Efficient Prototype Filter Realizations for Cosine-Modulated Filter Banks

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RÉSUMÉ

Cet article presente des implantations optimisées de filtres prototypes de bancs de filtres modulés à reconstruction parfaite, à retard quelconque. Les implantations proposées sont basées sur des "schémas de liftage". Ici, l'approximation des coefficients "de liftage" est faite de manière à ce que le filtrage n'emploie que peu d'operations de décalage et addition. Comme le "schéma de liftage" est robuste aux erreurs de quantification ainsi introduites, la propriété de recontruction parfaite du banc de filtres est conservée. Le prototype d'origine et celui obtenu par cette methode sont comparés en terme de complexité d'implantation et d'atténuation hors-bande.

ABSTRACT

This paper presents efficient realizations of prototype filters for biorthogonal cosine-modulated filter banks with arbitrary overall system delay. The implementation is based on lifting schemes. The lifting coefficients will be approximated such that the filter operations can be realized by a few shift and add operations. Since the lifting scheme is robust against coefficient quantization we do not loose the perfect reconstruction (PR) property of the filter bank when doing this approximation. The original and the approximated prototype filters are compared in terms of complexity and stopband attenuation.

1. INTRODUCTION

Cosine-modulated filter banks are known to provide a very efficient realization. They consist of two main stages: the polyphase filters of the prototype and a discrete cosine transform, see Figure 1 for an M-channel analysis bank. Herein, the filters $G_k(z)$, $k = 0, \dots, 2M-1$, denote the type-1 polyphase filters [1] of the prototype. Using fast DCT for the transform the main computational complexity remains with the multiplications during the filter operation [2].

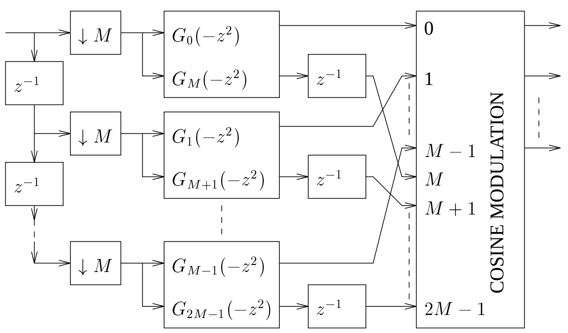

Figure 1: Cosine-modulated M-channel analysis filter bank

For biorthogonal cosine-modulated filter banks the overall system delay can be chosen independently of the filter length, thus being interesting for applications where a low delay of the filter bank and a higher stopband attenuation as in the paraunitary case are desirable. They have recently been studied in literature by several authors [3, 4, 5, 6, 7, 8].

Although many design methods for PR cosine-modulated filter banks have been developed within the last few years, see e.g. [1, 9, 6, 4], none of these methods takes into consideration the implementation cost due to the wordlengths of the filter coefficients. For paraunitary cosine-modulated filter banks an efficient realization method based on an approximation of the lattice coefficients with CORDIC rotations has been presented in [10].

We here extend the results of [10] to the biorthogonal case using the idea of lifting and dual lifting [11, 12, 8] in order to obtain a structure being robust against coefficient quantization. The aim of the coefficient quantization is to realize them by a small number of shift and add operations. The resulting structure is not only extremely regular but also designated for parallel processing.

For the sake of simplicity, we only consider biorthogonal cosine-modulated filters banks with a prototype of length $N = 2mM$ and an overall system delay of $D = 2sM + 2M - 1$ samples with $s \in \mathbb{N}$. In this case, the constraints on the type-1 polyphase components of the prototype filter write [7]

$$G_{2M-1-\ell}(z)G_\ell(z) + G_{M-1-\ell}(z)G_{M+\ell}(z) = \frac{z^{-s}}{2M} \qquad (1)$$
$$\ell = 0, \dots, 2M-1$$

and all polyphase components are of length m. The same prototype is used for the analysis and synthesis bank.

2. THE LIFTING SCHEME

The lifting scheme is a systematic way to construct biorthogonal wavelets [11, 12] or prototype filters for modulated filter banks [8]. One starts with short filters or simply the polyphase transform and successively increases the filter length. A single lifting step consists of increasing the length of one filter from a PR filter pair while keeping the PR property. Such a step is typically followed by a dual lifting step where the length of the second filter is increased. Overall, one alternates lifting and dual lifting to construct long biorthogonal wavelets or filters from short ones.

We here apply the lifting scheme to the polyphase filters $G_\ell(z)$ of the prototype filter. Thus the filter length can be increased while keeping the filter bank PR.

Writing (1) in matrix form, we obtain

$$\begin{bmatrix} G_\ell(z) & G_{M+\ell}(z) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} G_{2M-1-\ell}(z) \\ G_{M-1-\ell}(z) \end{bmatrix} = \frac{z^{-s}}{2M}. \qquad (2)$$

2.1. Zero-Delay Lifting

The identity matrix in (2) can be expressed as

$$\mathbf{AB}(\mathbf{AB})^{-1} = \begin{bmatrix} 1 & 0 \\ A(z) & 1 \end{bmatrix} \begin{bmatrix} 1 & B(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -B(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -A(z) & 1 \end{bmatrix}$$

corresponding to one lifting step and one dual lifting step. Choosing $A(z) = a_0 z^{-1}$ and $B(z) = b_0$ as in Figure 2, the filters $G''_*(z)$ are one tap longer than $G'_*(z)$. If $G'_*(z)$ cause a delay s' the same delay is obtained with the longer polyphase filters $G''_*(z)$.

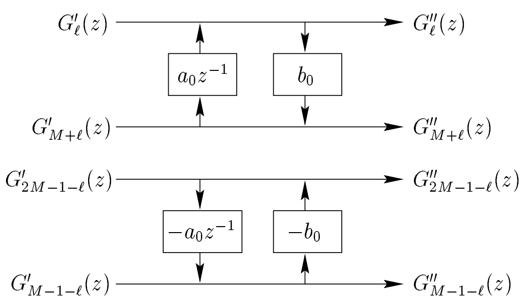

Figure 2: Lifting and dual lifting step with $\pm A(z) = \pm a_0 z^{-1}$ and $\pm B(z) = \pm b_0$, respectively

2.2. Maximum-Delay Lifting

It is also possible to obtain polyphase filters $G''_*(z)$ that are one tap longer than $G'_*(z)$ and increase the overall system delay by $4M$ samples using the structure in Figure 3.

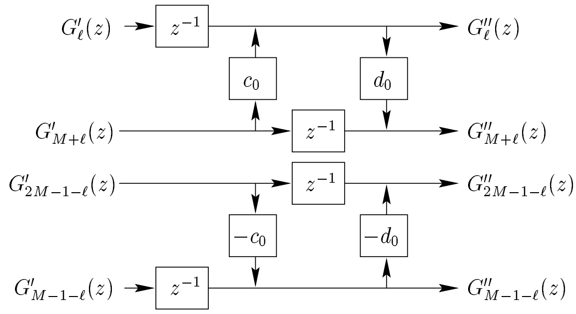

Figure 3: Maximum-delay lifting

If $G'_*(z)$ cause a delay s' the delay obtained with the longer polyphase filters $G''_*(z)$ is $s'' = s' + 2$, since the identity matrix in (2) is replaced by

$$z^{-2}\mathbf{CD}(\mathbf{CD})^{-1} = \begin{bmatrix} z^{-1} & 0 \\ c_0 & 1 \end{bmatrix} \begin{bmatrix} 1 & d_0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} z^{-1} & -d_0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -c_0 & z^{-1} \end{bmatrix}$$

2.3. Single-Delay Lifting

Combining matrices **A** and **D** or **C** and **B** results in the structures depicted in Figures 4 and 5. The filters $G''_*(z)$ are one tap longer than $G'_*(z)$ and the overall system delay is increased by $2M$ samples since $s'' = s' + 1$.

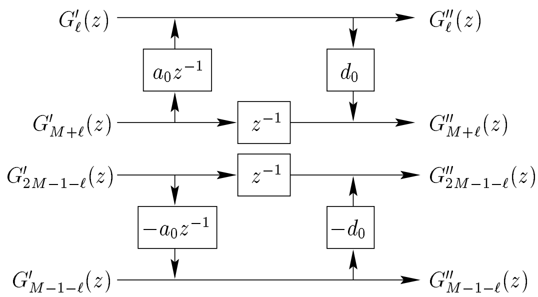

Figure 4: Single-delay lifting (type 1)

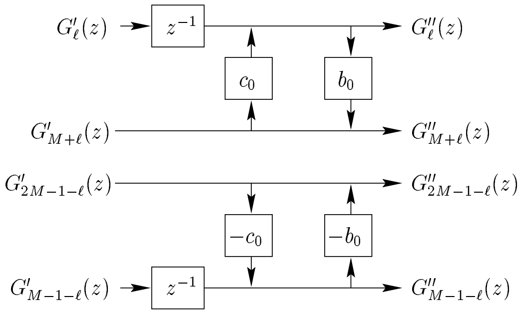

Figure 5: Single-delay lifting (type 2)

$$\begin{bmatrix} G_\ell(z) & G_{M+\ell}(z) \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix} \frac{1}{\sqrt{4M}} \mathbf{F}_0 \cdot \underbrace{\prod_{i_0=1}^{i_{0_{\max}}} (\mathbf{A}_{i_0}\mathbf{B}_{i_0})}_{\text{Zero-Delay Lift.}} \cdot \underbrace{\prod_{i_1=i_{0_{\max}}+1}^{i_{1_{\max}}} (\mathbf{A}_{i_1}\mathbf{D}_{i_1}) \cdot \prod_{i_2=i_{1_{\max}}+1}^{i_{2_{\max}}} (\mathbf{C}_{i_2}\mathbf{B}_{i_2})}_{\text{Single-Delay Lifting}} \cdot \underbrace{\prod_{i_3=i_{2_{\max}}+1}^{i_{3_{\max}}} (\mathbf{C}_{i_3}\mathbf{D}_{i_3})}_{\text{Max.-Delay Lift.}} \quad (3)$$

$$\begin{bmatrix} G_{2M-1-\ell}(z) \\ G_{M-1-\ell}(z) \end{bmatrix} = \underbrace{\prod_{i_3=i_{2_{\max}}+1}^{i_{3_{\max}}} (z^{-2}\mathbf{D}_{i_3}^{-1}\mathbf{C}_{i_3}^{-1})}_{\text{Max.-Delay Lift.}} \cdot \underbrace{\prod_{i_2=i_{1_{\max}}+1}^{i_{2_{\max}}} (z^{-1}\mathbf{B}_{i_2}^{-1}\mathbf{C}_{i_2}^{-1}) \cdot \prod_{i_1=i_{0_{\max}}+1}^{i_{1_{\max}}} (z^{-1}\mathbf{D}_{i_1}^{-1}\mathbf{A}_{i_1}^{-1})}_{\text{Single-Delay Lifting}} \cdot \underbrace{\prod_{i_0=1}^{i_{0_{\max}}} (\mathbf{B}_{i_0}^{-1}\mathbf{A}_{i_0}^{-1})}_{\text{Zero-Delay Lift.}} \cdot \mathbf{F}_0^{-1} \cdot \frac{1}{\sqrt{4M}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (4)$$

with

$$\mathbf{F}_0 = \begin{bmatrix} 1 & 0 \\ f_{0,0} & 1 \end{bmatrix} \begin{bmatrix} 1 & f_{0,1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ f_{0,2} & 1 \end{bmatrix},\ \mathbf{A}_i = \begin{bmatrix} 1 & 0 \\ a_{0,i}z^{-1} & 1 \end{bmatrix},\ \mathbf{B}_i = \begin{bmatrix} 1 & b_{0,i} \\ 0 & 1 \end{bmatrix},\ \mathbf{C}_i = \begin{bmatrix} z^{-1} & 0 \\ c_{0,i} & 1 \end{bmatrix},\ \mathbf{D}_i = \begin{bmatrix} 1 & d_{0,i} \\ 0 & z^{-1} \end{bmatrix},$$

2.4. Factoring Polyphase Filters into Lifting Steps

It has been shown in [8] that any set of four polyphase filters satisfying (2) can be expressed according to (3) and (4). $\mathbf{F}_0$ is a special initialization matrix ensures that the PR constraints are also satisfied for polyphase filters of length one, using a maximal number of free degrees. Given (3) and (4), the length m of the polyphase components and the delay s in (1) are determined by

$$m = i_{3_{\max}} + 1, \quad s = (i_{2_{\max}} - i_{0_{\max}}) + 2(i_{3_{\max}} - i_{2_{\max}}).$$

The matrices in (3) and their inverses in (4) contain the same coefficients. Therefore the structure is robust against coefficient quantization.

The succession of the terms in (3) is arbitrary while the order in (4) is the inverse. However, given a biorthogonal PR prototype filter in transversal form that shall be realized using the proposed structure with lifting steps described in Figures 2 to 5, we obtain different lattice coefficients for different orders of the terms in (3).

3. EFFICIENT PROTOTYPE REALIZATION

In order to obtain efficiently realizable prototype filters, we first have to design biorthogonal prototypes. This can be done by taking a paraunitary prototype filter, calculating the coefficients of the lifting scheme, adding zero-delay lifting steps that increase the polyphase filters up to the desired length and using a nonlinear optimization routine in order to optimize the lifting coefficients. A reasonable cost function is the stopband energy of the prototype filter. Alternatively, one can take prototype filters directly from literature.

Once, we have the transversal biorthogonal prototype filter, we have to decide on what lifting and dual lifting steps we want to implement and in which order, taking into consideration the constraints given by m and s. We then calculate the lifting coefficients as described in [8] and quantize them using signed binary number representation. This first quantization has to be done carefully, such that the frequency response of the prototype remains nearly the same as before quantization.

We then iteratively replace the least sensitive coefficient by a coarser approximation reducing the number of additions necessary for that coefficient by one (the least sensitive coefficient is the one that results in the smallest increase of the cost function when being replaced by the coarser approximation). This pruning is continued as long as the complexity in terms of shift and add operations is above the desired one or as long as the frequency response does not change significantly, depending on which constraint (complexity or frequency response) is the more important one.

4. EXAMPLES

In the following we consider an 8-channel cosine-modulated filter bank with an overall system delay of $D = 31$ samples.

Figure 6 shows for the unquantized case how the stopband energy can be decreased when increasing the prototype length.

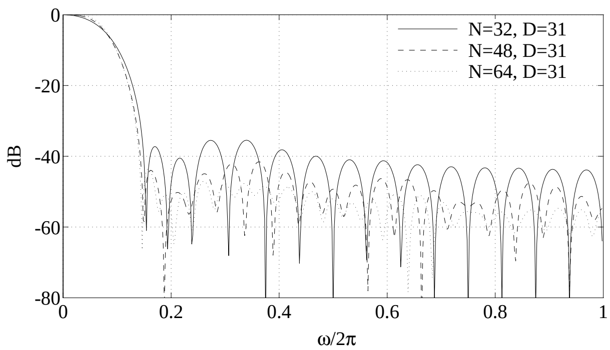

Figure 6: Magnitude responses of prototypes with different lengths resulting in the same filter bank delay

For the prototypes of length $N = 48$ and $N = 64$ and the order of lifting steps as in (3) we now present efficient realizations that were obtained by using the procedure described in the last section. Figure 7 shows the magnitude responses for $N = 48$ and $N = 64$, respectively, for different complexities.

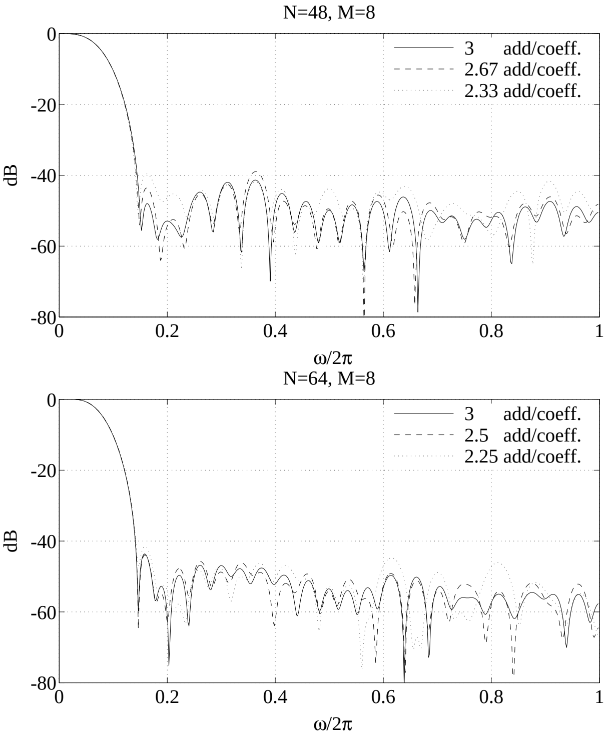

Figure 7: Magnitude responses of prototype realizations with different complexities

One interesting question is what prototype filter length is preferable when fixing the overall filter realization cost. Figure 8 shows the magnitude responses of a length-48 and a length-64 prototype filter that can both be realized with a total amount of 80 additions and yield an overall system delay of 31 samples.

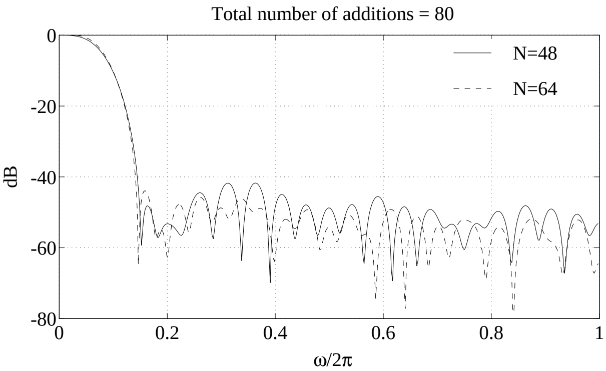

Figure 8: Comparison of prototype filter realizations with different lengths N but the same complexities

From Figure 8 it turns out that the length-64 prototype filter has a higher stopband attenuation although the number of additions per filter coefficient is lower than for the length-48 prototype (2.5 add/coeff. versus 3.33 add/coeff.). This is due to the fact that the frequency response in the case $N = 48$ cannot be better than the one given in Figure 6 for the unquantized case.

5. CONCLUSION

In this paper we have presented a simple procedure to find efficient realizations for biorthogonal prototype filters. Instead of a multiplication per coefficient we only have to perform a small number of shift and add operations. The resulting structure based on lifting schemes is highly regular and allows parallel processing of the polyphase filters. Although we do not necessarily find the best implementation since we do not optimize over the restricted parameter space for the lifting coefficients and although we cannot say what order of lifting steps is the most favorable, we have shown that powerful results can be obtained by a straightforward algorithm at low computational cost. Furthermore, we have pointed out, that it is not alway preferable to choose a short prototype filter when fixing the implementation cost.

6. REFERENCES

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