Indeterminacy and Spatiotemporal Data: Basic Definitions and Case Study

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Abstract

For some spatiotemporal applications, it can be assumed that the modeled world is precise and bounded, and that also our record of it is precise. While these simplifying assumptions are sufficient in applications like a land information system, they are unnecessarily crude for many other applications that manage data with spatial and/or temporal extents, such as navigational applications. This work explores fuzziness and uncertainty, subsumed under the term indeterminacy, in the spatiotemporal context. To better illustrate the basic spatiotemporal concepts of change or evolution, it is shown how the fundamental modeling concepts of spatial objects, attributes, and relationships and time points and periods are influenced by indeterminacy and how they can be combined. In particular, the focus is on the change of spatial objects and their geometries across time. Four change scenarios are outlined, which concern discrete versus continuous change and asynchronous versus synchronous measurement, and it is shown how to model indeterminacy for each. A case study illustrates the applicability of the paper's general proposal by describing the uncertainty related to the management of the movements of point objects, such as the management of vehicle positions in a fleet management system.

Keywords: spatiotemporal uncertainty, spatiotemporal indeterminacy, spatiotemporal fuzziness, moving objects, spatiotemporal data, trajectories

1. Introduction

Spatiotemporal applications have received substantial attention over the last years in both the research- and the application-oriented communities. Requirements analysis [27][36], models [11][38], data types [17], and data structures [23][24][29][31][32][37] are important topics in this area. Although considerable research effort and valuable results *do* exist, many studies and proposed approaches are based on the assumption that, in the spatiotemporal mini-world, objects have crisp boundaries, relationships among them are precisely defined; and accurate measurements of positions are assumed that lead to error-free representations.

However, reality differs; very often boundaries do not strictly separate objects but, rather, show a transition between them. Consider the example from an environmental system in which different climate zones, such as desert and prairie, are not precisely bounded. We encounter a transition, or fuzziness, between them. As another example, in a navigational system, the position of a moving vehicle, although considered as precise in nature, might not be known exactly, e.g., the GPS position of car A as of two minutes ago is known, but the precise, current position is not. This example is characterized by uncertainty (i.e., lack of knowledge or error) about its actual (current) position.

In this paper, we deal with *fuzziness and uncertainty* as related to spatiotemporal objects. More specifically, we start by pointing out the semantic differences between the two cases that constitute *spatiotemporal indeterminacy*: fuzziness, concerning "blurry" situations, and uncertainty, expressing the "not-exactly-known" reality. Our goal is to clarify these terms, observe their occurrence, study their impact on the spatial and temporal domains (i.e., spatial versus temporal fuzziness and uncertainty), as well as the combined effect, i.e., spatiotemporal fuzziness and uncertainty. We discuss how the basic spatiotemporal modeling concepts, such as spatial objects, attributes, relationships, time points, and time periods are influenced by indeterminacy. The approach of Dyreson et al. [13][14] on indeterminacy in the temporal domain is used as a vehicle to explore fuzziness and uncertainty in spatial, temporal, and spatiotemporal applications, as well as to point out their differences and similarities.

The contribution of this work is as follows. First, we integrate spatial and temporal indeterminacy in the spatiotemporal context, and, second, we show how both can be expressed by using any of the two mathematical theories: fuzzy set theory and probability theory. Third, we discuss the nature of spatiotemporal indeterminacy and offer a mathematical description of it. Fourth, the applicability of the indeterminacy proposal is illustrated by examining the moving point object example scenario. Here, we examine the contributing factors, the measurement error, and the sampling error more closely. This work is based on previous work on the capture of the fuzziness and uncertainty of spatiotemporal objects, and on the representation of moving point objects and the related uncertainty [28][30].

There are only few works on spatiotemporal indeterminacy. Work by Schneider [35] focuses on simple spatial and temporal uncertainty concepts and integrates them to describe spatial updates in a GIS database. However, the presented concepts are rather abstract and cannot immediately be applied. Moreira et al. [26] presents a data model for moving-point objects that is based on the decomposition of the trajectories of the objects into sections. In addition, so-called superset and subset semantics are proposed that aim to address uncertainty issues. A maximum error occurs when linearly approximating the position of an object in-between samples of its position, and this error is used in the process of query processing. However, this work is not connected to any specific application or technological context and thus does not cover the ranges of errors and the relationships between different error measures. Burrough et al. [8] present an approach that aims at describing the change of fuzzy features over time using a raster representation. Hadzilacos and Tryfona [20] consider the reasoning about the uncertainty of spatiotemporal positions at varying level of detail. They adjust the level of detail, or granularity, of the data according to a specific task or application context.

More work exists that concerns indeterminate temporal and spatial information individually. Dubois and Prade [13] take a probabilistic approach in handling indeterminacy of temporal information. On the other hand, research in the geography and surveying domain provides ways to describe and handle spatial indeterminacy. The concept of epsilon distances is introduced by Cheng and Molenaar [9], to quantify the cartographic error related to map production. Burrough et al. [6][7] study spatial uncertainty as related to soil boundaries. This work uses fuzzy set theory for soil classification.

Worboys [40][41] has considered spatial indeterminacy as it relates to resolution. Further, Vazirgiannis [39] uses fuzzy measures to better describe spatial relationships among determinate spatial objects. Schneider [33][34] takes a more pragmatic approach in that he models the spatial world in terms of spatial data types and expresses fuzziness as related to the data types and the operations on them. Bloch, e.g., [3], focuses on fuzzy distance measures and functions in the image processing context. Finally, Fischer [16] offers further readings on works on spatial indeterminacy beyond the exemplary ones presented here.

The rest of the paper is organized as follows. Section 2 covers a motivating example. Section 3 proceeds to briefly present the spatial and temporal concepts involved in the spatiotemporal application domain, explores the semantics, and offers mathematical formulations of indeterminate temporal and spatial concepts. Next, Section 4 discusses the spatiotemporal indeterminacy concepts, and Section 5 elaborates on the motivating example, the moving point objects case, to better illustrate and also to assess the feasibility of the indeterminacy concepts. Finally, Section 6 concludes with the future research plans.

2. Motivating Example – Moving Object Tracking

We can identify real-world entities whose extents and shapes are not relevant in a given application context, and, thus, can be modeled as point objects. As an application, consider the tracking of the continuous movements of cars, planes, people, etc. An application scenario is the optimization of transportation, especially in densely populated areas. An example fleet management project [4], conducted by Emphasis Telematics and the Research and Academic Computer Technology Institute, Greece, aims at designing what is termed an "Intelligent Fleet Management System." In this application, vehicles equipped with GPS devices transmit their positions to a central computer using either radio communication links or cellular phones. At the central site, the data is processed and utilized using data mining techniques.

To precisely capture the movement of such an object, we have to know its position at all times, i.e., on a continuous basis. However, GPS and telecommunications technologies only allow us to sample an object's position, i.e., to obtain the position at discrete instances of time, such as every few seconds. A first approach to representing the movements of objects would be to store the position samples. This would imply that we could not answer queries about the objects' movements at times in-between sampled positions. Rather, to obtain the entire movement, we have to interpolate the positions. The simplest approach is to use linear interpolation, as opposed to other methods such as polynomial splines [2]. In case we are dealing with point objects, the sampled positions then become the end points of line segments, which in turn constitute polylines. The movement of an object can be represented by a polyline in three-dimensional space (two spatial dimensions and one temporal dimension). In geometrical terms, the movement of an object is termed a *trajectory* (we will use "movement" and "trajectory" interchangeably) (cf. Figure 1(a)). Figure 1(b) shows the spatiotemporal space (the cube in solid lines) and several trajectories (the solid lines). The top of the cube represents the time of the most recent position sample. The wavy-dotted lines at the top symbolize the growth of the cube with time.

The trajectory data is affected by two sources of indeterminacy, (i) the measurement error and (ii) the sampling error. The accuracy and thus the quality of a measurement depends largely on the technique used—we will generally assume that GPS is used for position measurement. The movement is captured by position sampling at regular time periods. This introduces uncertainty about the position of the object in-between the measurements. So the accuracy of the representation of an object's movement is affected by the frequency with which position samples are taken, the sampling rate. This, in turn, may be set by considering the speed of the object and the desired maximum distance between consecutive samples.

Figure 2 illustrates how the movement is sampled at discrete time points (t_1 through t_9), approximated by using linear interpolation and how the measurement and the sampling error affect the representation.

In the following, we will develop a general framework for spatiotemporal indeterminacy that allows us to quantify and handle the above uncertainties. We first discuss the temporal and spatial uncertainty, to derive spatiotemporal uncertainty concepts, and then we finally relate them to the above example.

3. Temporal and Spatial Concepts and Indeterminacy

Several basic spatial and temporal concepts exist that are important in geo-referenced time-varying application environments. In the following, we briefly introduce these concepts and discuss how they are affected by indeterminacy and how this can be expressed mathematically. An introduction to the mathematical concepts used in the following can be found in Appendix A.

Categorizing spatiotemporal applications based on the type of data they manage reveals an interplay of temporal and spatial concepts. It is helpful to consider the following three categories.

(a) Applications dealing with *moving* objects, such as navigational applications; in these, objects are capable of continuously changing their positions across time. An example is a moving "car" on a road network.

(b) Applications involving objects located in space whose characteristics, as well as their location, may *change* across time. For example, in a land information system, "landparcels" change their locations by changing their shapes, but they do not "move."

(c) Applications dealing with objects that integrate the above two behaviors; for example, in environmental applications, "pollution" is measured as a *moving* phenomenon with properties and shape that *change* across time.

Based on these three types of applications, the following sections identify the relevant temporal and spatial modeling concepts and show how they are affected by indeterminacy. Based on these temporal and spatial concepts, spatiotemporal indeterminacy will be defined in Section 4.

3.1 Temporal Concepts

The literature reports on many different models of time. Some authors even propose taxonomies of time. In our work, we assume a linear ordered time line, isomorphic to a finite subset of the natural numbers. The elements of this set are termed *chronons*. Based on this fundamental definition, two basic time constructs are used to record facts and information of a database, namely *time points* and *time periods*. A time point t is located during a chronon, while a time period $[t_k, t_m]$, with t_k, t_m being time points and $t_k \leq t_m$ has duration and is defined as the chronons from the chronon during which t_k is located to the chronon during which t_m is located.

3.1.1 Indeterminate Time Points

A time point is *determinate* if it is known during which chronon it is located Figure 4(a) shows a determinate point I_1 located during chronon 2.[1] A time point is *indeterminate* if we do not know exactly during which chronon among a sequence of chronons it is located. An indeterminate time point is described by a *lower support*, an *upper support*, and a *probability function* [13]. The supports are chronons that delimit the location of the time point, e.g., for time point I_2 in Figure 4(b), the lower support is chronon 2 and the upper support is chronon 5, whereas the probability function tells us about where the time point is located within the range, e.g., a uniform distribution tells us that it is equally likely for the time point to be located during each of chronons 2 to 5.

The probability mass function, p_t, for the indeterminate time point t is defined for all chronons i:

$$p_t(i) = P[t = i] : i \in \{\mathbb{N} \times \mathbb{N}\} \tag{1}$$

[1] Dyreson and Snodgrass [14] state that we could either assume that chronons are the same size as time points, or that chronons are much bigger than time points, i.e., every chronon contains a large (possibly infinite) number of points. Assuming that chronons and time points are the same, we have to adopt the discrete model of time. If the model of time were then continuous or dense, we would be left with an infinite number of chronons. Since the model of time can possibly be continuous, we have to assume that chronons are much bigger than time points.

Here, $P[t = i]$ is the probability that time point t is located during chronon i. In our example that assumes a uniform distribution, e.g., $P[I_2 = 3] = 0.25$, the probabilities for chronons outside the lower support–upper support range are 0. Also, all indeterminate time points are considered to be independent, i.e.,

$$P[t_1 = i \wedge t_2 = j] = P[t_1 = i] \times P[t_2 = j] \tag{2}$$

Given that all probability distributions are fuzzy sets (cf. Appendix A), in using the probability mass function as basis, we obtain the following membership function:

$$\mu_t(i) = \lambda p_t(i) \tag{3}$$

In this formula, λ is an arbitrary scale factor that relates the membership grade to the probability of a point.

3.1.2 Indeterminate Time Periods

A time period is a subset of the time line bounded by two time points. Depending on whether the bounding points are determinate or indeterminate, we term the time period accordingly. In Figure 5(a), I_1 and I_2 are time periods that denote the indeterminate start and end point of an indeterminate time period. Possible periods can range from chronon 1 to chronon 8 (max), but have to range from at least 3 to at least 6 (min).

The indeterminate time period in Figure 5(a) can also be perceived as having a fuzzy boundary. In the following, we thus derive a membership function, $\mu_T(x)$, returning the degree to which an arbitrary chronon x is part of an indeterminate time period T. In Figure 5(a), it is obvious that at least chronons 3 through 6 are definitely part of time period T. Considering the time periods I_1 and I_2, if chronon 2 is within period I_1, so is chronon 3; and if chronon 1 is within, so are chronons 2 and 3. The same holds for chronons 6, 7, and 8 of I_2. Figure 5(b) gives the probability mass functions of I_1 and I_2, i.e., the probability for a chronon to be in T. Summing up the probability from “the outside to the inside” produces the probability density functions.

To derive the membership function $\mu_T(t)$, the time period T has to be split into three parts; (1) the “core” (chronons 4 and 5), (2) the periods I_1 and I_2, and (3) the outside world. A membership grade of 1 and 0 indicate definite and no membership in the time period, respectively. All chronons in the core have a grade of 1. The grade of the chronons in the periods is equal to the value of the probability density function. The following formula summarizes the membership function.

$$\mu_T(t) = \begin{cases} 1 & y \text{ in core} \\ \sum p(t) & y \in I_1 \cup I_2 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

For the case of arbitrarily small chronons, the probability density function for a given subset $A \subseteq T$ is computed as $Q(A) = \int_A p(t)\, dt$.

3.2 Spatial Concepts

Physical objects have spatial locations. In specific application environments, the objects’ position in space matter and then, these objects are called *spatial objects*, e.g., a moving “car” in a navigational system is a spatial object. The position is represented in terms of a geometry, which can be (of type) point, line, region, or any combination thereof [19]. The geometry of the position of a spatial object varies not only with the different type of physical object, but also with the application context, e.g., a city can be represented as region but also as a point.

Spatial objects can be related to one another based on their positions in space. A *spatial relationship* is a relationship among spatial objects, or more precisely, a relationship among the position of the objects involved. For example, two landparcels are neighbors, i.e., they have common borders.

Objects have attributes that characterize them. A spatial object may have, apart from descriptive attributes and its position, also *spatial attributes*, e.g., the "vegetation" of a "landparcel." Values of spatial attributes are determined by the object's position and the underlying physical space, not on the object itself. If the spatial object "landparcel" changes position, then the value of "vegetation" will change. Spatial attributes are also related to geometries in space, as they split space into parts within which the values of the spatial attributes remain the same; each part of space has (like, the objects' positions) geometry (of type) point, line, region, or a combination thereof. **Figure 3** shows a spatial object, two spatial attributes, and related geometries. Note that not all spatial objects have spatial attributes. This depends on the application requirements. For example, typically no spatial attribute is assigned to a moving car, while many (e.g., "vegetation," "soil type") are assigned to a landparcel. In geographic information systems, spatial attributes are also commonly referred to as *layers*.

3.2.1 Indeterminacy and Space

The nature of spatial indeterminacy, i.e., spatial fuzziness and uncertainty, is an important issue in geography and spatial information science. Fuzziness is a property of a geographic entity [18]. Furthermore, fuzziness concerns objects that cannot be precisely defined otherwise [15]. On the other hand, uncertainty results from limitations in the observation, i.e., the measurement process [18].

To illustrate the concept of indeterminacy as related to the above concepts, consider the example of different climate zones, e.g., desert and prairie. Zones do not have precise bounds, but, rather, *blurry* situations exist around the boundaries between zones. We can identify a location for which there is no precise knowledge whether it is in the desert or on the prairie, and we can find a location that is in-between. Consequently, the boundary between the two climate zones is *fuzzy*. However, for a forest partitioned into land parcels, we can clearly state which tree belongs to which land parcel. The boundaries between the land parcels are *crisp* and, thus, *certain*.

In contrast, let us consider the position of a moving vehicle whose position is not exactly known. This example is characterized by a *lack of knowledge* about the car's position. The car has a precise position—it is our lack of knowledge about the position that introduces *uncertainty*. Without further knowledge, we can only give the probable location of the car.

The above-mentioned examples indicate that the characteristic that distinguishes fuzzy and non-fuzzy locations is whether or not there is a *crisp boundary*. The concept of boundary introduces the *interior/exterior* notion, i.e., what is within the boundary and what is outside. Spatial fuzziness occurs (a) in the relationships among spatial objects and (b) in spatial attributes.

On the other hand, the characteristic that distinguishes uncertain and certain facts is whether or not there is *a lack of, or errors in our knowledge*, i.e., we do not have accurate knowledge about an otherwise precise location. As a result, spatial uncertainty can refer to the degree of knowledge we have about an object's position. Uncertainty about an object's position leads to uncertainty about the spatial relationship between this object and its neighbors, e.g., if the exact boundary of a land parcel is not known then the exact spatial relationships with its neighboring land parcels are not known either. Furthermore, uncertainty can exist for spatial attributes when knowledge about them is limited. Table 1 summarizes these results.

3.2.2 Indeterminate Geometries

Geometry is essential in defining the concepts of spatial object and spatial attribute. Further, spatial relationships are defined in terms of the positions and thus the geometries of spatial objects.

Points, lines, and *regions* are the most commonly used simple geometries in spatial applications. For the rest of the paper, we only consider points and regions with no holes and no disconnected parts, and we regard *lines* as a special case of regions.

Fuzziness is not applicable to points, since the concept of boundary and consequently of interior/exterior does not exist here. However, a point can be crisp or uncertain.

Because a region is established by its boundaries (something is inside/outside, or left/right), a region can be fuzzy, e.g., consider climate zones, whose boundaries are not crisp, but transitional. Regions can be uncertain. Consider the example of a land parcel with "not-exactly-known" (missing data) boundaries.

The following two sections focus on the mathematical treatment of the above concepts.

Indeterminate Points

We perceive *Space* as a set of points, homeomorphic to $\mathbb{N}^2$. The position of a point object (i.e., an object with geometry point) is determinate, if it can be given as a single point $p \in \mathbb{N}^2$. The position is indeterminate if it can only be expressed as a set of points, i.e., the exact position is unknown. A probability function describes the likelihood for each point to be the true position, e.g., a uniform distribution tells us that there is an equal chance for each point. The probability mass function, p_x, for an indeterminate point object *x* is:

$$p_x(i) = P[x = i] : i \in \{\mathbb{N} \times \mathbb{N}\} \tag{5}$$

Here, $P[x = i]$ is the probability that position *x* is mapped to point *i*, with *i* being a Cartesian coordinate.

As in the case of the time period, the probability that the position is outside the point set is 0. Further, all indeterminate positions are considered to be independent (cf. Section 3.1.1).

What applies to time points as shown in Section 3.1.1 can be applied to indeterminate points in the spatial context as well; probability distributions describing positional indeterminacy can be interpreted as fuzziness.

Indeterminate Regions

A *region* is a part of space bounded by a connected set of points, the boundary. A region can be determinate if the boundary points are determinate. Consequently, indeterminate points bound an indeterminate area. This definition is analogous to Section 3.1.2, which presented the concept of an indeterminate time period. The following example illustrates this.

Consider a map made up of two discrete regions, *A* and *B*, which share a common boundary. Repeated digitization of the map introduces errors such that we obtain a set of boundary points that lie more or less close to the actual boundary line. The distribution of the boundary points might take the form of a normal distribution. In Figure 6(a), we show the normal distribution of a particular boundary point. In the continuous case, the probability function will look as shown in Figure 6(b).

Analogously, an uncertain region can be described using a membership function. The membership function can be determined using an approach similar to the one for the temporal case (cf. Section 3.1.2). We split the underlying space into three parts, (i) the core of the area, (ii) the boundary region, and (iii) the outside. Consequently, a membership function for area *A* can be specified as follows.

$$\mu_A(i) = \begin{cases} 1 & i \in A \wedge i \notin C \\ \sum p(x) & i \in A \cap C \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

In the above formula, area *C* stands for the outside of area *A* and $p(x)$ is the probability mass function for a point being in area *A*. The argument of the membership function is a point and it returns a grade for the membership of this point in area *A*. The grade is 1 if the point is a definite member of the area and 0 if it is definitely not a member of the area. Otherwise the grade ranges between 1 and 0 (cf. Figure 7(a)).

Often a positional probability function is *unknown* or *complex*, e.g., there does not exist a unique probability function that describes the distribution of all points in the boundary, or we do not have "any information at all" about the boundary of a region. Consider here the transition between soil zones as described in Section 3.2.1. The boundary reflects the very nature of a phenomenon not being crisp and, thus, to give a probability function describing it is not possible. This illustrates the critical case for which fuzziness relieves uncertainty. We can still derive a valid membership function in *assuming a smooth and steady transition* from one zone to the other. A membership function for soil zones, as shown in Figure 7(b), could be characterized by the following formula (Schneider, 2001),

$$\mu_A(x,y)=\begin{cases}1 & \text{if } (x,y)\in A\\ 1-\dfrac{d_a}{d_a+d_b} & \text{if } (x,y)\notin A \wedge \ (x,y)\notin B\\ 0 & \text{otherwise}\end{cases} \tag{7}$$

where d_a and d_b are the distances from a point (x,y) to the core area of the soil zones *A* and *B*.

A formula for a distance *d* from an arbitrary point given by its coordinates (x,y) to an area *A* with the boundary B_A is as follows

$$d\left((x,y),B_A\right)=\min\left\{\text{dist}\left((x,y),(m,n)\right) \mid (m,n)\in B_A\right\} \tag{8}$$

where dist(*p*,*q*) is the Euclidean distance between two points $p,q\in\mathbb{R}^2$.

The underlying assumption above is that the transition between the climate zones is linear. The effects of other transitions on the membership function can be handled by changing the formula describing the membership grade for positions outside the core.

Other examples in this case are the boundary problem as experienced in the context of soil profiles, soil maps, and land evaluation classification [5][6].

4. Spatiotemporal Indeterminacy

Having covered spatial and temporal indeterminacy, we consider the combination, *spatiotemporal indeterminacy*. First we use examples and to then focus on the concept that is fundamental to spatiotemporal scenarios, namely *change*. Again, probability and fuzzy set theory are used to model indeterminate change.

To illustrate spatiotemporal indeterminacy, consider the application context of tracking the movement of vehicles, e.g., fleet management. The movement of such objects can be assessed using a sampling approach, i.e., we measure positions as discrete points in time. A representation that interpolates in-between the position samples is uncertain. For objects with extent, the change of location includes changes of their shapes, which have to be interpolated as well. Consider here the example of a coastline that bounds an island. Two processes influencing the coastline make an island an indeterminate region. The tides have (i) short-term effects, whereas (ii) over a longer period of time, a general drift affects the shoreline as well. If one is only interested in the general drift, the tidal effect can be modeled as a fuzzy boundary that changes with time (general drift).

Spatiotemporal indeterminacy can have more than one source, i.e., it can be the result of the combined effects of temporal and spatial indeterminacy. We proceed to consider possible scenarios for the context of spatiotemporal data.

In spatiotemporal applications, we are interested in spatial objects, relationships, and attributes over time; we are interested in recording their evolution, or *change,* in time. Thus, change is the most important concept in the spatiotemporal context, and will, in the following, serve as the basis for evaluating spatiotemporal indeterminacy. As stated in the literature [10][17][30], change (i) can either occur on a discrete or a continuous basis, and (ii) can be recorded in time points or in time periods. Table 2 illustrates the various *change* scenarios we can encounter in the spatiotemporal context.

4.1 Change Scenarios

The four scenarios in Table 2 are illustrated by using a 3-dimensional representation of the temporal change of geometry. Space (*x*- and *y*-coordinates in the horizontal plane) and time (time-coordinate in the vertical direction) are combined to form a three dimensional coordinate system. We use point geometry to keep the illustrations simple. However, the same four change scenarios apply to other geometries. To distinguish discrete from continuous changes, a continuous change of geometry from G_i to G_{i+1} is indicated by using an arrow in the spatial plane as opposed to a line in case of a discrete change.

In the change scenarios, the elements that can be indeterminate are *geometry*, *time point,* and *time period*. We consider each of the four change scenarios in Table 2 in turn.

Scenario 1 – discrete change of a geometry recorded in time points.

Here, geometry stays constant for some time and then changes instantly. The geometry is sampled at constant time periods *dt*. The geometry and/or the time point can be indeterminate.

Scenario 2 – continuous change of a geometry recorded in time points.

Here, we sample a constantly changing geometry at time periods *dt*. Knowing a geometry only at discrete time points has two implications, (i) recording geometries at time points means assessing a momentary situation without inferring anything about the geometry prior to or after the time point. Consequently, (ii) time and space are independent; not knowing the exact extent of the geometry does not affect the time period and vice versa.

In contrast, Scenarios 3 and 4 in Table 2 suggest that a *change function* of the form $C: t_x \rightarrow G_x$ exists that determines a geometry G_x for a time point t_x for a given time period $T_i = [t_i, t_{i+1}]$ with the respective, known geometries G_i and G_{i+1}.The change function *C* may be different for every time period.

Scenario 3 – discrete change of a geometry recorded in time periods.

The objective is to "begin" a new time period when a spatial change occurs, i.e., a new time periods start at the time points t_0 through t_4 in Scenario 3. The geometry is constant within a time period. In this case, spatial and temporal indeterminacy affect each other. Dealing with indeterminate spatial extents, e.g., measurement errors, implies that the time point at which a change occurs cannot be detected precisely. On the other hand, having an indeterminate temporal aspect, e.g., clock errors, introduces spatial indeterminacy.

Scenario 4 – continuous change of a geometry recorded in time periods.

This case is based on the fact that for a given time period $T_i = [t_i, t_{i+1}]$, there exists a change function that models the transformation from geometry G_i to geometry G_{i+1}. Each of these factors, i.e., (i) the time period, (ii) the geometry, and (iii) the change function, can be subject to indeterminacy.

In the simplest case, the geometries G_i and G_{i+1} and the time period T_i are *determinate*, and the change function returns a determine geometry G_x for a given time point $t_x \in T_i$. Here, we assume that the change function returns the geometry coinciding with the actual movement. Is this not the case, the change function *interpolates* in-between the geometries G_i and G_{i+1} and returns an indeterminate geometry. An example is to use linear interpolation, i.e., the two geometries G_i and G_{i+1} are considered to be the endpoints of a line.

If we further allow G_i and G_{i+1} to be *indeterminate*, our change function would in any case return an indeterminate G_x. In the following, we use the "~" symbol on top of the parameter to denote indeterminacy. This means that if geometry is described by a probability or membership function, this very function is subject to change in the time period T_i. Following the idea from before, we would have a change function that returns a probability or membership function for a given t_x (cf. Table 3(a)). However, by integrating the temporal component, we obtain a spatiotemporal probability or membership function, i.e., a function that changes with time. Table 3 summarizes this approach.

So far we always considered time to be determinate. We use time points to determine the start and the end of the current time period T_i, and to denote the time point t_x in question. In case t_i and t_{i+1} are indeterminate, we cannot state the precise beginning and end of the time period. Thus, the association of a geometry (indeterminate or not) to a time point becomes indeterminate. However, this affects mainly the change function and can be considered in adapting its form. The indeterminate time contributes some additional indeterminacy as to the determination of G_x for a given t_x. Table 4 adapts the approach shown in Table 3 to accommodate this aspect.

4.2 Summary

The following observations apply to the four spatiotemporal change scenarios of Table 2.

Change recorded in time points versus time periods differs in that in case of the former, we do not make any assumptions about the position of the geometry in-between recordings. For the latter case, additional knowledge is available that allows for an interpolation of the position samples in between measurements.

Recording change in time periods versus time points can be also referred to as synchronous versus asynchronous positional sampling. Considering the moving object context, in the synchronous case, the object transmits a new position only in case the old position and the respective change function do not allow us to derive the current position of the object, i.e., the moving object changed its speed, it turned, etc. Asynchronous sampling translates to measuring the object's position at fixed time intervals, e.g., every 30 seconds. Synchronous sampling, in contrast to asynchronous sampling, requires some object intelligence to determine when to measure the position.

The central element of spatiotemporal indeterminacy is the change function that manipulates geometries. Since the geometry of a location can be of type point, line, or region, the change function can be seen as a morphing algorithm that maps between different instances of geometries. The following section illustrates this approach for a particular case. We describe how to represent the movements of vehicles and how to quantify the errors associated with the chosen representation.

5. A Spatiotemporal Indeterminacy Scenario – the Moving Point Object Case Continued

Continuing the moving object case from Section 2, the following sections give details on how to quantify and describe the measurement and the sampling errors.

5.1 Measurement Error

An error can be introduced by inaccurate measurements. The accuracy and thus the quality of the measurement depend largely on the technique used. This work assumes that GPS is used for the measurement of positions.

Two assumptions are generally made when talking about GPS measurement accuracy. First, the error distribution, i.e., the error in each of the three dimensions and the error in time, is assumed to be *Gaussian*. Second, we can assume that the horizontal error distribution, i.e., the distribution in the x-y plane, is circular [25].

The error in a positional GPS measurement can be described by the probability function of Equation (9). The probability function is composed of two normal distributions in the two respective spatial dimensions. The mean of the distribution is the origin of the coordinate system. Figure 8 visualizes the error distribution. In addition to the mean, the standard deviation, σ, is a characteristic parameter of a normal distribution. Within the range of ±σ of the mean, in a bivariate normal distribution (2-dimensional), 39.35% of the probability is concentrated.

$$P_1(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (9)$$

A typical GPS receiver used in vehicle navigation systems has an error of 2 *m* (equal to 1σ). This measure refers to the standard deviation of a bivariate normal distribution centered at the receiver's true antenna position.

Which Case?

In the schema of Section 4, the sampling approach to capture the movement of an object is characterized by Scenario 4. Table 3 and Table 4 establish a foundation for obtaining a change function in-between measured positions. Table 4 gives function templates in case the times of sampling are not known precisely. However, GPS allows for highly precise timing and, thus, we neglect the effect of time. In Table 3, Case 1 gives a function template that applies when the sampled positions are known precisely. As we just saw, GPS measurements are accurate, but not precise. Thus, Case 2 of this table matches our problem. The following section shows how to establish a change function to determine the position of the moving object in-between sampling. We initially assume precise position samples.

5.2 Sampling Error

The movement is captured by measuring the object's position using a GPS receiver at regular time periods. Interpolating the movement in-between samples introduces an additional error in the representation of the movement. This section gives a model for the uncertainty introduced by this sampling error based on the sampling rate and the maximum speed of the object.

The uncertainty of the representation of an object's movement is affected by the frequency with which position measurements are made, the *sampling rate*. This, in turn, may be set by considering the speed of the object and the desired maximum distance between consecutive measurements. Let us consider an example that records the movements of school buses.[2]

Example 1

As a requirement to the application, the distance between two consecutive samples should be maximally 10 *m*. If the maximum speed of a bus is 120 *km*/*h*, this means that the position needs to be sampled at least 3.3 times per second. If a bus moves slower than its maximum speed, the distance between samples is less than 10 *m*. How do the position samples resemble the true movement of the object? Consider the three trajectories shown in Figure 9(a). Each is possible given the three measured positions P_1 through P_3. However, by just "looking" at the three positions, one would assume that the straight line best resembles the actual trajectory of the object.■

Since we did not have measurements in-between position samples, the best we can do is to *limit the possibilities of where the moving object could have been*. We constrain the trajectory of the object by what we know about the object's actual movement. Considering the trajectory in a time period $[t_1, t_2]$, delimited by consecutive samples, we know two positions, P_1 and P_2, as well as the object's maximum speed, v_m (cf. Figure 9(b)). If the object moves at maximum speed v_m from P_1 and its trajectory is a straight line, its position at time t_x will be on a circle of radius $r_1 = v_m(t_x - t_1)$ around P_1 (the smaller dotted circle in Figure 9(b)). Thus, the points on the circle represent the furthest away from P_1 the object can get at time t_x. If the object's speed is lower than v_m, or its trajectory is not a straight line, the object's position at time t_x will be somewhere within the area bounded by the circle of radius r_1. Similar assumptions can be made on the position of the moving object with respect to P_2 and t_2 to obtain a second circle of radius r_2.

These constraints on the position of the moving object mean that the object can be anywhere within the intersection of the two circular areas at time t_x. This intersection is shown by the shaded area in Figure 9(b). We use the term *lens* for this area of intersection. Since we do not have any further information, we assume a uniform distribution for the position within the lens, i.e., the object is equally likely anywhere within this lens shape.

Thus, the sampling error at time t_x for a particular position can be described by the probability function shown in Equation (10), where r_1 and r_2 are the two radii described above, s is the distance between the measured positions P_1 and P_2, and A denotes the area of the intersection of the two circles.

$$P_2(x,y) = \begin{cases} \dfrac{1}{A} & \text{for } x^2 + y^2 \le r_1^2 \wedge (x-s)^2 + y^2 \le r_2^2 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

Substituting $v_m(t_1 + t_x)$ and $v_m(t_2 - t_x)$ for the radii r_1 and r_2, respectively, the probability function shown in Equation (11) results. Its parameters are described in Table 5.

[2] This scenario is currently under investigation in collaboration with a company providing fleet management solutions.

$$P_2(x,y) = \begin{cases} \frac{1}{A} & \text{for } x^2 + y^2 \le (v_m(t_1 + t_x))^2 \wedge \\ & (x-s)^2 + y^2 \le (v_m(t_2 - t_x))^2 \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

Which Case?

Equations (10) and (11) describe the position of the moving object in-between position samples. Thus, this function is an instance of the function template as described in Case 1 of Table 3.

5.3 Combination of Error Sources – a Global Change Function

Table 3 gives a framework for change functions that incorporates indeterminate positions. In the context of this example, this translates to adapting Equation (10) such that the values for *x* and *y* are not precise, but are affected by the measurement error as described in Section 5.1. Although it seems trivial at first, this requires some heavy mathematical manipulation that is beyond the scope of this work.

A general mathematical framework suitable for this problem is *Kalman filtering* [21], which is a method for combining various error-prone measurements about the same fact into a single measurement with a smaller error. This mathematical framework stipulates a method of how to combine uncertainties to reduce the overall error. Assuming the measurements refer to position samples of a continuous movement in time, we can use *Kalman smoothing* to determine the positions at times that are in between the measured ones [1].

Examples of applying Kalman filtering to the domain of vehicle navigation include the integration of three independent positioning systems such as dead reckoning, map matching, and GPS, to determine the precise positions of moving vehicles [22].

6. Conclusions and Future Work

The work presented in this paper concerns spatial, temporal, and spatiotemporal indeterminacy. The term indeterminacy subsumes fuzziness that captures inherent imprecision and uncertainty that captures the lack of precise information. Fuzziness and uncertainty are first explored in the temporal context, where examples and definitions are given for indeterminate time points and time periods. The relationship between fuzzy set theory and probability theory is described in the temporal context, while both concepts are used to describe temporal indeterminacy. Next, spatial objects, relationships, and attributes are considered. In particular, this work explores indeterminacy in relation to geometric types, namely point and region, that are inherent to spatial objects and attributes. Again, the mathematical concepts of fuzzy set theory and probability theory are used to describe these phenomena; their differences and similarities are explored in the spatial context. Furthermore, the concept of change is introduced as the key spatiotemporal concept. Change can occur in discrete steps and on a continuous basis, and it can be recorded using regular sampling or in a change-driven fashion. Combining these concepts leads to four different change scenarios, which are affected by indeterminacy differently. The indeterminacy of change is formalized and combines the spatial and temporal concepts.

Finally, the rather general concepts are applied to an existing application area. We discuss uncertainty in the context of applications that involve moving-point objects. We give a change function that describes the positions of moving objects over time based on position samples. The change function is influenced by measurement and sampling errors.

The framework put forward in this paper is currently being applied in an extension to an existing fleet management system that will support spatiotemporal data analysis, including the study of certain operations on indeterminate data.

Future research plans include indeterminacy with respect to relationships among spatial, temporal, or spatiotemporal objects. Next, while the mathematical models presented are concrete enough to express the indeterminacy related to the temporal, spatial, and spatiotemporal domain, more detailed mathematical formulas are needed for specific applications. Finally, this work may be seen as a step towards the development of a more general framework of spatiotemporal data types and data structures that incorporate indeterminacy.

Acknowledgements

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Appendix A: An Introduction to Indeterminacy Measures

This section introduces the mathematical background to be able to express fuzziness and uncertainty in spatial, temporal, and spatiotemporal concepts. More specifically, *fuzzy set theory* and *probability theory* are covered.

A.1 Fuzzy Set Theory

Fuzzy set theory [42] is an extension and generalization to Boolean set theory. Let *X* be a classical (crisp) set of objects, called the universe. Membership in a classical subset *A* of *X* can be described by the characteristic function $\chi_A : X \to \{0,1\}$ such that for all $x \in X$ the following holds.

$$\chi_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases} \tag{12}$$

This function discriminates sharply between the members and non-members of set *A*. We can generalize this function by mapping the elements of set *X* not to the set {0,1}, but rather to the real interval [0,1]. Now, elements have no strict membership, but rather have a *degree of membership* in the set in question. Larger values indicate higher grades of membership. With *X* as the universe, the *membership function*

$$\mu_{\tilde{A}} : X \to [0,1] \tag{13}$$

returns for a given *element* of *X* the degree to which it belongs to the *fuzzy set.* $\tilde{A}$

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\} \tag{14}$$

All elements of *X* are evaluated towards a membership in $\tilde{A}$. Those elements that do “not at all” belong to the set have as degree of membership $\mu_{\tilde{A}}(x) = 0$, whereas the elements that “totally” belong to the set have as degree of membership $\mu_{\tilde{A}}(x) = 1$. Although fuzzy set theory seems sound and simple, it is difficult to actually apply it. A key problem is how to choose an appropriate membership function.

A.2 Probability Theory

With *X* again beoing the universe, the probability measure *P* is a mapping $2^X \to [0,1]$ that assigns a number *P*(*A*) to each *subset A* of *X*, and satisfies the Kolmogorov axioms [12]:

$$\begin{aligned} &P(X) = 1; P(\varnothing) = 0 \\ &P(A \cup B) = P(A) + P(B); \;\; iff \;\; A \cap B = \varnothing; \end{aligned} \tag{15}$$

Here, *P*(*A*) is the probability that an ill-known single-valued variable *y* ranging on *X* hits the fixed well-known set *A*. Given the case where the underlying domain of the universe *X* is discrete, the *probability mass function* $p(x) = P(\{x\})$ returns the probability for the single element $x \in X$. The probability density function returns the probability $Q(A) = \int_A p(x)dx$ for a given subset $A \in X$.

A.3 Differences Between Fuzzy Set and Probability Theory

Very often, fuzzy values are misunderstood to be probabilities, or fuzzy set theory is misunderstood as a new way to handle probabilities. These are misconceptions, since additivity is a minimum requirement of probabilities, i.e., all probabilities for alternative events have to sum up to one. This is not the case for membership grades. In mathematical terms, the membership function $\mu_{\tilde{A}}(x)$ is similar to $P(\{x\}) = p(x)$, except for the above condition, $\sum_{x \in X} p(x) = 1$ must hold, while this is not true for $\mu_{\tilde{A}}$.

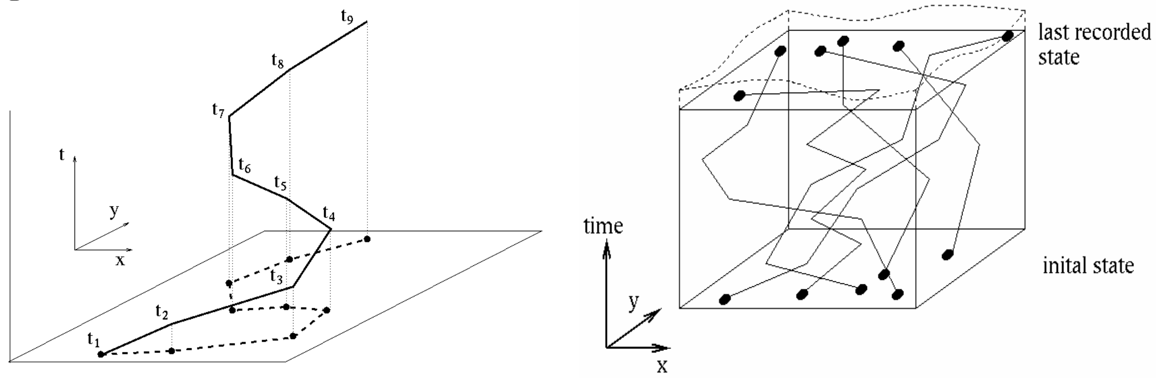
Further, a membership grade is defined only for the individual elements of a set, not for subsets. Probabilities can be given for any subset, i.e., also one element. However, all probability distributions are fuzzy sets. As fuzzy sets and logic generalize Boolean sets and logic, they also generalize probabilities.

In revisiting the indeterminate time point (cf. Figure 4), we can see that to describe the membership grade of a chronon with respect to a particular point, with 1 representing a certain membership, each of the chronons would have a membership grade of 1/4, which is equal to the probability that each of the chronons is the actual time point. Since the membership grade is equal to the probability, they add up to 1. However, in the case of a time period bound by indeterminate time points, we have regions that have a membership grade of 1 as well, i.e., they do not add up to 1. In this case, the membership function is not the "same" as the probability function, but the latter is used to derive the former.

Probability functions are used to describe uncertain positions. Consider here an unknown position, whose positional probability is scattered over a region, i.e., a set of points. If we state that the position is at one of the points of the region, then this statement is true, since all the probabilities scattered over the region add up to one. In other words, probability can also be defined for a set of points as opposed to only one point. On the other hand, if we, instead of giving a probability for a point, devise a membership grade, the sum of all membership grades over all the points in the region has no clear meaning. It is merely an arbitrary number. The membership grade of a point tells us about "the belief" that a point belongs to a particular set, e.g., the soil type desert.

Probabilistic concepts are related to what is the most likely position, i.e., where is the border for what is in and what is out. Fuzzy concepts are related to what belongs to what extent to a given set, i.e., what is "in" and what is "out." Fuzzy concepts refer to relative aspects whereas probabilistic concepts refer to absolute aspects of a spatial scenario.

Figures



(a) **(b)**

Figure 1: Movements and space

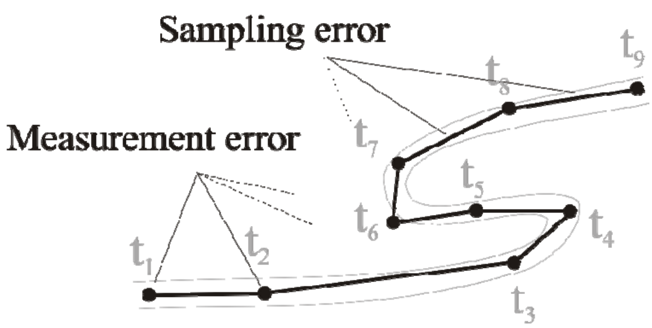


Figure 2: Overview of the error measures introduced by sampling movement

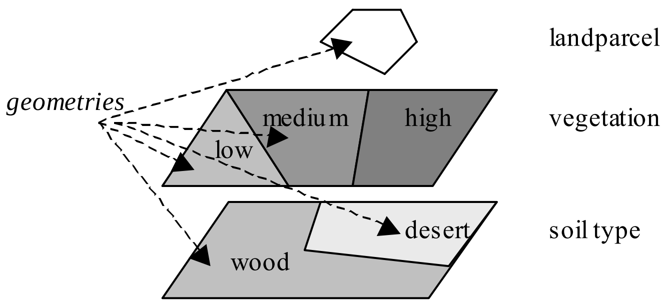


Figure 3: Spatial objects, space-depending attributes, and geometries in space.

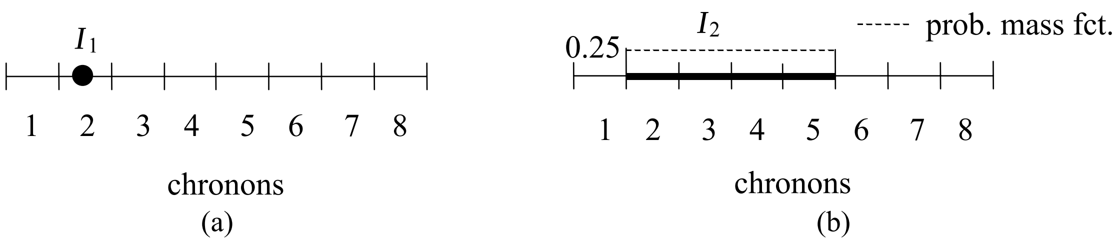


Figure 4: (a) Determinate (I_1) and (b) indeterminate (I_2) time points

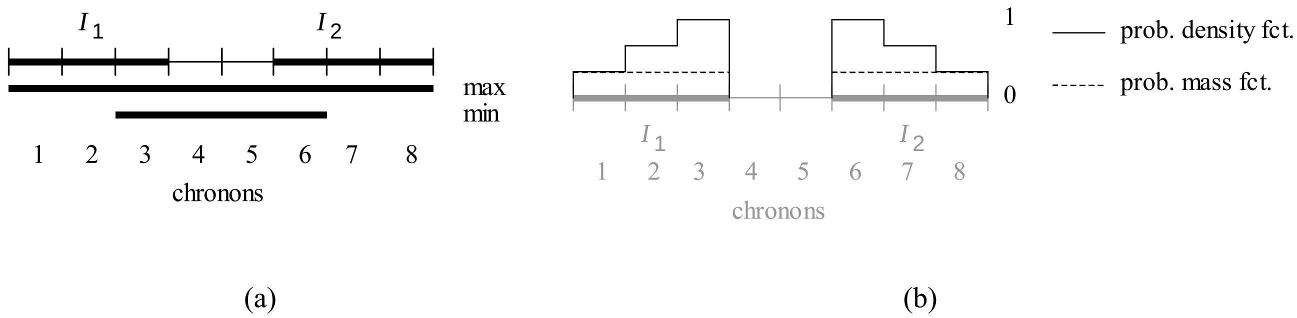


Figure 5: (a) Indeterminate time period, (b) probabilities of bounding time points

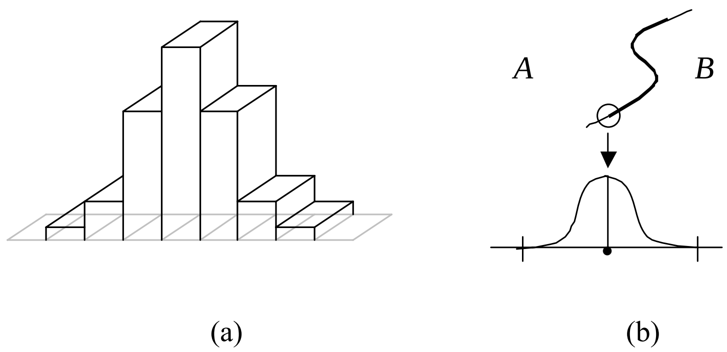


Figure 6: Boundary point probability

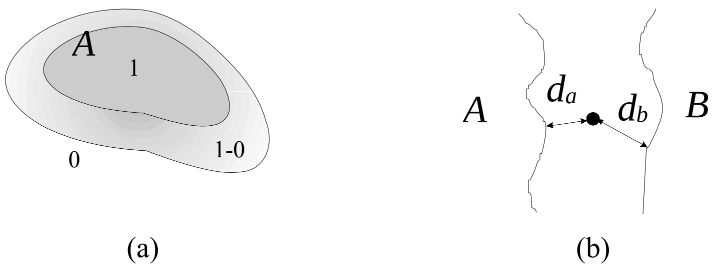


Figure 7: Boundary point probability

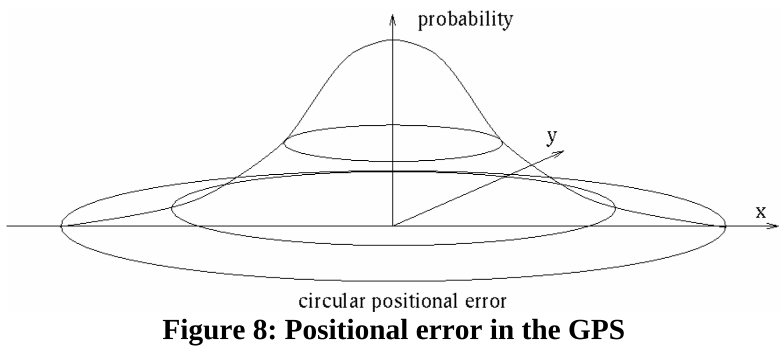
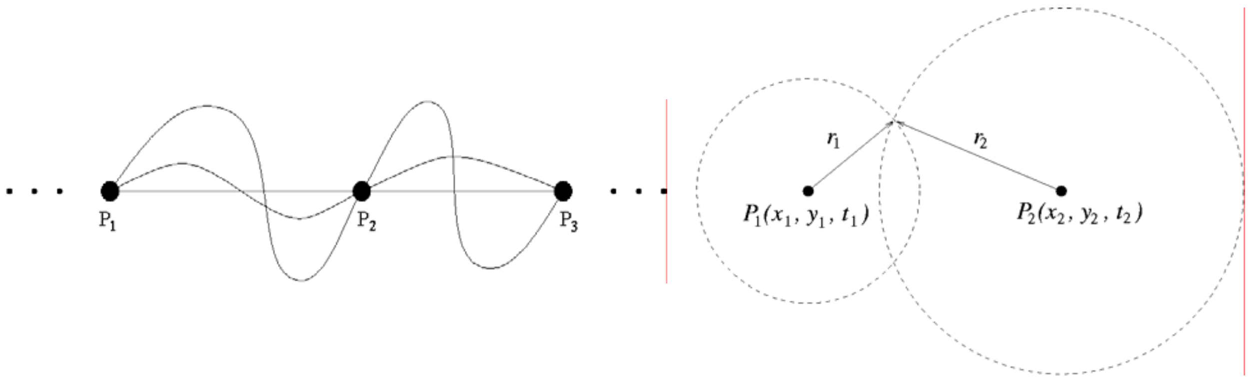


Figure 8: Positional error in the GPS



(a) (b)

Figure 9: Position samples: (a) possible trajectories and (b) associated uncertainty

Tables

Spatial Concepts/Indeterminacy	Fuzziness	Uncertainty
Object's position	–	√
Relationship among objects	√	√
Spatial attribute	√	√

Table 1: Spatial concepts and indeterminacy

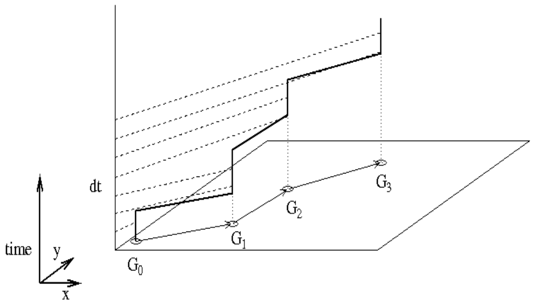
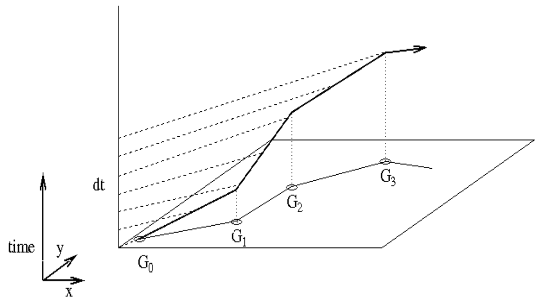
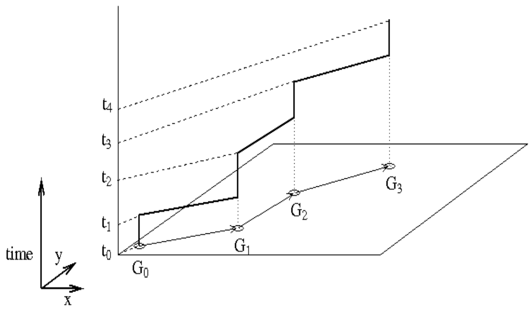
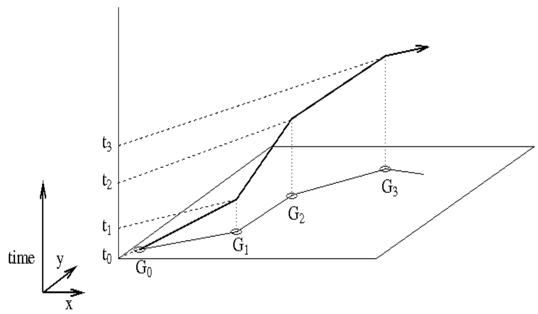
Change/ Time	Discrete	Continuous
Point	**1)** A geometry is recorded at a time point. The geometry can or cannot differ from the previously recorded one. We do not know when the change occurred. 	**2)** A geometry is sampled at *time points*. In between time points we have no knowledge about the geometry. 
Period	**3)** A geometry is valid for a given *time period*. After a change, a new time period starts. 	**4)** A geometry is sampled at time points, the starting and end points of the time period. Further, a time period is *assigned* a *"change" function* that models the positional change within the time period. 

Table 2: Four spatiotemporal change scenarios

Geometry (G_i, G_{i+1})	**Time (t_i, t_{i+1})**	**Change**
Determinate	Determinate	$C: t_x \to G_x$, where G_x, depending on the change function, is determinate or indeterminate ($\tilde{G}_x$)
Indeterminate	Determinate	(a) $C: t_x \to \tilde{G}_x$, where $\tilde{G}_x$ represents either a probability function, $P_x(i)$, or a membership function, $\mu_x(i)$ (b) $\mu_x(i,t)$ or $P_x(i,t)$

Table 3: Change scenarios without temporal indeterminacy

Geometry (G_i, G_{i+1})	**Time (t_i, t_{i+1})**	**Change**
Determinate	Indeterminate	$C:\tilde{t}_x \rightarrow \tilde{G}_x$
Indeterminate	Indeterminate	(c) $C:\tilde{t}_x \rightarrow \tilde{G}_x$, where $\tilde{G}_x$ is either a probability function, $P_x(i)$, or a membership function, $\mu_x(i)$ (d) $\mu_x(i,\tilde{t})$ or $P_x(i,\tilde{t})$

Table 4: Change scenarios incorporating temporal indeterminacy

v_m	maximum speed of the moving object
t_x	time for which the error distribution is computed
t_1	time of the first measured position
t_2	time of the second measured position
s	distance between the two positions, i.e., the length of the line segment
A	lens area, i.e., the area of the intersection of the two circles

Table 5: Parameters of the probability function, P_2, describing the sampling error