Relay Selection with Outdated Channel Estimates in Nakagami-m Fading

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***Abstract*—We study the effect of outdated channel estimates on decode-and-forward relay selection, when operating over Nakagami-m fading channels. Closed-form expressions for the exact outage probability are derived as function of the correlation between the actual and the estimated channel values. The diversity and coding gain are also studied, revealing a high dependence of diversity order on the aforementioned correlation coefficient. It is shown that when the channel estimates are not perfectly updated, the performance of relay selection is equivalent, in terms of diversity order, to the scheme where only a single relay is available.**

I. Introduction

Wireless relaying technology has been recently proposed as a method that promises significant performance improvement in wireless communications without any power increase [1], [2]. Among the most common relaying techniques that have been extensively analyzed in the literature is the so-called relay selection [3]–[9], where the system is able to select a single relay out of the set of available relays, in order to take advantage of the multiple paths available and thus achieve spatial diversity. It has been shown that activating only the relay with the strongest instantaneous end-to-end channel represents a bandwidth-efficient alternative to all-participate relaying, since, on one hand, the same diversity order is achieved, yet on the other hand the excessive bandwith usage that multiple relay activation entails is avoided [5].

Most of the literature dealing with relay selection in fading channels has assumed that the channel state information (CSI) is perfectly fed back to the terminal where the decision on relay selection takes place. This, however, may not be the case in practical scenarios where the channel changes rapidly enough, so that the CSI available at the selecting terminal is outdated, in the sense that it does not correspond to the actual time instance, but to a previous one. An exception to that are the works in [10], [11], where the diversity performance of relay selection in Rayleigh fading channels and outdated CSI was studied for decode-and-forward (DF) and amplify-and-forward relaying, respectively.

In an effort to examine the impact of outdated CSI in relay selection in more general fading scenarios, we consider here the case of Nakagami-m fading and study its effect on the performance of DF relay selection with outdated channel estimates. This model represents an extension of the work in [10], providing thus insight into the performance of relay selection with outdated CSI in more versatile fading scenarios. In particular, assuming a correlation coefficient ρ between the actual and the estimated channel values, we thoroughly examine the impact of ρ on the outage and diversity order performance of DF relay selection, when operating over Nakagami-m fading channels. Closed-form outage expressions are presented as a function of ρ, along with exact diversity and coding gain expressions. Numerical results manifest that the overall performance is highly dependent on ρ, in the sense that when ρ deviates from the perfect CSI case the diversity order of relay selection reduces to that of the scheme where only a single relay is available.

II. System Model

Let us consider a cooperative relaying system which consists of a single source terminal, S, N relays denoted by R_i, $i = 1, ..., N$, and a single destination terminal, D. The relays are assumed to operate in the half-duplex, as well as the DF mode [1]. Moreover, we assume independent and identically distributed (i.i.d.) fading in each of the links involved. Specifically, the fading amplitude h_{AB} in the link between two arbitrary nodes A, B, follows the Nakagami-m distribution, with probability density function (pdf)

$$f_h(z) = \frac{2\left(\frac{m}{\Omega}\right)^m}{\Gamma(m)} z^{2m-1} e^{-\frac{m}{\Omega}z^2}, \quad z \geq 0, m \geq 0.5 \qquad (1)$$

where $\Gamma(\cdot)$ is the Gamma function defined in [12, Eq. (8.310/1)] and $\Omega = E\left[|h_{AB}|^2\right]$ denotes the average squared amplitude of the A-B link. Let γ_{AB} represent the instantaneous signal-to-noise ratio (SNR) of the A-B link, i.e., $\gamma_{AB} = |h_{AB}|^2/N_0$, where N_0 is the additive white Gaussian noise (AWGN) power. Due to the i.i.d. fading assumption, the average SNR in each of the links involved is constant, and denoted by $\bar{\gamma}$. Moreover, we use the notation $f_X(\cdot)$ and $F_X(\cdot)$ to refer to the pdf and the cumulative density function (cdf) of the random variable (RV) X, respectively.

Because of the Nakagami-m fading distribution in each participating link, the pdf of the instantaneous SNR is Gamma distributed, i.e.,

$$f_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right), \quad \gamma \geq 0, m \geq 0.5. \qquad (2)$$

The corresponding cdf is given by

$$F(\gamma_{th}) = \Pr\{\gamma \leq \gamma_{th}\} = 1 - \frac{\Gamma\left(m, \frac{m}{\bar{\gamma}}\gamma_{th}\right)}{\Gamma(m)} \qquad (3)$$

where $\Gamma(\cdot,\cdot)$ denotes the upper incomplete gamma function defined in [12, Eq. (8.350/2)].

Relay Selection Process: Among the available relays, only a single relay is activated in each transmission session, based on the selection cooperation protocol [4]. In particular, the relay selection procedure is completed in two phases, as follows. In the first phase, the relays that can successfully decode the message form the so-called decoding set, denoted by $\mathcal{S}$. In the second phase, the destination collects the estimated CSI of the R_i-D links with $i : R_i \in \mathcal{S}$, and activates the relay with the strongest R_i-D channel.

Let $\hat{\gamma}_{R_iD}$ represent the estimated value of γ_{R_iD}, as seen at D. Hence, denoting with R_b the selected relay, it holds

$$b = \arg \max_{i:R_i \in \mathcal{S}} \hat{\gamma}_{R_iD}. \tag{4}$$

It is assumed that, because of feedback delay, $\hat{\gamma}_{R_iD}$ corresponds to a time instance which is not necessarily the same as the current time instance. Therefore, $\hat{\gamma}_{R_iD}$ and γ_{R_iD} represent in fact two correlated RVs taken from the same distribution, similarly to the outdated CSI model studied in [10], [11]. The level of CSI imperfection is reflected in the correlation coefficient between $\hat{\gamma}_{R_iD}$ and γ_{R_iD}, denoted by ρ. Hence, the joint pdf of $\hat{\gamma}_{RD}$ and γ_{RD} (where the indexes are dropped due to the i.i.d. fading assumption) is given by the bivariate gamma distribution [13]. Given the fact that $\hat{\gamma}_{RD}$ and γ_{RD} are identically distributed, its joint pdf is simplified through [12, Eq. (9.210/1)] to

$$\begin{aligned} f_{\hat{\gamma}_{RD},\gamma_{RD}}(x_1,x_2) &= \sum_{h=0}^{\infty} \frac{(m)_h\, m^{2m+2h} \rho}{h!\,(1-\rho)^{m+2h}} \\ &\times \prod_{i=1}^{2} \frac{x_i^{m+h-1} \exp\left\{-\frac{m x_i}{(1-\rho)\bar{\gamma}}\right\}}{\Gamma(m+h)\,\bar{\gamma}^{m+h}} \end{aligned} \tag{5}$$

where $(x)_y$ denotes the Pochhamer symbol defined in [12, pp. xliii].

III. Performance Analysis

The outage probability is defined as the probability that the overall SNR lies below a given threshold, denoted here by y. Hence, the outage probability of the scheme under consideration is expressed as [4]

$$P_{out} = \Pr\{\mathcal{S}=\emptyset\} + \sum_{l=1}^{N} \Pr\{\gamma_{R_bD} < y \,| |\mathcal{S}| = l\} \Pr\{|\mathcal{S}| = l\} \tag{6}$$

where $|\mathcal{S}|$ denotes the cardinality of $\mathcal{S}$ and $\emptyset$ is the empty set. It follows from (3) and the i.i.d. assumption on the fading on the S-R_i links that the second term within the summation in (6) can take the following form

$$\begin{aligned} \Pr\{|\mathcal{S}| = l\} &= \binom{N}{l} F^{N-l}(y)\,(1-F(y))^l \\ &= \binom{N}{l} \left(1 - \frac{\Gamma\left(m, \frac{m}{\bar{\gamma}} y\right)}{\Gamma(m)}\right)^{N-l} \left(\frac{\Gamma\left(m, \frac{m}{\bar{\gamma}} y\right)}{\Gamma(m)}\right)^{l}. \end{aligned} \tag{7}$$

Furthermore, by defining A_i as the event that the ith relay out of the l relays is selected, i.e., $A_i : i = b$, the cdf of γ_{R_bD} is given by

$$F_{\gamma_{R_bD}}(x) = \sum_{i=1}^{l} \Pr\{\gamma_{R_iD} \le x \,| A_i\} \Pr\{A_i\}. \tag{8}$$

Due to symmetry, $\Pr\{A_i\} = 1/l$, hence (8) can be written as

$$\begin{aligned} F_{\gamma_{R_bD}}(x) &= \Pr\{\gamma_i \le x \,| A_i\} \\ &= \int_0^{\infty} F_{\gamma_{R_iD}|\hat{\gamma}_{R_iD}}(x \,|\hat{\gamma}_{R_iD})\, f_{\hat{\gamma}_{R_iD}|A_i}(z \,| A_i)\, dz \\ &= \int_0^{x}\int_0^{\infty} f_{\gamma_{R_iD}|\hat{\gamma}_{R_iD}}(\omega \,|\hat{\gamma}_{R_iD})\, f_{\hat{\gamma}_{R_iD}|A_i}(z \,| A_i)\, dz d\omega. \end{aligned} \tag{9}$$

In order to evaluate (9), we first obtain the conditional PDF $f_{\gamma_{R_iD}|\hat{\gamma}_{R_iD}}(\gamma_{R_iD} \,|\hat{\gamma}_{R_iD})$ from the Bayes' theorem and (5) as

$$\begin{aligned} f_{\gamma_{R_iD}|\hat{\gamma}_{R_iD}}(x_1 \,| x_2) &= \frac{f_{\gamma_{R_iD},\hat{\gamma}_{R_iD}}(x_1,x_2)}{f_{\hat{\gamma}_{R_iD}}(\hat{\gamma}_{R_iD})} \\ &= \sum_{n=0}^{\infty} \frac{\rho^n m^{m+2n}}{n!(1-\rho)^{m+2n}} \frac{x_1^{m+n-1} x_2^n \exp\left(-\frac{m(x_1+\rho x_2)}{(1-\rho)\bar{\gamma}}\right)}{\Gamma(m+n)\,\bar{\gamma}^{m+2n}}. \end{aligned} \tag{10}$$

The conditional probability of $\hat{\gamma}_{R_iD}$, conditioned on A_i, is obtained as

$$\begin{aligned} f_{\hat{\gamma}_{R_iD}|A_i}(x_2 \,| A_i) &= \frac{\Pr\{\hat{\gamma}_{R_iD} = x_2 \cap A_i\}}{\Pr\{A_i\}} \\ &= \frac{1}{\Pr\{A_i\}} f_{\hat{\gamma}_{R_iD}}(x_2) \prod_{\substack{j=1 \\ j\neq i}}^{l} F_{\hat{\gamma}_{R_jD}}(x_2) = l f_{\hat{\gamma}_{R_iD}}(x_2)\, F^{l-1}_{\hat{\gamma}_{R_iD}}(x_2) \end{aligned} \tag{11}$$

Substituting (2) and (3) into (11), and applying the binomial expansion [12, Eq. (1.110)], we obtain

$$\begin{aligned} f_{\hat{\gamma}_{R_iD}|A_i}(x_2 \,| A_i) &= \frac{l \exp\left(-\frac{m x_2}{\bar{\gamma}}\right) m^m x_2^{m-1}}{\Gamma(m)\,\bar{\gamma}^m} \\ &\times \left(\sum_{j=0}^{l-1} \binom{l-1}{j} \left(\frac{-1}{\Gamma(m)}\right)^j \Gamma\left(m, \frac{m}{\bar{\gamma}} x_2\right)^j\right). \end{aligned} \tag{12}$$

Consequently, the cdf of γ_{R_bD} is inferred by substituting (10) and (12) into (9), yielding

$$\begin{aligned} F_{\gamma_{R_bD}}(x) &= l \int_0^{x}\int_0^{\infty} \sum_{n=0}^{\infty} \frac{\rho^n m^{2m+2n}}{n!(1-\rho)^{m+2n}} \\ &\times \frac{x_1^{m+n-1} x_2^{m+n-1} \exp\left(-\frac{m(x_1+\rho x_2)}{(1-\rho)\bar{\gamma}} - \frac{m x_2}{\bar{\gamma}}\right)}{\Gamma(m)\,\Gamma(m+n)\,\bar{\gamma}^{2(m+n)}} \\ &\times \left(\sum_{j=0}^{l-1} \binom{l-1}{j} \left(\frac{-1}{\Gamma(m)}\right)^j \Gamma\left(m, \frac{m}{\bar{\gamma}} x_2\right)^j\right) dx_2 dx_1 \end{aligned} \tag{13}$$

which after some elementary integrations and algebraic manipulations reduces to

$$\begin{aligned} F_{\gamma_{R_bD}}(x) &= l \int_0^{x} \sum_{n=0}^{\infty} \frac{\rho^n m^{2m+2n}}{n!(1-\rho)^{m+2n}} \frac{x_1^{m+n-1}}{\Gamma(m)\,\Gamma(m+n)\,\bar{\gamma}^{2(m+n)}} \\ &\times \exp\left(-\frac{m x_1}{(1-\rho)\bar{\gamma}}\right) \sum_{j=0}^{l-1} \binom{l-1}{j} \left(\frac{-1}{\Gamma(m)}\right)^j \\ &\times \left(\int_0^{\infty} x_2^{m+n-1} \exp\left(-\frac{m x_2}{\bar{\gamma}(1-\rho)}\right) \Gamma\left(m, \frac{m}{\bar{\gamma}} x_2\right)^j dx_2\right) dx_1. \end{aligned} \tag{14}$$

It is observed from (14) that in order to derive a closed-form expression for $F_{\gamma_{R_bD}}(x)$, the following integral needs to be solved

$$I(\mu,\alpha,m,\beta,j)=\int_0^\infty x^\mu \exp(-\alpha x)\,\Gamma^j(m,\beta x)\,dx. \quad (15)$$

For the case of $m\in\mathbb{Z}$, the integral in (15) is evaluated as it is illustrated in the appendix. Hence, $F_{\gamma_{R_bD}}(x)$ can be derived from (14) as

$$\begin{aligned}F_{\gamma_{R_bD}}(x)&=l\sum_{n=0}^{\infty}\frac{\rho^n m^{2m+2n}}{n!(1-\rho)^{m+2n}\Gamma(m)\Gamma(m+n)\bar{\gamma}^{2(m+n)}}\\&\times\left(\int_0^x x_1^{m+n-1}\exp\left(-\frac{mx_1}{(1-\rho)\bar{\gamma}}\right)dx_1\right)\\&\times\sum_{j=0}^{l-1}\binom{l-1}{j}\left(\frac{-1}{\Gamma(m)}\right)^j I\left(m+n-1,\frac{m}{\bar{\gamma}(1-\rho)},m,\frac{m}{\bar{\gamma}},j\right).\end{aligned} \quad (16)$$

which can be equivalently written, according to (45), as

$$\begin{aligned}F_{\gamma_{R_bD}}(x)&=l\sum_{n=0}^{\infty}\frac{\rho^n m^{m+n}}{n!(1-\rho)^{m+2n}\Gamma(m)\Gamma(m+n)\bar{\gamma}^{(m+n)}}\\&\times\left(\int_0^x x_1^{m+n-1}\exp\left(-\frac{mx_1}{(1-\rho)\bar{\gamma}}\right)dx_1\right)\\&\times\sum_{j=0}^{l-1}\binom{l-1}{j}(-1)^j g(m+n,\rho,j)\end{aligned} \quad (17)$$

where

$$\begin{aligned}g(x,\rho,j)&=\frac{\Gamma(j+1)}{\left(\frac{1}{(1-\rho)}+j\right)^x}\\&\times\sum_{\substack{\kappa_0,\kappa_1,\ldots,\kappa_{m-1}=0\\ \kappa_0+\kappa_1+\ldots+\kappa_{m-1}=j}}^{j}\left(\prod_{i=0}^{m-1}\frac{\left(\frac{1}{i!\left(\frac{1}{(1-\rho)}+j\right)^i}\right)^{\kappa_i}}{\Gamma(\kappa_i+1)}\right)\\&\times\left(x-1+\sum_{i=0}^{m-1}i\kappa_i\right)!\end{aligned} \quad (18)$$

It should be noted that in the case of Rayleigh fading $(m=1)$, (18) is reduced to

$$g(x,\rho,j)=\frac{(x-1)!}{\left(\frac{1}{1-\rho}+j\right)^x}. \quad (19)$$

Using [12, Eq. (3.351/1)] and [12, Eq. (8.352/6)], (17) takes its final form as shown below

$$\begin{aligned}F_{\gamma_{R_bD}}(x)&=l\sum_{n=0}^{\infty}\frac{\rho^n}{n!(1-\rho)^n\Gamma(m)\Gamma(m+n)}\\&\times\left(\Gamma(m+n)-\Gamma\left(\frac{my}{(1-\rho)\bar{\gamma}}\right)\right)\\&\times\sum_{j=0}^{l-1}\binom{l-1}{j}(-1)^j g(m+n,\rho,j)\end{aligned} \quad (20)$$

Consequently, a closed-form expression for the outage probability is obtained by combining (20), (7) and (6), yielding

$$\begin{aligned}P_{out}&=\left(1-\frac{\Gamma\left(m,\frac{m}{\bar{\gamma}}y\right)}{\Gamma(m)}\right)^N\\&+\sum_{n=0}^{\infty}\sum_{l=1}^{N}\sum_{j=0}^{l-1}l\binom{N}{l}\left(1-\frac{\Gamma\left(m,\frac{m}{\bar{\gamma}}y\right)}{\Gamma(m)}\right)^{N-l}\\&\times\left(\frac{\Gamma\left(m,\frac{m}{\bar{\gamma}}y\right)}{\Gamma(m)}\right)^l\frac{\rho^n(-1)^j}{n!(1-\rho)^n\Gamma(m)\Gamma(m+n)}\\&\times\left(\Gamma(m+n)-\Gamma\left(\frac{my}{(1-\rho)\bar{\gamma}}\right)\right)\\&\times\binom{l-1}{j}g(m+n,\rho,j).\end{aligned} \quad (21)$$

It is noted that, for practical SNR values, the infinite series in (21) converges after a finite number of terms, not greater than 100.

IV. High SNR analysis

A. Outage Expressions

The expression in (21) provides the exact outage probability of the system under consideration. However, due to its complicated form, it does not provide insight into the outage behavior under certain assumptions. To this end, we provide an approximate expression for the outage probability in the high SNR regime as follows.

It can be seen from (21) that for sufficiently high values of $\bar{\gamma}$, the term $n=0$ of the infinite series in the (21) dominates. Hence, (21) is reduced to

$$\begin{aligned}P_{out}&\approx\left(1-\frac{\Gamma\left(m,\frac{m}{\bar{\gamma}}y\right)}{\Gamma(m)}\right)^N\\&+\sum_{l=1}^{N}\sum_{j=0}^{l-1}l\binom{N}{l}\left(1-\frac{\Gamma\left(m,\frac{m}{\bar{\gamma}}y\right)}{\Gamma(m)}\right)^{N-l}\\&\times\frac{(-1)^j}{\Gamma^2(m)}\left(\frac{\Gamma\left(m,\frac{m}{\bar{\gamma}}y\right)}{\Gamma(m)}\right)^l\\&\times\left(\Gamma(m)-\Gamma\left(m,\frac{my}{(1-\rho)\bar{\gamma}}\right)\right)\\&\times\binom{l-1}{j}g(m,\rho,j).\end{aligned} \quad (22)$$

Furthermore, for sufficiently high values of $\bar{\gamma}$ the following expression holds

$$\begin{aligned}\frac{\Gamma\left(m,\frac{m}{\bar{\gamma}}y\right)}{\Gamma(m)}&=\frac{\Gamma(m)-\gamma\left(m,\frac{m}{\bar{\gamma}}y\right)}{\Gamma(m)}\\&\approx\frac{\Gamma(m)-\frac{\left(\frac{my}{\bar{\gamma}}\right)^m}{m}}{\Gamma(m)}=1-\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\end{aligned} \quad (23)$$

which is based on the series representation of the lower incomplete Gamma function [12, Eq. (8.354/1)] and as $x \to 0$

$$\gamma(s,x) \approx x^s/s. \tag{24}$$

Hence, (22) becomes

$$\begin{aligned} P_{out} \approx & \left(\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^N \\ & + \sum_{l=1}^{N}\sum_{j=0}^{l-1} l \binom{N}{l}\binom{l-1}{j}\left(\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^{N-l} \\ & \times \frac{(-1)^j}{\Gamma^2(m)}\left(1-\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^l \\ & \times \left(\Gamma(m)-\Gamma\left(m,\frac{my}{(1-\rho)\bar{\gamma}}\right)\right) g(m,\rho,j) \end{aligned} \tag{25}$$

One may notice that (25) in its current form does not provide insight into the behavior of the outage probability in the high SNR regime. However, (25) can be reduced to a simpler form, depending on whether or not ρ lies in the proximity of one, as follows.

Case of $\rho < 1$: In this case, the second argument of $\Gamma\left(m,\frac{my}{(1-\rho)\bar{\gamma}}\right)$ tends to zero as $\bar{\gamma} \to \infty$ and, according to (24), it holds that

$$\Gamma(m)-\Gamma\left(m,\frac{my}{(1-\rho)\bar{\gamma}}\right) \approx \frac{m^{m-1}y^m}{(1-\rho)^m\bar{\gamma}^m}. \tag{26}$$

Therefore, it follows that (25) reduces to

$$P_{out}(y) \approx \mathcal{C}_1 + \mathcal{C}_2 \tag{27}$$

where

$$\mathcal{C}_1 = \left(\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^N \tag{28}$$

and

$$\begin{aligned} \mathcal{C}_2 = & \sum_{l=1}^{N}\sum_{j=0}^{l-1} l\binom{N}{l}\left(\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^{N-l} \\ & \times \frac{(-1)^j}{\Gamma^2(m)}\left(1-\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^l \\ & \times \frac{m^{m-1}y^m}{(1-\rho)^m\bar{\gamma}^m}\binom{l-1}{j} g(m,\rho,j). \end{aligned} \tag{29}$$

It is easy to identify that in the high SNR regime and for $N > 1$, the dominant term in (27) is $\mathcal{C}_2$. Additionally, the maximum of the terms of $\mathcal{C}_2$ occurs for $l = N$ yielding

$$P_{out}(y) \approx N\sum_{j=0}^{N-1}\binom{N-1}{j}\frac{(-1)^j}{\Gamma^2(m)}\frac{m^{m-1}y^m}{(1-\rho)^m\bar{\gamma}^m} g(m,\rho,j) \tag{30}$$

which represents a high SNR expression for the outage probability for $\rho < 1$.

Case of $\rho \to 1$: Should this be the case, given that in the high SNR regime both $y/\bar{\gamma}$ and $(1-\rho)$ tend to zero in a linear fashion, we have

$$\lim_{\substack{\bar{\gamma}\to\infty \\ \rho\to 1}} \Gamma\left(m,\frac{my}{(1-\rho)\bar{\gamma}}\right) = \Gamma\left(m,\frac{my}{\lambda}\right) \tag{31}$$

where $0 < \lambda < \infty$ is given as

$$\lambda = \lim_{\substack{\rho\to 1 \\ \bar{\gamma}\to\infty}} \left[(1-\rho)\bar{\gamma}\right]. \tag{32}$$

Moreover, under the $\rho \to 1$ assumption we can further simplify (15) as follows.

- Let us assume $0 < x < \infty$, i.e., x is a finite positive real number. Then, using [12, Eq. (8.352/7)] we have for high SNR

$$\Gamma^j\left(m,\frac{m}{\bar{\gamma}}x\right) \approx \Gamma^j(m)\exp\left(-\frac{mj}{\bar{\gamma}}x\right). \tag{33}$$

- Let $x \to \infty$. In this case, (33) does not hold. However, it follows from L'Hospital's rule, as well as from the fact that λ is finite, that

$$\lim_{x\to\infty}\left[x^{m-1}\exp\left(-\frac{m}{\lambda}x\right)\Gamma^j\left(m,\frac{m}{\bar{\gamma}}x\right)\right] = 0. \tag{34}$$

Therefore, considering (33) and (34) we can express (15) in the high SNR region as

$$\begin{aligned} & I\left(m-1,\frac{m}{(1-\rho)\bar{\gamma}},m,\frac{m}{\bar{\gamma}},j\right) \\ & \approx \int_0^\infty x^{m-1}\exp\left(-\frac{mx}{(1-\rho)\bar{\gamma}}\right)\Gamma^j(m)\exp\left(-\frac{mj}{\bar{\gamma}}x\right)dx \\ & = \Gamma^{j+1}(m)\left(\frac{\lambda}{m}\right)^m. \end{aligned} \tag{35}$$

where λ is given in (32).

Consequently, (25) which is rewritten as

$$\begin{aligned} P_{out}(y) \approx & \left(\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^N \\ & + \sum_{l=1}^{N}\sum_{j=0}^{l-1} l\binom{N}{l}\binom{l-1}{j}\left(\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^{N-l} \\ & \times \frac{(-1)^j m^m}{\Gamma^{2+j}(m)\bar{\gamma}^m}\left(1-\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^l \\ & \times \left(\Gamma(m)-\Gamma\left(m,\frac{my}{(1-\rho)\bar{\gamma}}\right)\right) \\ & \times I\left(m-1,\frac{m}{(1-\rho)\bar{\gamma}},m,\frac{m}{\bar{\gamma}},j\right), \end{aligned} \tag{36}$$

can be further approximated, due to (31) and (35), by

$$\begin{aligned} P_{out} \approx & \left(\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^N \\ & + \sum_{l=1}^{N} l\binom{N}{l}\left(\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^{N-l}\left(1-\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^l \\ & \times \frac{\lambda^m}{\Gamma(m)\bar{\gamma}^m}\left(\Gamma(m)-\Gamma\left(m,\frac{my}{\lambda}\right)\right) \\ & \times \sum_{j=0}^{l-1}(-1)^j\binom{l-1}{j}. \end{aligned} \tag{37}$$

Moreover, it is easy to see that [12, Eq. (0.15.4)]

$$\sum_{j=0}^{l-1}(-1)^j\binom{l-1}{j} = \begin{cases} 1, & l=1 \\ 0, & \text{otherwise} \end{cases}. \tag{38}$$

Hence, (37) reduces to

$$P_{out}(y) \approx \left(\frac{m^{m-1}y^m}{\Gamma(m)\bar{\gamma}^m}\right)^N + N\left(\frac{m^{m-1}}{\Gamma(m)}\right)^{N-1}\frac{\lambda^m}{\Gamma(m)\bar{\gamma}^m} \times \left(\Gamma(m) - \Gamma\left(m, \frac{my}{\lambda}\right)\right)\left(\frac{y^m}{\bar{\gamma}^m}\right)^{N-1}. \quad (39)$$

As a final step for studying the outage performance of our system when $\rho \to 1$, we need to investigate the behavior of (39) with respect to λ. As can be seen from (32), λ reflects the effect of the outdated channel estimates in the dependance of the outage probability on the SNR, as the SNR becomes large. In practical high-SNR scenarios, however, it is assumed that $\bar{\gamma}$ is sufficiently large, yet it still takes a finite value, i.e., $\bar{\gamma} < \infty$. Furthermore, in the ideal asymptotic case of perfect CSI, one can assume that ρ is, informally speaking, closer to unity than the SNR is close to infinity, thus allowing us to assume $\lambda \to 0$. Hence, in this ideally asymptotic case in terms of perfectly updated CSI, (39) yields

$$P_{out}(y) \approx \left(\frac{m^{m-1}}{\Gamma(m)}\right)^N \left(\frac{y}{\bar{\gamma}}\right)^{mN}. \quad (40)$$

In fact, (40) presents the asymptotic outage expression in high SNR and perfect channel estimation.

B. Diversity and Coding Gain

An important result derived from the high SNR analysis of Section IV-A is the diversity and coding gain of the system under consideration. These results are summarized in the ensuing two corollaries.

Corollary 1 (Diversity Gain): The diversity gain of single relay selection over N relays, when operating over Nakagami-m fading channels with outdated channel estimates is given by

$$G_d = \begin{cases} mN, & \text{if } \rho = 1 \\ m, & \text{if } \rho < 1 \end{cases} \quad (41)$$

Proof: This result follows straightforwardly from (30) for the case of $\rho < 1$ and from (40) for the case of $\rho = 1$. ■

Interestingly, it is noted that the diversity order of relay selection highly depends on the correlation coefficient between the actual and the estimated channel values. To be more precise, it follows from (41) that, under the outdated CSI model considered here where ρ does not depend on SNR, *if $\rho \neq 1$ the diversity order of relay selection reduces to that of the scheme where a single relay is available*. This implies that, generally speaking, one can take advantage of the benefits of relay selection only provided that the channel estimates are perfectly updated in the decision terminal. Otherwise, even with the slightest deviation from the $\rho = 1$ case, performing relay selection does not offer any diversity improvement.

Corollary 2 (Coding Gain): The coding gain of single relay selection over N relays, when operating over Nakagami-m fading channels with outdated channel estimates is given by

$$G_c = \begin{cases} \left(\frac{m^{m-1}}{\Gamma(m)}\right)^{-1/m}, & \text{if } \rho = 1 \\ \left[\sum_{j=0}^{N-1}\binom{N-1}{j}\frac{N(-1)^j}{\Gamma^2(m)}\frac{m^{m-1}g(m,\rho,j)}{(1-\rho)^m}\right]^{-1/m}, & \text{if } \rho < 1 \end{cases} \quad (42)$$

Proof: Expressing the outage probability in the high SNR regime in terms of the coding and diveristy gain as $P_{out}(y) = (G_c SNR)^{-G_d}$, (42) follows straightforwardly from (30) for the case of $\rho < 1$ and from (40) for the case of $\rho = 1$. ■

One may note from (42) that, in contrast to the diversity gain, performing relay selection in cases where the channel estimates are not perfectly updated can still lead to a performance improvement, reflected in a coding gain increase.

V. Numerical Results

In this section, we provide some numerical results on the outage performance of relay selection with outdated channel estimates when operating over Nakagami-m fading channels. Interesting insights on the outage dependence on ρ and m are gained.

Fig. 1 illustrates the dependence of the overall outage probability on the level of CSI imperfection, as this is reflected through the correlation coefficient between the actual and the estimated channel values, ρ. The number of available relays is set to $N = 5$, the Nakagami-m parameter as $m = 2$ and the outage threshold SNR as $y = 3$. As can be seen, the value of ρ significantly affects the outage probability. In particular, it is noted that when ρ deviates from the perfect CSI case, the outage performance is severely degraded. This performance degradation is reflected in a diversity loss for any $\rho \neq 1$, which is evident even for values of ρ very close to unity. Moreover, as can be observed from the upper two curves of Fig. 1, the outage improvement due to an increase on the number of relays is small for relatively low values of ρ, without shifting the outage slope. This result is in accordance with (41) and (42) for $\rho < 1$.

The outage probability dependence of relay selection with outdated CSI on the Nakagami-m parameter is depicted in Fig. 2. We assume again five participating relays ($N = 5$), an outage threshold SNR of $y = 3$, and the correlation coefficient between actual and estimated channel values is $\rho = 0.5$. As expected from (41), it is seen that increasing m results in an outage probability decrease, but in all cases the negative slope of the outage curves in high SNRs equals m, since $\rho < 1$ holds.

The tightness of the high SNR approximation given in (25), as compared to the exact outage expression of (21), is depicted in Fig. 3, for several values of ρ, $N = 5$, $m = 1$, and $y = 3$. For the infinite series in (21) we considered the first 100 terms, as this number of terms is sufficient for convergence. One may observe that (25) represents a tight approximation of the exact outage probability for high SNR values. Moreover, it is seen that for small values of ρ this approximation becomes tight in the medium, or even in the low SNR regime.

Appendix

First, we use the alternative representation of the incomplete Gamma function [12, Eq. (8.352/4)] as

$$\Gamma(m, \beta x) = (m-1)!\exp(-\beta x)\sum_{i=0}^{m-1}\frac{\beta^i x^i}{i!}. \quad (43)$$

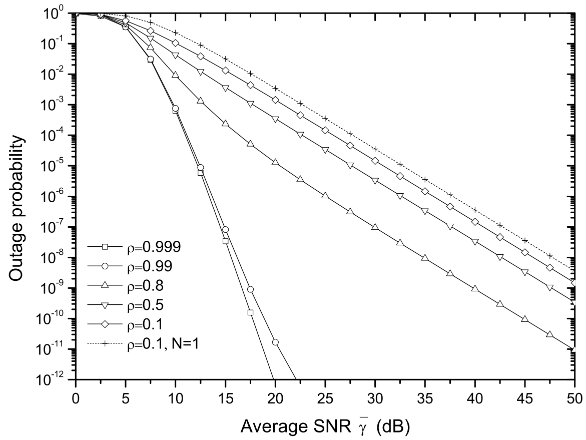

Fig. 1. Outage probability of relay selection with outdated channel estimates in Nakagami-m fading versus the average SNR, assuming $N = 5$, $m = 2$, $y = 3$, and several values of ρ.

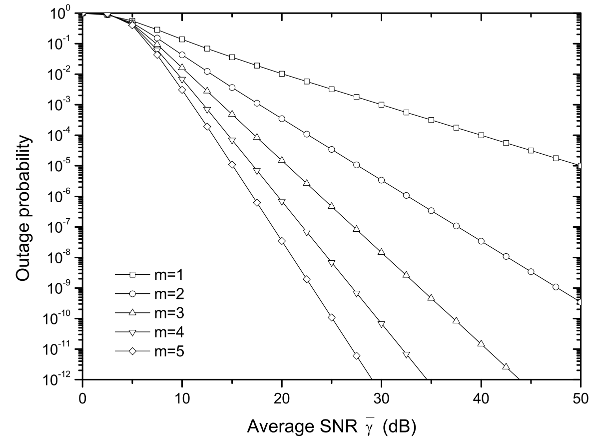

Fig. 2. Outage probability versus the average SNR for $\rho = 0.5$, $N = 5$, $y = 3$, and several values of the Nakagami-m shape distribution parameter, m.

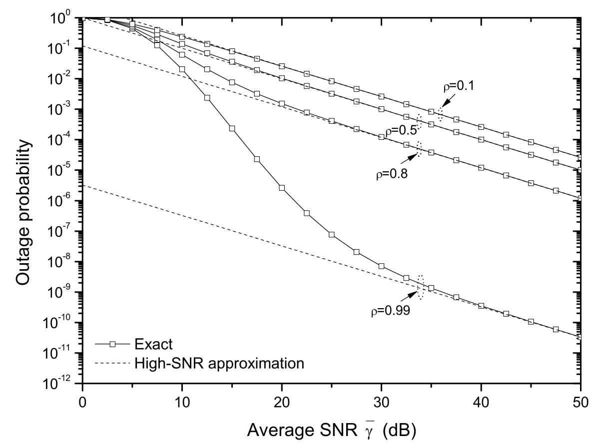

Fig. 3. Exact and approximated outage probability versus the average SNR, assuming $N = 5$, $m = 1$, $y = 3$, and several values of ρ.

Applying the multinomial theorem, we obtain

$$\Gamma^j(m,\beta x) = ((m-1)!)^j \exp(-j\beta x)\left(\sum_{i=0}^{m-1}\frac{\beta^i x^i}{i!}\right)^j$$

$$= \Gamma^j(m)\exp(-j\beta x)\sum_{\substack{n_0,n_1,\ldots,n_{m-1}=0\\ n_0+n_1+\ldots+n_{m-1}=0}}^{j}\Gamma(j)\prod_{i=0}^{m-1}\frac{\left(\frac{b^i}{i!}x^i\right)^{n_i}}{\Gamma(n_i)}. \tag{44}$$

Therefore, (15) yields

$$I(\mu,\alpha,m,\beta,j) = \sum_{\substack{n_0,n_1,\ldots,n_{m-1}=0\\ n_0+n_1+\ldots+n_{m-1}=0}}^{j}\Gamma^j(m)\Gamma(j+1)$$

$$\times\left(\prod_{i=0}^{m-1}\frac{\left(\frac{\beta^i}{i!}\right)^{n_i}}{\Gamma(n_i+1)}\right)\int_0^\infty x^{\mu+\sum\limits_{i=0}^{m-1}in_i}\exp(-(a+j\beta)x)\,dx$$

$$= \frac{\Gamma^j(m)\Gamma(j+1)}{(a+j\beta)^{\mu+1}}\sum_{\substack{n_0,n_1,\ldots,n_{m-1}=0\\ n_0+n_1+\ldots+n_{m-1}=j}}^{j}\left(\prod_{i=0}^{m-1}\frac{\left(\frac{\beta^i}{i!(a+j\beta)^i}\right)^{n_i}}{\Gamma(n_i+1)}\right)$$

$$\times\left(\mu+\sum_{i=0}^{m-1}in_i\right)!. \tag{45}$$

REFERENCES

[1] M. Dohler and Y. Li, "*Cooperative communications: Hardware, channel & PHY,*" Wiley & Sons, 2010.

[2] M. Uysal (Ed.), *Cooperative Communications for Improved Wireless Network Transmission: Frameworks for Virtual Antenna Array Applications*. IGI-Global, 2009.

[3] M. M. Fareed and M. Uysal, "On relay selection for decode-and-forward relaying", *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, p. 3341-3346, July 2009.

[4] E. Beres and R. Adve, "Selection cooperation in multi-source cooperative networks," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 118-127, Jan 2008.

[5] Y. Zhao, R.S. Adve and T.J. Lim, "Improving amplify-and-forward relay networks: optimal power allocation versus selection", *IEEE Trans. Wireless Commun.*, vol. 6, no. 8., pp. 3114-3123, Aug. 2007.

[6] V. Shah, N. B. Mehta, and R. Yim, "Relay selection and data transmission throughput tradeoff in cooperative systems", *IEEE Global Telecommunications Conference (Globecom)*, Honolulu, USA, Dec. 2009.

[7] R. Madan, N. B. Mehta, A. F. Molisch, and J. Zhang, "Energy-efficient cooperative relaying over fading channels with simple relay selection", *IEEE Transactions on Wireless Communications*, vol. 7, No. 8, Aug. 2008.

[8] H. Chen, J. Liu, L. Zheng, C. Zhai and Y. Zhou, "Approximate SEP analysis for DF cooperative networks with opportunistic relaying,"*IEEE Signal Processing Letters*, vol.17, no.9, pp.779-782, Sept. 2010

[9] J.-B. Kim and D. Kim, "Comparison of tightly power-constrained performances for opportunistic amplify-and-forward relaying with partial or full channel information,"*IEEE Commun. Letters*, vol.13, no.2, pp.100-102, Feb 2009

[10] J. L. Vicario, A. Bel, J. A. Lopez-Salcedo, and G. Seco, "Opportunistic relay selection with outdated CSI: Outage probability and diversity analysis," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 2872-2876, June 2009.

[11] D. S. Michalopoulos, H. A. Suraweera, G. K. Karagiannidis, and R. Schober, "Relay selection with outdated channel estimates, ", *IEEE Global Telecommunications Conference (Globecom)*, Miami, USA. To appear.

[12] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products,* New York, Academic Press, 7th edition, 2007

[13] J. Reig, L. Rubio and N. Cardona, "Bivariate Nakagami-m with arbitrary fading parameters," *Electron. Lett.*, vol. 38, no. 25, Dec. 2002.