Relay Beamforming in Cognitive Two-Way Networks with Imperfect Channel State Information

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***Abstract*—A cooperative bidirectional relaying scheme based on relay beamforming is proposed for an underlay cognitive relay network (CRN) which consists of two secondary transceivers and K cognitive relay nodes and a primary network (PN) with a transmitter and receiver, all equipped with single antenna. The result of incoordination between PN and SN is that the instantaneous channel gains of interference links between two networks are unavailable. So, it is reasonable to assume the availability of second-order statistics (SOS) of the channel coefficients between the relays and PN. Also the perfect channel state information (CSI) is assumed for SN itself. In this paper we find the beamforming coefficients based on requirements of PN or SN. To do so we propose signal-to-interference-plus-noise ratio (SINR) balancing and interference minimization approaches. Our simulation results show that the performance of the network can be degraded considerably by uncertainty about the channel coefficients.**

***Index Terms*—Cognitive two-way relay networks, relay beamforming, imperfect channel state information.**

I. Introduction

COGNITIVE radio (CR) has attracted much attention due to alleviating inefficient scarce spectrum resources utilization [1]. Under the hierarchical access model to spectrum sharing, unlicensed (secondary) users are able to access to a spectrum which originally allocated to licensed (primary) users via three main paradigms: interweave, underlay and overlay [2]. In the underlay approach, communication between SUs is acceptable in the presence of PUs over the same spectrum while the induced aggregate interference on primary receivers is below the predefined tolerable interference temperature (IT). In this work, we adapt the underlay approach due to its advantages from an implementation point of view [3].

Immense potential of cooperative communication to solve the problem of low power transmission and short coverage in underlay paradigm due to interference constraint has attracted great deal of attention. There are various cooperative schemes such as amplify and forward (AF), decode and forward (DF) [4] and etc. AF approach is of particular interest due to its inherent simplicity. Recently distributed relay beamforming [5], [6] has tremendous capability to alleviate interference using adjustable power in relays which is the best choice for satisfying the interference constraint in underlay CRNs.

In order to further increase the spectrum efficiency in underlay CRNs and from practical viewpoint in which the relays cannot receive and transmit simultaneously, two-way relaying can be applied. In [7], the problem of sum-rate maximization under constraints on interference on primary receiver for multi-antenna cognitive two-way relay network (CTRN) has been investigated but the mutual interference is not considered. Motivated by beamforming and two-way relaying in CRNs, we propose a novel framework based on beamforming for this network. To the best of our knowledge, the literature which deals with two-way cognitive relaying strategy is scarce.

The authors in [8], minimize the tolerable interference on primary receiver subject to two constraint on each transceiver QoS in CTRN with perfect CSI. The result of incoordination between PN and SN is that the instantaneous channel gains of interference links between two networks are unavailable. So, it is reasonable to assume the availability of second-order statistics (SOS) of the channel coefficients between the relays and PN. In section III, we propose a relay beamforming approach to maximize QoS requirements for the SN while satisfying IT constraint for the PN and in section IV, we repeat [8] with imperfect CSI. Using semidefinite relaxation, we turned them into convex feasibility problem which can be efficiently solved along with bisection search method. Our simulation results show that the performance of the network can be degraded considerably by uncertainty about the channel coefficients.

II. System Model

As depicted in Fig. 1, consider an underlay CTRN where a pair of SUs wishes to swap information through K randomly located stationary relaying nodes, in the presence of PN. The PUs and SUs are equipped with a single antenna all. Similar to [9] direct path between transceivers is not considered in this paper due to large path loss. Let us $\mathbf{f}_m = [f_{m1} f_{m2} \dots f_{mk}]^T$ for $(m = 1, 2)$ represents the flat fading reciprocal [10] channel vector between the relays and transceivers 1 and 2, where $(\cdot)^T$ represents the transpose operator. We also consider mutual interference between the PN and SN in this work. Hence $\mathbf{f}_p = [f_{p1} f_{p2} \dots f_{pk}]^T$ and $\mathbf{g}_p = [g_{p1} g_{p2} \dots g_{pk}]^T$ denote the interference channel vector from the relays to the PU-TX and PU-RX, respectively and h_{s1p}, h_{s2p} denote the interference links between SU-TX1, SU-TX2 and PU-RX. Also h_{ps_1}, h_{ps_2} represent the interference links between PU-TX and SU-TX1, SU-TX2. We assume that the relays be aware of SOS of the channels state information between the relays and PN. In other words, the correlation matrices of the channels $\mathbf{f}_p$ and $\mathbf{g}_p$ are known and given by $\mathbf{R}_{f_p} \triangleq E\{\mathbf{f}_p \mathbf{f}_p^H\}$ and $\mathbf{R}_{g_p} \triangleq E\{\mathbf{g}_p \mathbf{g}_p^H\}$. Here, $(\cdot)^H$ and $E\{\cdot\}$ denote the hermitian transpose and the statistical expectation, respectively. The two-way relaying, requires two consecutive equal-length separate time slots in which the relays work in

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amplify-and-forward fashion. In the multiple access phase, transceivers transmit their corresponding signals to the relays, simultaneously. These signals can be received by the PU-RX as well as the relays. The received signal at the i'th relay is given by

$$x_i = \sqrt{P} f_{1i} s_1 + \sqrt{P} f_{2i} s_2 + \sqrt{P_p} f_{pi} s_p^{(1)} + \upsilon_i \quad (1)$$

where the transmit power of transceivers and PU-TX are denoted as P and P_p, respectively. Let s_1, s_2 represent the information symbols transmitted by transceivers 1, 2 and $s_p^{(1)}$, $s_p^{(2)}$ represent the information symbols transmitted by PU-TX in the first and second time slot all with unit variance respectively. υ_i is the i'th relay noise with variance σ_υ^2.

During the broadcasting phase, the received signal at the i'th relay is manipulated by a beamforming weight w_i which can be expressed as $t_i = w_i x_i$. Since the knowledge of f_{mi} $(i = 1, \ldots, k)$ and s_m are available at m'th transceiver via channel estimation and training [11], thus m'th transceiver can subtracts the self interference in received signal and manipulate the remaining term to have

$$\tilde{y}_m = \underbrace{\sum_{i=1}^{k} \sqrt{P} f_{1i} f_{2i} w_i s_n}_{\text{desired signal}} + \underbrace{\sum_{i=1}^{k} f_{mi} w_i \upsilon_i + n_m}_{\text{total noise}} + \underbrace{\sum_{i=1}^{k} \sqrt{P_p} f_{mi} f_{pi} w_i s_p^{(1)} + \sqrt{P_p} h_{ps_m} s_p^{(2)}}_{\text{interference}} \ \text{for}(m = 1, 2). \quad (2)$$

Hence the received SINRs at the transceivers are given as [12]

$$\text{SINR}_m = \frac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{\mathbf{w}^H \mathbf{V}_m \mathbf{w} + P_p \sigma_{h_{ps_m}}^2 + \sigma_{n_m}^2} \ \text{for}(m = 1, 2) \quad (3)$$

where $\mathbf{w} \triangleq [w_1 w_2 \ldots w_k]^T$ is the beamforming weight vector and

$$\begin{cases} \mathbf{R} \triangleq P(\mathbf{f}_1 \odot \mathbf{f}_2)(\mathbf{f}_1 \odot \mathbf{f}_2)^H \\ \mathbf{V}_m \triangleq P_p(\mathbf{f}_m \mathbf{f}_m^H) \odot \mathbf{R}_{f_p} + \sigma_\upsilon^2 (\mathbf{F}_m \mathbf{F}_m^H) \ \text{for}(m = 1, 2) \end{cases}$$

where $\mathbf{F}_m = \text{diag}(\mathbf{f}_m)$ and $\odot$ stands for Schur-Hadamard multiplication. Also the received signal at the PU-RX in second interval can be expressed as

$$\begin{aligned} y_p^{(2)} &= \sqrt{P_p}\, h_p s_p^{(2)} + \sum_{i=1}^{k} g_{pi} w_i x_i + n_p \\ &= \underbrace{\sqrt{P_p}\, h_p s_p^{(2)}}_{\text{signal}} + \underbrace{\sum_{i=1}^{k} \sqrt{P_p} w_i g_{pi} f_{pi} s_p^{(1)}}_{\text{self interference}} + \underbrace{n_p}_{\text{noise}} \\ &+ \underbrace{\sum_{i=1}^{k} \sqrt{P} w_i g_{pi} f_{1i} s_1 + \sum_{i=1}^{k} \sqrt{P} w_i g_{pi} f_{2i} s_2 + \sum_{i=1}^{k} w_i g_{pi} \upsilon_i}_{\text{interference from secondary network}} \end{aligned} \quad (4)$$

where h_p denotes the flat fading channel coefficient between PU-TX and PU-RX. Using (4) we obtain the interference

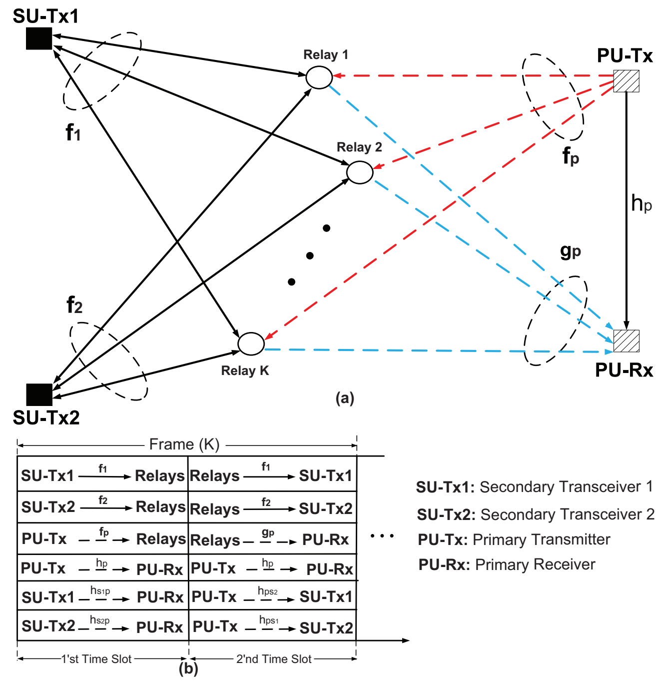

Fig. 1. (a) A proposed cognitive two-way relay network; (b) channel and interference links.

component power in second interval which consist of SN interference and self interference and can be written as:

$$P_I^{(2)} = \mathbf{w}^H \mathbf{D} \mathbf{w} \quad (5)$$

where $\mathbf{D} \triangleq \mathbf{U} + \mathbf{Y} + \mathbf{Z} + \mathbf{L}$ and

$$\begin{cases} \mathbf{U} \triangleq P(\mathbf{f}_1 \mathbf{f}_1^H) \odot \mathbf{R}_{g_p} & \mathbf{Y} \triangleq P(\mathbf{f}_2 \mathbf{f}_2^H) \odot \mathbf{R}_{g_p} \\ \mathbf{Z} \triangleq P_p \mathbf{R}_{f_p} \odot \mathbf{R}_{g_p} & \mathbf{L} \triangleq \sigma_\upsilon^2 \mathbf{R}_{g_p}. \end{cases}$$

Note that, the transmit power of transceivers should be adjusted such that the interference temperature constraint is not violated. So we choose transmit powers as follows

$$P = \min\{P_s^{max}, \frac{I_{th}}{\sigma_{h_{s_1p}}^2 + \sigma_{h_{s_2p}}^2}\} \quad (6)$$

where P_s^{max} and I_{th} are the maximum transmission power available at the transceivers and the maximum tolerable interference at the PU-RX, respectively.

III. SINR Balancing Approach

In this section, our goal is to find the beamforming weight vector $\mathbf{w}$ in order to balance the SINR of transceivers at the SN subject to an interference power constraint at the PN in first and second interval. Mathematically, the optimization problem can be represented as follows

$$\begin{aligned} \max_{\mathbf{w}} \quad & \min(\text{SINR}_1, \text{SINR}_2) \\ \text{s.t} \quad & P_I^{(2)} \le I_{th} \end{aligned} \quad (7)$$

Or equivalently

$$\begin{aligned} \max_{\mathbf{w}, t} \quad & t \\ \text{s.t} \quad & \frac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{\mathbf{w}^H \mathbf{V}_m \mathbf{w} + P_p \sigma_{h_{ps_m}}^2 + \sigma_{n_m}^2} \ge t \ \text{for}\ (m = 1, 2) \\ & \mathbf{w}^H \mathbf{D} \mathbf{w} \le I_{th}. \end{aligned} \quad (8)$$

Algorithm 1 Bisection Search Method

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Initialize:
    t_l ← 0    t_u    t ← (t_l + t_u)/2
while | t_u − t_l | ≥ ε do
    Check the feasibility of problem (9) with fixed t
    if the problem (9) is feasible: then
        t_l ← t    t ← (t_l + t_u)/2
    else
        t_u ← t    t ← (t_l + t_u)/2
    end if
end while
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Let us define $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$. Then, we can rewrite the problem as follows

$$\begin{aligned} \max_{\mathbf{X},t} \quad & t \\ \text{s.t} \quad & \text{tr}((\mathbf{R} - t\mathbf{V}_m)\mathbf{X}) \geq \tilde{t}_m \quad \text{for } (m = 1, 2) \\ & \text{tr}(\mathbf{D}\mathbf{X}) \leq I_{th}, \quad \text{rank}\, \mathbf{X} = 1, \mathbf{X} \succeq 0 \end{aligned} \tag{9}$$

where $\tilde{t}_m = (\sigma^2_{n_m} + P_p\sigma^2_{h_{ps_m}})t$ and $\mathbf{X} \succeq 0$ means that $\mathbf{X}$ is constrained to be a symmetric positive semidefinite matrix. The optimization problem in (9) due to the rank constraint is not convex. Let us ignore this constraint. As a result the problem in (9) due to the variation of t, becomes a quasi-convex. We can find the optimal value for t by using the bisection search method that in each step, we check the feasibility of (9) using the semidefinite programming (SDP) and then we decide for the next step based on algorithm (1). We start with an initial interval $[t_l, t_u]$. As the value of SINR is positive, we set t_l to be equal to zero and for the upper bound of interval, first we maximize the individual SINR's subject to interference constraints using Rayleigh-Ritz ratio inequality [6], then we choose the minimmum of individual maximized SINR's for t_u. Also we set the ϵ to be 10^{-3}. After solving the problem, we can check the rank constraint of matrix $\mathbf{X}$. Note that in [13], it is shown that for $m \leq 3$ in (9), the SDP relaxation has always a rank one solution.

IV. Interference Minimization Approach

In order to guarantee the interference constraint for PUs, we develop the interference minimization approach subject to two constraints on QoS of SUs when the transceiver powers (P_1 and P_2) and primary transmitter power (P_p) are given, and therefore, they are fixed. In other words, we solve the following optimization problem:

$$\begin{aligned} \min_{\mathbf{w}} \quad & P_I^{(2)} \\ \text{s.t} \quad & \text{SINR}_m \geq \gamma_m \quad \text{for } (m = 1, 2). \end{aligned} \tag{10}$$

Let us define $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$. Using (3), (5) then we can rewrite the problem as follows

$$\begin{aligned} \min_{\mathbf{w}} \quad & \text{tr}(\mathbf{D}\mathbf{X}) \\ \text{s.t} \quad & \text{tr}((\mathbf{R} - \gamma_m\mathbf{V}_m)\mathbf{X}) \geq \tilde{\gamma}_m \quad \text{for } (m = 1, 2) \\ & \text{rank}\, \mathbf{X} = 1, \mathbf{X} \succeq 0 \end{aligned} \tag{11}$$

where $\tilde{\gamma}_m = (\sigma^2_{n_m} + P_p\sigma^2_{h_{ps_m}})\gamma_m$. Using SDP relaxation technique, we can ignore the rank constraint in (11) and then it will be convex problem and can be efficiently solved using interior point method. Similarly to the previous section, (11) has always a rank one solution [13].

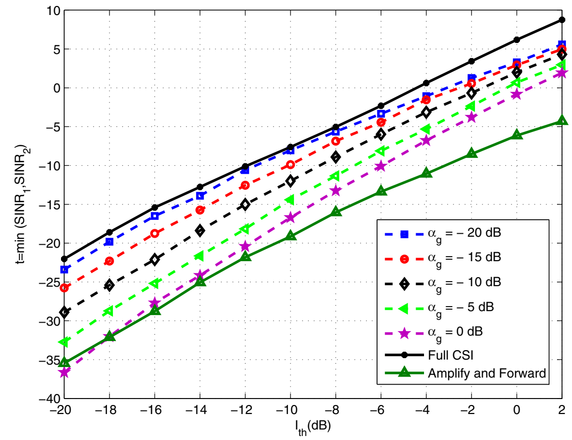

Fig. 2. The average values of the maximum achievable SINRs versus the interference temperature for different values of α_g and for simple AF scheme with full CSI.

V. Simulation Results

In the following simulations, we consider a SN with $K = 20$ relay nodes, and the secondary channel coefficients are generated independently as complex Gaussian random variables with unit variance in each simulation run. As an imperfect CSI is considered for the channels between the relays and PN, we model these channel coefficients as follows [6]

$$f_{pi} = \bar{f}_{pi} + \tilde{f}_{pi} \qquad g_{pi} = \bar{g}_{pi} + \tilde{g}_{pi} \tag{12}$$

where $\tilde{f}_{pi}$ and $\tilde{g}_{pi}$ are zero mean random variables which $\text{var}(\tilde{f}_{pi}) = \frac{\alpha_p}{1+\alpha_p}$, $\text{var}(\tilde{g}_{pi}) = \frac{\alpha_g}{1+\alpha_g}$ and $\bar{f}_{pi}$ and $\bar{g}_{pi}$ is the mean of f_{pi} and g_{pi} respectively and defined as follows:

$$\bar{f}_{pi} = \frac{e^{j\theta_i}}{\sqrt{1+\alpha_p}} \qquad \bar{g}_{pi} = \frac{e^{j\varphi_i}}{\sqrt{1+\alpha_g}} \tag{13}$$

where α_p and α_g are the uncertainty parameter of the mutual interference links which produce a tradeoff between the mean of channels and the variance of them. As we increase these parameters, the variance of the channels are increased while the mean of channels are decreased. θ_i and φ_i are uniform random variables which are chosen randomly from the interval $[0, 2\pi]$. Based on above definitions and knowledge of the SOS of the mutual interference channels, the channel covariance matrices are given by

$$\mathbf{R}_{f_p} = \bar{\mathbf{f}}_p\ \bar{\mathbf{f}}_p^H + \frac{\alpha_p}{1+\alpha_p}\mathbf{I} \qquad \mathbf{R}_{g_p} = \bar{\mathbf{g}}_p\ \bar{\mathbf{g}}_p^H + \frac{\alpha_g}{1+\alpha_g}\mathbf{I}.$$

All noise powers including relay noises, secondary and primary receiver noises are assumed to be 0 dBW. Throughout our numerical examples, the maximum transmit power of transceivers (P_s^{max}) and PU are also considered to be equal to 0 dBW.

Fig. 2 illustrates the average values of the maximum achievable SINRs versus the maximum interference power that

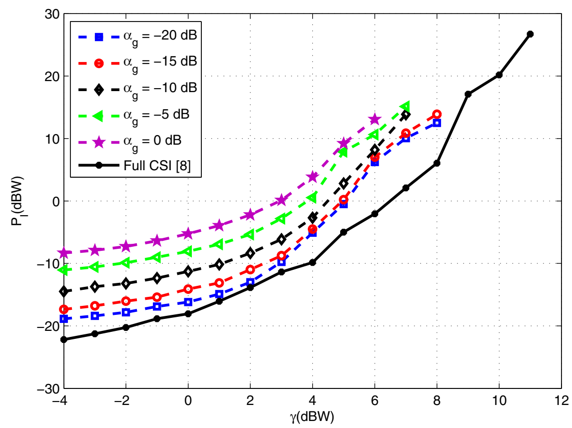

Fig. 3. The average values of the minimum interference power versus the SINR threshold for different values of α_g.

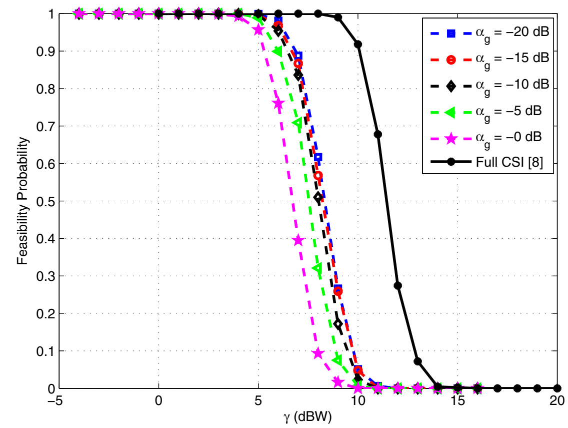

Fig. 4. Feasibility Probability of the problem (11) versus SINR threshold for different values of α_g.

primary receiver can tolerate for different values of α_g along with the assumption of full CSI for comparison. As expected, the uncertainty about the channel coefficients can be degrade the performance of the network considerably. In addition we have plotted the results for conventional AF scheme with full CSI and we have shown that the performance of the algorithm, we proposed, is much better than this scheme. This improvement is due to the use of beamforming in which the relays power can be adjusted, so the interference temperature constraint can be satisfied by low power consumption in relays. In the conventional AF scheme, relay coefficients can be calculated as follows:

$$\beta_i \triangleq \sqrt{\frac{I_{th}/|g_{pi}|^2}{K(P|f_{1i}|^2 + P|f_{2i}|^2 + P_p|f_{pi}|^2 + \sigma_v^2)}}. \quad (14)$$

We can see from the figure that in some cases, where the level of uncertainty increases, the conventional AF scheme with full CSI outperforms the results for $\alpha_g = 0\,dB$.

The equation (11) can be infeasible. When the number of simulation runs yields to infeasibility is larger than the half of the total simulation runs, the interference power computed by averaging over the feasible runs. Fig. 3 represents the minimum interference power versus the SINR threshold for different values of α_g along with the assumption of full CSI [8] for comparison. It is interesting to note that, the minimum interference power is very sensitive to α_g at higher values of target SINR and there exists a limit for achievable SINR. The problem (11) becomes infeasible, when SINR threshold exceeds these limits. Also, in Fig. 4 we plot the feasibility probability for different values of α_g. In high SINR threshold values, the number of infeasible runs increased and probability of feasibility decreased. Also we can see from this figure that increasing the uncertainty parameter decreases feasibility probability.

VI. CONCLUSION

In this paper, we designed two beamformers for an underlay CTRN assuming the SOS of the mutual interference channels are available for the relays. We study the SINR balancing technique where the smaller of the two transceiver SINRs is maximized while keeping the interference power below interference temperature. Also, in order to guarantee the interference constraint for PUs, we develop the interference minimization approach subject to two constraints on QoS of SUs. We herein have shown that these approaches lead to a semidefinite programming along with bisection search method.

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