Vehicle speed calculation from pedestrian throw distance

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Abstract: When a pedestrian is thrown through the air by the impact of a motor vehicle, the distance travelled by the pedestrian from the point of impact can be used to provide an estimate of the speed at which the vehicle was travelling. Three sets of data collected from real accidents are reviewed with the aims of assessing the value of this method and fixing the model used to estimate the speed. The data have been published by Stürtz and Suren [**1**], by Hill [**2**] and by Dettinger [**3**]. Each provides the vehicle speed estimated from skid marks and the pedestrian throw distance for a number of real traffic accidents. The parameters in a simple, physically motivated model are fitted to each dataset separately using a least squares method. It is found that the earlier dataset has a much less consistent relationship between throw distance and vehicle speed than the later sets. The two more recent sets of data are used to construct a model that provides an estimate of the vehicle speed based on the throw distance, the bounds within which, with 95 per cent certainty, the vehicle speed lies, and an absolute lower bound. The data are also used to assess a method considered by Wood for the estimation of vehicle speed from the distance between the final resting place of vehicle and pedestrian. It is concluded that this last method is not likely to be useful because of its sensitivity to errors in measurement.

Keywords: road traffic accident, pedestrian impact, pedestrian throw distance, vehicle speed

NOTATION

A	coefficient relating s to v: $v = A\sqrt{s}$
g	acceleration due to gravity
r	difference between estimates of v from skid marks and from pedestrian throw
s	total distance travelled by the pedestrian after impact
s_{v}	distance over which the vehicle skids to a halt after impact
v	speed of the vehicle at the time of impact
$v_{\min}$	minimum speed of the vehicle, estimated from pedestrian throw
v_{p}	speed of the pedestrian immediately after impact
v_{W}	speed of the vehicle, estimated by Wood's method
θ	angle from the horizontal at which the pedestrian is projected
μ	coefficient of friction between pedestrian and road
μ_{v}	coefficient of friction between vehicle and road
σ	r.m.s. value of r

1 INTRODUCTION

Methods for the reconstruction of road traffic accidents have advanced greatly over the last 25 years. Mathematical and physical models that allow the estimation of quantities such as the vehicle speed from measurements taken at the scene of an accident have been developed and are widely used in court proceedings. Although many other factors must be taken into account when a real accident is investigated, at least two methods have been established that allow the estimation of vehicle speed. These are the analysis of skid marks, and a method based on measurement of the distance that a pedestrian is thrown by impact with the vehicle. Reviews of some of these models are given by Smith [**4**, **5**], Lambourn [**6**] and Searle [**7**].

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A pedestrian struck by a motor vehicle in a traffic accident is often thrown a considerable distance by the force of the impact. Several studies [**1**–**3**] have shown that when the impact is square with the front of the vehicle, there is a strong correlation between the distance travelled by the pedestrian and the speed at which the vehicle was travelling. This relationship can be used by accident investigators to estimate the vehicle speed. As anti-lock braking systems become more popular and skid marks therefore become less common, the estimation of vehicle speed using pedestrian throw distance may become more important in traffic accident investigation.

Searle [**7**, **8**] has developed a simple theory of the trajectory of a pedestrian after impact, taking into account the flight through the air and subsequent bouncing and skidding. To use this theory directly, it is necessary to know the coefficient of friction between the pedestrian and the road and the angle to the horizontal at which the pedestrian was thrown off by the vehicle. These parameters are not usually known. Experimental data are therefore needed.

The problem is further complicated by other factors that may influence the distance that a pedestrian is thrown. The shape of the front end of the vehicle [**1**] and whether the vehicle was braking at the time of impact [**9**] have been suggested as complicating factors. Also, data collected from tests carried out using dummies and cadavers may produce results that are somewhat different from those of real accidents [**10**, **11**].

In this paper, three sets of data are considered, all collected from real accidents, and published by Stürtz and Suren [**1**], by Hill [**2**] and by Dettinger [**3**]. In each case, the speed of the vehicle involved in an accident has been calculated using the standard and well-tested method of measuring skid marks [**4**, **6**], and the distance travelled by a pedestrian after impact has been measured.

The data are compared with a simple model based on the work of Searle [**7**]. The more recent data sets of Hill and Dettinger fit the model better than the earlier data. Some simple equations, based on analysis of the more recent data, are given for the estimation of vehicle speed from pedestrian throw distance. It is shown that many of the complicating factors can be neglected and this simple model can be used with some certainty to place bounds on the speed of a vehicle based on a measurement of pedestrian throw distance.

Finally, using some additional information given by Hill, the accuracy of a method considered by Wood [**12**] for the estimation of vehicle speed from the distance between the final positions of the vehicle and the pedestrian is assessed. It has been suggested that this method might prove useful when the point of impact is not known, as is sometimes the case. Hill's data suggest that this method is not reliable.

2 RELATIONSHIP BETWEEN VEHICLE SPEED AND PEDESTRIAN THROW DISTANCE

Searle [**7**, **8**] has shown that when an object is projected at an angle θ to the horizontal with speed v_{p}, starting with its centre of mass at ground level, the total distance s it travels in a bouncing and skidding motion before it comes to rest is related to v_{p} by

$$v_{\mathrm{p}} = \frac{\sqrt{2\mu g s}}{\cos\theta + \mu \sin\theta} \tag{1}$$

where μ is the coefficient of friction between the object and the ground. If it is assumed that a pedestrian is projected at the speed at which the vehicle is travelling, or perhaps more slowly, it follows from minimizing v_{p} in the above equation that the minimum value of vehicle speed for a particular pedestrian throw distance s is

$$v_{\min} = \left(\frac{2\mu g s}{1+\mu^2}\right)^{1/2} \tag{2}$$

There is also an equation for the maximum speed, but this is not as reliable because of the many factors that may cause the pedestrian to be thrown at a speed lower than that of the vehicle.

Wood [**13**] has used a simple model to investigate the relationship between the vehicle speed v and the pedestrian throw distance s. He showed that when the initial height of the pedestrian's centre of mass is neglected, $v = A\sqrt{s}$, where A is a factor that depends on various parameters including the angle of the bonnet and the deceleration of the vehicle.

Since each of the equations relating v and s above have the form $v = A\sqrt{s}$, where A is a constant, discussion of angles of projection, coefficients of friction and other parameters can be avoided by simply finding the value of A that gives best agreement with the data. In real accidents, the parameters μ and θ are never known precisely. These and other uncertainties make the small improvements in accuracy yielded by an elaborate theory irrelevant. In this paper, the simple model described above is therefore used.

3 SOURCES OF DATA FROM TRAFFIC ACCIDENTS

The three sets of data considered in this paper have all been obtained from measurements taken after real traffic accidents. They are therefore expected to give more reliable models for accident investigation than those derived from tests with dummies in staged accidents. The work of Niederer *et al.* [**10**] and of Cesari and Ramet [**11**] shows that there are differences between the dynamics of impacts with real pedestrians and those of impacts with cadavers and dummies. One important cause of these differences is that pedestrians are usually moving (walking or running) when they are struck by vehicles.

Cases where the pedestrian was not struck squarely by the front of the vehicle or became snagged or trapped by the vehicle are not included in the datasets. For each accident, the data include a vehicle speed v_s calculated from skid marks and a pedestrian throw distance s.

The first dataset, published by Stürtz and Suren [**1**], and also mentioned by Appel *et al.* [**14**], contains data from 47 accidents in urban areas near Hanover and Berlin in Germany in 1973 and 1974. The data collected for accidents involving child pedestrians are not considered here, since the other two sources, published by Hill and Dettinger (see below), do not include such data, and a comparison is therefore not possible. The data are divided into classes according to the shape of the front of the vehicle. A pontoon-contour vehicle has the rectangular front end typical of passenger cars of this period, while a V-contour vehicle has a front which is more streamlined, as found on many sports cars. A box-contour vehicle has an almost vertical front surface, as in some heavy goods vehicles and vans. There are 19 accidents involving pontoon-contour vehicles, 12 involving V-contour vehicles and 4 involving box-contour vehicles. These data are shown in Fig. 1. The accidents involving box-contour vehicles have not been included in the analysis presented here because of the limited sample size.

The dataset published by Hill [**2**] contains data from 26 accidents in the Birmingham area of England during a three year period ending in 1987. Also given in Hill's paper are values of the coefficient of friction between the vehicle and the road, measured in tests.

Dettinger's paper [**3**] contains data from 12 accidents, compiled by the DEKRA Accident Research Agency in Stuttgart, Germany, in the years before 1996. The data from the papers of Hill and Dettinger are shown in Fig. 2.

4 ANALYSIS

For each dataset the best-fitting value of A in the model $v = A\sqrt{s}$ was determined by a least-squares fit to the values of v and $\sqrt{s}$. The residual

$$r = v - A\sqrt{s} \tag{3}$$

was also calculated for each accident, and the r.m.s. value σ of the residual, defined by

$$\sigma^2 = \frac{1}{N}\sum_{n=1}^{N} r_n^2 \tag{4}$$

where N is the number of accidents in a dataset and the subscript n labels individual accidents.

Inspection of the residuals confirmed that in all cases the distribution was roughly normal, with no obvious difference in the magnitude of residuals for low and high values of v. The authors therefore believe that σ provides a useful measure of the dispersion, or statistical spread, in the data. Table 1 summarizes the results of this analysis for each dataset. The value given for the width w of the 95 per cent band in Table 1 is based on 1.96σ.

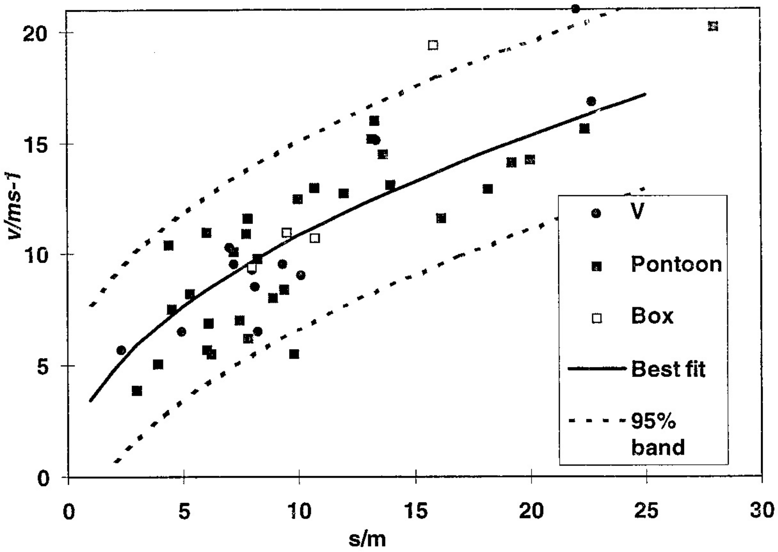

Fig. 1 Pedestrian throw distance s and vehicle speed v (estimated from skid marks) for the accidents described by Stürtz and Suren [**1**]. Also shown is the line $v = A\sqrt{s}$ with $A = 3.42$, the best-fit value for the pontoon- and V-contour data considered together, and the 95 per cent band for this combined set. Box-contour vehicles are not included in the data used to derive the best-fit and 95 per cent lines

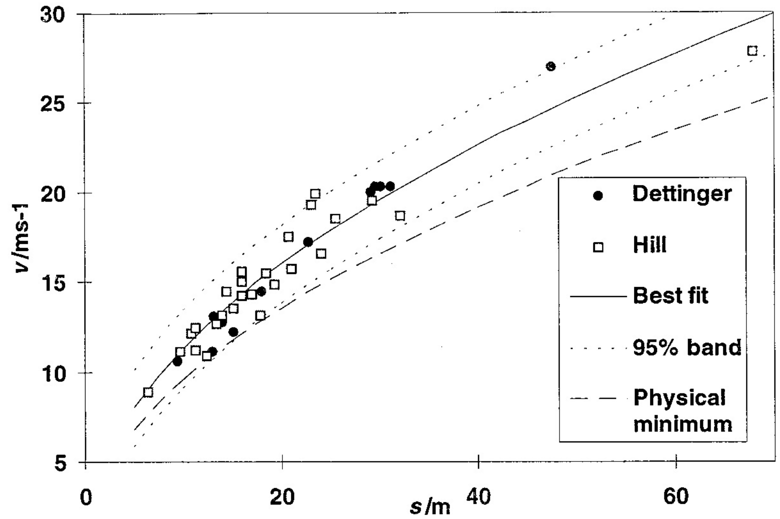

Fig. 2 Pedestrian throw distance s and vehicle speed v (estimated from skid marks) for the accidents described by Hill [2] and by Dettinger [3]. Also shown is the line $v = A\sqrt{s}$ with $A = 3.58$, the best-fit value for the Hill and Dettinger data considered together, and the 95 per cent band for this combined set

Assuming a normal distribution of residuals, the difference between the value of v given by the model and the true value of v would be expected to be less than w in 95 per cent of cases. It is important to note the difference between this 95 per cent band and the (much smaller) '95 per cent confidence interval' given by Stürtz and Suren [5] and also by Appel *et al.* [14]. The interval given by these authors is essentially the range within which, with 95 per cent certainty, the parameter A, and hence the best-fit line, lies. Clearly, a small width for this interval (as would occur in the case of very many data points, even if all had large residuals) does not ensure the accuracy of estimates derived from the model. It is therefore hard to see the relevance of this 'confidence interval'.

As is evident from Figs 1 and 2, all the datasets show a clear correlation between speed v and throw distance s. There is a difference of only 6 per cent between the largest and smallest values of A obtained, so all the datasets can be said to be fitted by roughly the same model. The most noticeable feature of the table is that the dispersion σ of the earlier data of Stürtz and Suren is twice as large as those of the more recent datasets published by Dettinger and by Hill. The two more recent sets also yield almost the same values of A, differing by only 2 per cent. Similarly, the values of A for V-contour and pontoon-contour vehicles in the data of Stürtz and Suren differ by only 1 per cent. This indicates that, even if the details of the dynamics of the impact between pedestrian and vehicle are affected by the vehicle shape [13], the pedestrian throw distance does not depend significantly on the geometry of the front of the vehicle.

Figure 3 shows the best-fit lines and 95 per cent bands for each of the datasets. It is clear that the datasets of Dettinger and of Hill have almost the same statistical properties, while the set of Stürtz and Suren (taking pontoon- and V-contour vehicles as a single class) is noticeably different.

The results given above suggest that there is an important difference between the earlier data of Stürtz and Suren and the later results of Dettinger and of Hill, which has resulted in a much wider dispersion in the earlier data. One possible explanation for this difference is the introduction of chalk guns for the accurate measurement of distances in skid tests [6]. The apparently poorer performance of the throw–distance model for the earlier data would then be due not to a failure of the model but to inaccuracies in the speeds v used to check the model.

If this is true, then the best way to obtain an accurate model of the dependence of throw distance on vehicle speed is to use only the more recent datasets of Hill and Dettinger. The best value of A for the combined data of Hill and Dettinger was therefore found. The resulting model is

$$v = 3.58\sqrt{s} \tag{5}$$

Table 1 Best-fitting value of A, the r.m.s. deviation σ from the model with this value of A, and the half-width of a 95 per cent band assuming a normal distribution (this half-width is 1.96σ)

Source	A ($\mathrm{m}^{1/2}$/s)	σ (m/s)	95% band half-width
Stürtz (pontoon only)	3.38	2.18	4.27
Stürtz (V only)	3.54	2.03	3.98
Stürtz (V+pontoon)	3.42	2.15	4.21
Hill	3.55	1.09	2.14
Dettinger	3.62	1.08	2.12
Hill+Dettinger	3.58	1.10	2.16

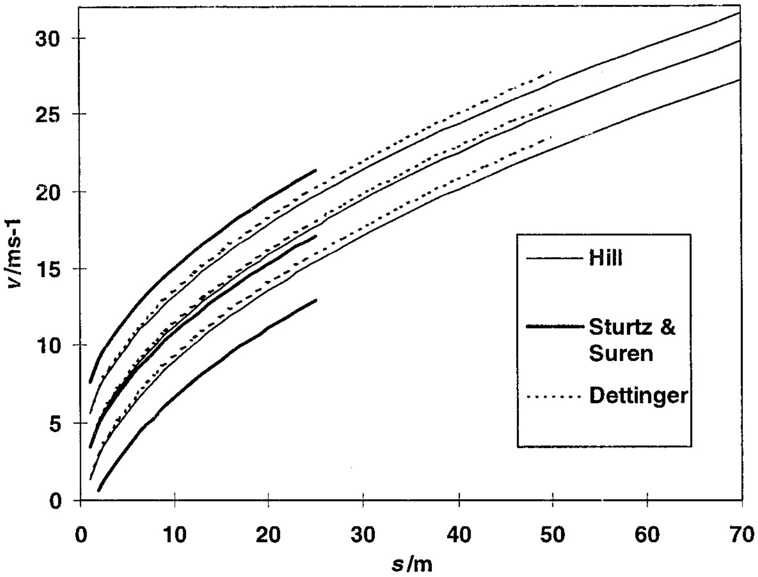

Fig. 3 Best-fit lines and 95 per cent confidence lines for each of the three datasets. The curves are shown only over the range of values covered by their respective datasets

The width of the 95 per cent band for this combined dataset is 2.16 m/s. Thus, equation (5) gives an estimate of the vehicle speed at the time of impact, with a 95 per cent probability that the speed lies between

$$v = 3.58\sqrt{s} - 2.16 \tag{6}$$

and

$$v = 3.58\sqrt{s} + 2.16 \tag{7}$$

where speed is measured in m/s and distance in m. The authors believe that this is the best available model for the estimation of vehicle speed from pedestrian throw distance.

It is also possible to give an absolute minimum for the vehicle speed. All the data points in the sets of Hill and of Dettinger lie on or above the line

$$v_{\min} = 3.03\sqrt{s} \tag{8}$$

Physical reasoning confirms this conclusion. The above equation for $v_{\min}$ is identical to the one obtained by putting $\mu = 0.7$, the smallest value found by Hill [2], into equation (2). The coefficient 3.03 in this equation is not very sensitive to the particular value of μ used, varying by only 3 per cent for the range of variation found by Hill, $0.7 \leqslant \mu \leqslant 0.94$. Figure 2 shows the best-fit line, the 95 per cent band and the $v_{\min}$ curve for the combined data of Dettinger and Hill.

5 WOOD'S METHOD

In the investigation of accidents, the point on the road where the impact occurred is sometimes not known exactly. The final positions of the car after it has skidded to a halt and of the pedestrian are usually known. Wood [**12**], following a suggestion by Burg and Rau [**15**], has shown how in this case an estimate of the vehicle speed can be obtained from the distance between the final position of the pedestrian and that of the vehicle. The idea behind the method is simple. The vehicle speed v is related to the pedestrian throw distance s by the equation $v = A\sqrt{s}$ and to the distance s_v skidded by the vehicle after impact before it comes to a stop by the equation [**4**]

$$v = (2\mu_v g s_v)^{1/2} \tag{9}$$

where μ_v is the coefficient of friction between the vehicle and the road. An estimate of the vehicle speed at impact is therefore given by

$$v_W = \left[\frac{s - s_v}{A^{-2} - 1/(2\mu_v g)}\right]^{1/2} \tag{10}$$

By measuring only the distance $s - s_v$ between the pedestrian and the vehicle, it is therefore possible to obtain an estimate of the vehicle speed. There is one obvious problem with this method: it is necessary to assume that the vehicle was braking at the moment of impact and that the wheels remained locked as it skidded to a halt. Even if this is true, measurement errors or the random element in the relationship between s and v may make the method unreliable. To assess this method, the data provided by Hill [**2**] are used, together with the model derived above ($A = 3.55$) from his data.

Included in Hill's tables are measured values of μ_v and s_v for each accident and the values of v calculated from skid marks. Since the pedestrian throw distance s is also given, it is possible to find the value of $s - s_v$ that would have been measured. From this value, v_W is estimated using equation (10) and compared with the value v from skid tests. Figure 4 shows v and the estimate v_W for each of the accidents in Hill's set. There is no obvious correlation between the two values, and in some cases the quantity inside the square root in equation (10) is negative (v_W has been replaced by zero in these cases for the figure). It is concluded that the method is not reliable.

The failure of this method is easy to explain. The pedestrian throw distance s and the vehicle skid distance s_v are often similar in magnitude, so the difference between them is relatively small. A small error in s or s_v can therefore lead to a large fractional error in $s - s_v$.

6 CONCLUSION

The surprisingly large difference in the statistical spread σ between the datasets considered here suggests that the method of estimating vehicle speed from pedestrian throw distance has become more accurate over the last twenty years. However, if it is true that the improvement is due to the introduction of chalk guns (or, more recently, electronic accelerometers) in skid tests, then the apparent improvement is an illusion. In fact, the relationship between pedestrian throw distance and vehicle speed has always been consistent: the only change has been that the data from skid marks used to check the relationship have improved in quality, making the estimates from throw distance appear more accurate. More research is needed to decide whether this is the true explanation.

The 95 per cent bands quoted here provide a better measure of the accuracy of the model than the 'confidence interval' given by Stürtz and Suren [1], as discussed in Section 4. The narrow interval used by these authors made the difference between the throw distances for pontoon- and V-contour vehicles appear more significant than it does in the present paper. Since the difference between the estimated speeds for pontoon- and V-contours is much smaller than the width of the 95 per cent band, the present authors do not believe that it is useful to make a distinction between pontoon- and V-contours for this purpose.

The pleasingly consistent relationship between throw distance and speed in the more recent data is maintained across two datasets collected in different countries at different times, so it is unlikely to have occurred by chance. The best-fit lines and confidence intervals described in this paper are simple and robust enough to be used in legal proceedings. The minimum speed given by equation (8) is also likely to be useful, being confirmed both by physical reasoning and experimental evidence. In criminal cases where a verdict must be reached beyond reasonable doubt, the minimum speed of equation (8) and the range of speeds given by equations (6) and (7) will probably be preferred, whereas in civil cases the balance of probabilities is used to reach a verdict and the best estimate of equation (5), together with the range from equations (6) and (7), will be more useful.

The model described in this paper has been obtained from accidents where the vehicle was braking and the pedestrian was not carried along on the vehicle. This latter condition is an important one. If the pedestrian travels for some distance on some part of the vehicle, then the distance from the point of impact to the pedestrian's final position will not be a true throw distance. In these circumstances the equations derived here should not be used. However, the condition that the vehicle should be braking is almost certainly not as important. The main difference between the effect of

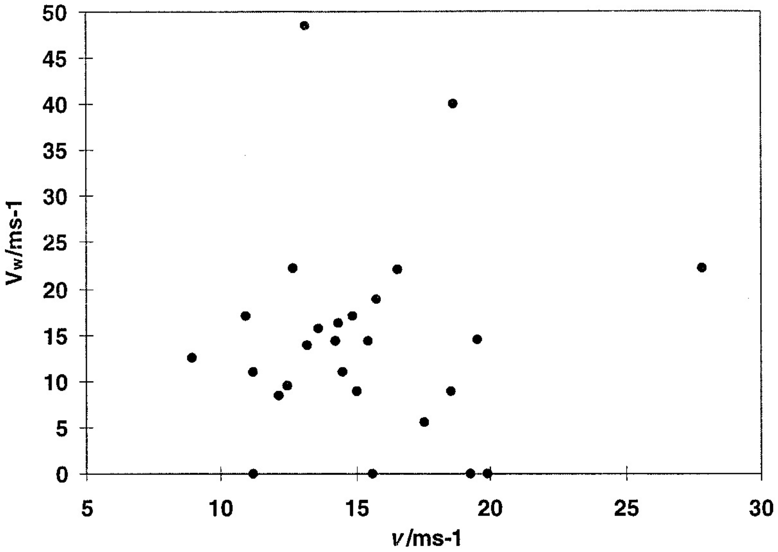

Fig. 4 Vehicle speeds estimated using skid marks, v, and using Wood's method, v_W, based on the data of Hill [2]

the impact of a braking vehicle and one that is not braking is that braking causes the front of the vehicle to dip. According to Wood [**13**], this will alter the dynamics of the pedestrian. However, the modification in the shape of the front of the vehicle is smaller than the difference between the V and pontoon classes of Stürtz and Suren [**1**], and this difference appears to have only a small influence on the distance that a pedestrian is thrown. Hence the model derived here is very likely to give a good indication of the speed of a non-braking, as well as a braking, vehicle.

In using the equations derived here it is of course important to make sure that the conditions for their validity are not in doubt. It must be ascertained that the impact of the pedestrian is square with the front of the vehicle and that the pedestrian is thrown into the air and not carried along or snagged by the vehicle. There may also be some uncertainty in the position on the road where the impact occurred; however, the presence of a scuff mark from the pedestrian's shoe often allows an accurate determination of this position. The method described here is only one of a range of techniques and sources of information that may be used to analyse an accident. In coming to a conclusion, it is important to evaluate all the available evidence.

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