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Design of Parameter Adaptive Fuzzy Controller for the Planar Double Inverted Pendulum

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Abstract. In this paper, The mathematical model is established using analytical dynamics method for planar double inverted pendulum, based on optimal control theory, parameter adaptive fuzzy controller is designed by use of variable fusion technology, thereby it can reduce the dimension of the input variables of the fuzzy controller, greatly decrease the number of the fuzzy rules, and the impact of the quantization factor on the control effectiveness is studied. The designed controller for system is applied in simulation experiments, the results show that the fuzzy controller can guarantee good control precision, fast response, control planar inverted pendulum stability.

Introduction

Inverted pendulum system has become a typical experiment platform to study various control theory and test control effects. Many researchers have been regarded as a typical object of study, constantly from which to explore new control strategies, related to scientific research gets a broad application in space technology and robotics. Inverted pendulum system controller based on the design of classical and modern control theory has been common, with the development of intelligent control algorithm research, the superiority of intelligent control method such as the neural network and fuzzy control has been generally recognized by the control community [1], scholars from various countries have undertaken extensive research on fuzzy control of inverted pendulum system.

Such as planar double inverted pendulum typical multi-variable system, with the complexity of the system, the total number of fuzzy rules will increase with the number of input variables index, namely, "the explosion of fuzzy rules is not conducive to real-time control”. With the help of the fusion functions which can merge state variables, this paper solves the fuzzy rule explosion problem. But state merger has greatly increased the difficulty of the development of fuzzy rules, make fuzzy control rules difficult to formulate and understand. In this paper, parameter adaptive fuzzy controller is designed based on fuzzy control theory [2]. It can change quantitative factors in real time on the basis of the error threshold (according to the need to set), has greatly improved the effect of the fuzzy controller.

Mathematical Model of Planar Double Inverted Pendulum

This object of study is Googol company GPIP2002 planar inverted pendulum. The mathematical model of planar double inverted pendulum is established by means of analytical dynamics method [3], Inverted pendulum system is mainly made up of controlled object, the direction of horizontal rails, servo motor, drive shaft and electric drive equipment. Control object is composed of the car, the downside pendulum, the upside pendulum, and two rotation shafts which connected to the car and pendulums, and shown in Fig.1. Based on the fulcrums of pendulum, coordinates $o_1x_1y_1z_1$ and $o_2x_2y_2z_2$ are established. In condition of ignore effects of air resistance and friction, etc. Planar inverted pendulum can seen as be comprised of car platform, shaft mass, the uniform downside pendulum and the uniform upside pendulum.

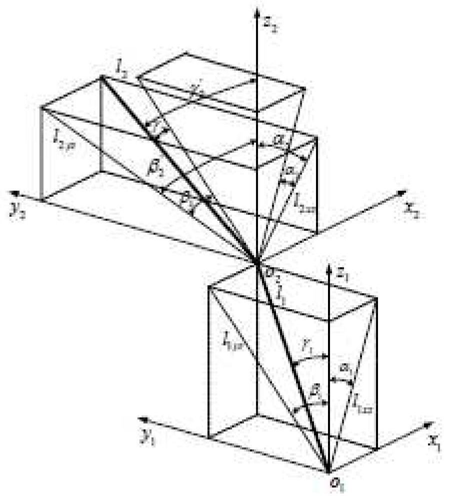


Fig. 1 Schematic of planar double inverted pendulum

The basic parameters are defined as follows: l_1, l_2 are the length of the downside pendulum and the upside pendulum respectively. m_1, m_2 are the quality of the downside pendulum and the upside pendulum respectively. m_3 is the quality of link mass which link to the pendulum. M_x, M_y are the quality of the platform motion part and swing bearing of X direction and Y direction respectively. Specific values are: l_1=0.2m, l_2=0.55m, m_1=0.06kg, m_2=0.13kg, m_3=0.27kg, g=9.8 m/s^2.
In this paper, differential equation of inverted pendulum system is established by the Lagrange equation[3], Lagrange operator as follows:

$$L(q,\dot{q}) = T(q,\dot{q}) - V(q,\dot{q}) \tag{1}$$

Where q is generalized coordinate of system? T is kinetic energy of the system. V is potential energy of the system. From the generalized coordinates q and L, Lagrange equations can be expressed as:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \tag{2}$$

Where i is system variable label, i =1,2,…, n, q= $\{q_1, q_2, q_3 \ldots\}$ is generalized variable, τ_i are system generalized external forces of along the direction of generalized coordinate. For planar double inverted pendulum, the generalized coordinates are: x, y, α_1, β_1, α_2, β_2. The total kinetic energy of the system is:

$$T = T_M + T_{m_1} + T_{m_2} + T_{m_3} \tag{3}$$

The total potential energy of the system is: $V = V_M + V_{m_1} + V_{m_2} + V_{m_3}$

Relevant parameters are used in (1), Since the external forces of the generalized coordinates α_1, β_1, α_2, β_2 is zero, and equations can be established by equation (2). $\ddot{\alpha}_1$, $\ddot{\beta}_1$, $\ddot{\alpha}_2$, $\ddot{\beta}_2$ are worked out. In the equilibrium position ($q=\dot{q}=0$), the Taylor series expansion of $\ddot{\alpha}_1$, $\ddot{\beta}_1$, $\ddot{\alpha}_2$, $\ddot{\beta}_2$ and linearization are executed, parameter values are brought into, decoupled state equations in X and Y directions of double inverted pendulums respectively are solved.

$$\begin{cases} \dot{X}_x = A_x X_x + B_x u_x \\ Y_x = C_x X_x \end{cases} \qquad \begin{cases} \dot{X}_y = A_y X_y + B_y u_y \\ Y_y = C_y X_y \end{cases}$$

Where control actions of X and Y direction are $u_x = \ddot{x}$, $u_y = \ddot{y}$, State variables are: $X_x = [x, \alpha_1, \alpha_2, \dot{x}, \dot{\alpha}_1, \dot{\alpha}_2]^T$, $X_y = [y, \beta_1, \beta_2, \dot{y}, \dot{\beta}_1, \dot{\beta}_2]^T$.

$$A_x = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 50.2235 & -14.7272 & 0 & 0 & 0 \\ 0 & -50.8908 & 49.4875 & 0 & 0 & 0 \end{pmatrix}, \quad B_x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -5.1248 \\ 5.1929 \end{pmatrix},$$

$$C_x = diag(1,1,1,1,1,1), \ A_y = A_x, \ B_y = B_x, \ C_y = C_x.$$

From state equations, linear models in X and Y directions of double inverted pendulums have been decoupled respectively, and can be controlled separately. Each system has one input, six outputs, the control object is greatly simplified. Controllability principle can be applied to verify the controllability of the system.

Design of Parameter Adaptive Fuzzy Controller

State Fusion Technology

Fuzzy controller generally outputs a control action by using the error and error change rate as the input. For multi-variable control system such as the inverted pendulum, the number of complete control rules is the number of input variables exponentially, so that control rules design become difficulties, the real-time implementation of control cannot guarantee. To solve these problems, combined with optimal control theory and fuzzy control strategy, using the state fusion technology, integrate multiple variables into synthesis error and synthesis error change rate, which makes the fuzzy controller design is greatly simplified.

Order $E = K_1 X_1 = [k_x, k_{\alpha_1}, k_{\alpha_2}][x_1, x_2, x_3]^T$, $EC = K_2 X_2 = [k_{\dot{x}}, k_{\dot{\alpha}_1}, k_{\dot{\alpha}_2}][x_4, x_5, x_6]^T$,

where K_1 and K_2 represent the weighting coefficient of each state variable respectively. According to the state feedback matrix $K = [k_x, k_{\alpha_1}, k_{\alpha_2}, k_{\dot{x}}, k_{\dot{\alpha}_1}, k_{\dot{\alpha}_2}]$ of LQR control [4], elements of matrix reflect weight of each variable in the controller output respectively. Such changes make the fuzzy controller input variables into two; reduce the difficulty of controller design, while retaining all system status information.

After Linearization of the mathematical models, linear optimal control theory is applied to design LQR controller for the state equation of X direction. Y direction uses the same control algorithm. For the form as fellow, state feedback control can be find:

$$u(t) = -Kx(t)$$

So as to minimize the performance index function

$$J = \frac{1}{2}\int_0^{T_i} [x^T(t)Qx(t) + u^T(t)Ru(t)]dt$$

Where Q is semi-definite matrix, R is positive definite matrix, Q and R are the weighting matrixes of state variables and input variables respectively. For the smallest performance index function, at first Hamilton function is constructed, derivation of this function was obtained and makes it equal to zero, which can determine the optimal control rate:

$$u(t) = -Kx(t) = -R^{-1}B^T Px(t)$$

Where P is the only positive definite symmetric solution which meet the Riccati equation PA+ATP-PBR-1BTP+Q=0.Where $Q = diag[300, 400, 500, 0, 0, 0]$, $R = 1$, the LQR state feedback gain matrix can be calculated by using MATLAB simulation.

Design of Fuzzy Controller

Parameter adaptive fuzzy controller is proposed in this paper, This controller is composed of two difference parameters fuzzy controllers, Single fuzzy controller is mainly used for rapid response and eliminate the large errors. If error quantization factor K_e increases, which is equivalent to narrowing the basic domain of the error, thereby increasing the control of the error variable; If the error change rate quantization factor K_{ec} increases, the overshoot can be reduced; If scale factor K_u of control decreases, the system oscillations reduces.

Assuming that e_0 is the large and small threshold (can be set according to need), when the system error is large, use the fuzzy controller (1), shrink K_e , K_{ec} and zoom K_u in order to achieve fast response, elimination of errors. When the system error is smaller, use the fuzzy controller (2), zoom K_e , K_{ec} and shrink K_u in order to reduce the overshoot, raise steady state accuracy [5], So that the parameter adaptive fuzzy controller can greatly improve the control performance.

Fuzzy controller input variables E , EC and output variables u domain are set to [-6, 6], membership functions is triangle, full overlap uniformly distributed, each variable is been described with seven fuzzy subsets {NB NM NS ZE PS PM PB}.

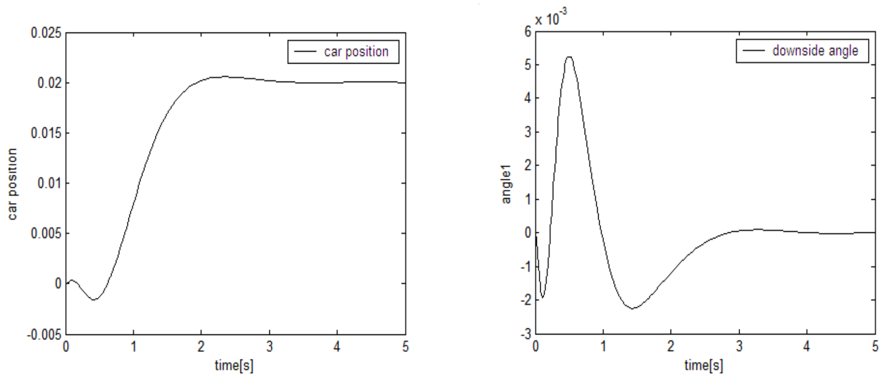
According to the domain of the fuzzy language variables, Mamdani-type fuzzy rules are been designed as shown in Table 1, defuzzification applied with gravity center method.

Table 1 Fuzzy conrol rule table of planar double inverted pendulum

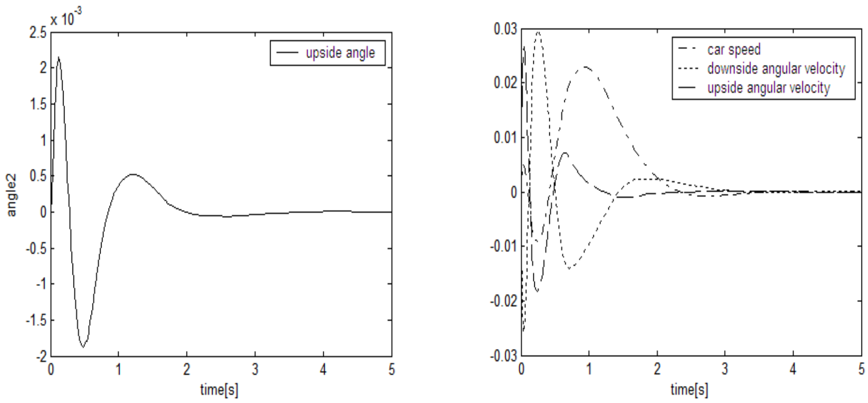
EC	E						
	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NM	NM	NS	ZE
NM	NB	NB	NM	NM	NS	ZE	PS
NS	NB	NM	NM	NS	ZE	PS	PM
ZE	NM	NM	NS	ZE	PS	PM	PM
PS	NM	NS	ZE	PS	PM	PM	PB
PM	NS	ZE	PS	PM	PM	PB	PB
PB	ZE	PS	PM	PM	PB	PB	PB

System Simulation Results

Apply the designed controller, a simulation model diagram in the SIMULINK is created, and carry out parameter adaptive fuzzy control simulation. Order K=[17.32, 89.04, 210.23, 18.72, 33.33, 36.38], K_1=[17.32, 89.04, 210.23], K_2=[18.72, 33.33, 36.38], when the system initial state is zero, there is the interference[0.03, 0, 0, 0, 0,0], the system state variables are shown in Fig. 2. The control input u is shown in Fig. 3.



(a) car position. (b) downside angle.



(c) upside angle. (b) state variables velocity.

Fig. 2 Simulation curves of state variables.

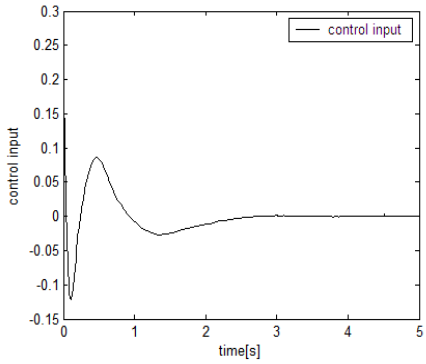


Fig. 3 Simulation curves of control input.

As can be seen from figures, the controller can control the inverted pendulum to achieve stability in the 3s, two swing angle and car displacement make a very small oscillation near the equilibrium position, the control input is also very smooth, so the parameter adaptive fuzzy controller can improve control accuracy and steady sate performance of control system, and achieve stable control.

Conclusion

In this paper, parameter adaptive fuzzy controller is proposed based on fuzzy control theory, by using SIMULINK, the simulation becomes intuitive, which make it is easy to modify the control parameters. The application of state fusion technology simplifies the design of fuzzy controller, which is conducive to the control algorithm and has important research value.

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