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ABSTRACT: This paper investigates how (1) links between strategic performance measures and (2) substitute and complementary characteristics of the agent's efforts affect the principal's use of performance measures in the agent's compensation contract. Increasing evidence indicates that firms use a business model or balanced scorecard approach for internal performance measurement, highlighting the links between performance measures and a "path" for improved performance. However, we know relatively little about why there exists wide variation in, and sometimes the absence of, incentive systems based on the business model approach. Using principal-agent analysis, we show that the directional impact of changes in performance measure interrelations on incentive weights depends on the relationship between the agent's tasks (i.e., substitute or complementary). For example, increases in performance measure interrelations do not necessarily require higher incentive weights on more sensitive and precise performance measures. Rather, if efforts are substitutes, the costs of effort are relatively higher, requiring the principal to induce lower levels of total effort by offering lower incentives. We also show that differences in the combination of performance measure interrelations and effort interactions affect profits in distinctly different ways. When efforts are substitutes for one another, increases in the sensitivities of profit to the other performance metrics, and thus to effort, may actually lead to lower profits.

Keywords: principal-agent theory, incentive compensation, balanced scorecard

1. Introduction

In this paper, we examine how the relations between both performance measures and tasks affect incentive contracts offered to employees. Managers typically use a set of performance measures to evaluate the effectiveness and efficiency of their business operations. These measures include both financial and non-financial measures and typically relate to one another in a causal manner, which aims to reflect a firm's business model. Kaplan and Norton (2001a) define this approach as a Balanced Scorecard. Kaplan and Norton (1996) claim that benefits exist for firms that better understand both the links between key performance measures and managers' ability to influence the realizations of these measures.

This *business model approach* to performance measurement focuses on the causal links between managerial actions, intermediate performance measures, and overall firm performance (Ittner and Larcker, 1998b). This has at least two contracting implications. First, contractible performance measures such as customer satisfaction and profit may be functionally related. Accordingly, in order to place contracting weights on these measures, contract designers must calibrate the weights for the interrelationships. Second, unobservable worker tasks such as customer service and employee training may interact. If contract designers use performance measures to induce related tasks, then they must also consider such interactions in their contract weight choices.

Interactive efforts (i.e., interacting tasks) can take the form of either complements or substitutes. If two types of effort are complements (substitutes) for one another, then the execution of one task impacts the employee's cost to perform the second task. In this case, the employee's marginal cost of effort decreases (increases), relative to her marginal cost of effort had the two tasks been unrelated. For example, a retail manager may focus efforts on both store organization and customer satisfaction. However, investing more effort into the organization and cleanliness of the stores may result in the customer requiring less help locating products and thus, enhance the customers' experience (i.e., the tasks are complements). Thus, due to the synergy between the tasks, the employee's marginal cost of customer satisfaction effort will decline. This lower marginal cost of effort translates into a lower wage paid to the agent.

Conversely, when two tasks are substitutes, an employee's performance of one task results in a higher marginal cost to perform the other task. Suppose that the employee is now responsible for training new employees, in addition to organizing the store and ensuring that customers are satisfied. The employee, however, may find it difficult to switch between teaching colleagues and helping customers; therefore the marginal cost of helping customers will be higher. Thus, these efforts are substitutes for one another, despite the fact that well-trained employees may eventually lead to satisfied customers (i.e., the performance measures are interrelated).

If a business model approach is adopted for contracting purposes, a difficult and important issue is how one should account for the net effect of performance measure interrelations and effort interactions when formulating the compensation contract. Incentive contracts based on internal performance measurement vary widely in practice (Kaplan and Norton, 2001b), and, in some cases, are absent despite the popularity of business model approaches such as the Balanced Scorecard for decision-making.

On the one hand, companies such as Sears detail the links between employee satisfaction, customer satisfaction, and profitability, and explicitly incorporate these measures into their employees' incentive contracts (Rucci, Kirn and Quinn, 1998).[1] On the other hand, Ittner, Larcker and Meyer (2003) provide evidence of incentive contracting weights based on a Balanced Scorecard implementation that was ultimately abandoned.[2] On balance, the empirical evidence is consistent with the view that incorporating a business model approach into incentive contracting is difficult and problematic. While standard principal-agent insights can explain some of the empirical phenomena, existing theory does not address the specific issues related to performance measure interrelation and effort interaction implied by the business model approach.

This paper investigates the manner in which (1) the *links between* strategic performance measures and (2) the substitute and complementary characteristics of the agent's tasks affect the principal's use of these measures in the agent's compensation contract. We model performance measure interrelationships and effort interactions using a principal-agent model that assumes linear compensation contracts, a negative exponential

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[1] In particular, Sears found that a 5-unit increase in employee attitude linked to a 1.3 unit increase in customer impression and ultimately translated into a 0.5% positive growth in revenues (Rucci, Kirn and Quinn, 1998).

[2] Further evidence on the use of the balanced scorecard is provided by Speckbacher, Bischof and Pfeiffer (2003).

utility function, and normally distributed error terms. The results indicate that the demand for multiple performance measures relates to the relative magnitudes of both the interrelationships between performance measures and the synergies between different efforts. We derive conditions under which measures that are *more* sensitive receive *less* emphasis for contracting, and *less* precise measures receive *more* contracting emphasis. The firm's contract designer must weigh the benefits from the availability of more precise and sensitive performance measures against any incremental costs associated with interactive effort and interrelated performance measures.

More importantly, the way in which the incentive contract trades off the costs and benefits derived from performance measure interrelation and effort interaction affects the resulting profits of the firm. We show that if efforts are complementary, higher (lower) performance measure interrelations increase (reduce) firm profits. Further, under certain conditions concerning the relative magnitudes of the effort and performance measure interaction, if effort is substitutable, higher (lower) performance measure interrelations actually *reduce* (increase) firm profits. When tasks relate to one another, inducing both tasks affects the firm's expected compensation costs. Additionally, using more sensitive and precise measures may reduce the risk premium payable by the principal. The firm must weigh the net impact of these effects in order to determine the "optimal" contract to offer its employees.

The remainder of the paper is organized as follows. Section 2 develops the basic analytical model. Section 3 analyzes the contract offered to the agent, and Section 4 analyzes the effect of the interrelationships on firm profits. Section 5 summarizes the results, describes areas of future research, and concludes the study.

2. Basic Model

Consider a risk-neutral principal (e.g., an owner) who hires a risk- and effort-averse agent (e.g., an employee). The agent performs two interrelated tasks. Her effort affects firm profit but is unobservable to the principal. The principal observes firm profit along with two intermediate performance measures at the end of the period. These measures hierarchically relate to one another; each measure's outcome affects the next measure's results.[3] One interpretation of "hierarchically related performance measures" corresponds to the performance measures and the links between these measures as portrayed in both the Balanced Scorecard and the business model approach to internal performance measurement.[4] Accordingly, we label the measures accounting profit, a customer measure, and an internal measure (x, y_c, y_i, respectively).[5] For ease of presentation, Figure 1 presents a pictorial view of the key relationships in the model.[6] We refer to these relationships in the following discussion.

----- Insert Figure 1 About Here. -----

The firm's accounting profit, x, is directly affected by the customer measure, y_c. Because the agent's effort and the internal measure, y_i, affect the customer measure, these variables indirectly impact accounting profit. In a retail setting, customer-related effort, a_c, includes helping customers find products and offering adequate service to customers. Examples of internal-oriented effort, a_i, include time spent organizing the shelves in the store or training other employees. Firm accounting profit is defined as[7]

$$x = \underline{x} + \beta_{xc}\, y_c + \varepsilon_x, \text{ where } \varepsilon_x \sim N(0,\sigma_x^2). \tag{1}$$

Accounting profit, (1), is indirectly impacted by the agent's effort.[8] The parameter β_{xc} reflects the relation

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[3] In addition to the Sears case, recent research provides empirical support for this assumption. Ittner and Larcker (1998a) highlight a linear relationship between non-financial measures and stock price; Banker, Potter and Srinivasan (2000), Nagar and Rajan (2005), and Chen (2006) document a link between current non-financial and future non-financial and financial performance measures; Banker, Konstans and Mashruwala (2001) find associations between various customer and employee-based non-financial performance measures; Campbell, Datar, Kulp and Narayanan (2008) document the link between skills, strategy implementation, and financial performance; and Smith and Wright (2004) document a link between product value attributes, customer loyalty, and financial performance. Bryant, Jones and Widener (2004), however, find evidence that a measure's outcome directly affects not only the next measure's result but also subsequent measures' results.

[4] Note that this interpretation does not represent an entire balanced scorecard. Rather, it reflects one "path" between the measures, consistent with a Kaplan and Norton (2001a) strategy-mapping approach to the Balanced Scorecard. We chose not, for example, to include a direct link from the internal measure to the firm's profit. Our primary intention is to highlight the tension between performance measure sensitivities and costs of effort.

[5] Alternatively, one can think of the three interrelated measures as net income, revenue, and cost of goods sold or as stock price, net income, and a non-financial measure.

[6] We thank Ella Mae Matsumura for suggesting this pictorial view.

[7] Cost parameters are excluded from the profit function, without loss of generality.

[8] Arya, Fellingham and Schroeder (2004) analyze measurement errors and production shocks to understand the use of aggregate performance measures in a sequential production setting. They find that aggregate performance measures outperform individual performance measures if measurement errors dominate production shocks. The authors structure the aggregate performance measure similar to the profit measure in our model. Our primary focus is on the effects of the interrelations between performance measures and the interactions between effort, rather than on the tradeoffs between production shocks and measurement errors.

between profit and the customer measure. Exogenous factors, depicted by $\underline{x}$, also impact profit. The non-financial customer and internal measures are defined as[9]

$$y_c = a_c + \beta_{ci} y_i + \varepsilon_c, \text{ where } \varepsilon_c \sim N(0,\sigma_c^2), \text{ and} \tag{2a}$$

$$y_i = a_i + \varepsilon_i, \text{ where } \varepsilon_i \sim N(0,\sigma_i^2). \tag{2b}$$

All error terms (i.e., ε_x, ε_c, and ε_i) are independent of one another. However, due to the interrelationship between the performance measures, the performance measures are correlated. We refer to the noise that relates specifically to a performance measure as "unique noise". We term all other noise "spillover noise." For example, σ_c^2 is the unique noise of the customer measure while σ_i^2 spills over from the internal measure into the customer measure.[10]

The principal offers the agent a contract, w, which depends on the three performance measures. For tractability, and consistent with prior literature (e.g., Feltham and Xie, 1994), we assume a linear compensation scheme. The compensation scheme consists of a fixed salary component, f, a profit-based component, and two components based on the customer and internal measures, respectively. The wage is represented as

$$w = f + v_x x + v_c y_c + v_i y_i. \tag{3}$$

Based on the compensation contract offered, the agent performs customer-related and internal-oriented actions to maximize her expected utility. She incurs a personal cost of effort, $\kappa(a_c,a_i)$, which consists of a cost for each type of effort and an additional (reduced) cost if the efforts are substitutes (complements). The degree of complementarity or substitutability between the efforts is represented by γ, where $|\gamma| < 1$. The agent's cost of effort is defined as

$$\kappa(a_c,a_i) = ½\,(a_c^2 + a_i^2) + \gamma\, a_c\, a_i. \tag{4}$$

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[9] The nature of the performance measures assumed in this paper is similar to other theoretical work. For example, Datar, Cohen Kulp and Lambert (2001) examine tradeoffs between performance measure sensitivity, precision, and congruity, Dikolli (2001) examines tradeoffs between sensitivity, precision, and an agent's employment horizon, while Christensen, Feltham, Hofmann and Sabac (2004) investigate tradeoffs between timeliness and precision. The model in this paper, however, focuses on the effect that performance measure interrelations and effort interactions (which are not explicitly modeled in the other work) have on the contracting weights.

[10] Note that we have transformed the standard agency model in which each type of effort affects one performance measure in order to analyze how the link between performance measures, and the magnitude of this link, affect the weight placed on each measure in the agent's contract.

When $\gamma > 0$, the agent's cost of effort is higher than the case in which the efforts are independent. Then, the efforts are substitutes; performing one task increases the marginal cost of performing the other task. For complementary efforts, $\gamma < 0$, i.e., performing one task lowers the marginal cost of performing the other task.

The agent has a negative exponential utility function with an Arrow-Pratt measure of absolute risk aversion, r. The utility function is represented by

$$u(w,a_c,a_i) = -\exp[-r(w - \kappa(a_c,a_i))]. \tag{5}$$

With normally distributed uncertainty and negative exponential utility, the agent's problem can be expressed as maximizing the certainty equivalent of the expected utility (Holmstrom and Milgrom (1987, 1994)). The certainty equivalent, which equals the mean value of compensation minus the agent's cost of effort and the risk premium, is expressed as

$$\begin{aligned} \mathrm{CE}(w,a_c,a_i) &= \mathrm{E}[w] - \kappa(a_c,a_i) \\ &\quad - \tfrac{1}{2}\, r\, \{v_i^2\sigma_i^2 + (v_c + \beta_{xc}v_x)[2\, \beta_{ci}\, \sigma_i^2\, v_i + (v_c + \beta_{xc}v_x)(\sigma_c^2 + \beta_{ci}^2\, \sigma_i^2)] + v_x^2\, \sigma_x^2\}. \end{aligned} \tag{6}$$

The principal solves the following problem

$$\max_{f, v_x, v_c, v_i, a_c, a_i} \quad \Pi = \mathrm{E}[x|a_c,a_i] - \mathrm{E}[w] \tag{7a}$$

subject to $\quad \mathrm{CE}(w,a_c,a_i) \geq U$, and (7b)

a_c and a_i maximize $\mathrm{CE}(w,a_c,a_i)$. (7c)

The principal selects the incentive weights to maximize firm profits less the agent's wage, subject to the agent meeting her participation and incentive compatibility constraints. The participation constraint (7b) ensures that the agent earns, in expectation, at least an amount equal to her next best alternative. Without loss of generality, we assume that the agent's expected wage net of effort cost in another position, U, is zero. Finally, the agent selects customer-related and internal-oriented actions (i.e., a_c and a_i) to maximize her expected utility. The incentive compatibility constraint (7c) reflects this condition.

To focus on the interrelationship between performance measures and the complementarity (substitutability) of the efforts, we concentrate on the case in which the interrelations between performance measures are positive (i.e. $\beta_{xc} > 0$ and $\beta_{ci} > 0$).[11] The following two sections present the model analyses.

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[11] In the remainder of the paper, we note instances in which a change in the signs of these sensitivities alters the results.

3. Contracting Implications of Interrelated Measures and Interactive Effort

3.1 Optimal Incentive Contract

Companies monitor many performance measures to evaluate both firm and managerial performance. Often, these performance measures affect one another in a sequential manner.[12] In this section, we investigate how the principal structures the compensation contract offered to the agent in the presence of interrelated performance measures and interactive effort.

The principal solves the problem expressed in (7a) to (7c). The principal can select f such that the participation constraint (7b) is binding. Moreover, we solve the incentive compatibility constraints (7c) to determine the agent's actions. Substituting these actions plus the fixed salary f^* into the objective function, and solving for the closed form solution to the weights in the compensation contract leads to the following proposition.

Proposition 1. The optimal weights assigned to the performance measures in the assumed linear contract are:

$$v_x^* = 0, \tag{8a}$$

$$v_c^* = D^{-1}\,\beta_{xc}\,[1 + r\,(1 - \beta_{ci}\,\gamma)\,\sigma_i^2]\text{ , and} \tag{8b}$$

$$v_i^* = D^{-1}\;r\,\beta_{xc}\,[(\beta_{ci} - \gamma)\,\sigma_c^2 - \beta_{ci}\,(1 - \beta_{ci}\,\gamma)\,\sigma_i^2]\text{ , with} \tag{8c}$$

$$D = 1 + r\,\sigma_i^2 + r\,\sigma_c^2\,[1 + r\,(1 - \gamma^2)\,\sigma_i^2].$$

Proof: All proofs are presented in the Appendix.

Given the assumption that effort only affects profit via the customer performance measure, zero weight is placed on profit in the agent's compensation contract. Holmstrom (1979) suggests that a performance measure should be added to the compensation contract only if the measure offers new information regarding the agent's actions. Given the current model assumptions, the profit measure adds noise to the customer measure without simultaneously adding more information.[13] Therefore, the principal uses the customer and

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[12] Although the performance measures may be related in a bi-directional manner, we assume unidirectional relationships. Additionally, the measures of profit, customer actions, and internal actions may reflect measurements over multiple accounting periods. In order to focus on the effect of the interaction rather than the time effect, we exclude time lags from the analysis.

[13] The customer measure is a sufficient statistic for profit, with respect to a_c and a_i.

internal measures to induce the agent's actions and does not place any weight on the profit measure.

The weight on the customer measure depends on the sensitivity of the profit measure to the customer measure, the sensitivity of the customer measure to the internal measure; the complementarity/substitutability between the agent's efforts; the agent's measure of risk aversion; and the noise in both the customer and internal measures. The customer measure induces the desired amount of customer-related effort and simultaneously induces the agent to perform internal-oriented effort. The weight on the internal measure is used to adjust the agent's internal-oriented effort either up or down to the desired amount. Note that the total weight placed on the internal measure is equivalent to the weighting that would be placed on the measure if each of the measures were redefined to reflect only one type of effort.[14]

Corollary 1. Suppose that all sensitivities and contract weights are non-negative; Table 1 summarizes the changes in the weights on the individual performance measures and in the incentive ratio $IR^* = v_c^*/v_i^*$, given changes in the exogenous variables.

----- Insert Table 1 About Here. -----

Including interactive effort generates comparative statics that differ from the cases studied in prior literature. Accordingly, the results in Corollary 1 illustrate that the formulation of compensation contracts requires careful consideration of the relationships between the measures and between tasks. For example, if efforts do not interact (i.e., γ=0), changes in the interrelation between the customer and internal measure (β_{ci}) will not affect the weight placed on the customer measure (v_c^*) and increases in the interrelation between profit and the customer measure (β_{xc}) lead to increases in the weight placed on the customer measure.[15] As profit becomes more sensitive to the customer effort, the principal will want to induce more customer effort

[14] Because the model implicitly assumes the interrelations between the performance measures are observable, it can be transformed into an equivalent model with separate performance measures that relate to unique efforts. Similarly, because effort interaction is observable, the model can be transformed to include independent effort cost. Appendix B sketches the proof for both transformations. Because our focus is on the incentive weights for performance measures that are included in a business model, our analysis proceeds with a consideration of the untransformed model.

[15] This is easily seen mathematically by inspection of v_c^*; if $\gamma = 0$, then the optimal incentive weight is independent of β_{ci} and increasing in β_{xc}.

by increasing the weight placed on the customer measure.[16] Although profit indirectly captures customer effort, the customer measure is a more "precise" measure of this effort.

Table 1, column 2 summarizes the effects of interrelated measures and interactive effort on the weight placed on the customer measure. If the agent's efforts are complements (i.e. $-1 < \gamma < 0$), the weight placed on the customer measure (v_c^*) increases with increases in the magnitude of the customer-internal measure interrelation (β_{ci}) and with increases in the magnitude of the financial/customer measure interaction (β_{xc}). As these interrelations grow stronger, profits become more sensitive to both customer and internal measures and therefore, to the agent's efforts. When the efforts are complementary, performing one task lowers the marginal cost of performing another task. Thus, it is more cost effective for the principal to induce the two actions together via the customer measure; increases in these interrelations lead to an increase in the weight placed on the customer measure.

When efforts substitute for one another (i.e. $0 < \gamma < 1$), increases in the customer-internal measure interrelation (β_{ci}) reduce the weight on the customer measure. Again, the increase in the interrelation translates into profit being more sensitive to the agent's internal effort. The principal desires more internal effort. However, the customer measure induces *both* types of effort, which is more costly (when efforts are substitutes) than inducing internal effort separately using the internal measure. Consequently, the principal lowers the weight placed on the customer measure. This does not necessarily imply that the principal increases the weight placed on the internal measure. The weight on the internal measure will increase only if the customer-internal measure interrelation is large or the unique internal noise is relatively small.

Interestingly, increases in the substitutability of effort can lead to either an increase or decrease in the weight placed on the customer measure. The direction of the effect depends on the magnitude of the customer-internal measure interrelation (β_{ci}). If efforts are already substitutable and become more substitutable, then with *low* interrelation (i.e., implying profit is relatively insensitive to internal effort) the principal will induce customer effort and "punish" the agent for providing increasingly costly internal effort.

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[16] The sensitivity of profit *both* to the internal measure and to internal effort equals $\beta_{xc}\beta_{ci}$. Accordingly, we refer to the sensitivity of profit to the internal measure and to internal effort analogously throughout the paper.

In other words, when interrelations are low, increases in the cost to induce both types of effort, γ, heightens the need for the principal to induce productive, profit-sensitive customer effort (by increasing the weight on the customer measure) and punishing relatively low-profit generating internal effort (by decreasing the weight on the internal measure).

This intuition is confirmed by the derivative of the weight on the internal measure with respect to γ. When the internal measure interrelation (i.e. β_{ci}) tends to zero, increases in the costs of effort will generate decreases in the internal measure weight.

A higher customer-internal measure interrelation reflects a higher sensitivity of profit to internal effort. In this case the principal wants to induce both types of effort optimally, rather than induce customer effort and punish internal effort. Thus, increases in the cost of effort through γ force the principal to consider the relative importance of the different efforts. At some point, the principal will reduce the weight on the customer measure to reflect a relatively greater importance of the internal effort, through β_{ci}.

Table 1, column 3 reports the effects of interrelated measures and interactive effort on the weight placed on the internal measure. The internal measure weight increases, unambiguously, as the relationship between profit and the customer measure increases. The increase in the sensitivity of profit to internal effort provides an incentive for the principal to induce more internal effort from the agent. However, as highlighted above, the weight on the internal measure moves ambiguously with changes in the link between the customer and internal measures.[17] If the efforts are substitutes and the link between the measures (β_{ci}) is high or the unique noise in the customer measure is high, the weight on the internal measure will increase. Under such circumstances, the principal prefers using the internal measure to motivate internal effort.

When profit is highly sensitive to internal effort, the intuition for the effect of changes in the substitutability of effort on changes in the weight on the internal measure is more complex. The comparative static indicates that the principal must weigh (1) the relative noise in the internal measure and its "parent," the customer measure; (2) the *magnitude* of the sensitivity of profit to the internal measure (i.e. through β_{ci});

[17] Even if $\beta_{ci} < 0$, the weight on the internal measure increases as β_{xc} increases. In such a situation, it can be shown that efforts must be complementary. Thus, the benefits from the complementary nature of the efforts exceed the cost that occurs due to the negative impact of the internal measure on profit.

and (3) whether the effort is substitutable or complementary. In some cases, these three effects offset each other, causing the principal to *increase* the weight on the internal measure if substitutability of effort increases and *decrease* the weight otherwise.

Table 1, column 4 reports the effects of interrelated measures and interactive effort on changes in the incentive ratio (IR^*). Changes in the incentive ratio characterize how the principal adjusts the *relative* weight on the two performance measures, given changes in the exogenous variables. While both incentive weight increase in the weight placed on the customer measure (β_{xc}), the relative weight is not affected by this increase. As this interrelation increases, profit becomes more sensitive to both customer and internal effort, without affecting the relative informativeness of the customer and internal measure. Thus, the incentive weights placed on both measures increase to the same degree, such that the relative weight is constant.

Interestingly, increases in the customer-internal measure interrelation can lead to either an increase or decrease in the weight placed on the customer measure relative to the internal measure. The direction of the effect depends on a subtle relation of the degree of complementarity or substitutability, the magnitude of customer-internal measure interrelation, and the unique noise of the internal measure (σ_i^2). In general, while the increase in the sensitivity of profit to internal effort provides an incentive to the principal to induce more internal effort, for sufficient effort interactions it is more cost-effective to use the customer measure more intensely as compared with the internal measure. However, given a low interrelation and a relatively noisy internal measure, it is more cost-effective to increase the relative weight on the customer measure for any degree of complementarity or substitutability. The key to the latter result is the relative noise of the two performance measures, and that for a low interrelation only a marginal spill-over of the internal measure's noise occurs.

Finally, the weight on the customer measure relative to the internal measure can either increase or decrease with changes in the effort interaction. For complementary and substitutable efforts, the direction of change depends on the customer-internal measure interrelation. With more substitutable (less complementary) efforts and low interrelations, the principal prefers to induce more customer effort and less internal effort, and this is achieved by increasing the weight on the customer measure and decreasing the weight on the internal measure.

Overall, these results illustrate that changes in the (relative) weights placed on the performance measures depend on the relationship between the efforts, the strength of the links between the measures, and the relative noise levels of the measures.

3.2 Forgoing the Use of Performance Measures

The weights on the customer and internal measures in Proposition 1 can theoretically be negative. For instance, this will arise if the customer measure is relatively insensitive to internal effort (i.e., β_{ci} tends to zero). In such a case the principal will use the incentive weights to punish the agent for performing (unproductive) internal effort.[18] In a similar vein, Smith (2002) analytically derives a negative weight on a customer satisfaction measure. He argues that in practice, a firm might choose to contract only on profit in such instances. In practical terms, it is difficult for contract designers to operationalize negative weights in incentive contracts. Consistent with this view and for ease of presentation, we assume the weights on the customer and internal measures are non-negative in the following corollaries.

Proposition 1 implies that there are ranges in which the customer measure or the internal measure will not be used in the contract (i.e., the v_c^* or v_i^* weight is equal to or less than zero). Corollaries 2, 3, and 4 identify these ranges.

Corollary 2. Assuming all sensitivities and contract weights are non-negative, the customer measure is not used when:

$$\gamma \geq (\beta_{ci} r \sigma_i^2)^{-1} (1 + r\sigma_i^2). \tag{9}$$

Given the non-negative assumption regarding the sensitivity to internal-oriented effort (i.e., β_{ci}), this condition never holds if the efforts are complementary. Therefore, with complementary efforts, the customer measure will always be used. The customer measure is ignored when the efforts are substitutable, and the degree of substitutability exceeds the above ratio. The substitutability of efforts results in a larger marginal

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[18] A natural question here is: if the principal does not want internal effort, why not simply place a zero weight on the internal measure? Theoretically, by providing an effective disincentive to the agent through a negative weight on the internal measure, the principal is able to infer more about the productive customer effort (because customer and internal efforts are intrinsically sensitive to each other through γ) than would be the case if the principal relied on the customer measure alone.

cost of inducing both efforts rather than inducing only one effort. When the efforts become "too" substitutable, it is no longer advantageous to induce both tasks; the cost exceeds the gain derived from customer-related effort. Consequently, the principal does not weight the customer measure, induces only internal-oriented effort, and earns profit based on the flow-through of this effort.

Corollary 3. Assuming all sensitivities and contract weights are non-negative, the principal does not place any weight on the internal measure, beyond that placed on it through v_c, when the following condition holds:

$$\gamma \geq (\sigma_c^2 - \beta_{ci}^2\sigma_i^2)^{-1}\,\beta_{ci}\,(\sigma_c^2 - \sigma_i^2). \tag{10}$$

When this condition holds, the principal prefers to use an aggregate measure that reflects both efforts rather than using the customer and internal measures separately, even if the tasks are substitutes. He weights the internal measure the same as the customer measure multiplied by the sensitivity of the customer measure to the internal measure. Note that this condition can hold when the efforts are either complementary or substitutable.

The range in which the internal measure is not used depends on the relative noise in the two measures. If the unique noise in the internal measure is high relative to the unique noise in the customer measure and the sensitivity of the customer measure to the internal measure (β_{ci}) is low, the principal will refrain from imposing more risk on the agent and use only the customer measure.

We often observe contracts that weight aggregate measures and "ignore" disaggregate measures (similar to this) in practice. For example, companies often base compensation on a percentage of profits. By doing so, they implicitly link compensation to a percentage of revenues less the same percentage of costs. Consequently, placing weight on the aggregate measure will induce the correct allocation of efforts.

When two different types of efforts are related, the cost of performing one task affects the cost of performing the other task. For example, if two tasks are complements, performing one task reduces the marginal cost of performing the other task. Consequently, inducing the first task via an aggregate performance measure will also induce some of the second task. At some point, the spillover effect will eliminate the demand for a separate performance measure of the second type of effort.

This elimination of the demand for a separate performance measure arises independent of the quality of the performance measures available. Thus, a "good" performance measure (e.g. high signal-to-noise ratio) of, for example, internal effort may be driven out by a "bad" (e.g. low signal-to-noise ratio) performance measure of customer effort, because the principal wants to induce customer effort.[19]

Corollary 4. Assuming all sensitivities and contract weights are non-negative, the principal places a combined weight of zero on the internal measure and gives no incentive for the internal action when the following holds:

$$\gamma \geq (r\sigma_c^{2})^{-1}\, \beta_{ci}\, (1 + r\sigma_c^{2}). \tag{11}$$

The above condition identifies a range in which the principal does not desire any internal oriented effort. The right-hand side of the expression in (11) is always positive, which implies that γ must be positive for the condition to hold. This in turn implies that for (11) to hold the efforts must be substitutes. The intuition here is that for a sufficiently high level of substitutes (i.e., sufficiently high γ), motivating both tasks is too costly for the principal. Thus, he only motivates the task of customer-oriented effort. Additionally, the larger β_{ci} is, the less likely (11) will hold. If a high β_{ci} causes (11) to not hold and yet efforts are substitutable, the principal will still motivate both tasks. Finally, internal-oriented effort is always induced if efforts are complementary, since performing one task lowers the agent's marginal cost of performing another; the principal always prefers the agent to perform both tasks (assuming that they both positively impact profit).

In summary, these results imply that the way in which efforts interact will directly affect how interrelated measures are included in incentive contracts. These effects are distinctly different from sensitivity, precision, and congruity effects found in prior research on contractible performance measures in multi-task settings. Thus, while the notion of contracting on relatively more precise and sensitive performance measures that ultimately map into profit seems appealing, the contract should be carefully designed to account for the interactions between the performance measures and between the agent's tasks.

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[19] To illustrate, consider the case in which the noise in the internal measure is close to zero. Even when β_{ci} is relatively low, the signal-to-noise ratio of the internal measure will be high, implying that the internal measure is a "good" performance measure. Yet, if γ is significantly larger than β_{ci}, Corollary 3 suggests that the "good" internal measure will be ignored, irrespective of how "bad" is the signal-to-noise ratio of the customer measure.

4. Profit Implications of Interrelated Measures and Interactive Effort

In this section, we examine the profit impact of contracting on interrelated performance measures in the presence of interactive effort. We aim to understand the conditions under which contracting on functionally related measures leads to incrementally profitable outcomes.

We first substitute the incentive weights from equations (8a), (8b), and (8c) into the principal's objective function to generate a profit function comprised of only exogenous parameters.[20] We then take the derivative of the profit function with respect to the parameters that represent the interrelationship between i) the profit and customer measures (i.e., β_{xc}) and ii) the customer and internal measures (i.e., β_{ci}). We analyze the results for both the complementary efforts and the substitute efforts separately.

Complementary effort arises where performing one type of effort reduces the marginal cost of performing an alternative type of effort. We define complementary effort formally as the cases where $-1 < \gamma < 0$. This definition implies that the overall cost of effort incurred by (and payable to) the agent is lower than would be incurred if the efforts were independent.

Taking the derivative of profit each with respect to β_{xc} and β_{ci}, respectively, gives the following results:

$$\partial\Pi/\partial\beta_{xc} = (1-\gamma^2)^{-1}D^{-1}\,\beta_{xc}\,[1-2\,\gamma\,\beta_{ci} + \beta_{ci}^{\,2} + r\,((\beta_{ci}-\gamma)^2\,\sigma_c^{\,2} + (1-\beta_{ci}\gamma)^2\,\sigma_i^2)] > 0, \tag{12a}$$

$$\partial\Pi/\partial\beta_{ci} = (1-\gamma^2)^{-1}D^{-1}\,\beta_{xc}^{\,2}\,[(\beta_{ci}-\gamma)(1 + r\,\sigma_c^{\,2}) - r\,\gamma\,(1-\beta_{ci}\gamma)\,\sigma_i^2]. \tag{12b}$$

Analyzing these results leads to the following Proposition.

Proposition 2. With complementary effort (i.e., $-1 < \gamma < 0$), any increase (decrease) in the given performance measure interrelation (i.e., β_{xc} or β_{ci}) will lead to an increase (decrease) in the profit of the firm.

Proposition 2 suggests that if efforts are complementary, the more sensitive the aggregate performance measures are to subordinate performance measures, the higher will be the profits of the firm. By assumption, higher sensitivities between the performance measures translate into greater profit effects and complementary efforts translate into a lower cost. These two effects both positively impact profits.

If efforts are substitutable and the agent selects one type of effort, the cost of selecting the other type of

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[20] A statement of the profit net of effort cost is provided in the proof to Proposition 2.

effort is marginally higher. The agent incurs additional costs to achieve the optimal levels of effort, given that both efforts are desired. Formally, substitutable efforts correspond to the cases where $0 < \gamma < 1$. Applying this restriction to equations (12a) and (12b), it is clear that there exist conditions under which profits may decrease as the interrelations increase. These conditions are highlighted in the following proposition.

Proposition 3. An increase in the interrelation between the customer and internal measure will lead to a decrease in firm profits, if effort is substitutable (i.e., $0 < \gamma < 1$) and $\beta_{ci} < \underline{\beta}(\gamma)$,

$$\text{where } \underline{\beta}(\gamma) = \frac{\gamma(1 + r(\sigma_c^2 + \sigma_i^2))}{1 + r(\sigma_c^2 + \gamma^2\sigma_i^2)} .$$

Contrary to standard intuition, Proposition 3 indicates that an increase in the sensitivity between the customer and internal performance measures may decrease profits, given substitutable efforts. For example, profits will decrease if the customer-internal measure interrelation is rather low. Increases in the sensitivity between performance measures alone lead to higher firm profits. However, the high cost associated with inducing both efforts simultaneously exceeds the profit increase. Consequently, overall profits decline.

------ Insert Figure 2 About Here. ------

In Figure 2, we plot the relationship between the firm's profits and the interrelation between the customer and the internal measures (β_{ci}). For illustration purposes we assume parameters of $\gamma = ½$; $\underline{x} = 0$; $\beta_{xc} = r = \sigma_j = 1$, $j = i, c$.[21] Note that for values where $\beta_{ci} < \underline{\beta}(\gamma) = 2/3$, the effect of interactive effort on the cost of effort outweighs the benefit from contracting on a relatively more precise performance measure that is functionally related to the profits of the firm. Accordingly, profits are decreasing in β_{ci} when this condition holds. Conversely, when $\beta_{ci} > \underline{\beta}(\gamma)$ (i.e., where $\beta_{ci} > 2/3$), profits are increasing in the interrelation between the customer measure and the internal measure. In this range, the benefits from contracting on a more precise disaggregate performance measure and inducing a more profitable action outweigh any incremental costs from the effort interaction.

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[21] The results and intuition generalize to other parameter values.

It is clear from Propositions 2 and 3 that this paper's key results center on the case of substitute efforts. It is under conditions where substitute efforts play a dominant role that the principal may find it in her best interests to reduce the emphasis placed on disaggregate performance measure regardless of the properties (i.e., sensitivity, precision, and congruence) of that measure. Assigning substitute tasks to employees is not uncommon. For example, managers or supervisors are often responsible for customer service as well as for (1) monitoring theft by either employees or customers, (2) training other employees, or (3) supervising/implementing physical changes to the logistics and presentation of the retail environment. Additionally, managers responsible for both customer satisfaction and employee satisfaction may encounter substitutable effort choices. A manager can conveniently keep an employee satisfied by devoting significant attention to coaching, mentoring, and demonstrating ways for subordinates to improve their performance. However, the ability of the manager to simultaneously attend to customers maybe compromised.

Overall, these results illustrate that while incorporating interrelated performance measures into incentive contracts can often increase firm profits, there are conditions under which such contracting translates into lower profits. Before firm management writes contracts for its employees, they must carefully evaluate how the performance metrics relate to one another (i.e., functional relationships) and the types of tasks that the employee is expected to perform (i.e., complements, substitutes, or independent).

5. Conclusion

It is common to find both performance measures and managerial tasks that interact with one another. The outcome of two different measures is often functionally related. Additionally, managers' actions may be complements or substitutes for one another. Both types of interactions affect the incremental information learned from the various performance measures, the risk placed on the manager through her employment contract and thus, the compensation contract offered to the manager. Some measures, such as profit, appear informative and are commonly used in compensation contracts. However, given the links between performance measures and the firm's beliefs about which performance measures the employee directly impacts, the profit measure may not be useful in contracting.

In this paper, we investigate the effect of interrelated performance measures and effort interactions on the compensation contract offered to the agent, independent of standard sensitivity, precision and congruity effects. We use a simplified principal-agent LEN model to focus on these effects. The findings indicate that the nature of these interrelationships and interactions affect the weights placed on the various measures in the assumed linear contract. In particular, these effects can change the magnitude of optimal weights used and whether or not a performance measure, which is informative about effort, will be used in the contract, independent of the sensitivity, precision, or congruity of the measures.

Moreover, we find that under certain conditions increases in performance measure interrelations that (ceteris paribus) translate into increased profits, can in fact lead to reductions in profit if, at the same time, the agent performs different types of substitutable efforts. Changes in the performance measure interrelations generate changes in the worker's effort allocation between different activities. If the tasks are substitutes, a change in the employee's effort allocation may lead to higher effort costs and therefore higher expected compensation costs. If the additional compensation costs outweigh the profit increases from the higher sensitivities, total profits will be reduced. Thus, if a firm's contract designers intend to change incentive weights based on changes in interrelations between performance measures, they must simultaneously consider the relationship between the agent's tasks.

Future research can further analyze and extend the model in this paper. We observe many companies that add the individual performance metrics together and weight this sum in the compensation contract. It may be possible to expand the current analysis to investigate conditions under which this simplified weighting scheme leads to the same level of profits as the incentive scheme currently studied. Additionally, there is often a non-linear relationship between performance measures. For example, increased customer satisfaction may not increase profit while declines in customer satisfaction (below a threshold) significantly harm the firm's profits. The model can be extended to analyze the implication of such non-linear relationships between performance measures.

Table 1
Comparative Statics of Optimal Incentive Weights

	(1)	*(2)*	*(3)*	*(4)*
Parameter j	$\partial v_x^*/\partial j$	$\partial v_c^*/\partial j$	$\partial v_i^*/\partial j$	$\partial IR^*/\partial j$
β_{xc}	$= 0$	> 0	> 0	$= 0$
β_{ci}	$= 0$	> 0 if $\gamma < 0$	> 0 if $\gamma > \gamma_{l1}$	> 0 if $\gamma \notin [\gamma_{l2},\gamma_h]$
γ	$= 0$	> 0 if $\beta_{ci} < \beta_{h1}$	> 0 if $\beta_{ci} > \beta_{h2}$	> 0 if $\beta_{ci} < \beta_{h3}$

where:[22]

β_{xc}, β_{ci}, and γ are defined as in Figure 1,

$\beta_{h1} \equiv [1 + r\sigma_i^2 + r\sigma_c^2 (1 + r (1 + \gamma^2)\sigma_i^2)]^{-1} 2 \gamma r\sigma_c^2 (1 + r\sigma_i^2)$,

β_{h2} is the $\beta_{ci} > 0$ that solves $(\beta_{ci}^2\sigma_i^2 - \sigma_c^2)[1 + r\sigma_i^2 + r\sigma_c^2 (1 + r (1 + \gamma^2)\sigma_i^2)] + 2 \gamma \beta_{ci} r^2\sigma_c^2\sigma_i^2 (\sigma_c^2 - \sigma_i^2) = 0$,

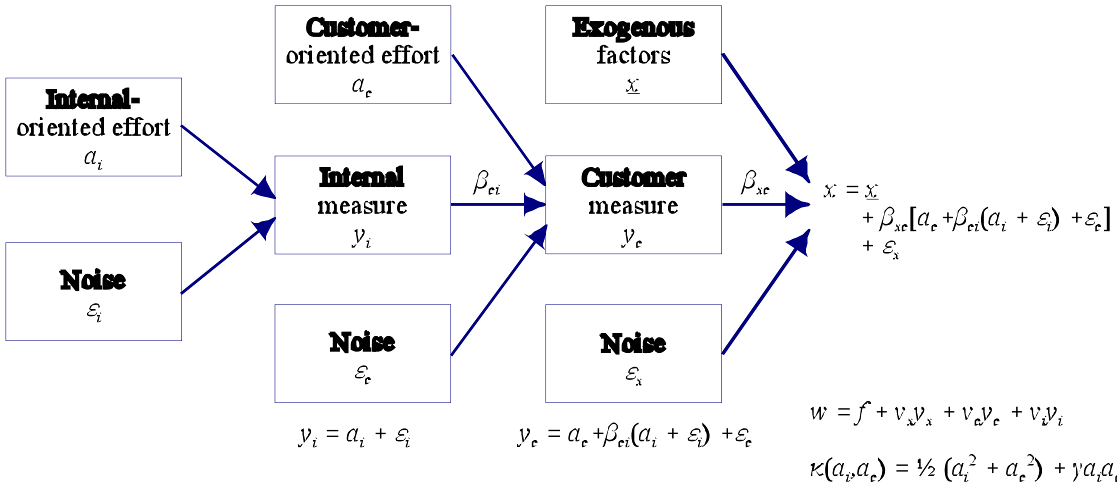
$\beta_{h3} \equiv ([(1 + r\sigma_i^2) \sigma_c^2] / [(1 + r\sigma_c^2) \sigma_i^2])^{1/2}$,

$\gamma_{l1} \equiv \frac{1}{2} \beta_{ci}^{-1} (\sigma_i^2 - \sigma_c^2) / \sigma_i^2$, and

γ_{l2} and γ_h solve $\sigma_i^2 [r \gamma (\sigma_c^2 + \beta_{ci}^2\sigma_i^2) - \beta_{ci} (1 + r\sigma_i^2)]^2 = (1 + r\sigma_i^2) [(1 + r\sigma_c^2)\beta_{ci}^2\sigma_i^2 + r\sigma_c^2 (\sigma_c^2 - \sigma_i^2)]$.

[22] See the proof to Corollary 1 for the derivation of the expressions representing the cutoff values.

Figure 1: Pictorial View of Model



Where:

γ ≡ magnitude of interaction between customer and internal effort

β_{xc} ≡ magnitude of relation between profit and customer measure

β_{ci} ≡ magnitude of relation between customer and internal measure

f ≡ fixed salary

v_x ≡ incentive weight on profit measure

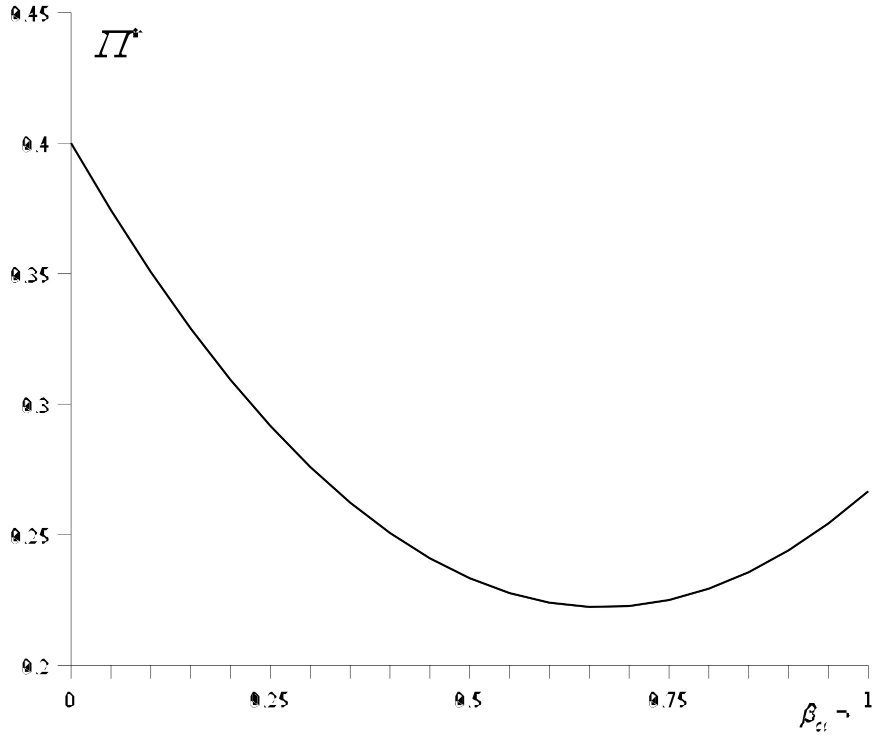
v_c ≡ incentive weight on customer measure

v_i ≡ incentive weight on internal measure

w ≡ agent's wage

$\kappa(a_i, a_c)$ ≡ agent's cost of effort

Figure 2: Graphical Illustration of Proposition 3



This graphs represents a plot of the relationship between firm profits (i.e., Π^*) and the interrelation between the customer measure and internal measure (i.e., β_{ci}), assuming the following parameters: $\gamma = ½$; $\underline{x} = 0$; $\beta_{xc} = r = \sigma_j = 1, j = i, c$.

Appendix A:
Proof of Propositions

Proof of Proposition 1

The agent's certainty equivalent expands to:

$$\mathrm{CE}(w,a_c a_i) = f + (v_x\,\beta_{xc} + v_c)(a_c + \beta_{ci} a_i) + v_i\,a_i - (½\,a_c^{\,2} + ½\,a_i^2 + \gamma\,a_c\,a_i)$$

$$- ½\,r\,\{v_i^2\sigma_i^2 + (v_c + \beta_{xc}v_x)[2\,\beta_{ci}\,\sigma_i^2\,v_i + (v_c + \beta_{xc}v_x)(\sigma_c^{\,2} + \beta_{ci}^{\,2}\,\sigma_i^2)] + v_x^{\,2}\,\sigma_x^{\,2}\}. \tag{A.1}$$

The first-order conditions on the agent's actions a_i and a_c are:

$$-a_i - \gamma\,a_c + v_i + \beta_{ci}v_c + \beta_{xc}\beta_{ci}v_x = 0,\text{ and} \tag{A.2a}$$

$$-\gamma\,a_i - a_c + v_c + \beta_{xc}v_x = 0. \tag{A.2b}$$

Solving these equations for a_i and a_c gives:

$$a_i^* = (1 - \gamma^2)^{-1}\,[v_i + (v_c + \beta_{xc}v_x)(\beta_{ci} - \gamma)],\text{ and} \tag{A.3a}$$

$$a_c^{\,*} = (1 - \gamma^2)^{-1}[-\gamma\,v_i + (v_c + \beta_{xc}v_x)(1 - \beta_{ci}\,\gamma)]. \tag{A.3b}$$

Substituting the solutions for a_i^* and $a_c^{\,*}$ into the objective function, taking first order conditions of the objective function with respect to v_x, v_c, and v_i, respectively, and solving simultaneous equations gives the weights placed on the performance measures, as presented in equations (8a), (8b), and (8c).

The total weight placed on the internal measure is:

$$v_i^{\,*} + \beta_{ci}\,v_c^{\,*} = D^{-1}\,\beta_{xc}\,[\beta_{ci} + r\,(\beta_{ci} - \gamma)\,\sigma_c^{\,2}]. \tag{A.4}$$

■

Proof of Corollary 1

Take the derivatives of each of the absolute weights with respect to the exogenous parameters. First,

$$\partial v_c^{\,*}/\partial\beta_{xc} = D^{-1}\,[1 + r\,(1 - \beta_{ci}\,\gamma)\,\sigma_i^2]. \tag{A.5}$$

Since $|\gamma| < 1$, the denominator is positive. With positive incentive weights (in particular, $v_c^{\,*} > 0$), the numerator is positive.

$$\partial v_c^{\,*}/\partial\beta_{ci} = -D^{-1}\,\gamma\,\beta_{xc}\,r\sigma_i^2. \tag{A.6}$$

Similar to equation (A.5), the denominator is positive. Consequently, if γ is negative, the derivative will be positive.

$$\partial v_c^*/\partial\gamma = -D^2\, r\, \beta_{xc}\, \sigma_i^2\, \{-2\,\gamma\, r\sigma_c^2\,(1 + r\sigma_i^2) + \beta_{ci}\,[1 + r\sigma_i^2 + r\sigma_c^2\,(1 + r\,(1 + \gamma^2)\sigma_i^2)]\}. \qquad (A.7)$$

In equation (A.7) the denominator is positive. Consequently, if the numerator is negative, the derivative will be positive. This occurs when

$$\beta_{ci} < \beta_{h1} \equiv [1 + r\sigma_i^2 + r\sigma_c^2\,(1 + r\,(1 + \gamma^2)\sigma_i^2)]^{-1} 2\,\gamma\, r\sigma_c^2\,(1 + r\sigma_i^2).$$

$$\partial v_i^*/\partial\beta_{xc} = D^{-1}\, r\ \ [(\beta_{ci} - \gamma)\,\sigma_c^2 - \beta_{ci}\,(1 - \beta_{ci}\,\gamma)\,\sigma_i^2]. \qquad (A.8)$$

The denominator in equation (A.8) is positive. With positive incentive weights (in particular, v_i^* >0), the numerator is positive.

$$\partial v_i^*/\partial\beta_{ci} = D^{-1}\, r\, \beta_{xc}\,[\sigma_c^2 - (1 - 2\,\beta_{ci}\,\gamma)\,\sigma_i^2]. \qquad (A.9)$$

The derivative is positive if the numerator is positive. This is the case if

$$\gamma > \gamma_{l1} \equiv \tfrac{1}{2}\,\beta_{ci}^{-1}\,(\sigma_i^2 - \sigma_c^2)\,/\,\sigma_i^2.$$

$$\partial v_i^*/\partial\gamma = D^{-2} r\beta_{xc}\{(\beta_{ci}^2\sigma_i^2 - \sigma_c^2)[1 + r\sigma_i^2 + r\sigma_c^2\,(1 + r\,(1 + \gamma^2)\sigma_i^2)] + 2\,\gamma\,\beta_{ci}\, r^2\sigma_c^2\sigma_i^2\,(\sigma_c^2 - \sigma_i^2)\}. \qquad (A.10)$$

While $\beta_{ci} = 0$ yields $\partial v_i^*/\partial\gamma < 0$, $\partial v_i^*/\partial\gamma > 0$ if β_{ci} sufficiently positive. Setting the expression in curly brackets equal to zero and solving for β_{ci} yields β_{h2}. Thus, $\partial v_i^*/\partial\gamma > 0$ if $\beta_{ci} > \beta_{h2}$.

The incentive ratio is given by

$$IR^* \equiv \frac{v_c^*}{v_i^*} = \frac{1 + r(1 - \beta_{ci}\gamma)\sigma_i^2}{r[(\beta_{ci} - \gamma)\sigma_c^2 - \beta_{ci}(1 - \beta_{ci}\gamma)\sigma_i^2]}.$$

$$\partial IR^*/\partial\beta_{xc} = 0. \qquad (A.12)$$

$$\partial IR^*/\partial\beta_{ci} = \frac{[(1 - 2\beta_{ci}\gamma)\sigma_i^2 - \sigma_c^2]\,(1 + r\sigma_i^2) + r\gamma^2\sigma_i^2(\beta_{ci}^2\sigma_i^2 + \sigma_c^2)}{r[(\beta_{ci} - \gamma)\sigma_c^2 - \beta_{ci}(1 - \beta_{ci}\gamma)\sigma_i^2]^2}.$$

Assuming positive incentive weights, the denominator is positive. Therefore, the sign of (A.13) depends on the sign of the numerator. Rearranging the condition for a positive numerator yields

$$\sigma_i^2\,[r\,\gamma\,(\sigma_c^2 + \beta_{ci}^2\sigma_i^2) - \beta_{ci}\,(1 + r\sigma_i^2)]^2 > (1 + r\sigma_i^2)\,[(1 + r\sigma_c^2)\beta_{ci}^2\sigma_i^2 + r\sigma_c^2\,(\sigma_c^2 - \sigma_i^2)]. \qquad (A.14)$$

First, assume that the right hand side of (A.14) is positive. Setting the left hand side equal to the right hand side and solving for γ yields γ_{l2} and γ_h. Then, (A.14) is true for $\gamma \notin [\gamma_{l2},\gamma_h]$. Note, however, that the upper and lower borders of the interval can take on values that are absolutely larger or equal to one, and that the interval can be empty, i.e., the numerator is positive for any $|\gamma| < 1$. The latter result follows, e.g., if the right hand side of (A.14) is negative, which is the case if $\sigma_i > \sigma_c$ and β_{ci} sufficiently small.

$$\partial IR^*/\partial\gamma = \frac{\sigma_c^2(1+r\sigma_i^2) - \beta_{ci}^2\sigma_i^2(1+r\sigma_c^2)}{r[(\beta_{ci}-\gamma)\sigma_c^2 - \beta_{ci}(1-\beta_{ci}\gamma)\sigma_i^2]^2}.$$

Similar to (A.13), assuming positive incentive weights, the denominator is positive. The numerator is positive if β_{ci} is sufficiently small. In particular, the numerator is positive if

$$\beta_{ci} < \beta_{h3} \equiv ([(1 + r\sigma_i^2)\,\sigma_c^2] \,/\, [(1 + r\sigma_c^2)\,\sigma_i^2])^{½}. \qquad ■$$

Proof of Corollary 2

The weight on the customer measure will be zero or negative when the numerator of (8b) is non-positive. Assuming that all parameters are nonzero, this occurs when $1 + r\,(1 - \beta_{ci}\,\gamma)\,\sigma_i^2 \leq 0$. Solving for γ, this occurs when $\gamma \geq (\beta_{ci} r\sigma_i^2)^{-1}(1 + r\sigma_i^2)$. ■

Proof of Corollary 3

The weight on the internal measure alone is zero or negative when the numerator of (8c) is non-positive. This occurs when $\gamma \geq (\sigma_c^2 - \beta_{ci}^2\sigma_i^2)^{-1}\,\beta_{ci}\,(\sigma_c^2 - \sigma_i^2)$. ■

Proof of Corollary 4

The internal-oriented task is not motivated when the combined weight on the internal and customer measure is zero or negative. This occurs when $v_i^* + \beta_{ci}\,v_c^* \leq 0$. From (A.4), the denominator of this sum is positive. Setting the numerator smaller than or equal to zero and solving for γ gives

$$\gamma \geq (r\sigma_c^2)^{-1}\,\beta_{ci}\,(1 + r\sigma_c^2). \qquad ■$$

Proof of Proposition 2

The principal's net payoff will be the gross profits derived by the firm minus the expected wage payable. Assuming the agent's participation constraint binds, the expected wage payable is equal to the costs of effort plus the risk premium payable to the agent. After substituting optimal effort levels and incentive weights, the net payoff reduces to the following expression:

$$\Pi^* = \underline{x} + ½\,(1-\gamma^2)^{-1}D^{-1}\,\beta_{xc}^2\,[1 - 2\,\gamma\,\beta_{ci} + \beta_{ci}^2 + r\,((\beta_{ci}-\gamma)^2\,\sigma_c^2 + (1-\beta_{ci}\gamma)^2\,\sigma_i^2)]. \qquad \text{(A.16)}$$

Taking the derivative of the above equation with respect to β_{xc} and β_{ci} (separately) generates (12a) and (12b). First, the derivative in (12a) is positive. Next, apply the assumption that $-1 < \gamma < 0$ to (12b) and note that this expression becomes unambiguously non-negative. ■

Proof of Proposition 3

Setting $\beta_{ci} = 0$ in (12b) yields $\partial \Pi^*/\partial \beta_{ci} < 0$ for any $0 < \gamma < 1$. On the other hand, the numerator in (12b) increases in β_{ci}. In particular, the numerator is positive (yielding $\partial \Pi^*/\partial \beta_{ci} > 0$) if

$$\beta_{ci} > \underline{\beta} \equiv \frac{\gamma(1 + r(\sigma_c^2 + \sigma_i^2))}{1 + r(\sigma_c^2 + \gamma^2 \sigma_i^2)}.$$

Hence, for $\beta_{ci} < \underline{\beta}$ and $0 < \gamma < 1$, $\partial \Pi^*/\partial \beta_{ci} < 0$. ■

Appendix B
Transformation of the Model

(A) Equivalent Model with Separate Performance Measures Relating to Unique Efforts

The interrelated performance measures can be redefined to eliminate the interaction. For example, let

$$x' = x - \beta_{xc}\, y_c, \tag{B.1a}$$

$$y_c' = y_c - \beta_{ci}\, y_i, \text{ and} \tag{B.1b}$$

$$y_i' = y_i. \tag{B.1c}$$

The agent's contract in terms of the adapted measures is $w = f + \upsilon_x\, x' + \upsilon_c\, y_c' + \upsilon_i\, y_i'$. Similar to the procedure described in the proof to Proposition 1, we can solve for the optimal incentive rates given the adapted measures. Alternatively, substituting the definition of the adapted measures, i.e., (B.1a), (B.1b), and (B.1c), into the agent's contract, and rearranging yields

$$w = f + \upsilon_x\, x + (\upsilon_c - \beta_{xc}\, \upsilon_x)\, y_c + (\upsilon_i - \beta_{ci}\, \upsilon_c)\, y_i.$$

Piecewise comparison yields the following conditions for an equivalent contract:

$$\upsilon_x = v_x, \tag{B.2a}$$

$$\upsilon_c - \beta_{xc}\, \upsilon_x = v_c, \text{ and} \tag{B.2b}$$

$$\upsilon_i - \beta_{ci}\, \upsilon_c = v_i. \tag{B.2c}$$

Since an optimal contract implies $v_x^* = 0$, the incentive weights for the adapted customer and internal measures are equivalent to the incentive weights for the original measures (i.e., $\upsilon_c^* = v_c^*$ and $\upsilon_i^* = v_i^*$).

(B) Equivalent Model with Independent Effort Costs

We redefine the effort levels to eliminate the interaction and express the principal's decision problem in terms of the redefined tasks. For example,

$$e_i = \lambda\, a_i + \tfrac{1}{2}\, \gamma / \lambda\, a_c \text{ and} \tag{B.3a}$$

$$e_c = \tfrac{1}{2}\, \gamma / \lambda\, a_i + \lambda\, a_c. \tag{B.3b}$$

The agent's cost of effort in the redefined tasks is $\kappa(e_c,e_i) = \tfrac{1}{2}\,(e_c^2 + e_i^2)$, i.e., no interaction exists for the redefined tasks. Substituting (B.3a) and (B.3b), and simplifying yields

$$\kappa(a_c,a_i) = 1/8\, (\gamma^2 + 4\, \lambda^4) / \lambda^2\, (a_c^2 + a_i^2) + \gamma\, a_c\, a_i.$$

In particular, $\kappa(a_c,a_i) = ½ (a_c^2 + a_i^2) + \gamma\, a_c\, a_i$, if, e.g., $\lambda = \underline{\lambda} \equiv (½ [1 + (1 - \gamma^2)^{½}])^{½}$. Note that $\gamma = 0$ results in $\underline{\lambda} = 1$, such that $e_j = a_j$, $j = i,c$.

Solving (B.3a) and (B.3b) for a_c and a_i yields

$$a_i = (4\, \lambda^4 - \gamma^2)^{-1}\, 2\, (2\, \lambda^3\, e_i - \gamma\, \lambda\, e_c) \text{ and} \tag{B.4a}$$

$$a_c = (4\, \lambda^4 - \gamma^2)^{-1}\, 2\, (2\, \lambda^3\, e_c - \gamma\, \lambda\, e_i). \tag{B.4b}$$

Substituting (B.4a) and (B.4b) into (2a) and (2b) yields performance measures in terms of the redefined tasks, i.e.,

$$y_c'' = (4\, \lambda^4 - \gamma^2)^{-1}\, 2\, (2\, \lambda^3\, e_c - \gamma\, \lambda\, e_i) + \beta_{ci}\, y_i + \varepsilon_c, \text{ where } \varepsilon_c \sim N(0,\sigma_c^2), \text{ and} \tag{B.5a}$$

$$y_i'' = (4\, \lambda^4 - \gamma^2)^{-1}\, 2\, (2\, \lambda^3\, e_i - \gamma\, \lambda\, e_c) + \varepsilon_i, \text{ where } \varepsilon_i \sim N(0,\sigma_i^2). \tag{B.5b}$$

Similar to the procedure described in the proof to Proposition 1, we can now solve for the optimal incentive rates given the redefined tasks ((B.3a) and (B.3b)) with *independent* effort costs (i.e., for $\lambda = \underline{\lambda}$) and using adapted measures ((B.5a) and (B.5b)). Hence, interactive effort essentially has a similar effect as using adapted aggregate performance measures, where the degree of interaction (γ) affects the sensitivity of the adapted measures to each action.

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