Adaptive wavelet filtering for bearing monitoring based on block bootstrapping and white noise test

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Abstract
This study describes a novel scheme of adaptive wavelet filtering for bearing monitoring based on block bootstrapping and white noise test. The scheme consists of three main steps. First, the vibration signal is decomposed into wavelet domain, and the correlations between the wavelet coefficients are measured by lag autocorrelations. Second, according to the intensity of correlation at each level, either the block bootstrapping or general bootstrapping procedure is adopted to produce new pseudo-samples from the original wavelet coefficient series. Finally, as actual signal and noise have different translating characters along the levels in wavelet domain, the optimal decomposition level is achieved through whitening test on the wavelet coefficients, and the accuracy of the test is also obtained by the pseudo-samples. The simulation and experimental results show that the proposed procedure can be used to adaptively determine the optimal decomposition level and obtain superior filtering capability.

Keywords
Wavelet filtering, optimal decomposition level, bootstrapping, block bootstrapping, whitening test

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Introduction
Bearing monitoring is a typical problem in condition monitoring of rotating machinery, as the signals collected from the equipments are always contaminated by noise. Signal filtering is an essential step for improving the quality of the signal before any further treatment can be made. Due to its good localization and multi-resolution features in the time–frequency domain, the wavelet transform has been widely used. The selection of wavelet decomposition level has a direct impact on the filtering effect. Various signals with different signal-to-noise ratios (SNRs) have corresponding suitable decomposition levels to obtain satisfying filtering results. If the decomposition level is too low, the SNR gain is very limited; if the level selected is too high, the computational burden is increased and the quality of the enhanced signal is low because the useful data have been greatly reduced.

Several schemes have been reported for this issue. Xu et al.² concluded that the wavelet part of the last level contained only the fundamental frequency signal, and accordingly the scale part almost had no fundamental frequency component. Therefore, they evaluated the optimal level according to the pattern of the minimal energy in the scale part. Some scholars determined the decomposition level based on the change of wavelet coefficient entropy along decomposition levels.³–⁵ In the absence of the consideration of the character of the wavelet coefficients, the physical interpretation of the obtained decomposition level was vague. Kaewpijit et al.⁶ utilized the correlation between the signal reconstructed from wavelet coefficients and the original signal to calculate the optimal decomposition level. However, the main obstacle was that the threshold of the correlation could not be easily resolved and it involved complicated calculation. Lau and Ngan⁷ obtained the optimal decomposition level in

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bearing fault diagnosis based on each signal source frequency, but in most situations, it was difficult to directly get these frequencies. The cross-validation method was used to select the optimal decomposition level in some literatures, which did not integrate the priori knowledge about useful signal and white noise. In addition, the correlation between wavelet coefficients was rarely considered. Zhang et al. and Yuan et al. took account of the different features of useful signal and white noise along wavelet decomposition levels, and they selected the optimal decomposition level based on whitening test. However, the condition of the test and the correlation between coefficients were not considered.

To improve the test’s robustness, assessing the accuracy of the test is another challenging problem. In most real cases, a large amount of data are not readily collected because of the measurement cost, time constraint, or other limitations. Thus, it is desirable to exploit the bootstrap method to improve the testing performance. Bootstrap is essentially a computational tool for statistical inference, which enhances the tests’ performance by creating new samples from given samples by some random resampling procedures. Härdle et al. verified that the bootstrapping method was more accurate than those based on the first-order asymptotic distribution theory. The bootstrapping method assumes that the variables are i.i.d. random variables. Though the wavelet transform has a decorrelation performance, the correlation may be shown between the coefficients and sometimes it is remarkable. In order to resolve this obstacle, the block bootstrapping approach is utilized in this study, and the optimal block length is obtained according to the mean square error (MSE) criterion.

A hybrid scheme of adaptive wavelet filtering for bearing monitoring is proposed in this article, which integrates the two powerful tools of white noise test and bootstrapping for the decision of the wavelet decomposition level under heavy noisy conditions. The white noise test is used to check the wavelet coefficients and decide the decomposition level primitively, and then the bootstrapping process is employed to assess the accuracy of the test and finally determine the decomposition level, which improves the robustness of the test.

The article is organized as follows. First, ‘The decorrelation of wavelet transform and bootstrap procedure’ is briefly introduced. Next, ‘The adaptive wavelet denoising method with optimal decomposition level’ is described. The bootstrapping approach in wavelet domain and the optimal block length of wavelet coefficients vector are also explained. ‘The proposed denoising scheme for bearing monitoring’ section presents the computational efficiency results of our optimal decomposition level method. This is realized through both simulation signal analysis and experimental results applied to a bearing. Further discussions are presented in this section, including other factors relating to the selection of the optimal decomposition level. Finally, our concluding remarks for this study are provided.

The decorrelation of wavelet transform and bootstrap

The decorrelation of wavelet transform

The primary purpose of the wavelet transform is to transform a signal from time domain to time–frequency domain. The actual signal and the noise have different characters in wavelet domain. Mallat and Hwang described the modular maximum trend of coefficients of the signal and the noise along different levels. In addition, Mallat and Zhong pointed out that the trend of the modular maximum of the wavelet coefficients is related with the Lipschitz index. After the signal is decomposed by wavelet functions, the wavelet coefficients of the white noise component are still white noise, while the power of the useful signal is pressed to a relatively few coefficients with large value, and the amplitude becomes larger along the levels. At the small level, the primary component is noise, and the coefficients appear as a white noise characteristic. With increasing decomposition levels, the useful component is gradually dominant and the wavelet coefficients appear as a non-white noise characteristic.

The discrete wavelet transform (DWT) can remarkably reduce the correlations between the wavelet coefficients, even though the signal has significant correlation. However, between the coefficients in each level, some correlation emerges and this correlation has not an obvious relation with the level size. In some levels, the correlation seems strong and in other levels it is weak, and can even be ignored. The ‘Practical application’ of section 4.2 is a good example for this case. So, the independent or entirely dependent hypothesis of the wavelet coefficients of each decomposition level is not true. In such circumstances, some formula used in the independent condition cannot be directly applied and some pretreatment technologies are necessary. In this article, the block bootstrapping procedure is employed.

Bootstrap

As is well known, the bootstrapping scheme pioneered in the seminal paper of Efron and Tibshirani was geared toward independent data. It resolves the problem such as the confidence interval for small sample
size, which is difficult to deal with, for the standard approach. In practice, the bootstrap method always has the limitation of the \textit{i.i.d.} assumption. This problem is originally noticed by Singh.\textsuperscript{20} Because the bootstrap method for the \textit{i.i.d.} data disturbs the sequence of the sample, any dependence between observations will be absent in the generated pseudo-samples. Thus, 10 years after Efron and Tibshirani’s pioneering finding, Künsch’s\textsuperscript{14} breakthrough indicated the starting point for the focus of research activity on this new area of block resampling and the block bootstrap method has become a common and important approach for weak-dependent samples. The block resampling scheme is based on the assumption that the blocks are \textit{i.i.d.}, which signifies that the segments are nearly independent. The segments chosen for bootstrapping can be either non-overlapping or overlapping. The non-overlapping block bootstrapping procedure is adopted in this article and Figure 1 is the schematic diagram.

**Adaptive wavelet denoising method with optimal decomposition level**

**DWT-based block bootstrapping procedure**

The idea behind DWT-based bootstrapping is based on the fact that the DWT acts as a decorrelating transform for a time series. Specifically, whereas the time series itself can exhibit a high degree of autocorrelation, the autocorrelation of its DWT coefficients can be remarkably reduced. Considering the statistic character of these wavelet coefficients, in the cases in which we know little about it and the number of samples is very limited, we offer the white noise test with DWT-based block bootstrapping.

As shown in Figure 2, the DWT-based block bootstrapping steps for a time series are mentioned as follows

Step 1: Given a time series \( x = \{x_1, x_2, \ldots, x_n\} \) of length \( 2^{J'} \), decompose it into the wavelet domain using one-step DWT to obtain the wavelet coefficient vector \( w = \{w_1, w_2, \ldots, w_{n/2}\} \) and the scaling coefficient vector \( v = \{v_1, v_2, \ldots, v_{n/2}\} \).

Step 2: Calculate the optimal block length \( \hat{b}_{\text{opt, BB}} \) of the vector \( w \) using the method mentioned in the next section. If \( \hat{b}_{\text{opt, BB}} = 1 \), turn to step 3, otherwise, if \( \hat{b}_{\text{opt, BB}} > 1 \), segment the wavelet coefficient vector \( w \) with the block length \( \hat{b}_{\text{opt, BB}} \) and turn to step 4.

Step 3: Utilizing the bootstrap procedure, do random sampling with replacement from the wavelet coefficient vector to create \( B \) numbers of wave-strapped subvectors \( w^{(b)} \) with dimensions similar to \( w \), where \( b = 1, 2, \ldots, B \). Turn to step 5.

Step 4: Utilizing the block bootstrapping procedure, do random sampling with replacement from the segmented wavelet coefficient vector to create \( B \) numbers of wave-strapped subvectors \( w^{(b)} \) with dimensions similar to \( w \), where \( b = 1, 2, \ldots, B \).

Step 5: Applying the white noise test to each \( w^{(b)} \), calculate the confidence interval of the statistic value. Then, compare the confidence interval with the

![Figure 1. The schematic diagram of wavelet-based block bootstrapping.](pic.sagepub.com)
white noise criterion and check whether the series is a white noise sequence. In this article, the white noise test is based on chi-squared test. A complete description about the white noise test is available in Percival et al.\textsuperscript{21}

### The optimal block length of wavelet coefficients vector

The blocks resampling technology is based on the assumption that the blocks are \textit{i.i.d.} and the selection of the optimal block length is a crucial issue. Politis and White\textsuperscript{22} employed spectral estimation via flat-top lag windows to derive plug-in estimates of the optimal block length which minimized the \textit{MSE} of block bootstrapping variance estimates. Then, Theunissen et al.\textsuperscript{23} verified that the block bootstrap results tally with the observation very well by experiment. Several years later, Patton et al.\textsuperscript{24} modified the formula cited by Politis and White\textsuperscript{22} and got a more accurate result. The optimal block length is obtained in this study using the formula mentioned by Politis and White\textsuperscript{22} and Patton et al.\textsuperscript{24}

Suppose the sequence \( W = \{W_1, W_2, \ldots, W_N\} \) is an observation from a real-valued wavelet coefficients sequence \( \{W_n, n \in \mathbb{Z}\} \) having mean \( \mu \), and autocovariance sequence \( R(q) \), where \( \mu \) and \( R(q) \) are unknown, the sample mean \( \bar{W}_N \) is asymptotically normal; so, the variance can be expressed as

\[
\sigma_N^2 = \text{var}(\sqrt{N\bar{W}_N}) = R(0) + 2\sum_{q=1}^{N-1}(1-q/N)R(q).
\]

Denote the variance of the sample produced by the block bootstrapping procedure as \( \sigma_{BB}^2 \). Here, \( b \) is the block length. Define the spectral density function as

\[
g(w) = \sum_{q=-\infty}^{\infty} R(q) \cos(wq).
\]

Let \( D_{BB} = 2g^2(0), G = \sum_{k=-\infty}^{\infty} |k| R(k) \) and minimize the \textit{MSE} of \( \sigma_{BB}^2 \), the optimal block length can be represented with

\[
b_{opt, BB} = \left( \frac{2G^2}{D_{BB}} \right)^{1/3} N^{1/3} \tag{1}
\]

Combining with the technology of ‘flat-top’ lag-window, the \( G, g(0) \) and \( D_{BB} \) can be accurately estimated, which are denoted as \( \hat{G}, \hat{g}(0) \), and \( \hat{D}_{BB} \), respectively. Then, the optimal block length of the block bootstrapping procedure is obtained by

\[
b_{\hat{opt}, BB} = \left( \frac{2\hat{G}^2}{\hat{D}_{BB}} \right)^{1/3} N^{1/3} \tag{2}
\]

### Adaptive wavelet denoising procedure

From the depiction of the process of wavelet decomposition and white noise test, we can now outline the steps of adaptive wavelet denoising with the optimal decomposition level

**Step 1:** Decompose the time series data into wavelet domain using one-step DWT. If the number of the wavelet coefficients is smaller than 30, stop decomposing and turn to step 4.

**Step 2:** Retain the scale coefficients, and take white noise test procedure to the wavelet coefficients with the block bootstrapping procedure. If the data are a white noise series, turn to step 3; otherwise, turn to step 4.

**Step 3:** Calculate the threshold of this level, and take the scale coefficients as a time series and turn to step 1. The threshold \( T_j \) of the \( j \)-th level is estimated by the formula given by Donoho\textsuperscript{1}

\[
\begin{align*}
T_j &= \sigma_j \sqrt{2 \log N} \\
\sigma_j &= \text{median}(|W|)/0.6745
\end{align*}
\tag{3}
\]
where $N$ is the number of the wavelet coefficients $W$ at the $j$th level.

Step 4: Give up the last step decomposition. Suppose the total decomposition times are $n$; then, the optimal decomposition level is $n$. Implement the wavelet denoising method with soft threshold to the wavelet coefficients.

Step 5: Reconstruct the signal with the scaling coefficients of level $n$ and the treated wavelet coefficients from level 1 to level $n-1$. The obtained signal is an estimation of the actual signal.

Figure 3 shows the flow chart of the procedure mentioned above.

The proposed denoising scheme for bearing monitoring

Numerical simulation

The DWT-based wavelet denoising with block bootstrap method is first tested on the standard simulated signals of ‘heavy sine’ and ‘blocks’ in Matlab toolbox. The contaminative signals are produced by adding white noise to original signals. The parameters are configured as follows: the $SNR/MSE$ values are 9.62/1.06, 27.31/1.06, 12.81/1.06, and 30.50/1.06, respectively; the significant level of the hypothesis test is 0.05; the boundary extended mode is first-order smoothness mode; the mother wavelet function is the Daubechie’s of five coefficients (db5); the number of sampling point is $2^{12}$; and the bootstrapping times are equal to 99. The aforementioned threshold selection criteria and the soft-threshold function are adopted and the optimal decomposition levels are obtained as 5, 4, 5, 2, respectively. Taking the $SNR$ and $MSE$ as the measurement criteria, Table 1 lists the denoised results of the above four signals with the obtained optimal decomposition levels.

The $SNR$ and the $MSE$ are formulated by Ribeiro et al. $^{25}$

\[
SNR = 10 \log_{10} \left[ \frac{\sum_{i=1}^{N} (s(i))^2}{\sum_{i=1}^{N} (s(i) - \hat{s}(i))^2} \right]^{2},
\]

\[
MSE = \frac{1}{N} \sum_{i=1}^{N} (s(i) - \hat{s}(i))^2
\]

(4)

As shown in Table 1, for the ‘heavy sine’ signal with $SNR/MSE = 9.62/1.06$, according to the autocorrelation of wavelet coefficients from level 1 to level 4, the optimal block length is 1 and the quantile corresponding to the autocorrelation lag and the significant level is 31.41. Using the bootstrap method, the 95% confidence intervals of chi-squared test are [18.91, 21.47], [18.17, 20.38], [18.71, 21.15], and [16.57, 19.04], respectively. So, the first four levels wavelet coefficients pass the white noise test, but for the wavelet coefficients at the fifth level, the optimal block length in block bootstrapping process is 35, and the quantile corresponding to the autocorrelation order (lag 13) calculated by the number of the wavelet coefficients at this level and the significant level is 22.36; the confidence interval is [52.01, 56.97]; so, the coefficients cannot pass the white noise test and the decomposition level is selected as 5. We can see that the $SNR/MSE$ of the signal is increased to 23.10/0.05 and continued decomposing leads to little improvement of the numerical results. When the decomposition proceeds to the seventh level, the $SNR/MSE$ even appears with a tendency of decline. Considering the denoising effect and the calculation costs, it is appropriate to select 5 as the decomposition level. Similarly, for the ‘heavy sine’ signal with $SNR/MSE = 27.31/1.06$, when the decomposition level is 3, the optimal block length is 4, and the coefficients...
Table 1. Result comparison between the optimal decomposition level and vicinal levels for the simulated signals, the boldface values corresponding to the optimal decomposition level (the whitening test significant level is 0.05; the wavelet function is db5; the number of sampling point is $2^{12}$; and the bootstrapping times are equal to 99).

<table>
<thead>
<tr>
<th>Contaminative signal (SNR/MSE)</th>
<th>Optimal block length</th>
<th>First level</th>
<th>Second level</th>
<th>Third level</th>
<th>Fourth level</th>
<th>Fifth level</th>
<th>Sixth level</th>
<th>Seventh level</th>
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<tbody>
<tr>
<td>Heavy sine (9.62/1.06)</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>10</td>
<td>7</td>
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<tr>
<td>Autocorrelation lag</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>13</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
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<td>31.41</td>
<td>31.41</td>
<td>31.41</td>
<td>31.41</td>
<td>31.41</td>
<td>22.36</td>
<td>14.07</td>
<td>9.49</td>
</tr>
<tr>
<td>Confidence interval</td>
<td>[18.91,21.47]</td>
<td>[18.17,20.38]</td>
<td>[18.71,21.15]</td>
<td>[16.57,19.04]</td>
<td>[52.01,56.97]</td>
<td>[27.93,35.08]</td>
<td>[16.28,19.70]</td>
<td></td>
</tr>
<tr>
<td>SNR/MSE</td>
<td>12.53/0.54</td>
<td>15.44/0.28</td>
<td>18.26/0.15</td>
<td>21.39/0.07</td>
<td>23.10/0.05</td>
<td>23.80/0.04</td>
<td>22.53/0.05</td>
<td></td>
</tr>
<tr>
<td>Heavy sine (27.31/1.06)</td>
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<td>1</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>24</td>
<td>8</td>
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<tr>
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<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>13</td>
<td>7</td>
<td>4</td>
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<td>31.41</td>
<td>22.36</td>
<td>14.07</td>
<td>9.49</td>
</tr>
<tr>
<td>Confidence interval</td>
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<td>[18.12,20.82]</td>
<td>[22.54,27.07]</td>
<td>[3.11,37.21]</td>
<td>[21.54,27.83]</td>
<td>[20.06,24.29]</td>
<td>[13.91,17.15]</td>
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<td>35.26/0.17</td>
<td>37.54/0.10</td>
<td>38.09/0.09</td>
<td>38.02/0.09</td>
<td>37.12/0.11</td>
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<td>2</td>
<td>3</td>
<td>15</td>
<td>11</td>
<td>7</td>
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<td>20</td>
<td>20</td>
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<td>22.36</td>
<td>14.07</td>
<td>9.49</td>
</tr>
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<td>[19.14,21.65]</td>
<td>[19.09,21.83]</td>
<td>[20.09,23.37]</td>
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<tr>
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<td>20.01/0.20</td>
<td>20.92/0.16</td>
<td>20.50/0.18</td>
<td>19.89/0.21</td>
<td>19.27/0.23</td>
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</tr>
<tr>
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<td>4</td>
<td>8</td>
<td>7</td>
<td>28</td>
<td>20</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>Autocorrelation lag</td>
<td>20</td>
<td>20</td>
<td>20</td>
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<td>20</td>
<td>13</td>
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<td>22.36</td>
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</tr>
<tr>
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<td>[42.29,50.32]</td>
<td>[8.98,11.15]</td>
<td>[9.66,12.29]</td>
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<tr>
<td>SNR/MSE</td>
<td>32.93/0.61</td>
<td>34.51/0.42</td>
<td>35.13/0.37</td>
<td>35.19/0.36</td>
<td>34.64/0.41</td>
<td>34.18/0.45</td>
<td>33.78/0.50</td>
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</table>
show weak correlation with the corresponding quantile 31.41. The confidence interval is [22.54, 27.07]; so, the coefficients at this level pass the white noise test. When the decomposition goes to the fourth level, the correlation between the coefficients is stronger and the corresponding quantile is 31.41. The confidence interval is [31.13, 37.21]; so, the coefficients at this level cannot pass the test and the optimal decomposition level should be selected as 4. Accordingly, the \( SNR/MSE \) is raised to 37.54/0.10. Figure 4 shows the comparison of the original, noised \((SNR = 9.62\) and 27.31\), and denoised signals with the optimal decomposition level. By a similar process, we can analyze the ‘blocks’ signals.

**Figure 4.** The comparison of the original ‘heavy sine,’ noised, denoised signals with the optimal decomposition level: (a) the noised signal with \( SNR = 9.62 \) and (b) the noised signal with \( SNR = 27.31 \).

**Figure 5.** A schematic of the experimental system.

Rolling element bearings are widely used in modern rotating machinery. Faults occurring in the bearings may lead to fatal breakdowns of machines. Therefore, it is significant to be able to accurately detect and diagnose the existence of the faults occurring in the bearings. Vibration-based analysis has been the most popular approach for the detection of the localized bearing defect.

**Practical application**

Rolling element bearings are widely used in modern rotating machinery. Faults occurring in the bearings may lead to fatal breakdowns of machines. Therefore, it is significant to be able to accurately detect and diagnose the existence of the faults occurring in the bearings. Vibration-based analysis has been the most popular approach for the detection of the localized bearing defect.
The vibration data used in this study have been obtained from the data set of the rolling element bearings under different bearing conditions. As shown in Figure 5, the test bearings are installed in a motor-driven mechanical system and support the motor shaft. An accelerometer is mounted on the motor housing at the drive end of the motor to acquire the vibration signals from the bearing. The vibration data are acquired with a 12k Hz sample rate, and each data set is made up of 480,000 points. Using electrical-discharge machining method, single point faults are introduced to the surface of the test bearing components, which are inner race, outer race, and ball. The fault diameter is 0.18 mm and the fault depth 0.28 mm. The drive end bearing type is 6205-2RS JEM SKF, deep groove ball bearing. The experimental rotating frequency is about 30 Hz. The test bearing geometry is listed in Table 2. As the data are collected under an ‘ideal’ laboratory condition, to simulate a real working environment, for the outer race fault, inner race fault, ball fault, and without fault conditions, we add white noise to the original signals and the noise levels are −10, −10, −17, and −5 db, respectively.

Each bearing element has its own characteristic frequency of defect. Those frequencies can be calculated from the kinematics relation. For a bearing with a stationary outer race, the characteristic bearing defect frequencies of inner race (BFIR), outer race (BFOR), and ball (BFB) are listed in Table 3, which are calculated using the following formula

\[
\text{BFIR} = \frac{N_b}{2} \left[ 1 + \left( \frac{B_d}{P_d} \cos \alpha \right) \right] f_r
\]

(5)

\[
\text{BFOR} = \frac{N_b}{2} \left[ 1 - \left( \frac{B_d}{P_d} \cos \alpha \right) \right] f_r
\]

(6)

\[
\text{BFB} = \frac{1}{2} \left( \frac{P_d}{B_d} - \frac{B_d}{P_d} \cos^2 \alpha \right) f_r
\]

(7)

where \(B_d\) is the ball diameter; \(P_d\) the pitch diameter; \(N_b\) the number of balls; \(\alpha\) the contact angle; and \(f_r\) the rotating frequency of the spindle.

Figure 6(b) shows the 1–20th lag autocorrelation of the vibration signal of bearing without fault in Figure 6(a), which indicates that a strong correlation emerges in the original signal. Then, the original signal is decomposed by wavelet transform with five levels and the mother wavelet function is ‘sym8’. Figure 6(c) depicts the 1–20th lag autocorrelation of the coefficients at each level. The strong correlation in the original signal is obviously reduced by the wavelet transform. However, we also find that some correlation is visible in some levels; so, at these levels, the traditional statistical methods cannot be directly applied. In order to assess the accuracy of the test, an appropriate block length is required and the pseudo-samples should be generated by the block bootstrapping instead of the general bootstrapping process.

The signals of bearing with different faults are then analyzed using the proposed algorithm. The parameters are configured as follows: the significant level is 0.05; the boundary extended mode is first-order smoothness mode; the wavelet function is ‘sym8’; the number of sampling point is 2^{12}; and the bootstrapping times are equal to 99. The aforementioned threshold selection criteria and the soft-threshold function are adopted. For example, for the signal of bearing with outer race fault, the obtained optimal decomposition level by the proposed method is 5. Table 4 lists the optimal block length, autocorrelation lag, quantile corresponding to the significant level, and the chi-squared test confidence interval for the wavelet coefficients of the vibration signal after decomposition at each level. As listed in Table 4, for the signal of practical application, according to the autocorrelation of wavelet coefficients from level 1 to level 4, the optimal block lengths are 2,1,1, and 1, respectively, and corresponding to the autocorrelation lag and the significant level, the quantile is 31.41. The weak dependence of the coefficients is shown in level 1; so, the chi-squared test method cannot be directly used to this level and the block bootstrapping procedure should be deployed first. Using the bootstrap method, from level 1 to level 4, the 95% confidence intervals of chi-squared test are [20.02, 25.12], [18.72, 21.43], [17.56, 20.30], and [18.01, 20.52], respectively; so, the first four levels of wavelet coefficients pass the white noise test. However, at the fifth level, the

<table>
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<th>Table 2. Bearing geometry information.</th>
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<td>Inside diameter (mm)</td>
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<th>Table 3. Bearing character frequencies (Hz).</th>
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Figure 6. The vibration signal of bearing without fault and its autocorrelation function (ACF) in time and wavelet domains: (a) the original vibration signal ($N = 4096$) and (b) the 1–20th lag autocorrelation of the original signal, with dashed lines indicating the 95% confidence interval for zero, $0 \pm 2/\sqrt{N}$, and (c) The 1–20th lag autocorrelation of the wavelet coefficients from level 1 to level 5, labeled as ACF-1–ACF-5, with dashed lines indicating the 95% confidence interval for zero, $0 \pm 2/\sqrt{N/2}$.
optimal block length and the quantile are 6 and 23.69, respectively, whereas the confidence intervals are [27.68, 34.64]; so, the coefficients cannot pass the white noise test. Therefore, the decomposition level should be selected as 5. In order to verify the validity of our statement, the envelope spectrums for the original signal with different decomposition levels are compared in Figure 7. As the original signal contains quantity of noise (Figure 7(a)), the composition of characteristic frequency 141 Hz is not obvious in Figure 7(b). The signal is reconstructed with four levels of decomposition and the envelope spectrum is shown in Figure 7(c); because the decomposition level is insufficient, the characteristic frequency information has no obvious improvement. Figure 7(d) is the result of five levels of decomposition, as the noise is properly removed and the useful signal is perfectly retained, the characteristic frequency is very striking. Figure 7(e) maps the envelope spectrum of the original signal with six levels of decomposition, and the composition of characteristic frequency is not apparent because of the excessive decomposition.

For rest of the cases, including the bearing with inner race fault, ball fault, and without fault, the results are listed in Tables 5 to 7, respectively; the original signals and the envelope spectrums of the original signal and the reconstructed signals with optimal decomposition level are shown from Figures 8 to 10, respectively. As shown in Table 5, the optimal decomposition level of bearing signal with inner race fault is 4. Figure 8(a) is the original signal with inner race fault and Figure 8(b) shows that the fault frequency (162 Hz) is submerged in strong noise background. However, in Figure 8(c), this frequency is more outstanding than its surroundings. From Table 6, the optimal decomposition level of bearing signal with ball fault should be 5. Figure 9(a) is the original signal with ball fault and the fault frequency (107 Hz) is not distinctly reflected in Figure 9(b) before filtering, but in Figure 9(c), the characteristic frequency is clearer, and the near-maximum amplitude is the fourth harmonics of modulation frequency (30 Hz). The signal of the bearing without fault is shown in Figure 10(a) and also processed using the proposed approach. As the SNR is high, the axial frequency and its second harmonics are shown in Figure 10(b). From Table 7 and Figure 10(c), we can see that the optimal decomposition level is 3 and the axial frequency and its second harmonics are standing out more. These results pinpoint the fact that the proposed method here is effective.

### Discussion

From the results of the experiments, we conclude that an advisable decomposition level is an important factor to obtain a promising denoising effect. If the level is too small, the noise cannot be weakened enough. However, if it is too large, the useful signal can be damaged, and furthermore, SNR/MSE increases a little or descends dramatically, and the calculation burden simultaneously increases.

Theoretically, when the wavelet coefficients at the nth level cannot pass the white noise test, the decomposition should be stopped, and the optimal decomposition level should be $n - 1$. However, the experimental results indicate that selecting $n$ as the optimal decomposition level leads to a better denoising effect in most cases. For example, for the signal of bearing with outer race fault mentioned above, taking 5 rather than 4 as the decomposition level leads to a more desirable filtering result.

The results in Table 8 show the optimal decomposition levels that are obtained by the white noise test method for the simulation signals mentioned in section ‘Numerical simulation’. We can observe that for these signals, the solutions are similar to the proposed schemes. However, the proposed method provides more accurate estimates compared to the previous one. Due to the application of bootstrapping procedure, the robustness of the algorithm is substantially enhanced.

We also study some other aspects directly relating to the subject discussed here. The first one is the selection of mother wavelet, which is another important issue in wavelet transform. Some resolutions were brought forward in the recent years, which deduced the proper mother wavelet functions should be similar to the

| Table 4. Result comparison between the optimal decomposition level and vicinal levels for the signal of bearing with outer race fault. The boldface values corresponding to the optimal decomposition level (the whitening test significant level is 0.05; the wavelet function is ‘sym8’ and the number of sampling point is 212; and the bootstrapping times are 99). |
|-------------|----------------|----------------|----------------|----------------|----------------|
|             | First level    | Second level   | Third level    | Fourth level   | Fifth level    |
| Optimal block length | 2              | 1              | 1              | 1              | 6              |
| Autocorrelation lag    | 20             | 20             | 20             | 20             | 14             |
| Quantile              | 31.41          | 31.41          | 31.41          | 31.41          | 23.69          |
| Confidence interval   | [20.02,25.12]  | [18.72,21.43]  | [17.56,20.30]  | [18.01,20.52]  | [27.68,34.64]  |
Figure 7. The vibration signal of bearing with outer race fault: (a) the original vibration signal, (b) the envelope spectrum of the original signal, (c) the envelope spectrum of the denoised signal with four levels of wavelet decomposition, (d) the envelope spectrum of the denoised signal with five levels of wavelet decomposition (the optimal), and (e) the envelope spectrum of the denoised signal with six levels of wavelet decomposition.
analyzed signals, while we focus on the effect of the selection of decomposition level with different mother wavelets. A further investigation indicates that, when a different mother wavelet is utilized, the obtained optimal decomposition level may be different. Take the ‘heavy sine’ signal with $\text{SNR}/\text{MSE} = 9.62/1.06$ for an example. When we adopt the Daubechies wavelets db2, db6, and db8, respectively, which are from low order to high order, and all other conditions remain the same, the obtained optimal decomposition levels are 7, 4, and 5 accordingly, and the obtained $\text{SNRs}$ are 25.23, 20.91, and 22.62, respectively. For db2, all the wavelet coefficients can pass the white noise tests from level 1 to level 7. Since the number of the eighth level coefficients is only 18, the seventh level should be selected as the optimal decomposition level. Furthermore, when the mother wavelet bior1.3 is selected, we find that the optimal decomposition level is 4. This result can be theoretically explained, because the wavelet coefficients reflect the similarity of the signal and the mother wavelet function. If we select different mother wavelets, we will obtain the different coefficients. If the signal is quite similar to the mother wavelet function, the obtained wavelet coefficients will be large; otherwise, it will be small. The coefficients affect the following white noise tests. Second, the selection of the optimal wavelet decomposition level is also affected by the boundary extended mode. For the former case, when the symmetric-padding is adopted, all the wavelet coefficients can pass the white noise tests from level 1 to level 7. Since the number of the eighth level coefficients is only 24, the seventh level should be selected as the optimal

| Table 5. Result comparison between the optimal decomposition level and vicinal levels for the signal of bearing with inner race fault, the boldface values corresponding to the optimal decomposition level (the whitening test significant level is 0.05; the wavelet function is ‘sym8’; the number of sampling point is $2^{12}$; and the bootstrapping times are equal to 99). |
|---------------------------------|------------------|------------------|------------------|------------------|
|                                 | First level      | Second level     | Third level       | Fourth level     |
| Optimal block length            | 1                | 1                | 1                | 17               |
| Autocorrelation lag             | 20               | 20               | 20               | 20               |
| Quantile                        | 31.41            | 31.41            | 31.41            | 31.41            |
| Confidence interval             | [18.55,21.41]    | [19.02,21.86]    | [18.19,21.67]    | [62.51,81.56]    |

| Table 6. Result comparison between the optimal decomposition level and vicinal levels for the signal of bearing with ball fault, the boldface values corresponding to the optimal decomposition level (the whitening test significant level is 0.05; the wavelet function is ‘sym8’; the number of sampling point is $2^{12}$; and the bootstrapping times are equal to 99). |
|---------------------------------|------------------|------------------|------------------|------------------|
|                                 | First level      | Second level     | Third level       | Fourth level     | Fifth level      |
| Optimal block length            | 2                | 1                | 1                | 4                | 23               |
| Autocorrelation lag             | 20               | 20               | 20               | 20               | 14               |
| Quantile                        | 31.41            | 31.41            | 31.41            | 31.41            | 23.69            |
| Confidence interval             | [20.79,23.57]    | [18.82,22.25]    | [17.53,20.41]    | [24.67,28.90]    | [39.94,45.07]    |

| Table 7. Result comparison between the optimal decomposition level and vicinal levels for the signal of bearing without fault, the boldface values corresponding to the optimal decomposition level (the whitening test significant level is 0.05; the wavelet function is ‘sym8’; the number of sampling point is $2^{12}$; and the bootstrapping times are equal to 99). |
|---------------------------------|------------------|------------------|------------------|
|                                 | First level      | Second level     | Third level       |
| Optimal block length            | 2                | 1                | 12               |
| Autocorrelation lag             | 20               | 20               | 20               |
| Quantile                        | 31.41            | 31.41            | 31.41            |
| Confidence interval             | [18.66,20.94]    | [19.14,22.10]    | [219.72,255.37]  |
decomposition level. The extended mode affects not only the decomposition level in the value of the wavelet coefficients, but also the autocorrelation of the wavelet coefficients. Third, the selection of the significant level is another important factor. Take the ‘heavy sinc’ signal with $SNR/MSE = 9.62/1.06$ for an example. When we adopt the significant levels 0.01, 0.05, and 0.5 respectively, the obtained optimal decomposition levels are 7, 5, and 7 accordingly, and the obtained SNRs are 22.95, 23.10, and 23.75, respectively. With different significant levels, the filtering results are not far out of line with those obtained by 0.05, though the optimal decomposition level is different. Therefore, in general, the significant level taken as 0.05 is acceptable.

In addition, the present algorithm is derived for the signal contaminated by the white noise. Especially, if the $SNR$ is lower, the effect will be more satisfactory. Although, for the most practical contaminated signal, the noise can generally be assumed as white noise; however, if the noise is not assumed as white noise in the

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**Figure 8.** The vibration signal of bearing with inner race fault: (a) the original vibration signal, (b) the envelope spectrum of the original signal, and (c) the envelope spectrum of the denoised signal with optimal decomposition level.
special case, this algorithm may be not so appropriate and further research will be conducted to solve this problem.

**Conclusions**

A novel denoising method based on the block bootstrapping white noise tests is presented for condition monitoring of bearings. The selection of the optimal wavelet decomposition level is very challenging. The results for both simulated signal as well as actual vibration signal experiments of bearing show the effectiveness of the proposed approach to determine a reasonable decomposition level while improving the $SNR/MSE$ and enhancing the effect of the wavelet threshold denoising. The experimental results and the theoretical analysis demonstrate that the optimal decomposition level is also related to several factors such as the selection of the mother wavelet and the boundary extended mode. It should arouse extensive attention, especially for the selection of the significant level. However, nowadays, it mainly depends on people’s experiences. From this point of view, how to select the appropriate solution is worthy of further research.

**Figure 9.** The vibration signal of bearing with ball fault: (a) the original vibration signal, (b) the envelope spectrum of the original signal, and (c) the envelope spectrum of the denoised signal with optimal decomposition level.
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**Figure 10.** The vibration signal of bearing without fault: (a) the original vibration signal, (b) the envelope spectrum of the original signal, and (c) the envelope spectrum of the denoised signal with optimal decomposition level.

**Table 8.** The obtained optimal decomposition level by white noise test.

<table>
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<tr>
<th>Signal (SNR/MSE)</th>
<th>Heavy sine (9.62/1.06)</th>
<th>Heavy sine (27.31/1.06)</th>
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References