LETTER

Multiuser MIMO Downlink Transmission Over Time-Varying Channels*

Kai ZHANG[†a)], *Student Member and* **Zhisheng NIU**[†b)], *Member*

SUMMARY The performance of multiuser MIMO downlink systems with block diagonalization (BD) relies on the channel state information (CSI) at the transmitter to a great extent. For time division duplex TDD systems, the transmitter estimates the CSI while receiving data at current time slot and then uses the CSI to transmit at the next time slot. When the wireless channel is time-varying, the CSI for transmission is imperfect due to the time delay between the estimation of the channel and the transmission of the data and severely degrades the system performance. In this paper, we propose a linear method to suppress the interferences among users and data streams caused by imperfect CSI at transmitter. The transmitter first sends pilot signals through a linear spatial precoding matrix so as to make possible that the receiver can estimate CSI of other users, and then the receiver exploits a linear prefilter to suppress the interference. The numerical results show that the proposed schemes achieve obvious performance enhancement in comparison to the scheme assuming perfect CSI at the transmitter.
key words: *– Multiuser MIMO, Time-varying channels, Time division duplex (TDD)*

1. Introduction

Previous work [1] has indicated remarkable spectral efficiency of single-user MIMO links. However, there is increasing interest in multiuser MIMO systems, especially in downlink broadcast scenarios [2]. It has been shown that Dirty paper coding (DPC) [5] achieves the sum-rate capacity of MIMO broadcast channel [3] [4]. However, because of high computational burden of successive encoding and decoding, the DPC schemes are difficult to implement in practical systems. A suboptimal strategy that can serve multiple users at a time like DPC, but with much reduced complexity, is block diagonalization (BD) [6], in which the base station (BS) performs linear spatial precoding enforcing a zero inter-user interference constraint. In BD, data streams from different users are multiplexed in the spatial domain.

The above work is based on the assumption that channel state information (CSI) is perfectly known at the transmitter. However, in many applications, this assumption is not reasonable, especially if the channels vary rapidly. For TDD systems, the transmitter estimates the CSI while receiving data at current time slot and then uses the CSI to transmit at the next time slot. When the wireless channel is time-varying, the CSI for transmission is imperfect due to the time delay between the estimation of the channel and the transmission of the data. For multiuser MIMO systems with BD, outdated CSI at transmitter results in not only self-interference of different data streams but also interference among users, and severely degrades the performance especially in the cases with highly moving users or long delay.

Recent work [7] [8] has considered imperfect CSI at the transmitter for single user MIMO systems, where self-interference of different data streams caused by imperfect CSI degrades the performance. In [7], the authors use perturbation analysis to evaluate the self-interference and propose a power allocation strategy to achieve specified performance. A new architecture is proposed in [8]. At first, pilot tones were sent through a matrix generated by the outdated CSI at the transmitter and secondly, ZF or MMSE receiver is implemented at the receiver to eliminate the self-interference. For multiuser MIMO downlink systems with BD, [9] has investigated the effects of imperfect CSI on the capacity. The performance of multiuser MIMO systems strongly depends on the correlation between the real and the estimated CSI at the transmitter.

In this paper, we analyze the effect of imperfect CSI on multiuser MIMO systems with BD, and propose a method to suppress the interferences among users and data streams caused by imperfect CSI with low complexity. First, the transmitter exploits a linear spatial precoding at the training stage so as to make possible that each user can estimate the so-called *equivalent channel matrix* of other users and therefore can exploit Multiuser detection. Based on the equivalent channel matrix, we propose a linear multiuser MMSE (Minimum Mean Square Error) receiver to suppress the interference of other users as well as the self-interference of different data streams and improve the system performance.

2. System Model

Consider a multiuser MIMO downlink system with BD, where a base station (BS) equipped with m transmit antennas transmits to K mobile users, each equipped with n_k $(k = 1, \cdots, K)$ antennas, on the assump-

[†]The authors are with Tsinghua-Hitachi Joint Lab on Ubiquitous IT, Department of Electronic Engineering, Tsinghua University, Beijing 100084, China.
a) E-mail: zhangkai98@mails.tsinghua.edu.cn
b) E-mail: niuzhs@tsinghua.edu.cn
*This paper has been partially presented at APCC2006

tion that $m \geq \sum_{k=1}^{K} n_k$. The channels are assumed to be be quasi-static** flat fading and denoted by $\mathbf{H}_k = \{h_{ij}(k)\}_{n_k \times m}$, where $h_{ij}(k)$ is the channel gain from the ith transmit antenna to the jth receive antenna of the kth user. Let $\mathbf{b}_k$ denote the $n_k \times 1$ transmit signal vector of user k. This signal vector is first multiplied by a $m \times n_k$ matrix $\mathbf{T}_k$ and then transmitted through m transmit antennas. At the receiver of user k, the received signal vector $\mathbf{y}_k$ ($n_k \times 1$) is processed by a linear filter $\mathbf{R}_k$ to generate the output data vector $\widetilde{\mathbf{b}}_k$ as

$$\widetilde{\mathbf{b}}_k = \mathbf{R}_k \left(\mathbf{H}_k \sum_{k=1}^{K} \mathbf{T}_k \mathbf{b}_k + \mathbf{n}_k \right), \tag{1}$$

where $\mathbf{n}_k$ denotes the AWGN term which are modelled as i.i.d. zero mean complex Gaussian random variables with σ_n^2 variance, and $\mathbf{b}_i$ satisfies $\mathrm{E}[\mathbf{b}_k \mathbf{b}_k^H] = I_{n_k}$, where E[.] is the expectation operator and superscript ^{H} is the conjugate transpose operation.

For multiuser MIMO downlink systems with BD, the BS separates the users in spatial domain by selecting a set of precoding matrices $\{\mathbf{T}_k\}_{k=1}^{K}$ which is generated by exploiting CSI at the transmitter under the constraint that users do not interference with others, i.e. $\mathbf{H}_j \mathbf{T}_i = 0 \, \forall j \neq i$ as in [2]. Define the $\widetilde{\mathbf{H}}_j$ as

$$\widetilde{\mathbf{H}}_j = [\mathbf{H}_1^T \ \cdots \ \mathbf{H}_{j-1}^T \ \mathbf{H}_{j+1}^T \ \cdots \ \mathbf{H}_K^T]^T, \tag{2}$$

where superscript ^{T} is used to indicate transpose operation. Under a rich scattering environment, we can assume the rank of $\widetilde{\mathbf{H}}_j$ is l_j, where $l_j = \sum_{k=1, i\neq j}^{K} n_k$. The singular value decomposition (SVD) of $\widetilde{\mathbf{H}}_j$ is

$$\widetilde{\mathbf{H}}_j = \mathbf{U}_j \begin{bmatrix} \widetilde{\mathbf{\Sigma}}_j & 0 \\ 0 & 0 \end{bmatrix} [\widetilde{\mathbf{V}}_j^{(1)} \, \widetilde{\mathbf{V}}_j^{(0)}]^H. \tag{3}$$

The right singular vectors $\widetilde{\mathbf{V}}_j^{(0)}$ form a basis for the null space of $\widetilde{\mathbf{H}}_j$, whose rank is $m - l_j$. Therefore, $\widetilde{\mathbf{H}}_j \widetilde{\mathbf{V}}_j^{(0)} = 0$, i.e., $\mathbf{H}_i \widetilde{\mathbf{V}}_j^{(0)} = 0 \, \forall i \neq j$. Due to the zero-interference constraint, $\mathbf{T}_j$ should lie in the null space of $\widetilde{\mathbf{H}}_j$. Thus, the precoding matrix is written as $\mathbf{T}_j = \widetilde{\mathbf{V}}_j^{(0)} \mathbf{A}_j$, where $\mathbf{A}_j$ is used for further optimization. For simplicity, here we assume $n_j = m - l_j$. In the cases of $n_j < m - l_j$, we can select a subspace of $\widetilde{\mathbf{V}}_j^{(0)}$ to obtain $\mathbf{T}_j$ and straightly extend our following work.

Define $\overline{\mathbf{H}}_j$ as $\mathbf{H}_j \widetilde{\mathbf{V}}_j^{(0)}$. Then $\hat{\mathbf{H}}_j$ is an $n_j \times n_j$ matrix, the SVD of which is

$$\overline{\mathbf{H}}_j = \mathbf{U}_j \mathbf{\Sigma}_j \mathbf{V}_j. \tag{4}$$

According to the SVD transmission in MIMO systems [1], the precoding matrix is then written as

$$\mathbf{T}_j = \widetilde{\mathbf{V}}_j^{(0)} \mathbf{A}_j = \widetilde{\mathbf{V}}_j^{(0)} \mathbf{V}_j \mathbf{P}_j, \tag{5}$$

where $\mathbf{P}_j$ is a diagonal matrix defining power allocation for user j according to water-filling:

$$[\mathbf{P}_j]_{k,k} = \max \left(0, \mu - \frac{\sigma_n^2}{[\mathbf{\Sigma}_j]_{k,k}^2} \right), \tag{6}$$

where $[\mathbf{\Sigma}_j]_{k,k}^2$ is the kth diagonal element of $\mathbf{\Sigma}_j$, and μ is from the power constraint $\sum_{j=1}^{K} \mathrm{Trace}(\mathbf{P}_j) = P_t$, where P_t is the total transmit power.

The received signal of user k is then processed by a linear filter $\mathbf{R}_k = \mathbf{U}_j^H$ and becomes

$$\widetilde{\mathbf{b}}_k = \mathbf{\Sigma}_j \mathbf{P}_j \mathbf{b}_k + \mathbf{U}_j^H \mathbf{n}_k \tag{7}$$

In many practical systems, water-filling power allocation puts a high demand on the linear range of transmit power amplifiers, which is extremely costly, especially for multiple antenna systems. Therefore we also consider the scheme with equal power allocation and have

$$\widetilde{\mathbf{b}}_k = \sqrt{\frac{P_t}{m}} \mathbf{\Sigma}_j \mathbf{b}_k + \mathbf{U}_j^H \mathbf{n}_k. \tag{8}$$

3. TDD Multiuser MIMO System with BD over Time-varying Channels

3.1 The impact of outdated CSI on MIMO downlink

In TDD systems, the reciprocity property of the channel is used to obtain CSI at the transmitter. Assume that the users transmit data to BS at time slot t. Pilot symbols are transmitted by different users prior to data transmission through channel $\mathbf{H}_k(t)$ $(1 \leq k \leq K)$. The BS then estimates the CSI by the pilot symbols. Although the channel estimation from pilot symbols is not perfect in practice and results in channel estimation errors, the impact of the errors on system performance is not severe, especially for the system with sufficient long pilot symbols [9]. Thus we assume the estimation at this stage is perfect for simplicity. In next time slot $t+1$, the BS uses the CSI estimated at time slot t for downlink transmission. The delay between the estimation of the channels and the transmission is the length of a time slot T_s. When the channel is time-varying, the CSI at BS is outdated and thereby leading (7) as

$$\widetilde{\mathbf{b}}_k(t+1) = \mathbf{U}_j^H(t+1) \left(\mathbf{H}_k(t+1) \sum_{i=1}^{K} \widetilde{\mathbf{V}}_i^{(0)}(t) \mathbf{V}_i(t) \mathbf{P}_i(t) \mathbf{b}_i(t+1) + \mathbf{n}_k(t+1) \right) \tag{9}$$

Representing the estimation errors as

$$\mathbf{H}_k^e(t+1) = \mathbf{H}_k(t+1) - \mathbf{H}_k(t), \tag{10}$$

we have

**Quasi-static means that the channel are constant over a frame length and changed independently between different frames.

$$\widetilde{\mathbf{b}}_k(t+1) = \mathbf{U}_j^H(t+1)\mathbf{H}_k(t+1)\widetilde{\mathbf{V}}_k^{(0)}(t)\mathbf{V}_k(t)\mathbf{P}_k(t)\mathbf{b}_k(t+1) + \mathbf{U}_j^H(t+1)\mathbf{H}_k^e(t+1)\sum_{i\neq k}\widetilde{\mathbf{V}}_i^{(0)}(t)\mathbf{V}_i(t)\mathbf{P}_i(t)\mathbf{b}_i(t+1) + \mathbf{U}_j^H(t+1)\mathbf{n}_k. \quad (11)$$

The first term in (11) includes inter-stream interference of user k, and the second term is the inter-user interference. From Jake's model [11], the correlation between the channel gains in two slots is $J_0(2\pi F_d T_s)$, where $J_0()$ is the zeroth-order Bessel function of the first kind and F_d is the Doppler frequency. When the channels are varying fast or the time delay T_s is large, the correlation is not high enough that the channel estimation errors are large and degrade the performance greatly.

3.2 Proposed Method

In order to reduce the degradation of the interferences, we propose a pilots architecture that is shown in Fig. 1. When the BS is transmitting data to users at time slot $t+1$, the pilot symbols are first processed by the precoding matrices $\{\mathbf{T}_i(t)\}_{i=1}^K$, which are generated by the BS from the outdated CSI. For user j, the channel is estimated as $[\mathbf{H}_j(t+1)\mathbf{T}_1(t), \cdots, \mathbf{H}_j(t+1)\mathbf{T}_m(t)]$. Defining the *equivalent channel matrix* from user i to user j as

$$\hat{\mathbf{H}}_{ij}(t+1) = \mathbf{H}_j(t+1)\mathbf{T}_i(t), \quad (12)$$

we get the received signal vector of user j as

$$\mathbf{y}_j(t+1) = \sum_{i=1}^{K}\hat{\mathbf{H}}_{ij}(t+1)\mathbf{b}_i(t+1) + \mathbf{n}_j(t+1). \quad (13)$$

By the proposed pilots architecture, each user can obtain the equivalent channel matrix of other users by channel estimation, and therefore can exploit Multiuser detection receiver to suppress the interferences. The performance can be further improved by using non-linear detection method, e.g., successive interference cancellation. For simplicity, we only consider linear method in this paper. We can obtain the linear filter of user j, $\mathbf{R}_j(t+1)$ from MMSE criteria as

$$\min \mathrm{Trace}(E[(\mathbf{R}_j(t+1)\mathbf{y}_j(t+1) - \mathbf{b}_j(t+1))(\mathbf{R}_j(t+1)\mathbf{y}_j(t+1) - \mathbf{b}_j(t+1))^H]) \quad (14)$$

$$\text{Subject to}: \quad \mathrm{Trace}(\mathbf{R}_j(t+1)\mathbf{R}_j(t+1)^H) = 1 \quad (15)$$

Then we have

$$\mathbf{R}_j = \hat{\mathbf{H}}_{jj}^H(\sum_{i=1}^{K}\hat{\mathbf{H}}_{ij}\hat{\mathbf{H}}_{jj}^H + \sigma_n^2 I_{n_j})^{-1}. \quad (16)$$

4. Numerical Examples

In the simulations, we consider a MIMO downlink system with a BS equipped with 6 antennas transmitting data to 3 users each equipped with 2 antennas. Jake's model is used to simulate the time-varying channels. We also assume independent fading among all the antennas. For comparison purpose, we simulate the following reference systems:

1. Open loop system: The BS only transmits data to one user at a time.
2. Multiuser MIMO with perfect CSI
3. Multiuser MIMO with outdated CSI
4. One stream scheme : We also consider the one stream scheme, where one stream is used for each user all the time. This scheme is proof to be optimal when the inter-user interference is large [10].

Monte Carlo simulations are used to compare the sum capacity of different reference systems. We generate large number of channel realizations and compute the SINR of each substream of each user. The interferences are assumed to be Gaussian, thereby leading the capacity of each substream can be computed by Shannon formula. Assuming a typical time-duplex delay $T_s = 1$ ms, we have different F_dT_s in scenarios with different F_d.

In Fig. 2, we plot the sum capacity of water-filling system with $F_dT_s = 0.02$ against SNR. The outdated CSI at BS greatly degrades the performance of multiuser MIMO downlink, especially for high SNR. For 25 dB cases, the capacity gap between the system with perfect and imperfect CSI is about 6 bps/Hz. The proposed method can obviously improve the performance and the improvement increases as the SNR increases. Fig. 3 plots the sum capacity of equal power system with $F_dT_s = 0.02$. The performance enhance achieved by the proposed method is larger in equal power systems. In both these two systems, one stream scheme is not better than other multiuser schemes, since the inter-user interference caused by outdated CSI is not that severe.

Fig. 4 plots the sum capacity of water-filling system with fixed SNR 15 dB. When F_dT_s is more than 0.08, the open loop transmission outperforms the multiuser systems with imperfect CSI. The proposed scheme can enlarge the application range of multiuser MIMO systems. When F_dT_s is less than 0.12, using multiuser system is still more efficient that the open loop scheme. Note that one stream scheme achieves better performance than the proposed scheme for large F_dT_s, but its performance is worse than the open loop scheme in such cases, therefore is still not suitable for practical application in such cases.

5. Conclusion

In this paper we have investigated the effect of outdated CSI at the transmitter on multiuser MIMO systems with BD. A pilots architecture and linear Mul-

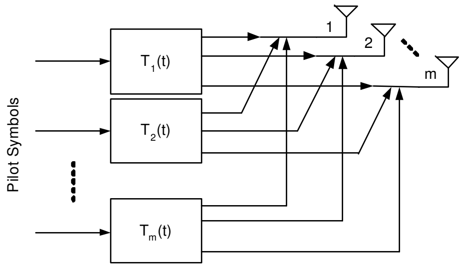

Fig. 1 Pilots architecture at time $t + 1$

tiuser MMSE receiver are proposed jointly to suppress the inter-stream and inter-user interferences at the receiver side. The simulation results have evaluated the efficiency of the proposed method. When the F_dT_s of the system is in a particular range, the proposed method can achieve better performance in comparison with other schemes.

6. Acknowledgement

The authors would like to express sincere thanks to Hitachi R&D Headquarter for their continuous supports.

References

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecom ETT*, vol. 10,no. 6, pp. 585-596, Nov. 1998.

[2] Q. H. Spencer, C. B. Peel, A. L. Swindlehurst, and M. Haardt, "An introduction to the multi-user MIMO downlink," *IEEE Commun. Mag.*, pp. 60-67, Oct. 2004.

[3] P. Viswanath and D. Tse, "Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality ," *IEEE Trans. Inf. theory*, vol. 49, pp. 1912-1921, Aug. 2003.

[4] G. Carie and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Trans. Inf. Theory*, vol.49, pp. 1691-1706, July 2003.

[5] M. Costa, "Writing on Dirty Paper," *IEEE Trans. Inf. Theory*, vol. 29, pp. 439C441, May 1983.

[6] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-Forcing Methods for Downlink Spatial Multiplexing in Multi-User MIMO Channels," *IEEE Trans. Sig. Proc.*, vol. 52, pp. 461-471, Feb. 2004.

[7] T. J. Willink, "Improving power allocation to MIMO eigenbeams under imperfect channel estimation," *IEEE Commun. Lett.*, vol. 9, pp. 622-624, July 2005.

[8] G. Lebru, J. Gao and M. Faulkner, "MIMO transmission over a time-varying channel using svd," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 757-764, Mar. 2005.

[9] G. Primolevo, O. Simeone and U. Spagnolini, "Effects of imperfect channel state information on the capacity of broadcast OSDMA-MIMO systems," in *prof. IEEE Workshop on Sig. Proc. Advances in Wireless Commun.*, pp. 546-550, July 2004.

[10] R. S. Blum, "MIMO capacity with interfernce," *IEEE J. Select. Areas Commun.*, vol. 21, pp. 793-801, June 2003.

[11] W. C. Jakes, *Microwave Mobile Communications*, IEEE Press, 1993.

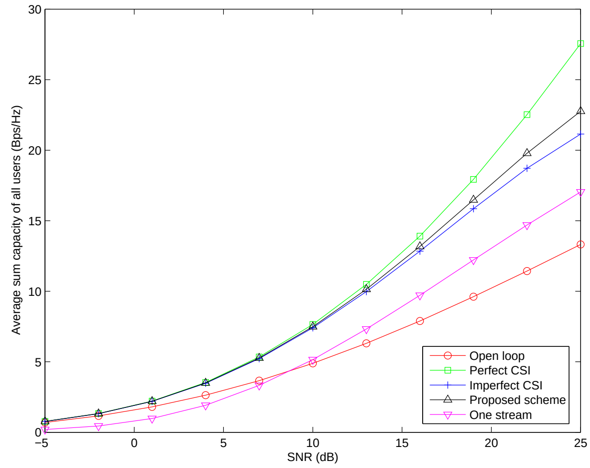

Fig. 2 Sum capacity of water-filling system with $F_dT_S = 0.02$

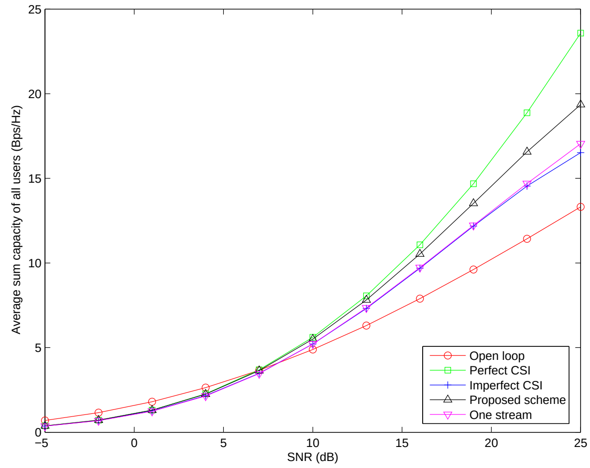

Fig. 3 Sum capacity of equal power system with $F_dT_S = 0.02$

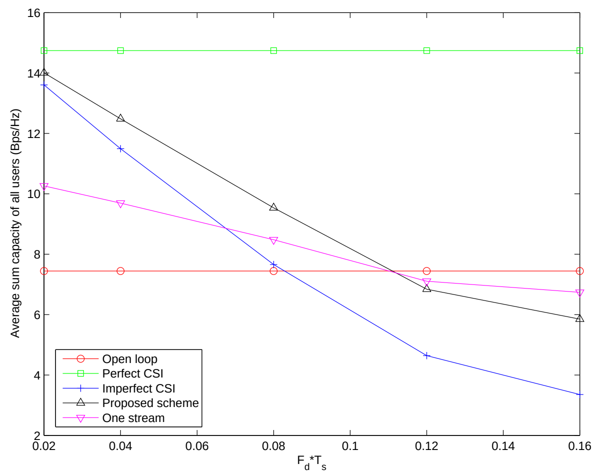

Fig. 4 Sum capacity of water-filling system with fixed SNR 15 dB