Sensing-Throughput Tradeoff for Cognitive Radio Networks

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***Abstract*— In a cognitive radio network, the secondary users are allowed to utilize the frequency bands of primary users when these bands are not currently being used. To support this spectrum reuse functionality, the secondary users are required to sense the radio frequency environment, and once the primary users are found to be active, the secondary users are required to vacate the channel within a certain amount of time. Therefore, spectrum sensing is of significant importance in cognitive radio networks. There are two parameters associated with spectrum sensing: probability of detection and probability of false alarm. The higher the probability of detection, the better the primary users are protected. However, from the secondary users' perspective, the lower the probability of false alarm, the more chances the channel can be reused when it is available, thus the higher the achievable throughput for the secondary network. In this paper, we study the problem of designing the sensing duration to maximize the achievable throughput for the secondary network under the constraint that the primary users are sufficiently protected. We formulate the sensing-throughput tradeoff problem mathematically, and use energy detection sensing scheme to prove that the formulated problem indeed has one optimal sensing time which yields the highest throughput for the secondary network. Cooperative sensing using multiple mini-slots or multiple secondary users are also studied using the methodology proposed in this paper. Computer simulations have shown that for a 6MHz channel, when the frame duration is $100ms$, and the signal-to-noise ratio of primary user at the secondary receiver is -20dB, the optimal sensing time achieving the highest throughput while maintaining 90% detection probability is $14.2ms$. This optimal sensing time decreases when distributed spectrum sensing is applied.**

***Index Terms*— Cognitive radio, sensing-throughput tradeoff, spectrum reuse, spectrum sensing, throughput maximization.**

I. INTRODUCTION

THE last decade has witnessed the increasing popularity of wireless services. Based on fixed spectrum allocation methodology, in many countries, most of the available radio spectrum has been assigned for various services. On the other hand, careful studies of the spectrum usage pattern have revealed that the allocated spectrum experiences low utilization. In fact, recent measurements by Federal Communications Commission (FCC) have shown that 70% of the allocated spectrum in US is not utilized. Furthermore, time scale of the spectrum occupancy varies from milliseconds to hours [1]. This motivates the concept of *spectrum reuse* that allows secondary users/network to utilize the radio spectrum licensed/allocated to the primary users/network when the spectrum is temporally not being utilized.

The core technology behind spectrum reuse is *cognitive radio* [3], [4], which consists of three essential components: (1) *Spectrum sensing*: The secondary users are required to sense and monitor the radio spectrum environment within their operating range to detect the frequency bands that are not occupied by primary users; (2) *Dynamic spectrum management*: Cognitive radio networks are required to dynamically select the best available bands for communications; and (3) *Adaptive communications*: A cognitive radio device can configure its transmission parameters (carrier frequency, bandwidth, transmission power, etc) to opportunistically make best use of the ever-changing available spectrum.

In December 2003, FCC issued a Notice of Proposed Rule Making that identifies cognitive radio as the candidate for implementing opportunistic spectrum sharing [2]. The IEEE then formed the 802.22 Working Group to develop a standard for wireless regional area networks (WRAN) [6], which is an alternative broadband access scheme operating in unused VHF/UHF TV bands. By doing so, it is required that no harmful interference is caused to the incumbent primary users, which, in the VHF/UHF bands, include TV users and the FCC part 74 wireless microphones [6].

Fig. 1 illustrates the topology of a WRAN system where the primary users are TV users and wireless microphones, and the secondary users include both WRAN base station (BS) and WRAN customer premise equipments (CPEs). The WRAN systems are designed to provide wireless broadband access to rural and suburban areas, with the average coverage radius of 33 km. The operating principle of WRAN is based on opportunistic access to temporarily unused TV spectrum. The fundamental objective for a WRAN system is to maximize the spectrum utilization of the TV channels when they are not used by the primary users. To protect the primary users, whenever the primary users become active, the WRAN system has to vacate that channel within a certain amount of time (say 2 seconds as specified by 802.22 working group). Thus spectrum sensing is of significant importance for cognitive radio systems. In 802.22 WRAN, each medium access control (MAC) frame consists of one sensing slot and one data transmission slot, thus periodic spectrum sensing

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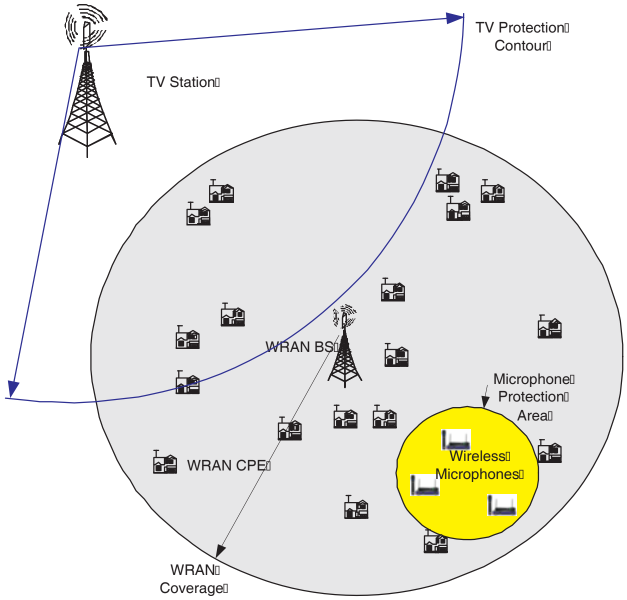

Fig. 1. A deployment scenario for IEEE 802.22 WRAN: the primary users are TV receivers and wireless microphones.

is carried out. Associated with spectrum sensing are two parameters: probability of detection and probability of false alarm. The higher the detection probability, the better the primary users can be protected. However, from the secondary users' perspective, the lower the false alarm probability, the more chances the channel can be reused when it is available, thus the higher the achievable throughput for the secondary users. Thus there could exist a fundamental tradeoff between sensing capability and achievable throughput for the secondary network.

In this paper, we study the fundamental problem of sensing-throughput tradeoff for cognitive radio networks. Particularly, we are interested in the problem of designing the sensing slot duration to maximize the achievable throughput for the secondary users under the constraint that the primary users are sufficiently protected. Using energy detection scheme, we prove that there indeed exists an optimal sensing time which achieves the best tradeoff. Cooperative sensing using multiple secondary users and diversity reception using multiple mini-slots are also studied based on the proposed tradeoff methodology.

While spectrum sensing is a conventional signal detection problem [16], [9], it has received growing attention recently in the area of wireless sensor networks [17], [18] and cognitive radio networks [12], [13], [14]. In sensor networks, the focus is on detecting the presence of an event using multiple distributed sensors. In cognitive radio networks, the criterion considered so far is in terms of protecting the primary user, i.e., maximizing the probability of detection under the constraint of probability of false alarm. In this paper, in order to formulate the sensing-throughput tradeoff problem, the objective turns out to be minimizing the probability of false alarm, under the constraint of probability of detection. We thus formulate the sensing-throughput tradeoff problem from this perspective.

In [20], the authors considered a different tradeoff problem from the one we are interested in this paper. In particular, in [20], when the present band used by the secondary users is detected to be busy due to the re-appearance of primary user, the secondary user then searches an out-band so that its transmission can be continued. The first problem studied in [20] is to minimize the average search time while targeting high chance that the secondary user can find at least one available out-band channel. Once the average search time is found, paper [20] continues to optimize the in-band sensing time in order to achieve maximum average throughput.

This paper is organized as follows. Section II presents the general model for spectrum sensing and reviews the energy detection scheme. The relation between probability of detection and probability of false alarm is also established in this section. In Section III, we study the sensing-throughput tradeoff problem, and prove the existence of optimal sensing time based on energy detection scheme. Section IV studies the sensing scheme based on multiple mini-slots which achieves time diversity. Distributed spectrum sensing using multiple secondary users is studied in Section V. Performance evaluation and comparisons are given in Section VI, and finally, conclusions are drawn in Section VII.

II. Spectrum Sensing Preliminaries

In this section, we first present the general model for spectrum sensing, then review the energy detection scheme and analyze the relationship between the probability of detection and the probability of false alarm.

A. General Model for Spectrum Sensing

Suppose that we are interested in the frequency band with carrier frequency f_c and bandwidth W and the received signal is sampled at sampling frequency f_s. When the primary user is active, the discrete received signal at the secondary user can be represented as

$$y(n) = s(n) + u(n), \tag{1}$$

which is the output under hypothesis $\mathcal{H}_1$. When the primary user is inactive, the received signal is given by

$$y(n) = u(n), \tag{2}$$

and this case is referred to as hypothesis $\mathcal{H}_0$. We make the following assumptions.

- (AS1) The noise $u(n)$ is a Gaussian, independent and identically distributed (iiid) random process with mean zero and variance $\mathsf{E}[|u(n)|^2] = \sigma_u^2$;
- (AS2) The primary signal $s(n)$ is an iid random process with mean zero and variance $\mathsf{E}[|s(n)|^2] = \sigma_s^2$;
- (AS3) The primary signal $s(n)$ is independent of the noise $u(n)$.

Denote $\gamma = \frac{\sigma_s^2}{\sigma_u^2}$ as the received signal-to-noise ratio (SNR) of the primary user measured at the secondary receiver of interest, under the hypothesis $\mathcal{H}_1$. We consider both real-valued Gaussian noise case and circularly symmetric complex Gaussian (CSCG) noise case. For the primary signal $s(n)$, we consider four scenarios: (1) BPSK modulated signal; (2) complex PSK modulated signal; (3) real-valued Gaussian signal and (4) CSCG signal.

Two probabilities are of interest for spectrum sensing: probability of detection, which defines, under hypothesis $\mathcal{H}_1$, the probability of the algorithm correctly detecting the presence of primary signal; and probability of false alarm, which defines, under hypothesis $\mathcal{H}_0$, the probability of the algorithm falsely declaring the presence of primary signal. From the primary user's perspective, the higher the probability of detection, the better protection it receives. From the secondary user's perspective, however, the lower the probability of false alarm, there are more chances for which the secondary users can use the frequency bands when they are available. Obviously, for a good detection algorithm, the probability of detection should be as high as possible while the probability of false alarm should be as low as possible.

B. Energy Detector

Energy detection is the most popular spectrum sensing scheme. Let τ be the available sensing time and N the number of samples (N is the maximum integer not greater than τf_s, and for notation simplicity, we assume $N = \tau f_s$). The test statistic for energy detector is given by

$$T(y) = \frac{1}{N}\sum_{n=1}^{N}|y(n)|^2. \tag{3}$$

Under hypothesis $\mathcal{H}_0$, the test static $T(y)$ is a random variable whose probability density function (PDF) $p_0(x)$ is a Chi-square distribution with $2N$ degrees of freedom for complex-valued case, and with N degrees of freedom for real-valued case. If we choose the detection threshold as ϵ, the probability of false alarm is then given by

$$P_f(\epsilon,\tau) = Pr(T(y) > \epsilon|\mathcal{H}_0) = \int_{\epsilon}^{\infty} p_0(x)dx. \tag{4}$$

Using central limit theorem (CLT), we have the following proposition.

Proposition 1: For a large N, the PDF of $T(y)$ under hypothesis $\mathcal{H}_0$ can be approximated by a Gaussian distribution with mean $\mu_0 = \sigma_u^2$ and variance $\sigma_0^2 = \frac{1}{N}\left[\mathsf{E}|u(n)|^4 - \sigma_u^4\right]$. Further,

- If $u(n)$ is real-valued Gaussian variable, then $\mathsf{E}|u(n)|^4 = 3\sigma_u^4$, thus $\sigma_0^2 = \frac{2}{N}\sigma_u^4$.
- If $u(n)$ is CSCG, then $\mathsf{E}|u(n)|^4 = 2\sigma_u^4$, thus $\sigma_0^2 = \frac{1}{N}\sigma_u^4$.

Next, we focus on the CSCG noise case for which the probability of false alarm is given by

$$P_f(\epsilon,\tau) = \mathcal{Q}\left(\left(\frac{\epsilon}{\sigma_u^2} - 1\right)\sqrt{\tau f_s}\right), \tag{5}$$

where $\mathcal{Q}(\cdot)$ is the complementary distribution function of the standard Gaussian, i.e.,

$$\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp\left(-\frac{t^2}{2}\right)dt. \tag{6}$$

Under hypothesis $\mathcal{H}_1$, denote $p_1(x)$ as the PDF of the test static $T(y)$. For a chosen threshold ϵ, the probability of detection is given by

$$P_d(\epsilon,\tau) = Pr(T(y) > \epsilon|\mathcal{H}_1) = \int_{\epsilon}^{\infty} p_1(x)dx. \tag{7}$$

Proposition 2: For a large N, the PDF of $T(y)$ under hypothesis $\mathcal{H}_1$ can be approximated by a Gaussian distribution with mean $\mu_1 = (\gamma+1)\sigma_u^2$ and variance

$$\sigma_1^2 = \frac{1}{N}\left[\mathsf{E}|s(n)|^4 + \mathsf{E}|u(n)|^4 - (\sigma_s^2 - \sigma_u^2)^2\right], \tag{8}$$

if $s(n)$ and $u(n)$ are both circularly symmetric and complex-valued, and

$$\sigma_1^2 = \frac{1}{N}\left[\mathsf{E}|s(n)|^4 + \mathsf{E}|u(n)|^4 - (\sigma_s^2 - \sigma_u^2)^2 + 2\sigma_s^2\sigma_u^2\right], \tag{9}$$

if $s(n)$ and $u(n)$ are both real-valued. Furthermore,

- If $s(n)$ is complex PSK modulated and $u(n)$ is CSCG, then $\sigma_1^2 = \frac{1}{N}(2\gamma+1)\sigma_u^4$;
- If $s(n)$ is BPSK modulated and $u(n)$ is real-valued Gaussian, then $\sigma_1^2 = \frac{2}{N}(2\gamma+1)\sigma_u^4$;
- If $s(n)$ and $u(n)$ are both CSCG, $\mathsf{E}|s(n)|^4 = 2\sigma_s^4$ and $\mathsf{E}|u(n)|^4 = 2\sigma_u^4$, then $\sigma_1^2 = \frac{1}{N}(\gamma+1)^2\sigma_u^4$;
- If $s(n)$ and $u(n)$ are both real-valued Gaussian, $\mathsf{E}|s(n)|^4 = 3\sigma_s^4$ and $\mathsf{E}|u(n)|^4 = 3\sigma_u^4$, then $\sigma_1^2 = \frac{2}{N}(\gamma+1)^2\sigma_u^4$.

Proof: The proof is again based on CLT. Detailed proof is given in Appendix A.

Remark: For the four cases of primary user's signal and additive noise we considered, the complex counterpart yields half the variance as that of the real-valued case. This can be understood by considering the fact that the complex case, in fact, provides twice the samples as compared to the real-valued case.

We focus on the complex-valued PSK signal and CSCG noise case. Based on the PDF of the test static, the probability of detection can be approximated by

$$P_d(\epsilon,\tau) = \mathcal{Q}\left(\left(\frac{\epsilon}{\sigma_u^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{2\gamma+1}}\right). \tag{10}$$

For a target probability of detection, $\bar{P}_d$, the detection threshold ϵ can be determined by

$$\left(\frac{\epsilon}{\sigma_u^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{2\gamma+1}} = \mathcal{Q}^{-1}(\bar{P}_d). \tag{11}$$

From (5), on the other hand, this threshold is related to the probability of false alarm as follows:

$$\mathcal{Q}^{-1}(P_f) = \left(\frac{\epsilon}{\sigma_u^2} - 1\right)\sqrt{\tau f_s}. \tag{12}$$

Therefore, for a target probability of detection $\bar{P}_d$, the probability of false alarm is related to the target detection probability as follows:

$$P_f = \mathcal{Q}\left(\sqrt{2\gamma+1}\,\mathcal{Q}^{-1}(\bar{P}_d) + \sqrt{\tau f_s}\gamma\right). \tag{13}$$

On the other hand, for a target probability of false alarm, $\bar{P}_f$, the probability of detection is given by

$$P_d = \mathcal{Q}\left(\frac{1}{\sqrt{2\gamma+1}}(\mathcal{Q}^{-1}(\bar{P}_f) - \sqrt{\tau f_s}\gamma)\right). \tag{14}$$

Finally, for a given pair of target probabilities $(\bar{P}_d, \bar{P}_f)$, the number of required samples to achieve these targets can be

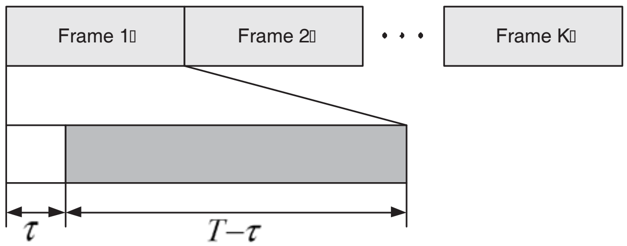

Fig. 2. Frame structure for cognitive radio networks with periodic spectrum sensing (τ: sensing slot duration; $T-\tau$: data transmission slot duration.)

determined from (11) and (12) by cancelling out the threshold variable ϵ. The minimum number of samples is given by

$$N_{\min} = \frac{1}{\gamma^2}\left[\mathcal{Q}^{-1}(\bar{P}_f) - \mathcal{Q}^{-1}(\bar{P}_d)\sqrt{2\gamma+1}\right]^2. \quad (15)$$

III. Sensing-Throughput Tradeoff

In the previous section, the relationship between probability of detection and probability of false alarm has been established. In this section, we study the fundamental tradeoff between sensing capability and achievable throughput of the secondary networks. Using energy detection scheme, we will prove that there indeed exists the optimal sensing time with which the highest throughput for the secondary network is achieved, yet the primary users are sufficiently protected.

A. Problem Formulation

Fig. 2 shows the frame structure designed for a cognitive radio network with periodic spectrum sensing where each frame consists of one sensing slot and one data transmission slot. Suppose the sensing duration is τ and the frame duration is T. Denote C_0 as the throughput of the secondary network when it operates in the absence of primary users, and C_1 as the throughput when it operates in the presence of primary users. For example, if there is only one point-to-point transmission in the secondary network and the SNR for this secondary link is $SNR_s = P_s/N_0$, where P_s is the received power of the secondary user and N_0 is the noise power. Let P_p be the interference power of primary user measured at the secondary receiver, and assume that the primary user's signal and secondary user's signal are Gaussian, white and independent of each other. Then $C_0 = \log_2(1+SNR_s)$ and $C_1 = \log_2\left(1+\frac{P_s}{P_p+N_0}\right) = \log_2\left(1+\frac{SNR_s}{1+SNR_p}\right)$, where $SNR_p = P_p/N_0$. Obviously, we have $C_0 > C_1$. Note if the primary user's signal is non-Gaussian, the above formula for C_1 can be treated as the lower bound of achievable rate for secondary link when the primary user is active.

For a given frequency band of interest, let us define $P(\mathcal{H}_1)$ as the probability for which the primary user is active, and $P(\mathcal{H}_0)$ as the probability for which the primary user is inactive. Then $P(\mathcal{H}_0)+P(\mathcal{H}_1)=1$.

There are two scenarios for which the secondary network can operate at the primary user's frequency band.

- Scenario I: When the primary user is not present and no false alarm is generated by the secondary user, the achievable throughput of the secondary link is $\frac{T-\tau}{T}C_0$.
- Scenario II: When the primary user is active but it is not detected by the secondary user, the achievable throughput of the secondary link is $\frac{T-\tau}{T}C_1$.

The probabilities for which Scenario I and Scenario II happen are $(1-P_f(\epsilon,\tau))P(\mathcal{H}_0)$ and $(1-P_d(\epsilon,\tau))P(\mathcal{H}_1)$, respectively. If we define

$$R_0(\epsilon,\tau) = \frac{T-\tau}{T}C_0(1-P_f(\epsilon,\tau))P(\mathcal{H}_0), \quad (16)$$

and

$$R_1(\epsilon,\tau) = \frac{T-\tau}{T}C_1(1-P_d(\epsilon,\tau))P(\mathcal{H}_1), \quad (17)$$

then the average throughput for the secondary network is given by

$$R(\tau) = R_0(\epsilon,\tau) + R_1(\epsilon,\tau). \quad (18)$$

Obviously, for a given frame duration T, the longer the sensing time τ, the shorter the available data transmission time $(T-\tau)$. On the other hand, from (13), since $\mathcal{Q}(x)$ is a monotonically decreasing function of x, for a given target probability of detection, $\bar{P}_d$, the longer the sensing time, the lower the probability of false alarm, which corresponds to the case that the secondary network can use the channel with a higher chance. The objective of sensing-throughput tradeoff is to identify the optimal sensing duration τ for each frame such that the achievable throughput of the secondary network is maximized while the primary users are sufficiently protected. Mathematically, the optimization problem can be stated as

$$\max_{\tau} \quad R(\tau) = R_0(\epsilon,\tau) + R_1(\epsilon,\tau) \quad (19)$$

$$\text{s.t.} \quad P_d(\epsilon,\tau) \geq \bar{P}_d, \quad (20)$$

where $\bar{P}_d$ is the target probability of detection with which the primary users are defined as being sufficiently protected. In practice, the target probability of detection $\bar{P}_d$ is chosen to be close to but less than 1, especially for low SNR regime. For instance, in IEEE802.22 WRAN, we choose $\bar{P}_d = 0.9$ for the SNR of -20dB. It is pointed out that if the primary users require 100% protection in its frequency band, it will then be not allowed for the secondary usage in that frequency band. Also, we suppose the activity probability $P(\mathcal{H}_1)$ of primary users is small, say less than 0.3, thus it is economically advisable to explore the secondary usage for that frequency band. Since $C_0 > C_1$, the first term in the right hand side of (18) dominates the achievable throughput. Therefore the optimization problem can be approximated by

$$\max_{\tau} \quad \tilde{R}(\tau) = R_0(\epsilon,\tau) \quad (21)$$

$$\text{s.t.} \quad P_d(\epsilon,\tau) \geq \bar{P}_d. \quad (22)$$

For a given sensing time τ, according to (10), we may choose a detection threshold ϵ_0 such that $P_d(\epsilon_0,\tau) = \bar{P}_d$. We may also choose a detection threshold $\epsilon_1 < \epsilon_0$ such that $P_d(\epsilon_1,\tau) > \bar{P}_d$. Obviously, $P_f(\epsilon_1,\tau) > P_f(\epsilon_0,\tau)$. Thus from (16) and (17), we have $R_0(\epsilon_1,\tau) < R_0(\epsilon_0,\tau)$ and $R_1(\epsilon_1,\tau) < R_1(\epsilon_0,\tau)$. Therefore, the optimal solution to (21) and (22) is achieved with equality constraint in (22). Finally, $R_0(\epsilon_1,\tau)+R_1(\epsilon_1,\tau) < R_0(\epsilon_0,\tau)+R_1(\epsilon_0,\tau)$, thus the optimal solution to (19) and (20) is also achieved when the equality constraint in (20) is satisfied.

B. Energy Detection Scheme

When energy detector is applied, using (13) and choosing $P_d = \bar{P}_d$, we have

$$\tilde{R}(\tau) = C_0 P(\mathcal{H}_0)\left(1-\frac{\tau}{T}\right)\left(1-\mathcal{Q}\left(\alpha+\sqrt{\tau f_s}\gamma\right)\right). \quad (23)$$

where $\alpha = \sqrt{2\gamma+1}\mathcal{Q}^{-1}(\bar{P}_d)$. Thus, from (23), we can see that the achievable throughput of the secondary network is a function of the sensing time τ.

Theorem 1: Under the assumptions (AS1) - (AS3), if the primary signal is complex-valued PSK and the noise process is CSCG, for any target probability of detection, there exists an optimal sensing time which yields the maximum achievable throughput for the secondary networks.

Proof: It can be verified from (23) that

$$\begin{aligned}\frac{\tilde{R}'(\tau)}{C_0 P(\mathcal{H}_0)} &= \frac{\gamma\sqrt{f_s}(1-\frac{\tau}{T})}{2\sqrt{2\pi\tau}}\exp\left(-\frac{(\alpha+\sqrt{\tau f_s}\gamma)^2}{2}\right)\\ &\quad -\frac{1}{T}+\frac{1}{T}\mathcal{Q}(\alpha+\sqrt{\tau f_s}\gamma). \quad (24)\end{aligned}$$

Obviously,

$$\lim_{\tau\to T}\tilde{R}'(\tau) < C_0 P(\mathcal{H}_0)\left(-\frac{1}{T}+\frac{1}{T}\mathcal{Q}(\alpha)\right) < 0, \quad (25)$$

$$\lim_{\tau\to 0}\tilde{R}'(\tau) = +\infty. \quad (26)$$

In deriving (25), we have used the fact that $\mathcal{Q}(x)$ is a decreasing function and upper bounded by 1. Eqs (25) and (26) mean that $\tilde{R}(\tau)$ increases when τ is small and decreases when τ approaches T. Hence, there is a maximum point of $\tilde{R}(\tau)$ within interval $(0, T)$.

Furthermore, from (18), we have

$$R'(\tau) = \tilde{R}'(\tau) - \frac{1}{T}C_1 P(\mathcal{H}_1)(1-\bar{P}_d). \quad (27)$$

Thus, again

$$\begin{aligned}\lim_{\tau\to T}R'(\tau) &< \lim_{\tau\to T}\tilde{R}'(\tau) < 0, \quad (28)\\ \lim_{\tau\to 0}R'(\tau) &= \lim_{\tau\to 0}\tilde{R}'(\tau) - \frac{1}{T}C_1 P(\mathcal{H}_1)(1-\bar{P}_d)\\ &= +\infty. \quad (29)\end{aligned}$$

Hence, there is a maximum point of $R(\tau)$ within interval $(0, T)$.

In Appendix B, we further show that $R(\tau)$ is concave for the range of τ in which $P_f(\tau) \le 0.5$. This makes the maximum point of $R(\tau)$ to be unique in this range. If the optimal sensing time falls into this range, which should be the case for most frequency reuse scenarios, efficient search algorithms can then be developed. Otherwise, exhaustive search is needed in order to find the optimal sensing time. Finally, while *Theorem 1* assumes that the signal is complex-valued PSK and the noise is CSCG, the fact that there exists the optimal sensing time within the interval $(0, T)$ is also valid for other cases (real-valued or CSCG signal).

IV. MULTI-SLOT SPECTRUM SENSING

In this section, we are interested in the case when the sensing slot in each frame is split into multiple discontiguous mini-slots. Let M be the number of mini-slots, τ_1 the sensing time for each mini-slot, and again T the duration for each frame. We fix the the total sensing time in each frame to $\tau = M\tau_1$, and the number of samples for each mini-slot is $N_1 = N/M$ (without loss of generality, we assume that N_1 is an integer.)

Let us consider the following hypotheses for the ith mini-slot:

$$\mathcal{H}_1 : y_i(n) = h_i s_i(n) + u_i(n), \quad (30)$$

$$\mathcal{H}_0 : y_i(n) = u_i(n), \quad (31)$$

and make the following assumptions:

- The channel coefficients h_i's are zero-mean, unit-variance complex Gaussian random variables;
- The noises are independent of each other for each of the M mini-slots;
- The signal power and noise power are constant over the M mini-slots, i.e., $\mathsf{E}[|s_i(n)|^2] = \sigma_s^2$, and $\mathsf{E}[|u_i(n)|^2] = \sigma_u^2$ for all i;
- The primary user is either active or inactive for all the M mini-slots.

The objective of multi-slot spectrum sensing is, using the available data measurements from the M mini-slots, to decide whether the primary user is active or not.

The decision can be made through two methods: data fusion and decision fusion.

- *Data Fusion:* Process the measurements from all mini-slots jointly and then make the final decision based on the calculated statistics;
- *Decision Fusion:* Process each mini-slot's data separately, and make individual decisions. The final decision is then made by fusing the individual decisions.

For time varying channels, multi-slot spectrum sensing is expected to achieve time diversity for sensing the presence of primary user even with the help of a single secondary user.

A. Data Fusion

Let $T_i(y)$ be the test statistic for the ith mini-slot:

$$T_i(y) = \frac{1}{N_1}\sum_{n=1}^{N_1}|y_i(n)|^2, \quad (32)$$

Using data fusion, the test statistic used for final decision is then represented as

$$T(y) = \sum_{i=1}^{M} g_i T_i(y), \quad (33)$$

where $g_i \ge 0$ is the weighting factor associated with the ith mini-slot. Without loss of generality, we assume that $\sum_{i=1}^{M} g_i^2 = 1$.

Proposition 3: For a large N, the PDF of $T(y)$ under hypothesis $\mathcal{H}_0$ can be approximated by a Gaussian distribution with mean $\mu_0 = \sigma_u^2\sum_{i=1}^{M} g_i$ and variance $\sigma_0^2 = \frac{1}{N_1}\sum_{i=1}^{M} g_i^2\left[\mathsf{E}|u(n)|^4 - \sigma_u^4\right]$.

Proposition 4: For a large N, the PDF of $T(y)$ under hypothesis $\mathcal{H}_1$ can be approximated by a Gaussian distribution with mean $\mu_1 = \sigma_u^2 \sum_{i=1}^M g_i(|h_i|^2\gamma + 1)$ and variance

$$\sigma_1^2 = \frac{1}{N_1}\sum_{i=1}^M g_i^2[|h_i|^4 \mathsf{E}|s(n)|^4 + \mathsf{E}|u(n)|^2 - (|h_i|^2\sigma_s^2 - \sigma_u^2)^2], \quad (34)$$

if the primary signal and noise are both circularly symmetric and complex-valued, and

$$\sigma_1^2 = \frac{1}{N_1}\sum_{i=1}^M g_i^2[|h_i|^4 \mathsf{E}|s(n)|^4 + \mathsf{E}|u(n)|^2 - (|h_i|^2\sigma_s^2 - \sigma_u^2)^2 + 2|h_i|^2\sigma_s^2\sigma_u^2], \quad (35)$$

if the primary signal and noise are both real-valued.

Again, let us consider complex PSK modulated signal and CSCG noise, we then have

$$\sigma_0^2 = \frac{\sigma_u^4}{N_1}, \quad (36)$$

$$\sigma_1^2 = \frac{\sigma_u^4}{N_1}\left(1 + 2\gamma\sum_{i=1}^M g_i^2|h_i|^2\right). \quad (37)$$

Mimicking the process of deriving (13) and (14), we have the following relation between P_d and P_f:

$$P_f = \mathcal{Q}\left(f_1(g_1, \cdots, g_M; P_d)\right), \quad (38)$$

$$P_d = \mathcal{Q}\left(f_2(g_1, \cdots, g_M; P_f)\right), \quad (39)$$

where

$$f_1(g_1, \cdots, g_M; P_d) = \beta_1\mathcal{Q}^{-1}(P_d) + \gamma\sqrt{N_1}\sum_{i=1}^M g_i|h_i|^2, \quad (40)$$

$$f_2(g_1, \cdots, g_M; P_f) = \frac{1}{\beta_1}\left(\mathcal{Q}^{-1}(P_f) - \gamma\sqrt{N_1}\sum_{i=1}^M g_i|h_i|^2\right), \quad (41)$$

where $\beta_1 = \sqrt{1 + 2\gamma\sum_{i=1}^M g_i^2|h_i|^2}$. Since $\mathcal{Q}(x)$ is a decreasing function of x, from the primary user's interest, for a target probability of false alarm $\bar{P}_f$, we want to design the optimal g_i by maximizing the probability of detection, which gives:

$$\min_{g_1,\cdots,g_M:\sum_{i=1}^M g_i^2=1} f_2(g_1, \cdots, g_M; \bar{P}_f). \quad (42)$$

From the sensing-throughput tradeoff perspective, we wish a target probability of detection $\bar{P}_d$ to be achieved for each frame, thus the optimal g_i is designed to achieve minimum probability of false alarm:

$$\max_{g_1,\cdots,g_M:\sum_{i=1}^M g_i^2=1} f_1(g_1, \cdots, g_M; \bar{P}_d). \quad (43)$$

We are now interested in the low SNR regime where $\gamma \to 0$, thus $\beta_1 \to 0$. In this case, Eqs (40) and (41) are approximated by

$$f_1(g_1, \cdots, g_M; P_d) \approx \mathcal{Q}^{-1}(P_d) + \gamma\sqrt{N_1}\sum_{i=1}^M g_i|h_i|^2, \quad (44)$$

$$f_2(g_1, \cdots, g_M; P_f) \approx \mathcal{Q}^{-1}(P_f) - \gamma\sqrt{N_1}\sum_{i=1}^M g_i|h_i|^2. \quad (45)$$

Theorem 2: Suppose the low SNR regime is of interest. For a target probability of detection $\bar{P}_d$, the optimal g_i achieving minimum probability of false alarm is given by

$$g_i = \frac{|h_i|^2}{\sqrt{\sum_{i=1}^M |h_i|^4}}. \quad (46)$$

Proof: For low SNR regime, (43) is equivalent to the following optimization function:

$$\max_{g_1,\cdots,g_M:\sum_{i=1}^M g_i^2=1} \sum_{i=1}^M g_i|h_i|^2. \quad (47)$$

Using Cauchy-Schwarz inequality, we obtain the optimal combining coefficients given by (46).

On the other hand, the following theorem describes the optimal weighting coefficients from the primary user's perspective.

Theorem 3: Suppose the low SNR regime is of interest. For a target probability of false alarm $\bar{P}_f$, the optimal g_i achieving maximum probability of detection is also given by (46).

When the channel coefficients are unknown, a simple way for data fusion is to choose $g_i = \frac{1}{\sqrt{M}}$. In this case, we obtain:

$$P_f = \mathcal{Q}\left(\beta_2\mathcal{Q}^{-1}(P_d) + \gamma\sqrt{\frac{N_1}{M}}\sum_{i=1}^M |h_i|^2\right), \quad (48)$$

$$P_d = \mathcal{Q}\left(\frac{1}{\beta_2}\left(\mathcal{Q}^{-1}(P_f) - \gamma\sqrt{\frac{N_1}{M}}\sum_{i=1}^M |h_i|^2\right)\right), \quad (49)$$

where $\beta_2 = \sqrt{1 + \frac{2\gamma}{M}\sum_{i=1}^M |h_i|^2}$.

Finally, let us consider a special case where $h_i = 1$ for all i, and g_i is chosen as $g_i = \frac{1}{\sqrt{M}}$. From (38), for a target $\bar{P}_d$, the probability of false alarm is given by:

$$P_f = \mathcal{Q}\left(\sqrt{2\gamma+1}\mathcal{Q}^{-1}(\bar{P}_d) + \gamma\sqrt{MN_1}\right). \quad (50)$$

Since $MN_1 = N$, the use of M mini-slots for the case of static channel does not provide any performance gain when data fusing is applied.

From (38), for a fading channel environment, in order to achieve same probability of detection for each frame, the detection threshold will change from one frame to another, thus the probability of false alarm will vary. Let us denote, for a given sensing duration τ, $\tilde{P}_f(\tau)$ as the average probability of false alarm over all frames. We then define the normalized achievable throughput as

$$B(\tau) = \frac{T-\tau}{T}(1 - \tilde{P}_f(\tau)). \quad (51)$$

B. Decision Fusion

Denote $P_d^{(i)}$ and $P_f^{(i)}$ as the probability of detection and probability of false alarm at the ith mini-slot, respectively. By choosing the detection threshold as ϵ_0, the probability of detection for the ith time slot is

$$P_d^{(i)} = \mathcal{Q}\left(\left(\frac{\epsilon_0}{\sigma_u^2} - \gamma|h_i|^2 - 1\right)\sqrt{\frac{N_1}{2\gamma|h_i|^2+1}}\right), \quad (52)$$

and the probability of false alarm for the ith slot is given by

$$P_f^{(i)} = \mathcal{Q}\left(\left(\frac{\epsilon_0}{\sigma_u^2} - 1\right)\sqrt{N_1}\right). \quad (53)$$

Once the decision is made for each time slot, there are different rules available for making final decision on the presence of the primary user.

1) Optimal Decision Fusion Rule: Let I_i be the binary decision from the ith time slot, where $I_i \in \{0, 1\}$ for $i = 1, \cdots, M$. The optimal decision fusion rule is the Chair-Varshney fusion rule [16], which is a threshold test of the following statistic:

$$\Lambda_0 = \sum_{i=1}^{M}\left[I_i \log\frac{P_d^{(i)}}{P_f^{(i)}} + (1 - I_i)\log\frac{1 - P_d^{(i)}}{1 - P_f^{(i)}}\right] + \log\frac{P(\mathcal{H}_1)}{P(\mathcal{H}_0)}. \quad (54)$$

If $\Lambda_0 \geq 0$, then the primary user is present; otherwise, there is no primary user.

2) "Logic-OR (LO)" Rule: LO rule is a simple decision rule described as follows: if one of the decisions says that there is a primary user, then the final decision declares that there is a primary user. Mathematically, define $\Lambda = \sum_{i=1}^{M} I_i$, if $\Lambda \geq 1$, then the primary user is present; otherwise, there is no primary user.

Assuming that all decisions are independent, the probability of detection and probability of false alarm of the final decision are, respectively,

$$P_d = 1 - \prod_{i=1}^{M}(1 - P_d^{(i)}), \quad (55)$$

$$P_f = 1 - \prod_{i=1}^{M}(1 - P_f^{(i)}). \quad (56)$$

3) "Logic-AND (LA)" Rule: LA rule works as follows: if all decisions says that there is a primary user, then the final decision declares that there is a primary user. Mathematically, define $\Lambda = \prod_{i=1}^{M} I_i$, if $\Lambda = 1$, then the primary user is present; otherwise, there is no primary user. Again, assuming that all decisions are independent, the probability of detection and probability of false alarm of the final decision are, respectively,

$$P_d = \prod_{i=1}^{M} P_d^{(i)}, \quad (57)$$

$$P_f = \prod_{i=1}^{M} P_f^{(i)}. \quad (58)$$

4) Majority Rule: Another decision rule is based on majority of the individual decisions. If half of the decisions or more say that there is a primary user, then the final decision declares that there is a primary user. Mathematically, define $\Lambda = \sum_{i=1}^{M} I_i$, if $\Lambda \geq \lceil \frac{M}{2} \rceil$, where $\lceil x \rceil$ denotes the smallest integer not less then x, then the primary user is declared to be present; otherwise, there is no primary user. Assuming that all decisions are independent, and supposing that $P_d^{(1)} = \cdots P_d^{(M)} = P_{d,0}$ and $P_f^{(1)} = \cdots P_f^{(M)} = P_{f,0}$, the probability of detection and probability of false alarm of the final decision are given by,

$$P_d = \sum_{j=0}^{M - \lceil \frac{M}{2} \rceil} \binom{M}{\lceil \frac{M}{2} \rceil + j} (1 - P_{d,0})^{M - \lceil \frac{M}{2} \rceil - j} P_{d,0}^{\lceil \frac{M}{2} \rceil + j}, \quad (59)$$

$$P_f = \sum_{j=0}^{M - \lceil \frac{M}{2} \rceil} \binom{M}{\lceil \frac{M}{2} \rceil + j} (1 - P_{f,0})^{M - \lceil \frac{M}{2} \rceil - j} P_{f,0}^{\lceil \frac{M}{2} \rceil + j}, \quad (60)$$

respectively, where $\binom{c}{k} = \frac{c!}{k!(c-k)!}$.

V. Distributed Spectrum Sensing

In this section, we consider cooperative spectrum sensing using multiple distributed secondary users. The tradeoff methodology proposed in the previous section can be applied directly. Define the hypotheses for the ith receiver as:

$$\mathcal{H}_1 : y_i(n) = h_i s(n) + u_i(n), \quad (61)$$

$$\mathcal{H}_0 : y_i(n) = u_i(n), \quad (62)$$

for $i = 1, \cdots, M$. We make the following assumptions:

- The channel coefficients h_i's are zero-mean, unit-variance complex Gaussian random variables;
- The noises are independent of each other for the M receivers. Further, the noise power is constant over the M receivers, i.e., $\mathsf{E}[|u_i(n)|^2] = \sigma_u^2$, for all i.

A. Data Fusion

Suppose the channel coefficients from the primary user to each receiver are known. Using maximal ratio combining,

$$y(n) = \sum_{i=1}^{M}\frac{h_i^*}{\sum_{i=1}^{M}|h_i|^2} y_i(n). \quad (63)$$

The hypotheses (61) and (62) are equivalent to:

$$\mathcal{H}_1 : \quad y(n) = \sqrt{\sum_{i=1}^{M}|h_i|^2} s(n) + u(n), \quad (64)$$

$$\mathcal{H}_0 : \quad y(n) = u(n), \quad (65)$$

where $\mathsf{E}[|u(n)|^2] = \sigma_u^2$. Obviously, it is equivalent to sense the primary user with SNR $\gamma \sum_{i=1}^{M}|h_i|^2$ and sampling size N. From (13) and (14), for a target probability of detection, $\bar{P}_d$, the probability of false alarm becomes:

$$P_f = \mathcal{Q}\left(\beta_3 \mathcal{Q}^{-1}(\bar{P}_d) + \sqrt{N}\gamma\sum_{i=1}^{M}|h_i|^2\right), \quad (66)$$

where $\beta_3 = \sqrt{2\gamma\sum_{i=1}^{M}|h_i|^2 + 1}$. On the other hand, for a target probability of false alarm, $\bar{P}_f$, the probability of detection is given by

$$P_d = \mathcal{Q}\left(\frac{1}{\beta_3}(\mathcal{Q}^{-1}(\bar{P}_f) - \sqrt{N}\gamma\sum_{i=1}^{M}|h_i|^2)\right). \quad (67)$$

When the channel coefficients are unknown, similar to (32) and (33), a simple way for data fusion is to choose $g_i = \frac{1}{\sqrt{M}}$. In this case, we obtain:

$$P_f = \mathcal{Q}\left(\beta_4 \mathcal{Q}^{-1}(P_d) + \gamma\sqrt{\frac{N}{M}}\sum_{i=1}^{M}|h_i|^2\right), \quad (68)$$

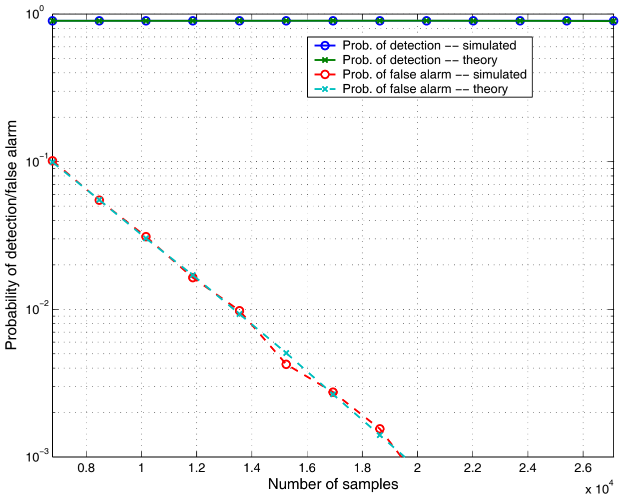

Fig. 3. Comparison of simulated and theoretical probability of detection and probability of false alarm: $\bar{P}_d = 0.9$, $SNR_p = -15$dB.

$$P_d = \mathcal{Q}\left(\frac{1}{\beta_4}\left(\mathcal{Q}^{-1}(P_f) - \gamma\sqrt{\frac{N}{M}}\sum_{i=1}^{M}|h_i|^2\right)\right), \quad (69)$$

where $\beta_4 = \sqrt{1 + \frac{2\gamma}{M}\sum_{i=1}^{M}|h_i|^2}$.

B. Decision Fusion

Suppose that $P_{d,i}$ and $P_{f,i}$ are the probability of detection and probability of false alarm from the ith secondary user, respectively. Similar to the multiple mini-slot case, the probability of detection and probability of false alarm of the final decision can be derived accordingly, from which the sensing-throughput tradeoff can be carried out.

VI. Computer Simulations

In this section, computer simulation results are presented to evaluate the sensing-throughput tradeoff for energy detection technique with various data/decision fusion schemes. The primary user is assumed to be a QPSK modulated signal with bandwidth of 6MHz. The sampling frequency is the same as the bandwidth of the primary user. The additive noise is a zero-mean CSCG process. We are interested in low SNR_p regime, and choose $P(\mathcal{H}_1) = 0.2$, and the target probability of detection as $\bar{P}_d = 0.9$.

A. One Secondary User for Spectrum Sensing

First, we compare the probability of detection and probability of false alarm calculated from Monte Carlo simulations and the theoretical results derived in this paper. 20000 Monte Carlo simulations are performed. In the simulations, for each sample size and under hypothesis $\mathcal{H}_1$, we first find out the detection threshold to achieve the target detection probability based on the 20000 test statistics, then apply this threshold to the hypothesis $\mathcal{H}_0$ and derive the probability of false alarm. Fig. 3 shows the comparison between simulated and theoretical probabilities for $\bar{P}_d = 0.9$ and $SNR_p = -15$dB. It is seen that the target probability of detection is achieved, and the probability of false alarm decreases with the increase of the number of samples.

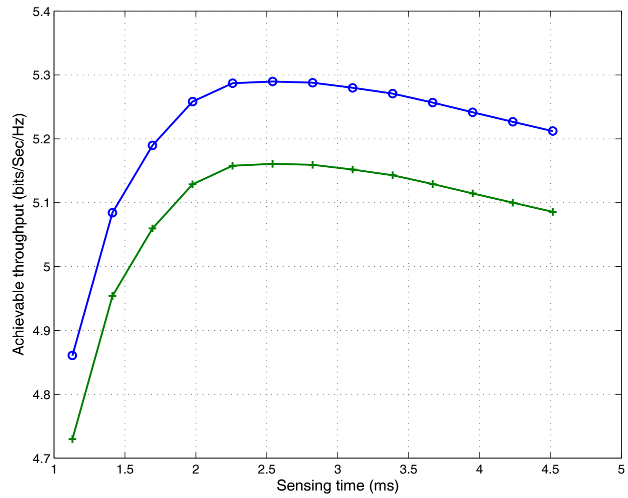

Fig. 4. Achievable throughput for the secondary network ("o" for $R(\tau)$, "+" for $\tilde{R}(\tau)$): $T = 100ms$, $SNR_p = -15$dB.

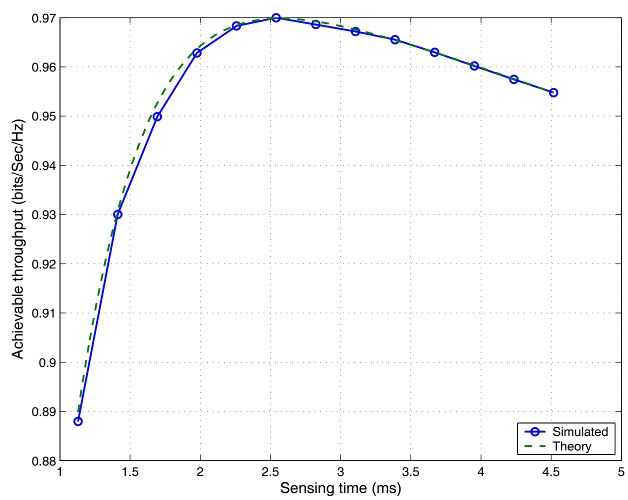

Fig. 5. Normalized achievable throughput for the secondary network: $T = 100ms$, $SNR_p = -15$dB.

Next, suppose the SNR for secondary transmission is $SNR_s = 20$dB, thus $C_0 = \log_2(1 + SNR_s) = 6.6582$ and $C_1 = \log_2\left(1 + \frac{SNR_s}{1+SNR_p}\right) = 6.6137$. We also choose the frame duration $T = 100ms$. Fig. 4 shows the achievable throughput versus the sensing time allocated to each frame for the secondary network. In this figure, two results are shown: $R(\tau)$ of (18), and $\tilde{R}(\tau)$ of (23). It is seen that for both quantities, the maximum is achieved at the sensing time of about $2.55ms$. Fig. 5 illustrates the normalized achievable throughput for the secondary network, which is defined as $B(\tau) = \left(1 - \frac{\tau}{T}\right)(1 - P_f)$. Again, it reveals a maximum point at the sensing time of about $2.55ms$. Furthermore, the simulated results match to the theoretical results very well. Because of this consistency, in the following evaluations, we will only consider the theoretical results.

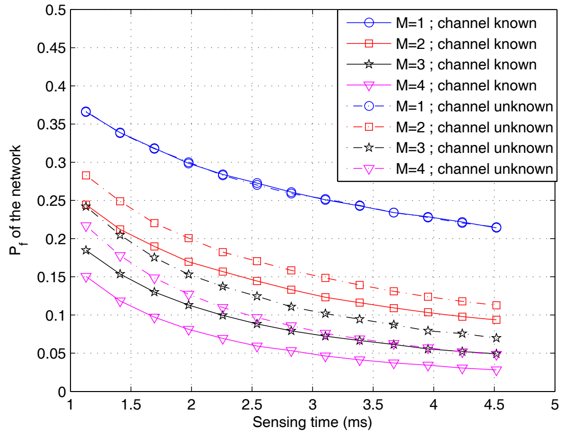

Fig. 6. The average probability of false alarm $\tilde{P}_f(\tau)$ using multi-slot spectrum sensing.

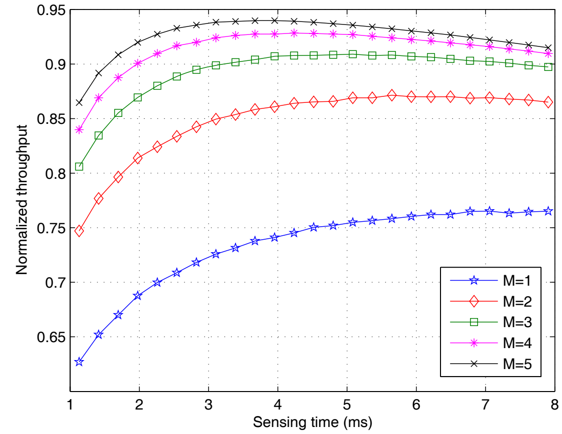

Fig. 7. Normalized throughput for multi-slot spectrum sensing when the fading coefficients are known.

B. Multi-Slot Spectrum Sensing

Instead of using one sensing slot in each frame, we now consider spectrum sensing using multiple mini-slots. The total sensing time in each frame is divided equally among the multiple mini-slots. The SNR_p's for primary user at different mini-slots are independent, and each follows an exponential distribution with mean SNR_p of -15dB. The target probability of detection for each frame is 90%, the frame duration is chosen to be $T = 100ms$, and the number of frames simulated is 100000. The fading coefficients are either completely known or completely unknown. When these coefficients are known, the weighting factors for fusing the test statistics of the mini-slots are computed from (46). When the fading coefficients are unknown, the weighting factors are chosen to be the same for all the mini-slots, i.e., $g_i = \frac{1}{\sqrt{M}}$.

Fig. 6 shows the average probability of false alarm $\tilde{P}_f(\tau)$ using different numbers of mini-slots as a function of the total sensing time τ. From this figure, it is seen that, for a given total sensing time, increasing the number of mini-slots improves the performance of the spectrum sensing especially when one sensing slot is split into two mini-slots. This is due to the effect of diversity reception. In fact, in order to achieve the same probability of detection for each frame, the detection threshold will vary from one frame to another, and the diversity reception helps to increase the received SNR of the primary user, thus to reduce the probability of false alarm. However the gain in improvement reduces when the number of mini-slots is getting larger. This figure also shows that the performance of data fusion without knowing the fading coefficients can approach that with known fading coefficients when the total sensing time is long enough.

Fig. 7 and Fig. 8 illustrate the normalized throughput for the secondary network with the fading coefficients known and unknown, respectively. As the number of mini-slots increases, the maximum normalized throughput increases, but the required sensing time to achieve the maximum normalized throughput decreases. With only one sensing slot and fading coefficients known, the normalized achievable throughput is 0.763 with a total sensing time of $7ms$, but when five mini-slots are used, the normalized achievable throughput is 0.94 at a total sensing time of $3.6ms$. Using five mini-slots and when the fading coefficients are unknown, the normalized achievable throughput is 0.925 at a total sensing time of $4.75ms$. Fig. 9 is plotted to compare the maximum normalized throughput between the cases when the fading coefficients are known or unknown. It is seen that when the number of mini-slots is large, there is a constant gap between the maximum normalized throughputs for the two cases.

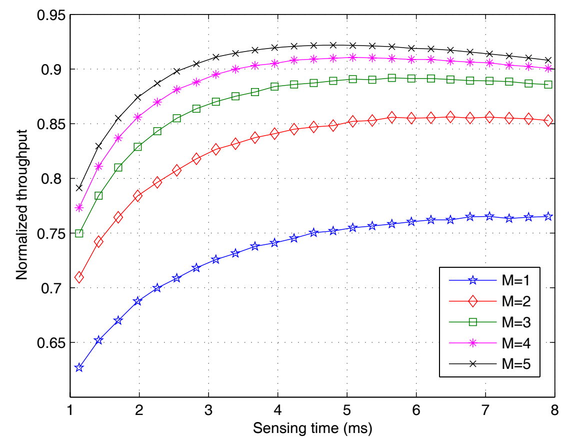

Fig. 8. Normalized throughput for multi-slot spectrum sensing when the fading coefficients are unknown.

C. Distributed Spectrum Sensing

Next, we consider the case when multiple secondary users are used to sense the presence of primary user. For simplicity, we assume that the received SNRs for primary user at each secondary user are all equal. Fig. 10 shows the optimal sensing time versus the number of sensing users when various decision fusion schemes are applied and $SNR_p = -15$dB. It is seen that for each fusion scheme, the optimal sensing time decreases when the number of sensing terminals increases. Fig. 11 illustrates the maximum normalized throughput of the secondary network with different number of sensing users. It

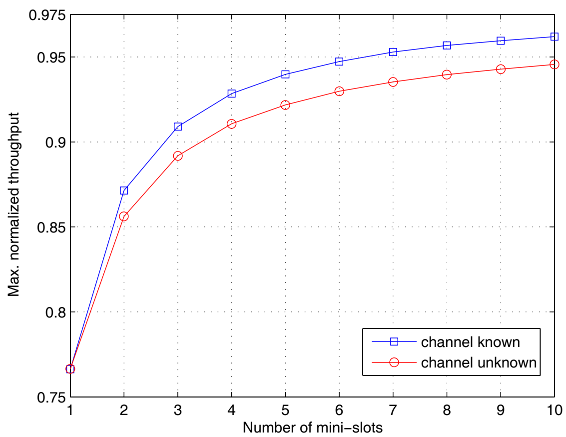

Fig. 9. Maximum normalized throughput for multi-slot spectrum sensing.

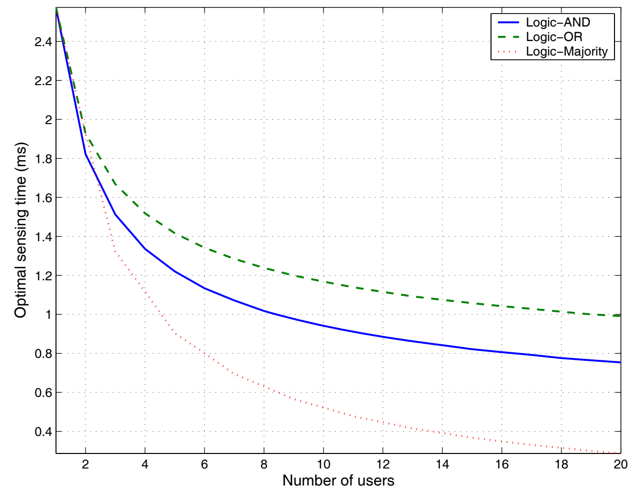

Fig. 10. Optimal sensing time for distributed spectrum sensing with various decision rules: SNR_p = -15dB, $T = 100ms$.

is also seen that this normalized throughput increases with the increase of number of sensing users. Fig. 12 shows the optimal sensing time for various frame duration and receive SNR_p using logic-AND decision fusion rule. If only one secondary user is used, the optimal sensing time increases from $2.55ms$ to $14.2ms$, when the SNR_p drops from -15dB to -20dB. When four secondary users are used, the optimal sensing time increases from $1.5ms$ to $9.5ms$, when SNR_p drops from -15dB to -20dB. Further, if the frame duration increases, the optimal sensing time also increases, for a given received SNR of primary user at the sensing users.

VII. Conclusions

In cognitive radio networks, the interests of primary users and secondary users are contradictory. In this paper, we consider MAC frame structure design supporting periodic spectrum sensing and formulate the sensing-throughput tradeoff problem by considering both users' interests. Particularly, we study the problem of designing the sensing slot duration to maximize the achievable throughput for the secondary users under the constraint that the primary users are sufficiently protected. Using energy detection scheme, we have proved that there indeed exists an optimal sensing time which achieves the best tradeoff. Cooperative sensing has also been studied based on the proposed tradeoff methodology. Computer simulations have shown that for a frame duration of $100ms$, and the SNR of primary user of -20dB, the optimal sensing time achieving the highest throughput while maintaining 90% detection probability is $14.2ms$. This optimal sensing time decreases to $9.5ms$ when 4 distributed secondary users cooperatively sense the channel using Logic-AND decision fusion rule.

Appendix A: Proof of Proposition 2

From (1), under hypothesis $\mathcal{H}_1$, if both $s(n)$ and $u(n)$ are complex-valued, we have

$$\sigma_1^2 = \mathsf{E}\left[|T(y)-\mu_1|^2\right] \quad (70)$$

$$= \mathsf{E}\left[\left(\frac{1}{N}\sum_{i=1}^{N}|s(n)+u(n)|^2-(\sigma_s^2+\sigma_u^2)\right)^2\right] \quad (71)$$

$$= \frac{1}{N}\mathsf{E}[(|s(n)|^2+|u(n)|^2+s(n)u^*(n) + s^*(n)u(n)-\sigma_s^2-\sigma_u^2)^2]. \quad (72)$$

If $s(n)$ is circularly symmetric, then it can be represented as $s(n) = s_r(n) + js_i(n)$ with $\mathsf{E}[s_r^2(n)] = \mathsf{E}[s_i^2(n)] = \frac{\sigma_s^2}{2}$, and $\mathsf{E}[s_r(n)s_i(n)] = 0$. Thus we have $\mathsf{E}[s^2(n)] = 0$, and similarly $\mathsf{E}[u^2(n)] = 0$. Using the fact that $s(n)$ and $u(n)$ are independent and with zero mean, we arrive at (8) by expanding (72).

On the other hand, if both $s(n)$ and $u(n)$ are real-valued, we have

$$\sigma_1^2 = \mathsf{E}\left[(T(y)-\mu_1)^2\right] \quad (73)$$

$$= \mathsf{E}\left[\left(\frac{1}{N}\sum_{i=1}^{N}(s(n)+u(n))^2-(\sigma_s^2+\sigma_u^2)\right)^2\right] \quad (74)$$

$$= \frac{1}{N}\mathsf{E}[(s^2(n)+u^2(n)+2s(n)u(n) -\sigma_s^2-\sigma_u^2)^2]. \quad (75)$$

Eq. (9) can then be derived by using the zero-mean and mutual independence property of $s(n)$ and $u(n)$.

Appendix B: Concavity of $R(\tau)$

Proposition 5: Under the assumptions (AS1) - (AS3), if the primary signal is complex-valued PSK and the noise process is CSCG, then $P_f(\tau)$ is decreasing and convex for the range of τ in which $P_f(\tau) \le 0.5$.

Proof: From (13), and notice $\alpha = \sqrt{2\gamma+1}\mathcal{Q}^{-1}(\bar{P}_d)$, we have:

$$P_f(\tau) = \mathcal{Q}(\alpha+\sqrt{\tau f_s}\gamma). \quad (76)$$

Differentiating $P_f(\tau)$ with respect to τ gives:

$$P_f'(\tau) = \frac{dP_f(\tau)}{d\tau} = -\frac{\gamma\sqrt{f_s}}{2\sqrt{2\pi}}\tau^{-1/2}\exp(-(\alpha+\gamma\sqrt{\tau f_s})^2/2). \quad (77)$$

For $\tau > 0$, it is clear that $P_f'(\tau) < 0$ so $P_f(\tau)$ is decreasing in τ. Furthermore, when $P_f(\tau) \le 0.5$, from (76) we have

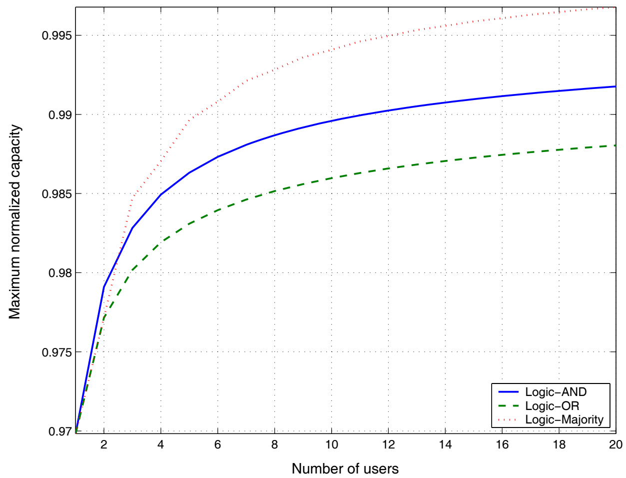

Fig. 11. Normalized achievable throughput for distributed spectrum sensing using various decision rules: $SNR_p = -15$dB, $T = 100ms$.

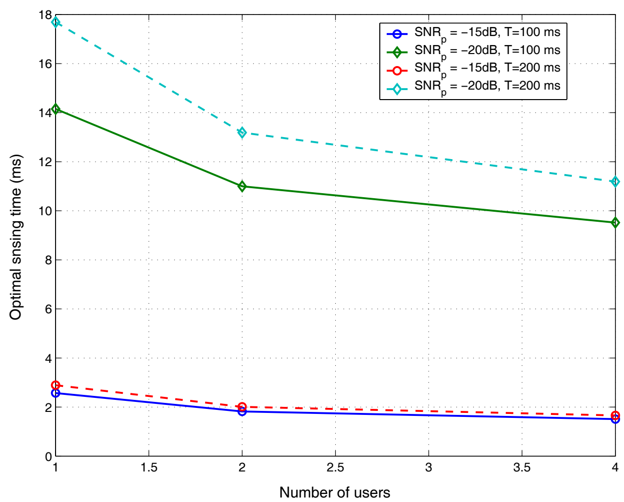

Fig. 12. Optimal sensing time for distributed spectrum sensing using logic-AND decision fusion rule.

$\alpha+\gamma\sqrt{\tau f_s} \geq 0$. This, together with (77), implies that $P_f'(\tau)$ is monotonically increasing in τ when $P_f(\tau) \leq 0.5$, i.e., $P_f(\tau)$ is convex in τ when $P_f(\tau) \leq 0.5$.

Proposition 6: For the range of τ such that $P_f(\tau)$ is decreasing and convex in τ, $\tilde{R}(\tau)$ is concave in τ, $0 \leq \tau \leq T$.

Proof: Let

$$\tilde{R}(\tau) = C_0 P(\mathcal{H}_0)\frac{T-\tau}{T}(1 - P_f(\tau)), \tag{78}$$

then

$$\tilde{R}'(\tau) = C_0 P(\mathcal{H}_0)\Big(\frac{P_f(\tau)}{T} - \Big(1 - \frac{\tau}{T}\Big) P_f'(\tau) - \frac{1}{T}\Big). \tag{79}$$

When $P_f(\tau)$ is decreasing and convex in τ, $P_f'(\tau)$ is negative and increasing in τ. Further, $P_f(\tau)$ is decreasing in τ. Therefore, from (79), it follows that $\tilde{R}'(\tau)$ is decreasing in τ, $0 \leq \tau \leq T$, which further implies $\tilde{R}(\tau)$ is concave in τ.

From Propositions 5 and 6, it follows that $R(\tau)$ is concave for the range of τ in which $P_f(\tau) \leq 0.5$. This further implies that there is a unique maximum point of $R(\tau)$ within this range.

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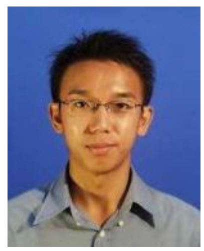

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