Stress wave effects in a finite element analysis of an impulsively loaded articular joint

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A dynamic contact finite element formulation was used to study transient stresses in the impulsively loaded rabbit knee, an established experimental model of mechanically induced osteoarthrosis. The computations were used to test the hypothesis that stress wave propagation and reflection, from juxtarticular interfaces of material property discontinuity, could be responsible for markedly increased levels of transient local cartilage stress. The finite element results demonstrated intuitively credible stress wave propagation and interfacial reflection phenomena. However, the magnitude of these waves was not nearly large enough to appreciably alter the quasi-static stress distributions otherwise prevailing. Thus, local stress wave reflection from interfaces of modulus discontinuity (for example the cartilage/subchondral plate) probably does not contribute appreciably to the heightened tissue sensitivity to impulsive loading experimentally observed in this animal model.

1 INTRODUCTION

Osteoarthrosis (OA), a form of joint wear and tear affecting over one-third of the adult population (1), is characterized by both deterioration of the articular cartilage and by alterations in the bony architecture of the joint. It is a chronic, painful process which results in a significant loss of mobility for many afflicted patients. It is generally believed that OA is not the result of any single cause, but involves both mechanical and biological factors (2). It has been suggested that the initiating factor in cartilage damage may be mechanical and the result of sub-structural disorganization and micro-damage (3, 4).

Numerous experimental animal models have been reported in the literature, with a wide variety of protocols for inducing tissue response similar to that seen in OA (5–9). Such models are useful for study of the initiation and progression of cartilage damage at the tissue level. Interest in mechanical damage to the cartilage microstructure as an initiating factor in cartilage deterioration has led to the development of several trauma-based models (10–15).

The results obtained from OA models based specifically on impulsive loading (15–17) suggest that the transient nature of impulsive loading is especially detrimental to the joint tissues. However, none of these experiments provide any direct information regarding the local stress to which juxtarticular tissues are subjected. This is unfortunate, since the immediate loading environment is most likely the primary stimulus for local tissue response in mechanically induced OA. Knowledge of the local stress would allow the comparison of damage thresholds from any one such model with those inferred from others.

The present investigation involves a dynamic contact finite element analysis of an impulsively loaded rabbit knee, an established experimental model of mechanically induced osteoarthrosis (15, 18, 19). That experimental model involves subjecting the hind limb of a rabbit to repetitive impulsive loads of roughly one and one-half times body weight, at one loading cycle per second for forty minutes daily, throughout the duration of typically a several-week test period (19). The dynamic force is delivered through a cam/follower arrangement (Fig. 1). During loading, the animals are suspended on a spring-balanced table with holes for the hind extremities. Of specific interest in the present analysis are the results of experiments using two different cam geometries to load two distinct rabbit populations. The two cams delivered the same peak force levels but differed by nominally an order of magnitude in their rate of load application to the rabbit limb. Those knees receiving the more highly impulsive loadings usually went on to develop rapid OA, while the other less impulsively loaded rabbit knees usually did not (19).

Previous finite element analysis of this animal model (20) indicated that inertially derived alteration of quasi-static juxtarticular stress patterns was not an adequate explanation for observed differences in the occurrence of OA in these two groups. The potential development of transient local stress peaks in the rabbit limb as a result of constructive interference between propagating or reflected stress waves is an alternative explanation for the observed experimental disparity. It is well established (21) that impact loads can cause appreciable compressive stress waves to propagate through long bones and that these waves are altered by the local mechanical properties of the bone (for example fracture callus, osteoporosis). Previous work (22) has suggested that propagating stress transients in the lower limb may be implicated as a factor contributing to osteoarthritic deterioration. The present authors therefore hypothesized that stress wave propagation/reflection phenomena in the immediate region of articular contact might contribute to alterations in the local stress distributions in the impulsively loaded joints, since adjacent material regions possess grossly disparate mechanical properties.
2 METHOD

A mid-compartment sagittal plane section of a rabbit knee was stylus-digitized and zoned to create a 2618 degree-of-freedom plane strain model. The mesh (shown in Fig. 2a) contained 1390 isotropic continuum elements and 45 dynamic contact elements. Eight distinct element layers were designated beneath each of the articulating surfaces, with the six most superficial layers representing articular cartilage, and the deepmost two layers representing calcified cartilage and subchondral bone respectively. Contact at the surface was assumed frictionless and the meniscus was neglected. All materials were modelled as being isotropic and linearly elastic. Figure 2b shows the specific designation of material property regions within the mesh. Impulsive loading was supplied through prescribed upward displacements of the distal section of the tibial segment. The inertia of the distal section of the tibia segment. The inertia of the distal-most row of cortical bone elements (shaded parts in Fig. 2a).

Finite element analysis was performed using the implicit dynamic solution option of the FEAP algorithm (23). Newmark temporal discretization and linearization of the resulting non-linear equations was employed at each point in time, using Newton's method. Unlike standard gap elements, the FEAP contact elements feature a more rigorous treatment of impact (as well as release) conditions, aimed at accurately accounting for stress wave propagation and local vibrational phenomena critical to the impulsive loading considered in the present analysis. The contact elements modify the conventional equations of motion by introducing non-linear virtual work terms of the form

\[ \delta \left( \int_A \hat{F}^T \delta \hat{g} \, dA \right) = \int_A \delta \hat{F}^T \hat{g} \, dA + \int_A \delta \hat{g}^T \hat{F} \, dA \]

where \( \hat{F} \) is a vector of nodal contact forces and \( \delta \hat{g} \) denotes displacements between prospective pairs of contact nodes distributed over a potential contact surface \( A \). Thus, the global stiffness matrix is modified to reflect the appropriate contact state whenever impact or separation occurs.

Treatment of impact-release transients is included in the finite element formulation through modification of the standard Newmark formulas, to address otherwise discontinuous velocities and accelerations between the two contacting bodies. When prospective contact nodes are computed to be coming into contact (impacting), the algorithm first makes the displacements of paired contact nodes compatible, thereby enforcing impenetrability of the contacting surfaces. Then the impact conditions are imposed; that is contact forces are computed and applied, and the incompatible velocities and accelerations previously computed by the algorithm for the contacting nodes are adjusted. The appropriate values of velocity and acceleration on the contact surface are deduced from a local wave propagation analysis involving the theory of propagating singularity surfaces (24).

Release conditions, applied at the end of a period of contact between paired nodes, stem from these same concepts. Thus, postrelease velocities are computed based on prerelease data, once release is detected. The contact forces are simultaneously set equal to zero. Accelerations at the previously contacting nodes are then adjusted so that satisfaction of the equations of motion is maintained. This dynamic contact algorithm has been shown to produce results in precise agreement with established analytical solutions for a variety of simple-geometry impact problems [for example the impact of two identical elastic bars, impact of two dissimilar elastic bars, impact of an elastic bar against a rigid surface (25)].

For simple longitudinal wave propagation in a homogeneous isotropic continuum, the extensional wave velocity \( U \) is described by \( U = \sqrt{(E/\rho)} \), where \( E \) is Young's modulus and \( \rho \) is the mass density of that material. For subchondral bone, the corresponding velocity is roughly 1600 m/s, and for articular cartilage, roughly 200 m/s. When a travelling compressional wave encounters a material boundary, the stress amplitude reflected is directly proportional to the difference in characteristic velocities for the two materials (26):

\[ A_r = \frac{v_2 - v_1}{v_2 + v_1} A_i \]

where \( A_r \) is the amplitude of the reflected wave, \( A_i \) is the amplitude of the incident wave and \( v_1 \) is the extensional wave velocity in the \( j \)th material as the wave travels from material 1 to material 2. Clearly, at a boundary between articular cartilage and subchondral bone, a major portion (roughly 80 per cent) of a propagating wave would thus be expected to be reflected, leading to...
Finite element mesh developed for detailed study of stress wave propagation in the immediate juxtarticular region. An input step displacement pulse was applied at the distal tibia $s_{DT}(t)$ while the proximal femur was constrained against transverse ($x$-direction) displacement. The shaded element rows in the distal tibia and the proximal femur represent elements which were assigned elevated mass densities.

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**Fig. 2a** Finite element mesh developed for detailed study of stress wave propagation in the immediate juxtarticular region. An input step displacement pulse was applied at the distal tibia $s_{DT}(t)$ while the proximal femur was constrained against transverse ($x$-direction) displacement. The shaded element rows in the distal tibia and the proximal femur represent elements which were assigned elevated mass densities.

**Fig. 2b** The designation of material property regions within the finite element mesh.
the potential for transiently increased stress in the articular cartilage.

By this same argument, the majority of the initial stress wave propagating up the cancellous bone of the tibial metaphysis (velocity roughly 1000 m/s) would be reflected at the tibial subchondral plate, in essence never even entering the articular cartilage. However, the portion of these propagating waves that did enter the cartilage layers would be prone to remaining within that region, reflecting back and forth between the tibial and femoral subchondral plates. While this simple one-dimensional treatment of stress wave propagation is useful in visualizing the potential progression of events, it is unnecessarily limiting for the problem posed in the present analysis. The FEAP finite element code provides a more general two-dimensional treatment of wave propagation. In order to study impulsive stress wave propagation through the articular cartilage and across the osteochondral junction, a highly impulsive (10 μs) step displacement of 0.5 mm was applied to the distal tibia boundary. A subsequent finite element time increment of 10 μs was used to study the propagation of the resulting stress waves. The objective of the analysis was to ascertain whether or not the associated interfacial stress wave reflections were of a magnitude potentially responsible for cartilage damage.

3 RESULTS

The results from an analysis of stress wave propagation in the rabbit knee provide insight into the dynamic nature of stress transfer across the joint. When interpreting these results, it is important to remember that the input tibial displacement pulse used here to focus on stress wave propagation was smaller in amplitude and substantially more abrupt in rate of application—than the pulses delivered by the experimental cams. Longitudinal propagation of a typical compressive stress wave is illustrated in Fig. 3a. During proximal transmission up the tibial cortex (celerity nominally 80 m/s), all stress waves were observed to broaden and distort. Owing to bending effects, the dominant cortical stresses were tensile along some tracts (as in this instance) and compressive along others (as indicated by the inset heavy arrows). Special large stresses (an order of magnitude yet larger than those seen in the other cortical elements) were computed in the distal-most tibial element row, presumably owing to the use of lumped-mass elevated
density levels (shaded elements in Fig. 2a) to represent coupled masses (for example rabbit foot, thigh, and torso) not explicitly zoned in the model.

The articular cartilage layers per se were consistently observed to be a site of transient stress elevation, compared to the underlying and overlying metaphyseal cancellous bone. During early load uptake, the cartilage saw severalfold stress elevations (Fig. 3a), while later in time the elevations were less remarkable (Fig. 3b). The rapid (80 m/s) stress wave accounting for general compression of the tibial member was followed by a much slower-travelling (celerity nominally 7 m/s) stress wave eventually \((t = 4 \text{ ms})\) associated with upward femoral acceleration relative to the tibia. Though somewhat difficult to discern, the slower-travelling wave is evident in Fig. 3b, in the region marked with the dark arrow, where the local peak in longitudinal compressive stress has begun to advance up the tibial cortex.

Several smaller-magnitude longitudinal compression waves were noted reflecting back and forth within the tibial member (Fig. 3c and d). These waves were noted specifically within the articular cartilage and its underlying subchondral and epiphyseal bone. In particular, in Fig. 3c for the region designated with the dark arrow, it became clear that, as time progressed, a wave of compression was travelling in a proximal direction through the epiphyseal bone. Figure 3d shows a small-amplitude compressional stress wave propagating back through the tibial member in the opposite direction. It is logical to conclude that this represents a small wave reflection, probably from the tibial subchondral plate.

To visualize more clearly the smaller-magnitude stress waves propagating longitudinally through the tibial member, they are considered as being perturbations of a much more slowly varying (that is quasi-static) and substantially larger stress field. A baseline quasi-static stress measure, \(\sigma_s(t)\), was subtracted from the transient global stress distributions computed at each point in time (Fig. 4). The longitudinal stress in the tibial subchondral plate was chosen to serve as \(\sigma_s(t)\). Since cortical stress levels were much larger than those in the subchondral plate, the perturbed cortical stress in the tibial member (that is stress in excess of \(\sigma_s(t)\)) was very nearly equal to the full cortical stress (compare cortical regions of Figs 4a and 3b). However, within the immediate tibial juxtarticular region (basically the region in the tibia for which \(Y > \text{roughly 13 mm}\)), the perturbed stress field differed substantially from the unperturbed stress field (juxtarticular regions of Figs 4a and 3b). Focusing directly upon perturbed stresses in the juxtarticular region (Fig. 4b) one can appreciate a compressive stress wave propagating up through the epiphyseal bone towards the articular surface. Closer scrutiny of the perturbed stress distribution in the articular cartilage proper (Fig. 4c) revealed a yet smaller amplitude stress wave travelling though the femoral cartilage from the articular surface to the femoral subchondral plate.

4 DISCUSSION

The complex nature of the dynamic in vivo loading of the rabbit knee obviously dictated that several simplifying assumptions be made to provide for a tractable analysis. These include the use of a planar geometry, the absence of soft tissue (especially ligamentous and meniscal) structures and the use of lumped masses and simplified boundary/support conditions to model influences remote from the juxtarticular regions. Contact was assumed to be ideally frictionless, and the mechanical properties of the tissues modelled were assumed to be those of linear isotropic elastic continua. For the highly impulsive loading considered, these material assumptions appear to be appropriate, but inclusion of rate-dependent material properties might contribute to alteration of the juxtarticular stress distribution.

The present observation, that transiently amplified stresses at the articular surface accompany stress wave propagation, seems to contradict the traditionally accepted notion that articular cartilage serves as a cushioning layer. This is consistent with experimental work (13) which showed that articular cartilage does not significantly contribute to force attenuation in the impulsively loaded joint. By virtue of its rate-dependent response to loading, cartilage may serve a somewhat greater force-attenuating role in mildly abrupt loadings than it does in highly impulsive loading situations. Both of the experimental cam loadings, and certainly the presently simulated step impulse, produce much more rapidly varying rates of loading than anything experienced in typical activities of daily living.

Greatly elevated levels of compressive stress were noted in the distal-most tibial element row during system response to the applied step displacement pulse. The cause for these large-magnitude stress readings (shown in Fig. 3b) was most likely a combination of the nearby boundary constraint and acceleration of the lumped mass represented within that element row. These large inertial stresses eventually propagated along the cortices, but their magnitude attenuated very substantially during passage through the broad, relatively compliant metaphyses.

For the purposes of the present discussion, only axially travelling stress waves have been presented. In

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Fig. 4 A more detailed look at the small-amplitude stress waves propagating in the finite element model, obtained when a quasi-static baseline stress measure was subtracted from the transient global stress distributions computed at each time step:

(a) Looking at these perturbed stresses [that is stress in excess of \(\sigma_s(t)\)], there are indications of distinct waves travelling through the cortical region.

(b) Focusing directly upon perturbed stresses in the juxtarticular region demonstrates a compressive stress wave propagating up through the epiphyseal bone towards the articular surface (heavy arrow).

(c) Closer scrutiny of the perturbed stress distribution in the articular cartilage proper revealed a yet smaller amplitude stress wave travelling through the femoral cartilage from the articular surface to the femoral subchondral plate.
general, there exist transversely travelling stress waves in the model as well, and the FEAP finite element code does provide a limited consideration of these stress waves. However, in the relatively small amplitude of axially propagating stress waves observed in the present analysis, it can be assumed that the transversely travelling stress waves would be of limited consequence. A more detailed consideration of the distributions of transverse stress in the impulsively loaded finite element model has been previously presented (20).

This model was developed to test the hypothesis that stress wave reflection and/or reinforcement was a major contributing factor to increased levels of cartilage stress. However, the amplitudes of all of the stress waves observed propagating and reflecting through the epiphyseal region were much smaller than the quasi-static baseline stress characteristic of the joint loading. In addition, these small stress waves were discernible only for an input displacement pulse much more abrupt than those characterizing the experimental cans. Thus, the finite element results tend to refute the hypothesis that stress wave propagation and reflection are important in the extension-splinted rabbit model. Although the finite element analysis demonstrated intuitively credible patterns of stress wave propagation and interfacial reflection, the magnitude of these effects was not nearly large enough to appreciably alter the prevailing (baseline) quasi-static stress distributions.

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REFERENCES