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An Improved Extreme Point Enumeration Technique for Assignment Problem

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Introduction:

Assignment Problem is a special type of LPP, in which a number of jobs are to be performed by an equal number of persons, where each person can perform each job with varying efficiency. The main objective is to assign a single job to a single person at a time in such a way that total cost of assignment is minimum. The Hungarian Mathematician D. Konig had given the procedure for optimum solution in early 1950s. Since then several researchers who have made significant contributions in this area are Flood (1953), Dwyer (1955), Kuhn (1955) etc. In 1978, M.C. Puri discussed an extreme point enumeration technique for assignment problem. This procedure starts from an extreme point (infeasible) and moves from one extreme point to another extreme point in such a way that feasibility and optimality for the problem are achieved simultaneously.

Mathematical Model:

Suppose there are *n* jobs to be performed and *n* persons are available for doing these jobs. Assuming that each person can do each job at a time, though with varying degree of efficiency. One job is assigned to one person only. No job is to be left undone and each person is to be used.

		Job					
		1	2	.	.	.	n
	1	c_{11}	c_{12}	.	.	.	c_{1n}
	2	c_{21}	c_{22}	.	.	.	c_{2n}
Person	.	.		.	.	.	.
	.	.	.	.	.	.	.
	.	.	.	.	.	.	.
	n	c_{n1}	c_{n2}	.	.	.	c_{nn}

The mathematical model for such a problem is:

$$Min.Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$s.to. \sum_{i=1}^{n} x_{ij} = 1 \ , \quad j = 1, 2, ..., n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \ , \quad i = 1, 2, ..., n \qquad (1)$$

$$x_{ij} = 0 \ or \ 1$$

$$x_{ij} = \begin{cases} 1 & if \ i^{th} \ person \ is \ assigned \ to \ j^{th} \ job \\ 0 & Otherwise \end{cases} \qquad (2)$$

c_{ij} is the cost of i^{th} person to perform j^{th} job.

The assignment problem is clearly a particular type of zero-one integer programming problem but its structure is of very special type. This special nature of the problem is given as:

$$\begin{aligned} &\text{If } x_{kl} = 1 \text{ then } x_{il} = 0, \quad i = 1,2,...,n \quad i \neq k \\ &\text{If } x_{pq} = 1 \text{ then } x_{pj} = 0, \quad j = 1,2,...,n \quad j \neq q \end{aligned} \tag{3}$$

one person gets one job only, it follows that there should be only n x_{ij}'s which are unity each and remaining zeros and (3) shows that in the set of n x_{ij}'s no two i's and j's should be same, which is the feasibility criterion for the problem.

Solution Procedure:

The following steps are involved to obtain the feasible solutions

1. Arranged the values of c_{ij} in ascending order
2. Select the minimum value of c_{ij}. Let $c_{ij} = c_{rs}$ (say)
3. Set $x_{rs} = 1$ and cross out all elements having row index r and column index s
4. If $r \neq s$, then select the next minimum elements having the row index s, say c_{st},
5. Otherwise select the minimum c_{ij} from the remaining uncrossed elements.
6. Set the selected minimum element of step 4 say $x_{uv} = 1$ and cross out all elements having row index u and column index v
7. Repeat the Steps 4 & 5 until all the feasible assignments are done, which is also a feasible solution.

Note: For finding the other feasible solutions we have to select the next minimum of arranged c_{ij} not having the row index as in step 2. Retain the solution having the minimum cost termed as optimum solution.

Numerical Illustration:

Find the optimal assignments of four jobs and four persons when the cost of assignment is given as:

Person\Job	**I**	**II**	**III**	**IV**
1	12	30	21	15
2	18	33	9	31
3	44	25	24	21
4	23	30	28	14

Solution: First arrange the costs of the matrix in ascending order of their magnitude

c_{ij}	x_{ij}	x_{23}	x_{11}	x_{44}	x_{34}
09	x_{23}	1	1	1	0
12	x_{11}	0	1	1	0
14	x_{44}	0	0	1	0
15	x_{14}	0	0	0	0
18	x_{21}	0	0	0	0
21	x_{13}	0	0	0	1
21	x_{34}	1	1	0	1

23	x_{41}	1	0	0	1
24	x_{33}	0	0	0	0
25	x_{32}	0	0	1	0
28	x_{43}	0	0	0	0
30	x_{12}	1	0	0	0
30	x_{42}	0	1	0	0
31	x_{24}	0	0	0	0
33	x_{22}	0	0	0	1
44	x_{31}	0	0	0	0
$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$		83	72	**60**	98

The total minimum cost is 60. And the corresponding assignment is:

$$1 \Rightarrow I$$
$$2 \Rightarrow III$$
$$3 \Rightarrow II$$
$$4 \Rightarrow IV$$

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