Blind symbol rate detection for low-complexity multi-rate receivers

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Abstract— **In this contribution we investigate the performance of two novel symbol rate detectors when applied to a sub-optimal low-complexity multi-rate receiver. From the Maximum Likelihood (ML) criterion, two blind rate detection algorithms are derived and modified to operate on said low-complexity receiver. We compare these algorithms with a cyclic correlation-based scheme from literature in terms of their rate detection error probability. The proposed ML-based algorithms turn out to significantly outperform the correlation-based algorithm, both for a conventional receiver and the low-complexity receiver.**

I. Introduction

Next generation wireless devices will be expected to support different standards and different types of content. This has encouraged extensive research in the field of so-called Software Defined Radios (SDR) [1]. In these devices, the use of analog components is minimized in favor of Digital Signal Processing (DSP), controlled by (changeable) software programs. Physical layer aspects of software radio design regarding filter design, analog-to-digital conversion (ADC) and sample rate conversion can be found in [2]–[4]. This contribution is devoted to the aspects of rate detection and sample rate conversion for SDR and capitalizes on the results from [5].

In multi-rate systems with burst transmission, the symbol rate is constant within a frame, but can vary from one frame to the next. The transmitter can decide to change the symbol rate depending on various criteria, such as the channel conditions [6], dynamic bandwidth constraints, content type, etc. The receiver can obtain the value of the symbol rate in two ways: either this value is sent from the transmitter to the receiver through a separate channel, or the receiver (which knows the possible values of the symbol rate), estimates the symbol rate, based on the incoming packet. The latter strategy is known as rate detection (RD). Clearly, the former strategy leads to some overhead in terms of bandwidth, and requires the presence of a separate channel. Hence, in some scenarios, rate detection is preferred.

The problem of blind rate detection has already been treated extensively for DS-CDMA systems for 2nd and 3rd generation wireless devices. In DS-CDMA, the symbol rate can be changed by fixing the chip rate and varying the length of the spreading codes or by fixing the length of the spreading codes and varying the chip rate. These techniques are known as Variable Spreading Factor (VSF) and Variable Chip Rate (VCR), respectively [7]. Technical literature has thus-far mainly focused on VSF-CDMA (see for example [8]–[10] and references therein). However, the considered rate detection algorithms are very application-specific, are often developed specifically for BPSK, and, more importantly, are derived under the assumption that all channel parameters (such as channel gains, propagation delays etc.) are known. The techniques from this paper can be applied to VCR-CDMA, while the more general techniques from the companion paper [5] can be applied to any multi-rate system. Apart from the abovementioned work for DS-CDMA, most blind rate detectors exploit the cyclostationarity of the received signal [11], [12]. Although such detectors have some very attractive properties, they fail to operate properly when the excess bandwidth decreases or when the SNR is not sufficiently high. Since the data will generally be protected by an error-correcting code, a low SNR operating point can be assumed, so that these algorithms are no longer suitable. Another type of symbol rate detector was proposed in [13]: the received signal was filtered using an analog filter bank. Through an ad-hoc criterion, the most likely signal bandwidth was determined. The authors reported an estimation accuracy of 99.5% (i.e., a rate detection error probability of 0.005).

In this paper, we will investigate the problem of symbol rate detection for linear modulation in the presence of unknown channel parameters, for low-complexity multi-rate receivers. We will consider two algorithms from [5]: one based on a low SNR approximation of the maximum likelihood (ML) criterion and one based on the Expectation-Maximization (EM) algorithm. These algorithm will be applied to a particular receiver, designed for multi-rate transmission [14]. These algorithms will further be compared to a cyclic correlation-based approach from literature in terms of their rate error probability performance. Additionally, the impact of the proposed algorithms on the overall BER performance will be discussed.

II. System Model

Let us consider a general multi-rate transmission scheme. The transmitter forms packets as follows: a (possibly coded)

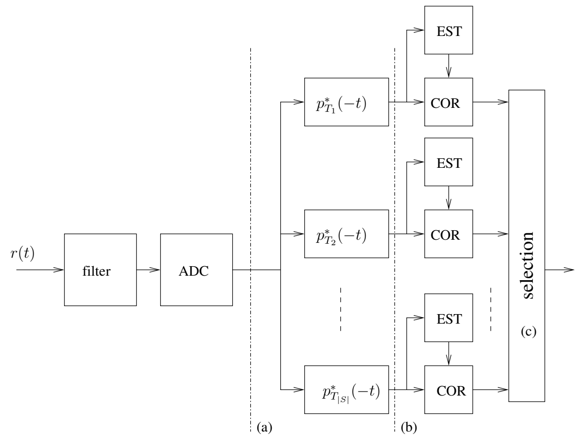

Fig. 1. Generic multi-rate receiver consisting of a bank of matched filters parameter estimation (EST) and correction (COR)

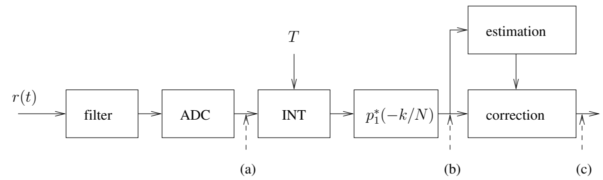

Fig. 2. Low-complexity multi-rate receiver consisting of an interpolator (INT), followed by a fixed matched filter. The interpolator requires knowledge of T.

bit sequence is mapped onto a sequence of K data symbols[1] taken from some constellation Ω. The transmitted (baseband) signal is given by

$$s_T\left(t\right)=\sqrt{E_s}\sum_{k=0}^{K-1}a_k p_T\left(t-kT\right) \tag{1}$$

where E_s denotes the energy per transmitted symbol, $\mathbf{a}=[a_0,\ldots,a_{K-1}]$ is the vector of data symbols and $p_T\left(t\right)$ is the transmit pulse corresponding to symbol rate $1/T$. We assume that $p_T\left(t\right)$ is a square-root cosine roll-off pulse[2], so that $g_T(kT)=\delta_k$ where $g_T\left(t\right)=\int p_T^*\left(-u\right)p_T\left(t-u\right)du$. The symbol interval T belongs to a finite set of equiprobable values: $T\in S=\{T_{min},\ldots,T_{max}\}$. The receiver is assumed to know the set S. For a flat, quasi-static channel, the complex envelope of the received signal can be expressed as

$$r\left(t\right)=hs_T\left(t-\tau\right)+n\left(t\right) \tag{2}$$

where $h=A\exp\left(j\theta\right)$ denotes the complex channel gain, τ the propagation delay and $n\left(t\right)$ a complex AWGN process with spectral density N_0. We model the phase θ as uniformly distributed in $(0,2\pi)$; the probability density function of the magnitude is arbitrary (e.g., in case of fading, a Rayleigh distribution is appropriate), while τ is uniformly distributed in $[-\Delta,+\Delta]$, for some constant Δ.

The receiver is fully digital: the signal $r\left(t\right)$ is band-limited through analog filtering and sampled at a fixed (i.e., independent of T) rate $1/T_s$:

$$r\left(kT_s\right)=hs_T\left(kT_s-\tau\right)+n\left(kT_s\right) \tag{3}$$

with $E\left[n\left(kT_s\right)n^*\left(lT_s\right)\right]=N_0T_s\delta_{k-l}$. A corresponding receiver is shown in Fig. 1: for each symbol rate, a different matched filter is required. Rate detection can take place prior to matched filtering (shown as (a) in Fig. 1), after matched filtering, but prior to estimation of τ and h (shown as (b) in Fig. 1) or, finally, after matched filtering, estimation and correction (shown as (c) in Fig. 1). In our case, correction refers to compensation of the carrier phase, timing correction and sample rate conversion (SRC) [4].

To avoid the need for a bank of matched filters, we will consider a different receiver structure (depicted in Fig. 2), first proposed in [15] and extended in [14]. In this low-complexity sub-optimal receiver, the matched filter is *preceded* by a digital interpolator. The interpolator performs sample rate conversion (i.e, it has input rate $1/T_s$ and output rate N/T, $N\in\mathbb{N}$), so that the matched filter taps become *independent* of the symbol rate. To see this, observe that $p_T\left(k\frac{T}{N}\right)=p_1\left(k\frac{1}{N}\right)/\sqrt{T}$. This is in stark contrast to a conventional receiver where the matched filter taps need to be changed according to the symbol rate. Again, symbol rate detection can take place at the three locations mentioned above (shown again as (a), (b) and (c) in Fig. 2).

III. Symbol rate detection

The main goal of the receiver is to recover the data symbols. In order to do this, the receiver requires reliable estimates of h, τ, and T. Generally, estimation algorithms for h and τ require knowledge of T. We will consider three rate detection algorithms.

Let us first start from the Maximum Likelihood (ML) principle. We denote by $\mathbf{r}$ the expansion of $r\left(t\right)$ onto a suitable basis. The ML estimate of the symbol interval is obtained by maximizing the likelihood function [16]:

$$\hat{T}_{ML}=\arg\max_{T\in S}p\left(\mathbf{r}\left|T\right.\right) \tag{4}$$

with

$$p\left(\mathbf{r}\left|T\right.\right)=\mathrm{E}_{\mathbf{a},\tau,h}\left[p\left(\mathbf{r}\left|\mathbf{a},\tau,h,T\right.\right)\right] \tag{5}$$

where $\mathrm{E}_{\mathbf{a},\tau,h}\left[.\right]$ denotes the averaging over all possible data sequences, τ and h. Taking into account the AWGN noise, we can write

$$\begin{aligned}&p\left(\mathbf{r}\left|\mathbf{a},\tau,h,T\right.\right)\\&=C\exp\left(-\frac{1}{N_0}\sum_k\left|r\left(kT_s\right)-hs_T\left(kT_s-\tau\right)\right|^2\right)\\&=C'\exp\left(\frac{2E_s}{N_0}\Re\left\{h^*\sum_{k=0}^{K-1}a_k^*y_T\left(kT+\tau\right)\right\}\right)\end{aligned}$$

[1] K is assumed to be independent of the symbol rate $1/T$. The proposed algorithms are easily adapted if this is not the case.

[2] In the case of DS-CDMA, $p_T(t)$ represents the chip pulse.

where C and C' do not depend on T and $y_T(t) = \int p_T^*(-u)\, r(t-u)\, du/\sqrt{E_s}$. The quantities $y_T(kT+\tau)$ are obtained by applying the received signal to a filter, matched to the transmit pulse corresponding to symbol rate $1/T$. Note that, for the low-complexity receiver, the output of the filter $\{p_1(l/N)\}$ at time $kT+\tau$ is equal to

$$\tilde{y}_T(kT+\tau) = y_T(kT+\tau)\sqrt{T}.$$

Unfortunately, averaging in (5) w.r.t. the unknown data symbols is generally intractable. We will now consider three practical rate detection algorithms. The first one is a cyclic correlation-based algorithm from [11]. Secondly, we review two algorithms from [5], which were designed for the receiver from Fig. 1. These algorithms are then modified so that they can be applied to our low-complexity multi-rate receiver. These algorithms correspond to performing rate detection at points (a), (b) and (c), respectively in Figs. 1 and 2.

A. Cyclic correlation based algorithm

This algorithm performs rate detection immediately after sampling [11]. It is computationally very efficient and requires no knowledge of h or τ. As it operates at the same point in the receivers from Fig. 1 and 2, the algorithm can be applied without modification from its form in [5]. Therefor, we omit additional mathematical details. The interested reader is referred to the original papers.

B. Low-SNR method

The full mathematical details are again given in [5]. The general idea is as follows: expanding $p(\mathbf{r}\,|\mathbf{a},\tau,h,T)$ in a Taylor series yields the following rate detection algorithm:

$$\hat{T} = \arg\max_{T\in S}\left\{\sum_{k=0}^{K-1}\int_{-\Delta}^{+\Delta}|y_T(kT+\tau)|^2\, d\tau\right\}.$$

Applying suitable approximations, this can be transformed into:

$$\hat{T} = \arg\max_{T\in S}\left\{\sum_{k=0}^{\lceil((K-1)T+\Delta)N/T\rceil}\frac{1}{T}\,|\tilde{y}_T(kT/N)|^2\right\}.$$

This algorithm also requires no knowledge of τ or h. However, it is computationally more complex than the cyclic correlation approach from [11]: the incoming signal has to be processed by the interpolator for all possible symbol rates. The resulting S sample sequences are applied to the matched filter. Only after considering the outputs of the matched filter, a decision is made w.r.t. the symbol rate.

C. EM approach

The last approach is based on the Expectation-Maximization (EM) algorithm [17]. The EM algorithm is a method that iteratively solves the problem of ML estimation of a parameter b. The algorithm makes use of the so-called *complete data* $\mathbf{z}$. The complete data is related

Algorithm 1 EM algorithm for rate detection

1: **for** $\tilde{T} = T_{max}$ to T_{max} **do**
2: perform sample rate conversion with $T = \tilde{T}$
3: perform matched filtering. This yields $\tilde{y}_{\tilde{T}}\left(l\tilde{T}/N\right)$
4: estimate τ and h using a standard algorithm. This yields $\hat{\tau}$ and $\hat{h}$
5: compute the soft symbol decisions $\tilde{a}_k$, $k = 0,\ldots,K-1$
6: $Q\left(\tilde{T}\middle|\tilde{T}\right) = \Re\left\{\frac{\hat{h}^*}{\sqrt{\tilde{T}}}\sum_{k=0}^{K-1}\tilde{a}_k^*\tilde{y}_{\tilde{T}}\left(k\tilde{T}+\hat{\tau}\right)\right\}$
7: **end for**
8: $\hat{T} = \arg\max_{\tilde{T}} Q\left(\tilde{T}\middle|\tilde{T}\right)$

to the observation $\mathbf{r}$ through some many-to-one mapping $\mathbf{r} = g(\mathbf{z})$. However, when b can take on values in a discrete set S only, the EM algorithms suffers from convergence problems [18]. To avoid this, we propose the following solution [19]:

$$\hat{b} = \arg\max_{b\in S} Q(b|\, b), \tag{6}$$

where

$$Q(b|\, b) \quad = \quad \mathrm{E}_{\mathbf{z}}\left[\log p(\mathbf{z}|\, b)\,|\mathbf{r}; b\right] \tag{7}$$

This approach has been shown to have very good performance in other detection problems [20] and does not require an initial estimate of b. Note that (6) can be applied only when S is finite.

How to apply the EM algorithm to the rate detection problem is shown in Algorithm 1. For a given trial value of T, sample rate conversion and matched filtering is performed. This is followed by estimation of τ and h. This estimation can be performed by means of a classical algorithm (see [16], [21], [22]) or by means of a more sophisticated EM algorithm (see [23]–[25]). The next step (computation of soft symbol decisions) requires some additional explanation: when determining $b = T$ and selecting as complete data $\mathbf{z} = [\mathbf{r},\mathbf{a}]$, it can be verified that [23] $Q\left(\tilde{T}\middle|\tilde{T}\right)$ is given by step 6 in Algorithm 1 where $\tilde{a}_k$ is the *a posteriori expectation* of a_k:

$$\tilde{a}_k = \sum_{a\in\Omega} a \times p\left(a_k = a\middle|\,\tilde{T},\mathbf{r},\hat{\tau},\hat{h}\right). \tag{8}$$

If we can obtain the a posteriori symbol probabilities $p\left(a_k = a\middle|\,T,\mathbf{r},\hat{\tau},\hat{h}\right)$, the symbol rate estimate can be easily computed. In the case of uncoded transmission, computation of the a posteriori symbol probabilities is trivial: $p\left(a_k = a\middle|\,\tilde{T},\mathbf{r},\hat{\tau},\hat{h}\right) = C\exp\left(-\frac{1}{N_0}\left|\frac{1}{\sqrt{\tilde{T}}}\tilde{y}_{\tilde{T}}\left(k\tilde{T}+\hat{\tau}\right)-\hat{h}\sqrt{E_s}a\right|^2\right)$ with C a normalizing constant. In the case of coded transmission, the a posteriori symbol probabilities can be computed using a MAP detector [26], [27]. As the latter approach will drastically increase the overall computational complexity,

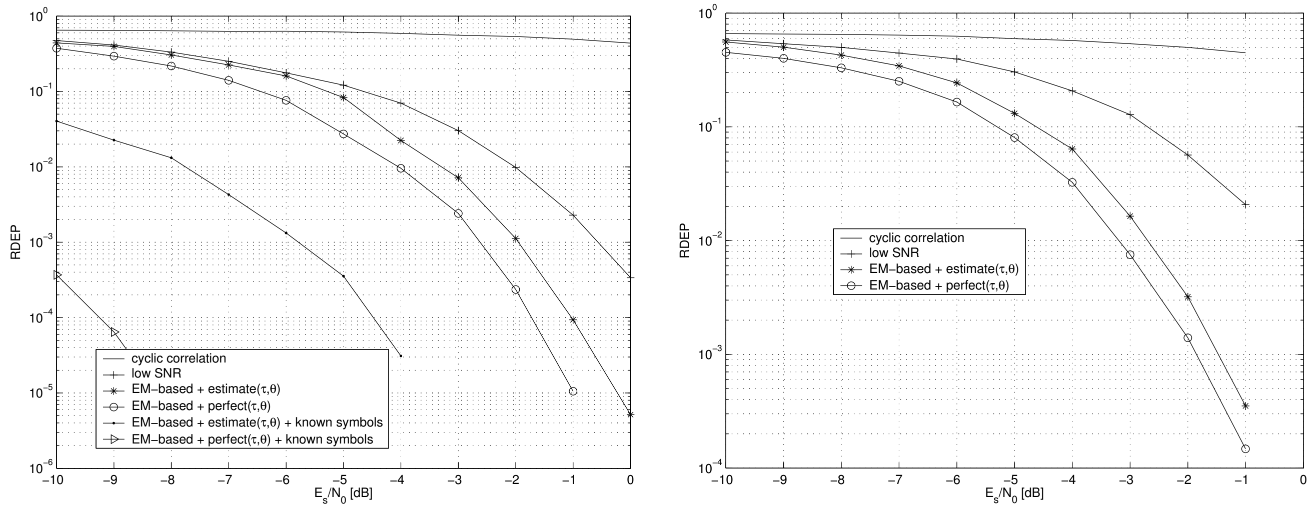

Fig. 3. Rate detection error probability vs. SNR for $K = 128$. Left: generic multi-rate receiver from Fig. 1. Right: low-complexity multi-rate receiver from Fig. 2.

it is preferred to treat the data symbols as uncoded during the rate detection process.

Clearly, the EM algorithm is more complex than the low-SNR method proposed in the previous section. As an additional disadvantage, it requires estimates of both τ and h. However, we will show that the EM-based algorithm has superior performance.

IV. Effect of rate detection errors on BER performance

The performance measure we consider is the RD error probability (RDEP), i.e., the fraction of frames for which the symbol rate is not correctly detected. As a detection error results in the loss of an entire frame, the RDEP should be sufficiently low. More specifically, if we denote the BER of a (coded or uncoded) system under perfect symbol rate detection by BER_0, then the BER in the presence of occasional RD errors is upper-bounded by

$$\begin{aligned} BER_{RD} &< BER_0\,(1 - RDEP) + 1 \times RDEP \\ &\approx BER_0 \left(1 + \frac{RDEP}{BER_0}\right). \end{aligned} \tag{9}$$

Consequently, in order to obtain a low BER degradation due to RD, the ratio $RDEP/BER_0$ should be below 1, in which case we obtain $BER_{RD} < 2BER_0$. A similar reasoning can be applied to frame error rate performance.

V. Numerical results

We have carried out computer simulations for BPSK transmission, with $p_T(t)$ a square-root cosine roll-off pulse with roll-off $\alpha = 0.5$. We set $S = \{T_{min}, 2T_{min}, 4T_{min}\}$, $T_s = 0.19T_{min}$, $N = 4$ and consider a frame lengths $K = 128$. As the data may be coded, a low-SNR operating point is assumed. However, we will treat the data as being uncoded during the rate detection process. It is important to note that the results obtained can be applied to determine the BER performance of any coded system by considering (9).

We consider both the generic multi-rate receiver from Fig. 1 and the low-complexity receiver from Fig. 2. We assume $A = 1$ and A is known to the receiver. Furthermore, different scenarios are considered for the EM-based algorithm: the propagation delay τ and the carrier phase θ are either known to the receiver (this situation will be referred to as $perfect(\tau, \theta)$) or estimated using conventional algorithms from [28] and [29], respectively (to be referred to as $estimate(\tau, \theta)$).

In Fig. 3 we show the RDEP for the three RD algorithms. On the left-hand side, we show the performance for the receiver from Fig. 1: in the SNR-region under consideration, the cyclic-correlation based approach exhibits very poor performance. The low-SNR algorithm has fairly good performance, but is outperformed by the EM-based algorithms, even when τ and θ are estimated. With perfect knowledge of τ and h, an additional gain of around 0.5 dB is visible. Finally, we have included results for the EM-based algorithm when the data symbols are perfectly known (i.e., $\tilde{a}_k \equiv a_k$). These results would give us an indication of the performance gains obtainable by exploiting code properties. In that case, we see from the left part of Fig. 3 that when τ and h are estimated a gain of around 5 dB is visible, compared to the low-SNR method. With perfect knowledge of τ, h *and* the data symbols, a gain of around 10 dB, compared to the low-SNR method, can be achieved. This illustrates that for some coded systems, it may be necessary to exploit code properties during estimation and/or rate detection to attain low BER degradations.

The right part of Fig. 3 shows the corresponding results for the low-complexity multi-rate receiver from Fig, 2. Comparing these results with the left part of Fig. 3, it is clear that both receiver structures yield similar results: the EM-based algorithms outperform the low-SNR method, with the cyclic correlation approach giving rise to large degradations. Note however, that the low-complexity multi-rate receiver suffers from a performance penalty compared to the generic multi-rate receiver, especially for the low-SNR approach.

VI. Conclusion

We have investigated two novel symbol rate detection algorithms for multi-rate transmission. One is derived from a low-SNR approximation of the likelihood function, while the other is based on the EM algorithm. These algorithms are applied to a generic multi-rate receiver as well as a sub-optimal low-complexity receiver, designed especially for multi-rate transmission. The conclusions are the same for both receivers: the EM-based algorithm has the higher complexity as it requires a posteriori probabilities of the data symbols and estimates of all channel parameters. On the other hand, the EM-based detector achieves superior performance compared to the low-SNR algorithm. We have shown that both proposed algorithms perform fairly well at the low-SNR operating point of powerful codes, whereas a cyclic correlation-based detector from literature is totally unreliable at low SNR. The low-complexity multi-rate receiver incurs a small performance penalty compared to the conventional generic multi-rate receiver.

Acknowledgement

This work has been supported by the Interuniversity Attraction Poles Program P5/11- Belgian Science Policy. The first author would like to thank V.S. Grigorov for providing simulation results.

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