

Power Anomalies in Testing Mediation

David A. Kenny¹ and Charles M. Judd²

¹University of Connecticut and ²University of Colorado Boulder

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Abstract

Two rather surprising anomalies relating to statistical power occur in testing mediation. First, in a model with no direct effect for which the total effect and indirect effect are identical, the power for the test of the total effect can be dramatically smaller than the power for the test of the indirect effect. Second, when there is a direct effect of a causal variable on the outcome controlling for the mediator, the power of the test of the indirect effect is often considerably greater than the power of the test of the direct effect, even when the two are of the same magnitude. We try to explain the reasons for these anomalies and how they affect practice.

Keywords

causality, hypothesis testing, intervention, power, mediation

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Mediation is a topic of enduring interest in the social and behavior sciences. Although a great deal has been written about mediation, most of that discussion is about the estimation of the mediation model and extensions thereof. When there are discussions of statistical power, they are mostly focused on the power of the overall test of mediation (Fritz & MacKinnon, 2007; Hayes & Scharkow, 2013; MacKinnon, Lockwood, Hoffman, West, & Sheets, 2002). Our interest in this article is also on power in mediation models, but we focus more on the relative power of different tests in the models. We explore first the relative power of the total and indirect effects when the direct effect equals zero, and, second, we explore the relative power of the direct and indirect effects. For each, we found some unexpected results, or what we call *power anomalies*. After we document these two anomalies, we present a rationale for why they occur and explore their implications.

The Basic Mediation Model

We assume the typical three-variable mediation model in which X is the causal variable, M is the mediator, and Y is the outcome (see Fig. 1). The model is captured by the following linear models:

$$M = i_M + aX + U$$

$$Y = i_Y + bM + c'X + V$$

The variables U and V are residuals or disturbances, i_M and i_Y are intercepts, and a , b , and c' are effects to be estimated. (In actual applications, the models are typically more complicated, but the anomalies that are the focus of this article are most easily seen in this simple context.) The mediator, M , represents the mechanism by which the causal variable is expected to affect the outcome. Some mediators are proximal to X and are essentially measures of compliance or adherence. For instance, X might be a drug intervention, and M might be a measure of the concentration of the drug in participants' blood. In other cases, M might be relatively distal from X , such as when it represents a preliminary symptom of the disease (Y). In mediation, the effect of X on Y controlling for M , or c' , is called the *direct effect*, and the effect of X on Y due to M , or ab , is called the *indirect effect*. The total effect of X on Y is typically labeled c , and this total effect equals the sum of the indirect and direct effects: $c = ab + c'$. In general, given a constant indirect effect, ab , with a more proximal mediator, a is greater than b , and with a more distal one, a is less than b .

We are assuming that the mediation model is correctly specified. By this we mean that the estimated causal

Corresponding Author:

David A. Kenny, University of Connecticut, Department of Psychology, 406 Babbidge Rd., Unit 1020, Storrs, CT 06269-1020
E-mail: david.kenny@uconn.edu

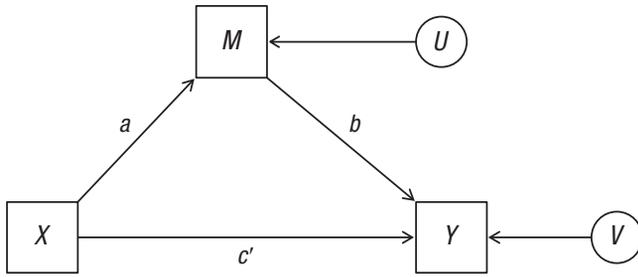


Fig. 1. Schematic showing the basic mediation structure, in which X is the causal variable, M is the mediator, and Y is the outcome variable. The effect of X on Y controlling for M , or c' , is the *direct effect*, and the effect of X on Y due to M , or ab , is the *indirect effect*. U and V are residuals or disturbances.

effects in the model are just that and that all omitted effects and covariances are zero. The violation of these assumptions can lead to seriously mistaken conclusions (Bullock, Green, & Ha, 2010; Judd & Kenny, 1981, 2010).

We also assume throughout that a and b are positive in sign, but c' may be positive or negative. For a and b to both be positive, sometimes one of the variables, usually either M or Y , needs to be rescaled. For instance, in the case of an intervention designed to reduce smoking, M might be peer pressure and Y smoking. Given the expected relationships (the intervention increases peer pressure to stop smoking), it makes sense to reverse Y so that higher values mean not smoking rather than smoking. We have also standardized X , Y , and M , and so a , b , c , and c' are standardized coefficients, although this is not essential for our conclusions.

The Relative Power of the Total and Indirect Effects Given Complete Mediation

Assume that there is complete mediation, and so c' equals zero in the population. In this case, the total effect is exactly equal to the indirect effect, or c equals ab . We assume that the direct effect, c' , is still estimated, and in the sample, its estimate would not be exactly zero. Because, in this case, the two values of c and ab in the population are the same, one might think that the power of testing that c equals zero ought to be the same as the power of testing that ab equals zero. Surprisingly, however, it turns out that the power to test c is often dramatically less than the power to test the indirect effect, ab . Sometimes 75 times as many cases are needed in order to have equal power for the test of c as for the test of ab .

In this section, we provide estimates of power for the total effect, c , and the indirect effect, ab , and compare the two. We assume that X , M , and Y are standardized continuous variables. We have standardized a and b to equal small, medium, and large effect sizes of .1, .3, and

.5, respectively, which produce values of ab and c ranging from .01 (when a and b both equal .1) to .25 (when a and b both equal .5). We then determine how many cases are needed for the test of ab and c to have 80% power. We make the usual assumption that U and V have normal distributions. For the test of ab , we used the joint-significance method, which has been shown to obtain power comparable with bootstrapping methods (Hayes & Scharkow, 2013) as well as other methods (Fritz & MacKinnon, 2007; MacKinnon et al., 2002). These other methods are preferable to the joint-significance method that we utilize because they can provide asymmetric confidence intervals, but given that our emphasis is on power and that this method has been shown to have power comparable with these other methods and is considerably more amenable to precise power calculations, we rely on the joint-significance approach. We compute the power for the test of a and the power for the test of b , and because those two statistics are independent, we simply multiply the two. All power calculations in this article can be obtained using the R package program PowMedR, which can be downloaded at <http://www.davidakenny.net/progs/PowMedR.txt>.

As Table 1 shows, the sample size required to achieve power of 80% to test c when standardized a and b both equal .3 is 966, but the sample size required for the test of ab is 114. Power is much higher in the test of the indirect effect than it is in the test of the total effect, even though in this case they both equal the very same value. Table 1 shows that the benefits of increased power of ab versus c are much greater when both a and b are small. The relative advantage is about 75 to 1 when both are .1, but “only” about 3 to 1 when both are .5. These findings parallel simulation results found in Rucker, Preacher, Tormala, and Petty (2011), who found that very often the direct effect can be statistically significant even though the total effect is not. This power advantage was perhaps first noted by Cox (1960).

Table 1. Sample Size (N) Required to Achieve 80% Power for the Test of the Null Hypothesis That c and ab Each Equal Zero When the Direct Effect (c') Is Zero (i.e., $c = ab$)

Effect				N	
a	b	ab	c	c	ab
.1	.1	.01	.01	78,485	1,030
.1	.3	.03	.03	8,718	781
.1	.5	.05	.05	3,136	781
.3	.1	.03	.03	8,718	859
.3	.3	.09	.09	966	114
.3	.5	.15	.15	345	84
.5	.1	.05	.05	3,136	1,041
.5	.3	.15	.15	345	111
.5	.5	.25	.25	122	41

The results in Table 1 also suggest a slight tendency for the power advantage for the test of ab over c to be larger when the mediator is distal (i.e., $b > a$) rather than proximal (i.e., $a > b$). As noted by Hoyle and Kenny (1999), the power in the test of the indirect effect with more proximal mediators is less than its power with more distal mediators because high colinearity of X and M increases the standard error of path b . To reinforce this point, in Table 2, we present the required sample sizes for the test of ab and of c when c' equals zero in cases in which the indirect effect ab (and c) is set at .0225 but the relative sizes of a and b , respectively, vary. The values in this table suggest two conclusions. First, the power advantage of testing ab over c declines as the mediator becomes either more proximal or more distal. Thus, the power advantage is greater when a and b are relatively equal.¹ Second, if a and b are unequal, the power advantage of ab over c is larger with a more distal mediator (i.e., when b is larger than a). It should be noted that when a is very large (about .8 or higher), the excessive colinearity between X and M can result in the power of c being greater than the power of ab (Cox, 1960). However, for reasonable values of a (less than about .75), the power of ab is always greater than c .

Note that the power advantage of ab over c is so great that even if c' is greater than zero, there may still be more power in the test of ab over c even though c is now greater than ab (O'Rourke & MacKinnon, 2013). For instance, if a equals .3, b equals .3, c' equals .06, and N is 200, the power of the test of ab (which equals .09) is .981 and of c (which equals $.15 = .09 + .06$) is .565. Of course, if there is inconsistent mediation, ab and c' having different signs, there would almost certainly be even less power in the test of c than for ab .

The finding that c has relatively low power, relative to the test of the indirect effect was perhaps first noted by MacKinnon et al. (2002). They found that using all four of Baron and Kenny's (1986) steps had low power, largely

because Step 1, the test of c , has low power, especially when c' equals zero. Shrout and Bolger (2002) explicitly acknowledged the low power in the test of the total effect. Also Rucker et al. (2011; see especially Table 1) conducted a simulation that showed similar results.

In the presence of complete mediation, an implication of what we found is that one might not uncover a statistically significant total effect but might still have sufficient power to detect a significant indirect effect. In other words, one might find significant mediation even when there is no overall effect to be mediated. What seems like a contradiction is not really one, a point echoed by several other researchers (Hayes, 2009; Shrout & Bolger, 2002; Rucker et al., 2011).

The Relative Power of the Indirect and Direct Effects

The anomaly that we just discussed concerns the relative power of the test of the indirect effect versus the total effect when the two are equal in the population, that is, there is no direct effect or c' . In this section, we consider a different power anomaly that arises concerning the relative power of the indirect effect or ab and the direct effect, or c' . This is an important issue because these two values are used to establish that M fully mediates the X -to- Y relationship. Full or complete mediation is typically claimed when ab is significant but c' is not. We shall see that this strategy for claiming complete mediation is often inadvisable, a point raised previously by several investigators (Hayes, 2009; Preacher & Kelley, 2011; Rucker et al., 2011).

For Table 3, we fixed the values of a and b and hence ab , and then determined the value of c' (and N) such that the tests of both ab and c' would have equal power of .80. Because ab is fixed, the question is the size of c' relative to ab . Table 3 dramatically shows that there is a relatively large power differential in that there is more power in the test of ab than the test of c' . Unless one has distal mediation ($b > a$) and relatively large effect sizes of .5 or more, there is much more power in the test of ab than in the test of c' .

In Table 4, we set ab equal to c' and then derived the sample size (N) necessary for the power of the test of ab to be equal to .80. Then we give the power of the test of c' , given that N . Also in Table 4, we compute an estimate of the probability that ab is statistically significant and c' not significant by computing the product of the probability that ab is significant ($\sim .80$) times the probability that c' is not significant. This probability is likely an underestimate because it is based on the erroneous assumption that the two events, ab being significant and c' not being significant, are independent. Most likely the two probabilities are positively correlated because estimates of b

Table 2. Sample Size (N) Required to Achieve at Least 80% Power for the Test of the Null Hypothesis That c and ab Each Equal Zero for Distal ($b > a$) and Proximal ($a > b$) Mediation When the Direct Effect (c') Is Zero

Effect			N	
a	b	ab	c	ab
.15	.15	.0225	15,500	458
.2	.1125	.0225	15,500	644
.1125	.2	.0225	15,500	618
.3	.075	.0225	15,500	1,530
.075	.3	.0225	15,500	1,392
.5	.045	.0225	15,500	5,163
.045	.5	.0225	15,500	3,873

Table 3. Values of c' Necessary for the Tests of c' and ab to Have at Least 80% Power for Given Values of a and b (and ab)

a	b	c'	ab	$100 \times ab/(ab + c')^a$
.100	.100	.087	.010	10.3%
.071	.141	.070	.010	12.5%
.141	.071	.070	.010	12.5%
.300	.300	.259	.090	25.8%
.212	.424	.192	.090	31.9%
.424	.212	.212	.090	29.8%
.500	.500	.397	.250	38.6%
.354	.707	.233	.250	51.8%
.707	.354	.352	.250	41.5%

^aIf c' and ab have equal power, then this value should equal 50%. Values less than 50% indicate that the test of ab is more powerful than the test of c' .

and c' are negatively correlated, given the assumptions that we have made. As Table 4 shows, unless there is distal mediation with large values of a and b , ab is often significant but c' is not, as often as 75% of the time, despite the fact that ab and c' are equal. However, except for the case of potent distal mediators, the chances of finding a significant direct effect and a nonsignificant indirect effect are slight.

At the start of this section, we said that the power difference in the test of ab and the test of c' makes it difficult to claim complete mediation. This is because we may find a significant indirect effect and a nonsignificant direct effect simply because the test of the former is generally more powerful than the test of the latter. Accordingly, we suggest that claims of complete mediation mandate more than simply a nonsignificant direct effect.

Why?

A common factor in both anomalies is that the test of the indirect effect or ab has more power than one might think. That is, the test of ab very often has more power than tests of both c and c' . Part of the reason why this may seem anomalous derives from the mistaken presumption that ab is a regression coefficient. It is not. It is the product of two coefficients, and it does not behave as a regression coefficient. One aspect of that difference is in power.

Logically, we would think that tests of ab and c (when c' is zero) should have equal power because they test the same hypothesis. However, statistical results are not always "logical." Consider some simple examples when comparing the means of three groups, 1, 2, and 3. Imagine that the means are ordered, such that the mean of Group 1 is less than the mean of Group 2, which is less than the mean of Group 3. We all know that is possible for there to be no significant differences between the means of Groups 1 and 2 and also between the means of Groups 2 and 3, and yet it may well be that the means of Groups 1 and 3 differ significantly. Although this can happen statistically, using the canons of logic, something seems out of kilter here. It can also happen, of course, depending on sample sizes, that the mean of Group 2 differs significantly from both of the other two means, but the means of Groups 1 and 3, which are actually further apart, are not significantly different. Again, the point is that statistical results do not always follow the canons of logic.

Our explanation hinges on the fact that the indirect effect is not just one effect but rather is the product of two effects. Consider the case when a and b both equal .3. Assuming as before that c' is zero, both the total effect and the indirect effect equal .09. When one tests the null hypothesis in the case of testing the total effect, power is

Table 4. Power of the Test of c' When ab Equals c' and the Test has 80% Power, With Corresponding Probabilities That Either the Direct or Indirect Effect Is Significant and the Other Is Not

Parameter				Power		Probability	
a	b	c'	ab	c'	ab	ab significant and c' not ^a	c' significant and ab not ^a
.100	.100	.010	.010	.06	.80	75.1%	1.2%
.071	.141	.010	.010	.07	.80	74.6%	1.4%
.141	.071	.010	.010	.07	.80	74.5%	1.4%
.300	.300	.090	.090	.16	.80	67.6%	3.1%
.212	.424	.090	.090	.25	.80	60.4%	4.9%
.424	.212	.090	.090	.22	.80	62.3%	4.4%
.500	.500	.250	.250	.36	.80	51.6%	6.9%
.354	.707	.250	.250	.87	.80	10.3%	16.9%
.707	.354	.250	.250	.51	.80	39.4%	10.2%

^aThe values in these columns are likely underestimates of the probability, as explained in the text.

determined in part by the fact that .09 is relatively close to zero. In contrast, when one tests the null hypothesis that ab equals 0, one is in essence simultaneously testing two effects, with a and b both equal to .3. Clearly, .3 is a lot further away from zero than is .09. Thus, it is easier to obtain a statistically significant indirect than total effect in this case. This explanation is bolstered by the earlier noted result that the power benefit is much greater when a and b are both small than when they are both large.

A simple metaphor might help at this point. It might be very hard to throw a ball 70 m in one throw. However, it is a lot easier to throw the ball 70 m in two throws. The single test of c is like the single throw, whereas the test of ab is like the double throw. This metaphor even helps one understand that the power gain is greatest when a and b are nearly equal and there is less of an advantage when they are relatively different. That is, one might be able to make two throws of 35 m each, but not one throw of 10 m and another of 60 m.

Implications

We believe that our two results have several important implications for research practice. First, consider the finding that the power of the test of the total effect is much less than the power of the test of the indirect effect. As a result of this fact, researchers may often fail to find evidence that X causes Y , but they will find evidence that M mediates the X -to- Y relationship.

There would seem to be a rather obvious implication: If it is plausible that M mediates the X -to- Y relationship, then one might test the total effect by using the indirect effect (Cox, 1960; O'Rourke & MacKinnon, 2013). Although such advice would seem reasonable, we have some concerns about its wisdom. First, how would one know for certain that c' is zero? We have already shown that, generally, the test of c' has low power relative to the test of ab , and so statistical tests of that assumption would seem inadvisable. Thus, if such a claim is made, it should be based on understanding that M is *the* mechanism through which X affects Y . Second and even more important, to use the indirect effect as a substitute for the total effect makes assumptions that may be invalid and untestable. If X is manipulated, then the total effect is a validly estimated causal effect. However, the indirect effect requires that many additional assumptions (e.g., uncorrelated U and V in Figure 1) must be made. Thus, we urge caution in substituting the test of the indirect effect for the test of the total effect.

That said, we can think of two situations in which there may be some benefit in testing ab over c . First, consider the X variable as a randomized intervention. In this research, a central question is whether the intervention is efficacious. It may well be that the total effect is greater

than zero, but its effect size is very small. Note that if the outcome is survival or cost, these seemingly "small" effects might still be very important. If one has a good understanding of the mechanism (i.e., one knows the mediator, and causal mediation assumptions seem plausible), one should expect to have much greater power at measuring the indirect effect than the total effect.

A second idea might be cases in which the measurement of Y is costly. For instance, Y might involve some expensive and invasive laboratory analysis. One might then measure Y on only a smaller subset of the entire sample. The total effect of X on Y would not have much power for the reasons extensively presented in this article and because Y was measured on only a subset of the sample. However, the indirect effect should have relatively high power, because the entire sample can be used to measure a , and because the effect size for b would be larger than c . Note that this strategy would be especially beneficial if the mediator is distal (i.e., closer to Y than to X) because that would increase the power of b , which would likely have much less power than a .

Eaton, Kalichman, Kenny, and Harel (2013) adopted a version of this strategy. They examined a randomized intervention called Project EXPLORE, designed to reduce the likelihood of HIV infection in gay men over a period of 4 years. The intervention was a large-scale behavioral intervention of 10 counseling sessions addressing factors associated with risks of unprotected anal sex. Effects were likely to be very small. Eaton et al. argued that complete mediation of the effect of intervention by the mediator, not having unprotected anal sex, was plausible. In their longitudinal logistic model, they were able to show an intervention effect that was missed in the original investigation.

The last implication discussed for the first anomaly is that one can extend the logic to multiple mediators. That is, if there were a causal chain $X \rightarrow M_1 \rightarrow M_2 \rightarrow Y$, there might be a power advantage of the indirect effect with both mediators over the indirect effect with a single mediator, either M_1 or M_2 . This conjecture deserves further investigation.

Our second anomaly suggests that researchers need to be very cautious about claims of complete mediation, especially when mediators are not distal. We think that it is especially problematic to trim nonsignificant values of c' and report them as zero without at least reporting their value with a standard error or confidence interval. Because there is usually much more power in the test of ab than of c' , it becomes important to be very careful about claims of complete mediation. If such claims are to be made, it is not just sufficient to show that c' is nonsignificant. A credible power analysis needs to be performed to show that there is sufficient power to detect meaningful values of c' . We suspect that if such a procedure were

undertaken, mediation studies would need to have very large sample sizes. We should note that several investigators make the stronger recommendation that researchers should in general avoid making claims about full or partial mediation (Hayes, 2009; Preacher & Kelley, 2011; Rucker et al., 2011).

Conclusion

Our article has several key limitations. First and foremost, we presume that the mediation model is correctly specified. Power analyses for a model that is misspecified are of little or no value. Second, the power calculations still require the usual ordinary least squares assumption of linearity, normality, homogeneity of variance, and no clustering. Third, in testing the indirect effect, we used the joint-significance method and did not perform bootstrapping. We relied on this method because it has been shown to have approximately the same power as approaches that are more distributionally appropriate but computationally difficult. However, because the test of the indirect effect is slightly more powerful when done with bootstrapping, most of our conclusions would only become stronger.

We hope that our elaboration of these two power facts can be useful to mediation researchers. We have explored some of their implications, but we expect and hope that other investigators will explore further implications.

Author Contributions

Both D. A. Kenny and C. M. Judd wrote the manuscript, and D. A. Kenny wrote the power program.

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Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

Note

1. As Hoyle and Kenny (1999) showed, power for the test of a fixed value of ab is actually maximal when a is slightly smaller than b .

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