 respectively) than those for the 1-day profits.   We have one less observation in Table 4 than in Tables 2 and 3 since we cannot calculate profits for the very last observation, 4/30/98, in our data set. In one of our 1-day, two of our 5-day, and three of our 10-day holding periods in the short-term sample, one of the four options did not trade on the day the position was closed so we do not have a price quote. All were far-from-the-money. If the unobserved option was far out-of-the money we assigned it a value of zero and if far in-the-money a value equal to its intrinsic value. These have virtually no impact on the profit figures in Table 4.

18. Since successive 5 and10 day holding periods overlap, the observations are not fully independent. The reported t-values are adjusted for this overlap.

19. In our data sets, ΔT = 1, 5, or 10 and is the same for all portfolios i so does not have a subscript.

20. Evidence points toward the former explanation. As reported below the portfolio's B-S delta at the end of the 1-day holding period is often far from zero. When we estimate equation 8 for the subsample of portfolios whose end-of-period delta and gamma are less than .2 in absolute

terms, the estimated coefficients b and c are no longer significantly different from zero.

21. It is also possible that traders trying to follow this strategy in substantial magnitudes would face price pressures, that is their buying would drive prices up and their selling would drive prices down reducing profits even further (and flattening the smile).