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# Teaching fundamentals of measurement-based load modelling approach through practical examples

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**Abstract** The paper presents a computer program designated for teaching load modelling fundamentals, answering an increasing need for education of electrical engineers in this area. The program is based on data obtained during numerous field tests that encompass voltage changes using on-load tap changers. In this way students apply a measurement-based load modelling approach and analyse load behaviour from many practical examples during voltage changes on different days of the week and over different time periods. The program allows future engineers to become familiar with some important fundamental issues of load modelling such as: identification of load model parameters, selection of proper load model, determination of representative load characteristics and adequacy of the usage of an approximate load model.

**Keywords** distribution system; electrical engineering education; load modelling

Load model parameters are input data for power systems analysis. Therefore the calculation results depend on the accuracy of these parameters. Furthermore, load modelling is becoming extremely important with the appearance of large numbers of new load devices, dispersed generators and a deregulated energy market. This has been recognised by researchers, who have made significant efforts to model the concerned load. For example, in Ref. 1 are presented results of about 250 experiments performed in transformer stations; some results of this comprehensive study are published in Ref. 2. On the other hand, Refs 3 and 4 present results of load modelling based on continuous measurements with recording devices placed at different load buses during several years. Some scientific papers focus on developing new procedures for identification of load model parameters.[5–7] These procedures use artificial neural networks or a combination of identification methods such as genetic algorithms and non linear optimisation methods.

Regarding the importance of load modelling, and the limited study material available for load modelling during Bachelor's studies, this paper introduces a program developed for students to learn some very important, basic issues of a measurement-based load modelling approach through practical examples. As a data base for teaching, the data obtained from field tests in transformer stations described in Ref. 1 are used. These field tests encompassed the series of transformer turn ratio changes that caused secondary voltage changes in the range from 0.95 to 1.1 p.u. During the tests recording equipment stored voltage and real and reactive power values every second. These values are used as input data for the program, which registers the start and end of every experiment, processes the data through several steps, and presents the results of all steps through corresponding figures, tables and output statements. Also,

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the students are encouraged to analyse results of load modelling issues in more detail and to vary the program criteria in order to become more familiar with the study matter.

## Most frequently used load models

Static load models describe real and reactive power dependence on voltage and frequency. Many static load models have been developed and most of them are presented in Ref. 8. Very frequently used static load models are exponential and polynomial models and their variants.[8–14] In many cases, however, frequency dependence is neglected due to the relatively narrow range of frequency variation. Therefore, the most frequently used static load models are:

the polynomial model of the form

$$P = P_n\left(p_1\left(\frac{U}{U_n}\right)^2 + p_2\frac{U}{U_n} + p_3\right), Q = Q_n\left(q_1\left(\frac{U}{U_n}\right)^2 + q_2\frac{U}{U_n} + q_3\right), \tag{1}$$

the exponential model

$$P = P_n\left(\frac{U}{U_n}\right)^{k_{pu}}, Q = Q_n\left(\frac{U}{U_n}\right)^{k_{qu}} \tag{2}$$

and the linear model

$$P = P_n\left(a_0 + a_1\frac{U}{U_n}\right), Q = Q_n\left(b_0 + b_1\frac{U}{U_n}\right). \tag{3}$$

The polynomial model (1) is also called the *ZIP* model since it consists of constant impedance (*Z*), constant current (*I*) and constant power (*P*) load components. In this model $P_n$ and $Q_n$ are real and reactive power at rated voltage $U_n$. Parameters $p_1$ and $q_1$ represent the relative participation of constant impedance load, $p_2$ and $q_2$ the relative participation of constant current load and $p_3$ and $q_3$ the relative participation of constant power load in the total load. The linear model (3) is a special case of the polynomial model, i.e. the first-order polynomial.

The exponential model (2) is even more frequently used than the polynomial model, especially in power system stability analyses. This model features parameters $k_{pu}$ and $k_{qu}$ that represent the derivatives of real and reactive power with respect to voltage in the vicinity of $U_n$ as stated in Ref. 14:

$$k_{pu} = \mathrm{d}P/\mathrm{d}U|_{U=1}, k_{qu} = \mathrm{d}Q/\mathrm{d}U|_{U=1}. \tag{4}$$

## The LMprogram

### Input data analysis

The bases of measurement-based load modelling approach are field measurements. For the LMprogram these essential input data are obtained through field measurements during a series of experiments of voltage changes. The voltage changes were performed by manual changes of turn ratio of several transformers in the distribution network of the city of Niš. On every transformer selected for the experiments a group of field tests was performed. Each group of tests implies sequences of turn ratio changes of a particular transformer in one season, but on different days of the week and periods of the day. Figure 1 depicts a simplified one-line diagram of equipment connections during one group of field tests for which results are presented in this paper. During these field tests the voltage was changed by transformer $T_1$ and a digital data acquisition device was connected to existing current (CT) and voltage transformers (VT) on the secondary side of one of two transformers 35/10 kV feeding ten feeders (F) whose load is predominantly residential (91% of total load).

The first step of the LMprogram is input data reading. Input data are nine Excel files, each containing voltage and real and reactive power values during one series of experiments. Experiments were performed in summer in three day periods – morning, afternoon and night, on three days: a working day, Saturday and Sunday.

Afterwards, voltage and real and reactive power values during every voltage change are separated. The program encompasses detection of the beginning and the end of every voltage change caused by the change of transformer turn ratio. Experiments were performed during normal operation of the power system, thus regular voltage changes that were recorded and averaged every second were up to 0.1%. Therefore, the voltage change greater than 0.25% of rated voltage between two near by averaged values is selected to be the indicator that an experiment began within particular second.

Every voltage change consisted of several successive manual changes of transformer tap ratio. In each consecutive position of on-load tap changer the voltage remained almost the same for approximately 5 s due to inertia of the load tap changer, as seen in Fig. 2 for one experiment performed on Sunday afternoon. Thus,

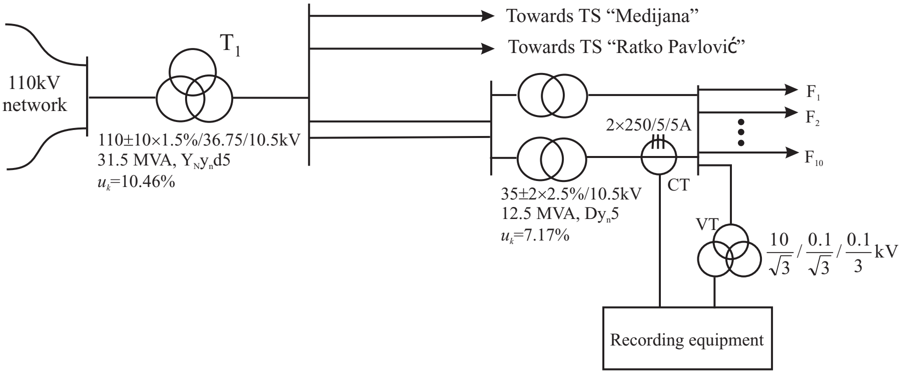

Fig. 1 *Simplified experimental scheme.*

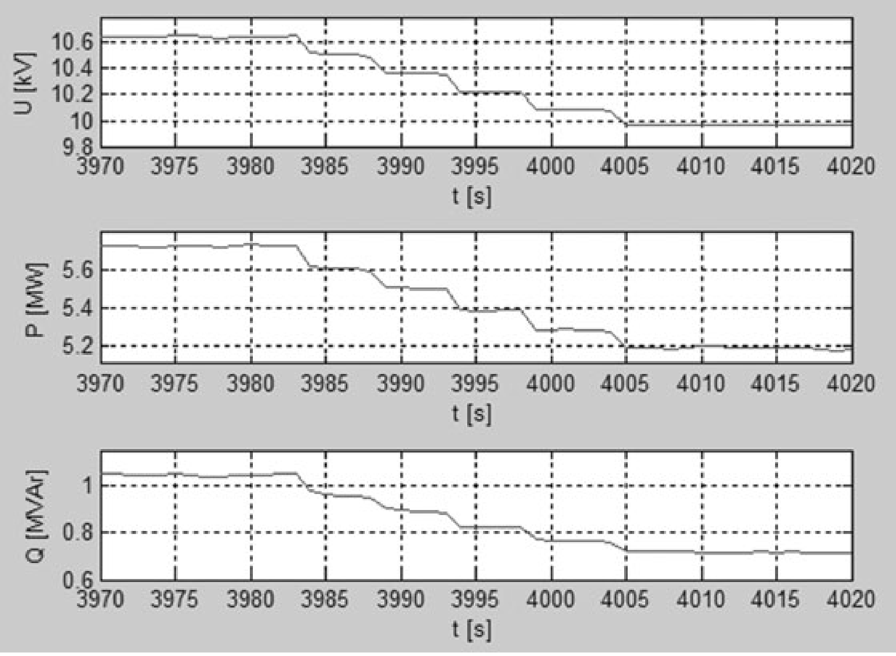

Fig. 2 *Voltage, real and reactive power during one voltage change experiment.*

the end of an experiment can be detected by the analysis of the voltage values during time periods longer than 6 s, after every single consecutive voltage change. The indication of the end of an experiment used in LMprogram is almost constant voltage without any variation greater than 0.25% during up to 15 s (due to possible delays in manual load tap changes caused by the personnel of the transformer station). Figure 2 also presents real and reactive power during the considered experiment. Both real and reactive power change immediately with voltage change and keep the new value almost constant. This is also the case in all other experiments and static load models such as (1)-(3) can be used for modelling of the short-term dynamic of the examined load.

The responses of real and reactive power to the voltage changes are modelled using Curve Fitting Toolbox – the collection of graphical user interfaces (GUIs) and M-file functions for fitting that operate in the MATLAB computing environment. The command *cftool* opens the graphical user interface called Curve Fitting Tool. It enables the user to select data for the *X* and *Y* axes and to open Fit Editor. The fit name can be entered and the type of fit like exponential, polynomial, power, rational etc., can be selected by Fit Editor. Parameters of any selected fitting function are identified by using a least squares method that minimises the function

$$J = \sum_{i=1}^{N} (P_m(t_i) - P_l(t_i))^2, \qquad (5)$$

or by using a nonlinear least squares method that implies trust-region, Levenberg-Marquardt or Gauss-Newton algorithm. In eqn (5) $P_m(t_i)$ is the measured real power value at instant $t_i$, $P_l(t_i)$ is the value of real power at the same instant according to the model and $N$ is the number of samples. A similar objective function that should be minimised relates the reactive power response to voltage change to measured values and to values according to the model, $Q_m(t_i)$ and $Q_l(t_i)$, respectively. The analysis of all input measured data from the concerned group of field tests performed according to the experimental scheme from Fig. 1 showed that a quadratic polynomial function describes power-voltage relationships very well since all fittings are obtained with pretty large correlation coefficient, greater than 0.9 for real, and greater than 0.8 for reactive power.

Afterwards, all input data are normalised. Voltage values are normalised by rated voltage, i.e. $U_n = 10$ kV. Real power values are normalised by the value of real power obtained by substitution of rated voltage value in the quadratic polynomial $P_l(t_i)$. Reactive power values are normalised by reactive power value obtained in a similar way, using the quadratic polynomial $Q_l(t_i)$ and value $U_n$.

### Selection of proper load model

After the normalisation, real and reactive power behaviour during the experiments is modelled with the most frequently used static load models: polynomial, linear and exponential in order to select a proper load model. The parameters of different models are identified using a least squares method in the case of polynomial and linear fits, and nonlinear least squares method in the case of exponential fit.

The adequacy of examined load models can be proven visually using the Curve Fitting Tool by comparison of residuals from different load models, and statistically. For statistical comparison of different load models, the squared correlation coefficient

$$R-\text{square}=1-\frac{\sum_{i=1}^{N}(p_m(t_i)-p_l(t_i))^2}{\sum_{i=1}^{N}(p_m(t_i)-\bar{p}_m)^2} \qquad (6)$$

and sum of squares due to errors, also called summed square of residuals,

$$SSE=\sum_{i=1}^{N}(p_m(t_i)-p_l(t_i))^2, \qquad (7)$$

are used. In eqns (6) and (7) $p_m(t_i)$ and $q_m(t_i)$ are normalised measured real and reactive power values, $\bar{p}_m$ denotes mean value of $p_m(t_i)$ and $p_l$ is real power according to the model. Analogous equations exist for squared correlation coefficient and sum of squares due to errors relating to reactive power.

Figure 3 presents normalised reactive power data from Fig. 2. Since these data relate to measured values from the sixth experiment of voltage changes (performed on Sunday afternoon), the data set is named qm6 vs. um6. On the same figure,

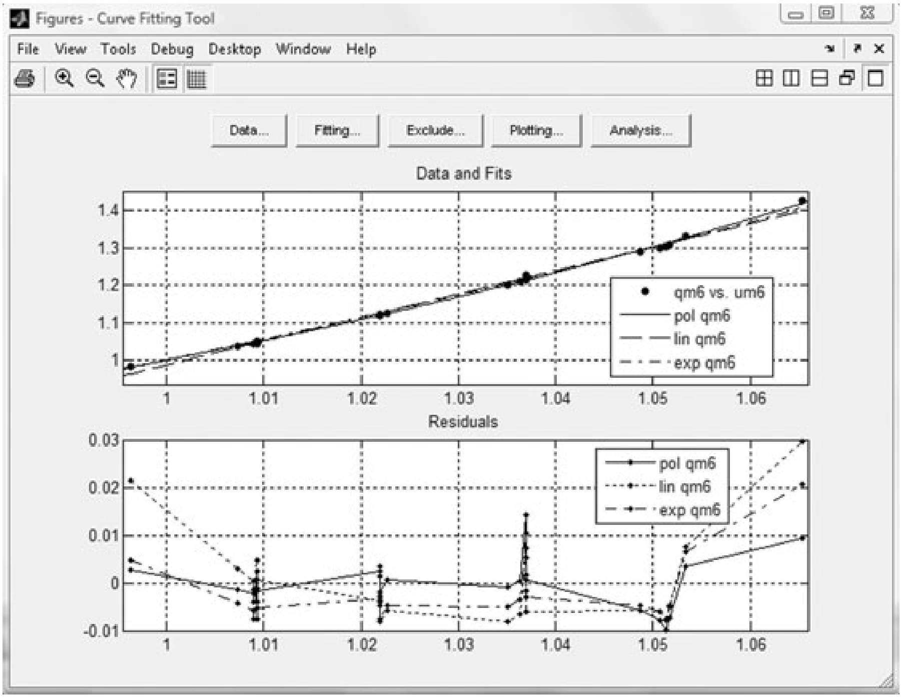

Fig. 3 *Reactive power data with quadratic polynomial (pol qm6), linear (lin qm6) and exponential (exp qm6) fits and corresponding residuals.*

TABLE 1 *Sum of squares due to errors and squared correlation coefficient of certain fits*

| Name | Data set | Type | *SSE* | *R*-square |
|---|---|---|---|---|
| pol qm6 | qm6 vs. um6 | Polynomial | 6.462929906674491E-4 | 0.997932518… |
| lin qm6 | qm6 vs. um6 | Polynomial | 0.001923113547481931 | 0.993847988… |
| exp qm6 | qm6 vs. um6 | Custom Eq… | 0.001089146521145399 | 0.996515836… |

quadratic polynomial, linear polynomial and exponential fits, pol qm6, lin qm6 and exp qm6, respectively, are presented together with corresponding residuals. The first two fittings are obtained by the functions that belong to the polynomial type and these can be selected from the fitting editor. The function for third fitting should be entered by the user as custom equation $y = x\hat{}kqu$. The analysis of the residuals shows that the smallest values correspond to the polynomial fit, up to 0.014 p.u., somewhat larger residuals are obtained for exponential fit (up to 0.021) and the largest ones relate to linear fit – up to 0.03 p.u. It indicates that polynomial fit is the best one, while linear fit yields the largest errors.

Furthermore, Table 1 contains squared correlation coefficients and sum of squares due to errors of all three fits from Fig. 3. The values from the table confirm that the quadratic polynomial model is better for modelling of reactive power than two other models, because it yields a larger squared correlation coefficient and smaller sum of squares due to errors in comparison to linear and exponential fits.

The analogue analysis for real power during the experiment from Fig. 2 reveals that all three fits: quadratic polynomial, linear polynomial end exponential, represent power-voltage behaviour almost equally well. Residuals of these fits are very small, smaller than 0.003 p.u. Sum of squares due to errors and squared correlation coefficients are also almost the same, *R*-squares differ at fourth decimal place and all are larger than 0.998, while *SSE*s are smaller than $3.5 \times 10^{-5}$ and differ from each other at sixth decimal place. However, it is found that the quadratic polynomial fit is a little bit better than two other fits, because it has somewhat larger squared correlation coefficient and smaller sum of squares due to errors.

It is suggested to students that they select data from other experiments from relevant series of experiments using the Curve Fitting Tool, imply different fittings, compare the residuals and the values of *SSE* and *R*-square obtained from these fittings, and discuss the results. However, LMprogram is designed to process data from all experiments of the series of voltage changes. Thus, the output file regarding one series of experiments contains: serial number of the experiment, identified parameters of all three examined load models (polynomial, linear and exponential) for this experiment both for real and reactive power, together with *R-square* and *SSE* of all fittings. On the basis of goodness of fit statistics (values of *R*-square and *SSE*) the proper load model, i.e. the best model of real and reactive power, for every experiment and for the series of experiments, is suggested. Also, the largest value of the correlation coefficient and the smallest sum of squares due to errors obtained for the series of experiments are listed, as well as selected load model. Therefore, the end of the output file regarding experiments performed on Sunday afternoon finishes with the states:

```
Proper load model of real power is quadratic polynomial model for 8
from 8 consecutive experiments. For this model the smallest value
of R is 9.981324e-001 and the largest value of SSE is
2.064755e-004.
Proper load model of reactive power is a quadratic polynomial model
for 8 from 8 consecutive experiments. For this model the smallest
value of R is 9.968787e-001 and the largest value of SSE is
5.489243e-003.
```

## Representative load characteristics

The series of experiments are performed in day periods when the load composition can be regarded to be the same and when the load level is almost constant, as stated in Ref. 1. Therefore, the differences between load model parameters obtained from even two subsequent experiments can be explained by the influence of white noise. In this sense, the question arises what are representative load characteristics of real and reactive power for one series of experiments, i.e. one period of considered day? These representative characteristics are obtained by LMprogram using all normalised measured voltage and power values from the series of experiments as input data for parameter identification. The least squares method is applied and parameters of representative quadratic polynomial real and reactive power characteristics are obtained.

Figure 4 presents measured voltage and real and reactive power values during experiments on Sunday afternoon, quadratic polynomial fits and corresponding residuals obtained by the Curve Fitting Tool. As seen from the figure, residuals concerning real power fit do not exceed 0.02 p.u., while most of those for reactive power are less than 0. 05 p.u. Goodness of fit statistics of real power are described by $SSE = 0.0172$ and $R$-square $= 0.9622$ ($R = 0.9809$), which indicates that the relation real power-voltage is very tight. An even larger correlation coefficient, $R = 0.9923$, is obtained for a quadratic polynomial fit of reactive power, while $SSE$ is larger, 0.1069.

The output file that summarises the results of parameter identification of representative load characteristics regarding Sunday afternoon contains parameters of the quadratic polynomial model for real and reactive power, maximum residual values and correlation coefficients. Also, it specifies the voltage range representative load characteristics are valid:

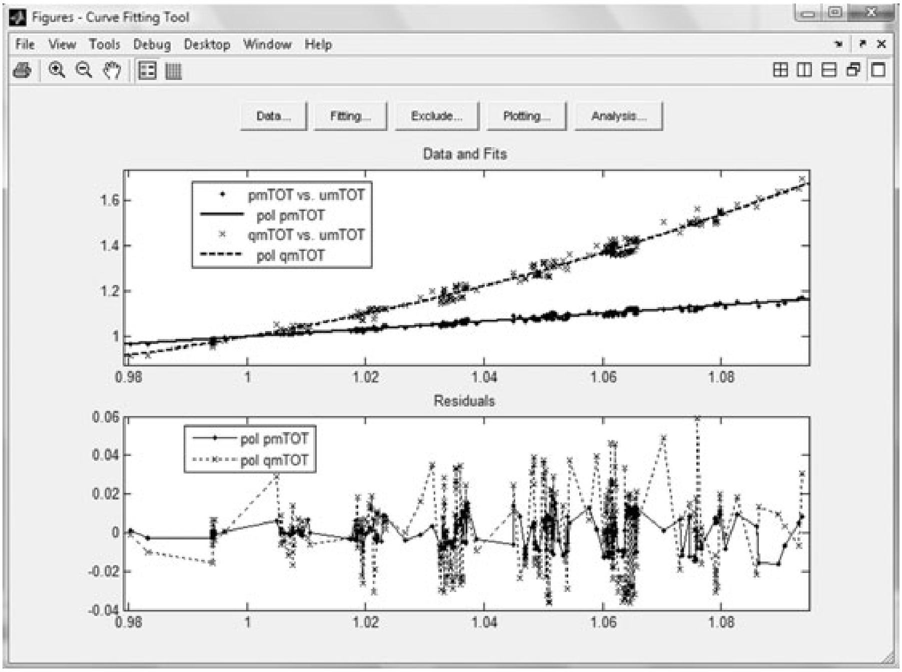

Fig. 4 *Normalized voltage (umTOT) and real and reactive power values (pmTOT and qmTOT), corresponding fits (pol pmTOT and pol qmTOT) and their residuals during a series of experiments.*

```
Parameters of quadratic polynomial model of real power are:
p1=1.167590e+000, p2=-7.137231e-001 and p3=5.462541e-001. Maximum
residual value is 1.901499e-002p.u. and correlation coefficient is
9.809289e-001. The model is valid in the voltage range from
9.803333e-001 to 1.093667e+000p.u.
Parameters of quadratic polynomial model of reactive power are:
q1=2.818180e+001, q2=-5.191781e+001 and q3=2.473617e+001. Maximum
residual value is 5.896279e-002p.u. and correlation coefficient is
9.922548e-001. The model is valid in the voltage range from
9.803333e-001 to 1.093667e+000p.u.
```

Students should process data from other series of experiments performed in different day periods and different days, identify the parameters of representative load characteristics in the same way as has been done for Sunday afternoon. Also, they should discuss the goodness of fit statistics that confirms the thesis that one representative characteristic can be adopted for one day period of a relevant day.

The whole procedure regarding determining of representative load characteristics can be summarised through the flow chart presented in Fig. 5. It is shown that nine characteristics for the concerned load from Fig. 1 are very close to each other, as well as in the cases of other groups of experiments in the distribution network of Niš. Therefore, proper conclusions regarding the possibility of grouping of these

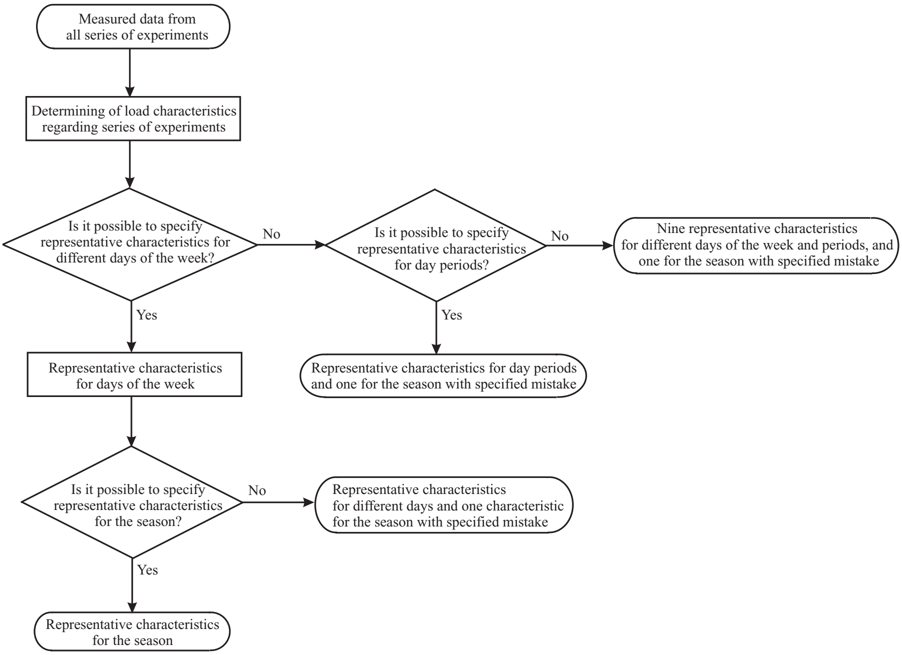

Fig. 5 *Flow chart for determining of representative characteristics.*

characteristics according to days or day periods cannot be found without an adequate calculating procedure.

Thus, LMprogram is designed in a way to enable students to answer the following question firstly: Is it possible to specify a representative load characteristic of real (and reactive) power for one day, i.e. for a working day, for Saturday and for Sunday? For this purpose, daily load characteristic of a certain day is obtained using a least squares method on data regarding three characteristics of day periods, morning, afternoon and night, of concerned day. If deviations of characteristics for morning, afternoon and night from the characteristic of the day are less than 5%, this daily characteristic is called the representative daily characteristic. Parameters of this characteristic are yielded among other program results.

After identification of representative daily load characteristics, one representative characteristic of real and one of reactive power for the season are tried to be specified. Seasonal characteristics are determined on the basis of all real or reactive power characteristics obtained from nine series of experiments in the season. If deviations of these nine characteristics from adequate real or reactive power seasonal characteristic are less than 5%, the seasonal characteristic will be representative for the whole season. If these deviations are greater than 5%, the seasonal characteristic will also be specified with the awareness of the biggest mistake the proposed characteristic introduces.

If it is not possible to specify representative characteristics for days, the program will group load characteristics according to day periods and try to find representative characteristics for every period – morning, afternoon and night. It encompasses determination of characteristics of day periods, i.e. mean characteristics, by application of a least squares method to three characteristics of the corresponding day period. Afterwards, maximum deviations of three characteristics of a certain day period from the corresponding mean characteristic are determined. If at least one deviation is greater than 5%, nine characteristics of real or reactive power (for three days and three day periods) will be specified. If the deviations are less than 5%, characteristics of day periods are called representative characteristics of morning, afternoon and night, and their parameters will be given as the output of LMprogram. Either nine characteristics or three representative characteristics for three day periods are determined, seasonal characteristics of real and reactive power, obtained on the bases of nine characteristics for different day periods and days, are specified, too. The biggest errors, i.e. largest deviation of nine representative characteristics from these seasonal characteristics, are denoted.

Thus, representative characteristics can be valid for the whole season – all days and day periods, for a particular day – working day, Saturday and Sunday, and for certain day period – morning, afternoon and night. For every representative characteristic, the voltage range in which this characteristic is valid is specified, too.

Here are the results of the procedure for determining representative characteristics applied to a group of field tests from Fig. 1. In the case of real power it is possible to specify representative characteristics for different days of the week and then representative seasonal characteristics. Thus, the final output data concerning real power are:

```
Parameters of representative seasonal characteristics of real power
are: p1=7.218316e-001, p2=6.329916e-002 and p3=2.138229e-001. The
model is valid in the voltage range from 9.526667e-001 to
1.099267e+000p.u.
```

In the case of reactive power, it is found that it is possible to specify representative characteristics for different days of the week, so maximum deviations of certain characteristics from working day, Saturday and Sunday characteristics are below 3%. Therefore, the parameters of three representative characteristics and seasonal characteristic are specified as follows:

```
Parameters of representative characteristics of reactive power are:
for working day q1=1.411774e+001, q2=-2.441464e+001,
q3=1.129397e+001, for Saturday q1=2.704197e+001 q2=-4.982022e+001,
q3=2.377605e+001, and for Sunday q1=2.688390e+001 q2=-4.925895e+001,
q3=2.337852e+001.
Parameters of seasonal characteristics of reactive power are:
p1=2.174821e+001, q2=-3.932087e+001 and q3=1.857181e+001. The model
is valid in the voltage range from 9.526667e-001 to
1.099267e+000p.u. This characteristic introduces the errors up to
5.671621e+000%.
```

## Approximate load model

As shown in the example of a series of experiments during Sunday afternoon, the polynomial load model is the best one for modelling the examined load. This statement is approved for other series of experiments, and for other groups of experiments in the city of Niš. Therefore, load modelling with an exponential model, which is even more frequently used, is just an approximation.

The adequacy of the usage of an approximate exponential load model is discussed for the example of seasonal characteristics of examined load. The parameters of exponential model are obtained using derivative of polynomial seasonal characteristics according to (4) for the voltage of 1 p.u. Obtained parameters are applied on the voltage within the range polynomial characteristics are valid, and real and reactive power values according to exponential model are generated.

Figure 6 presents polynomial and approximate exponential seasonal characteristics of real and reactive power and deviations of exponential characteristics from corresponding polynomial characteristic in percentiles of adequate polynomial characteristic value. Since real power characteristic is almost linear, the exponential characteristic matches polynomial characteristic very well, with nearly zero maximum deviation. Reactive power characteristic is pretty nonlinear and maximum deviation of adequate approximate characteristic is about -8%. Since an approximated characteristic is regard as acceptable for deviations smaller than 5%, the exponential characteristic of reactive power is applicable over a narrower voltage range than the examined voltage range. Thus, LM program yields:

```
Exponential load model can be used in the voltage range from
9.526667e-001, to 1.0993e+000 p.u. for real power, and in the range
from 9.527e-001 to 1.0691e+000p.u. for reactive power.
```

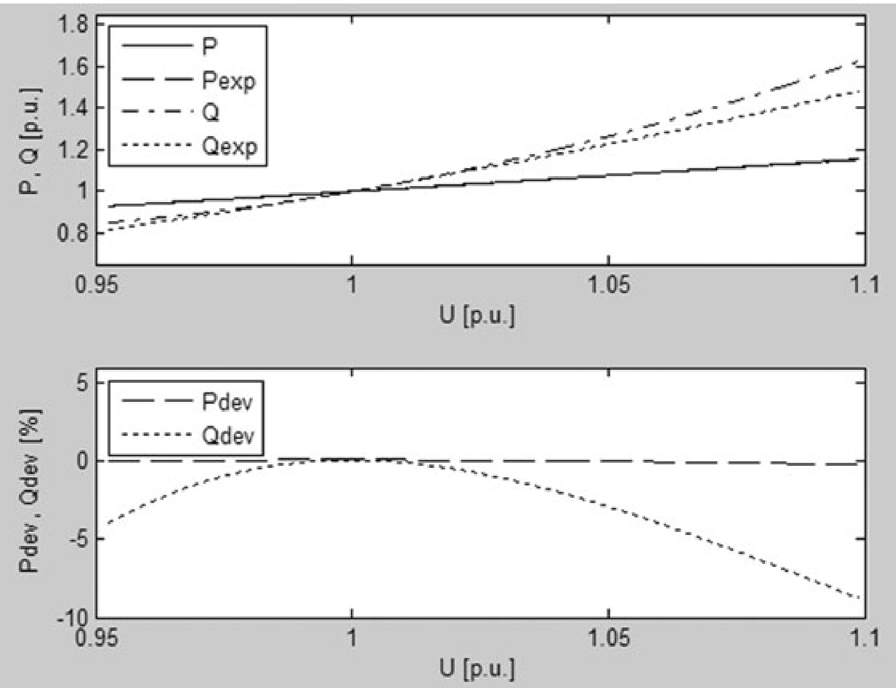

Fig. 6 *Real and reactive power seasonal characteristics, approximate exponential characteristics and corresponding deviations.*

The presented program is universal and students can implement it on the input data from other groups of experiments performed on the distribution network of the city of Niš. In this way one can compare load modelling results obtained for the load on different voltage levels and in different seasons.

The proposed didactic tool is planned to be one laboratory exercise of a new course of electrical power engineering studies at the Faculty of Electronic Engineering. Since the lecture about load modelling will include the basis of the measurement-based load modelling approach and most frequently used load models among other issues, the condition to attend laboratory exercises with implementation of LMprogram should provide correct answers to the questions regarding the foregoing items. Afterwards, at the end of every part of LMprogram: input data analysis, selection of proper load model, representative load characteristics and approximate load model, the students will have to explain all the program outputs (figures, tables and statements) required in order to understand the studied matter.

## Conclusion

This paper presents LMprogram, software that is designed for students of electric power engineering. The use of the presented program can help students to understand a measurement-based load modelling approach and study it from many practical examples, i.e. field experiments that yield data regarding load in different days and day periods. The results are figures, tables and detailed output data that concern very important load modelling issues: selection of a proper load model, procedure for determination of representative load characteristics and specification of the voltage range in which approximate load model is valid.

For the predominantly residential load at a 10 kV voltage level that is examined in the paper it is stated that the quadratic polynomial load model is the proper one. One representative seasonal characteristic of real power can be used for modelling of real power component of the load in all days and day periods of the relevant season, while the seasonal characteristic of reactive power introduces errors larger than 5%. Approximate exponential model for real power is valid in the examined voltage range from almost 0.95 to 1.1 p.u., while this model for reactive power can be used in a narrower voltage range.

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