Compact Confidential Transactions for Bitcoin

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Abstract

An enhancement is suggested to make Bitcoin transaction amounts hidden to all but the sender and receiver. In each transaction, the output amounts are encrypted with the public keys of the respective receivers. Only the transaction fee is publicly revealed, to allow miners to prioritise transactions. A homomorphic commitment for each transaction proves that the sum of the transaction inputs matches the sum of its outputs. A short Non-Interactive Zero-Knowledge Proof (NIZKP) for each output also convinces all verifiers that the sum does not overflow. Address construction includes an additional public view key to allow senders to encrypt output values. This approach practically resolves a core privacy issue in Bitcoin[1], but without overwhelming implementation complexity. The required commitments are an order of magnitude smaller than those proposed for Confidential Transactions[2], and do not depend on ring signatures.

1 Introduction

As Bitcoin[1] continues to generate interest in both academic and business circles, an increasing attention is being given to its pseudonymous properties. The question arises with regards to the levels of anonymity that can be achieved by a cryptocurrency, and the implementation cost of such protection.

1.1 Anonymity

Anonymity was a low priority in the Bitcoin implementation[3]. A brief look at the source code shows that a node's network address is openly shared with

*BM-2cWtzfLzPdzRcv8CDFTTRSHaSNbFfC3C6V
1SumKArxoEJ1HoGibmj8ygw1DZWYBvjmM
QR code generated with `http://bitcoinqrcode.org/`

other nodes, and at least one public internet service. Some early versions of the software could pay to a specific network address, and connected to a public Internet Relay Chat room to discover peering nodes.

Many in the cryptocurrency community consider anonymity to be a necessary property of electronic cash. Some users have been willing to risk their coins in inconvenient or fee-based anonymising mixers[4][5][6]. Despite evident need, no cryptocurrency has succeeded in providing strong user anonymity. This is because increased anonymity often implies an implementation cost for the software and usability cost for the user.

1.2 Fungibility

It has long been established that fungibility is a critical aspect assuring the free circulation of cash[7]. Because electronic cash transactions connect specific inputs and specific outputs, the fungibility of a specific output can be questioned. The fungibility of a coin can be influenced by accusations aimed at the source address, or even based on implied the wealth of its owner. At least one developer has attempted to add a coin black list into an operating system distribution[8]. Other attempts at implying the value of coins based on their source[9] continue to be made[10][11][12][13][14][15]. If coins are made more anonymous, they are also made more fungible, and thus more suitable for commerce.

2 Background

2.1 Mixers

There have been a number of trustless coin mixing algorithms (including CoinJoin, Dash[16] and CoinShuffle[17]) aimed at bringing anonymity to Bitcoin transactions. A big benefit of CoinJoin is that it requires no change to the Bitcoin transaction protocol, and has worked from Bitcoin's inception. Coin mixing does require extra software, and bigger transactions, with only a modest gain in privacy. A significant taint[18] could apply to all coins spent through a mixer, and does not necessarily improve fungibility.

2.2 Related Work

Stealth Addresses[19] hide a transaction's destination address. The sender performs an Elliptic Curve Diffie-Hellman handshake between the receiver's public key (available on the blockchain), and an internally generated nonce, to derive a new key known only to the sender, but detectable by the receiver given the nonce. The public part of the nonce is included in the transaction, and by testing every such transaction output nonce on the blockchain against its own keys, the receiver is able to find the relevant derived keys. CryptoNote[20], and subsequently Monero[21] make use of ring signatures to also hide the transaction's source address.

Some works have shown that it is possible to construct decentralised ledgers which preserve stronger anonymity. Both Zerocoin[22][23] and Zerocash[24] begin by minting anonymised tokens, rather than protecting the base coins. Of known proposals, only Zerocash and CoinWitness[25] take the step of also hiding transaction values. Hiding these values is critical because it ensures that fungibilty of a coin must remain independent of the amount in its owning address. If the value amounts are public, it becomes possible to target identity discovery resources towards wealthy senders. There is considerable complexity in new minting and pouring transactions.

Zerocoin provably deposits base coins to an accumulator token, and as the order of deposit is not necessarily similar to the order of withdrawal, ownership is obscured. There are down sides to this approach, firstly that Zerocoin transactions are costly in terms of size. The full Zerocash paper is over 50 pages long, so achieving a fault-free software implementation of such a complex specification would be resource intensive. It would be challenging to convince those outside the academic sphere of its correctness. The trust placed in the implementation of zkSNARKs and initial trusted set-up are a high cost of these systems. CoinWitness relies even more on novel concepts, which are yet to mature.

Recently, and parallel to the development of this paper, a concept of Confidential Transactions[2] has been introduced, where the transaction value is hidden by Pedersen commitment. These commitments depend on ring signatures for overflow prevention. Some information about the exponent is revealed to keep the proof size manageable. Even so, the storage required remains an order of magnitude larger than for the method proposed in this paper.

2.3 Privacy and Transmission

While securing coins mathematically, few proposals give practical consideration to real world methods of de-anonymising users. If the network node originating a transaction can be traced using time-correlation, or simpler methods, the transaction sender's network identity can be discovered, regardless of how well their coin addresses are mathematically hidden. Often, "the first node to inform you of a transaction is likely to be the source of it"[3]. Tackling the network transmission problem alone, would be a significant practical step toward anonymous transactions, without any changes to the transaction logic.

Anoncoin[26] and others have taken ad-hoc approaches in this direction by interfacing with TOR and I2P. Their approach attempts to hide the physical network address of all nodes, but for transactions this does no better than obfuscate their origin. Transmission through popular routing systems such as TOR is still subject to flow-correlation[27], Sybil and other attacks.

This paper does not aim to solve the transmission problem, but to make it less relevant. Zerocash employs complexity to hide all aspects of a transaction. However, address hiding does not necessarily anonymise a user, and privacy given by hiding the value may be of more benefit.

2.4 Homomorphic Schemes

It has been has suggested[28] that using homomorphic encryption could effectively hide transaction values. Partial homomorphic encryption schemes such as that of Paillier[29] and others, have been tested thoroughly and commitment based Non-Interactive Zero Knowledge Proofs (NIZKP) have been peer reviewed for a long[30] time. The schemes have shown us that provably correct calculation can be efficiently perfomed on variables, without knowing their actual value. The additive homomorphic property seems sufficient to verify that the sum of the outputs do not exceed the sum of the inputs in a cryptocurrency transaction. However, as homomorphic schemes are commonly constructed moduli some number, they wrap to zero after exceeding this modulus, so it is also necessary to check transaction outputs for overflow.

The Paillier[29] probabilistic encryption, results in a different ciphertext for every encryption of the same plaintext. An improvement to the system prevents ciphertext expansion[31]. However, despite the improvement, the Elliptic Curve Cryptography is thought to achieve an order of magnitude higher level of security for the same bit length[32]. ECC addition is deterministic rather than probabilistic.

2.5 Compact Proofs

Non-Interactive Zero Knowledge Proofs are arguments about numeric values. Definitive proofs tend to have a large storage requirement. Fortunately, Chan, Frankel and Tsiounis[33] (and later Boudot[34] and Zhengjun[35]) show how to construct compact proofs that an encrypted number lies within an interval, without revealing the number itself. This is achieved by allowing for an expansion factor between the condition under which the proof is generated, and what it proves. While a narrow interval must be known to construct the proof, the verifier can only be convinced of a much widened interval. It is sufficient to prove that encrypted numbers are small enough to prevent an overflow of their sum.

Compact proofs can be constructed for Elliptic Curves[36] in a similar way to how they are constructed for the Discrete Logarithm cryptosystems.

3 Construction

3.1 View and Spend Keys

While a single raw public key could be used as the address (and serve as both the spend and view key), this would prevent the user from applying independent security policies to the control and visibility of coins. The separation is necessary to authorise wallet software to report balances, without giving it the power to spend. Dual keys have been useful in the Monero[21] project. In the new scheme, the receiver's public view key is used by the sender to encrypt the transaction output value. The spend key is required, but not sufficient, to spend coins. The previous output value must also be known, so that proofs about the next output value can be constructed. The spend value can of course be cached in wallet software, once it is revealed with the help of the secret view key. The separation of control and visibility should be extended to support multi-signature transactions, where visibility can be granted to a subset of participants.

3.2 Address

If privacy is the priority, it is sufficient to construct a Bitcoin address and distribute it in concatenation with the public view key. The view key does not need to be included in the blockchain. However, this makes payments deniable. That is, a sender could feign payment by switching the public view key and burning the coins. Moreover, a receiver can claim the coins were burned, when they were not.

If the participants value un-deniability over view key un-linkability, the public view key can optionally be included in the spend script. Then sender will be able to prove that the coins are recoverable by the receiver, by presenting only the transaction and destination address. An un-deniable, but less private confidential address can then be constructed:

$$keyHash = Ripemd160(spendScript|Ripemd160(pubViewKey))$$

$$transactionAddress = keyHash|Checksum(keyHash)$$

$$undeniableAddress = base58(pubViewKey|transactionAddress)$$

3.3 Value Commitment

Rather than the value itself, only commitment to an output value is be stored in the transaction output. The value commitment is a point on the Elliptic Curve, which is calculated by a point multiplication of the curve generator by the value:

$$commitment = value * G$$

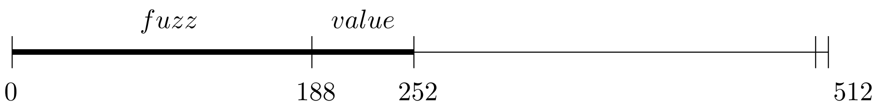
The coin value needs no more than 64 bits ($valuebits$), and a reasonably secure point representation will provide more than this (e.g. 512 bits). The remaining bits carry no useful information, and some are set randomly with a uniform distribution to blind the value, making it resistant to brute force attacks:

$$fuzz \sim U(0, 2^{fuzzbits})$$

$$fuzzedValue = value * 2^{fuzzbits} + fuzz$$

$$commitment = fuzzedValue * G$$

The bit representation can be visualised from low bits on the left to high bits on the right:



3.4 Proof of Sum

The probabilistic behaviour is only simulated, so commutative and deterministic properties of Elliptic Curve math are retained. This means an implicit proof of sum is available with no storage cost over that of the commitments. Non-coinbase transactions refer to a previous transaction for their input commitments, and the fee is public. For coinbase transactions, the input is public. For each transaction input, the commitment is made available in the block chain. Checking the following equality verifies the sum magnitude of the outputs matches the sum magnitude of the inputs:

$$\sum_{i=1}^{inputs} vinCommit_i + coinbase * G = \sum_{j=1}^{outputs} commitment_j + fee * G$$

A large output magnitude could cause the calculation to exceed the group order, causing an overflow. This sum proof allows an overflow on the output side of the equation.

3.5 Proof of Overflow Free Sum

To prove to all network participants that the sum does not overflow, it is also necessary to construct a proof that each output value is both positive and small enough in magnitude that their sum can not overflow. A proof based on the sum of squares[38] is constructed. As the fuzz is random, $fuzzedValue$ is not a fixed constraint, and an expensive search to find three squares of the exact large integer is avoided[1].

3.5.1 Substitute Commitments

The sender can heuristically find two integers, the sum of whose squares is sufficiently close to $fuzzedValue$:

$$x_1^2 + x_2^2 + \Delta = fuzzedValue$$

$$x_1 = \lfloor\sqrt{fuzzedValue}\rfloor$$

$$x_2 = \lfloor\sqrt{|fuzzedValue - x_1^2|}\ \rfloor$$

The sender may re-generate the fuzz until $0 < \Delta < 2^{fuzzbits/2}$, so that a high magnitude of Δ does not imply a higher value output. The verifier will then know the relationship between the values, but knowing Δ will not help her to compute $fuzzedValue$, nor x_k. Δ can be revealed[2] leaving an equation in three unknowns. Reversing the following equation would not reveal $fuzzedValue$:

$$\Delta = fuzzedValue - \lfloor\sqrt{fuzzedValue}\rfloor - \lfloor\sqrt{|fuzzedValue - \lfloor\sqrt{fuzzedValue}\rfloor^2|}\ \rfloor$$

The upper 8 bits are reserved to handle overflow up to some maximum number of outputs per transcation, $maxOutputs = 2^{reservedbits-1} = 128$.

x_1 has half the bits of $fuzzedValue$, with the lower $fuzzbits/2$, or 96 bits, still random. Due to the heuristic, x_2 is smaller and weaker[3] than x_1, but retains $fuzzbits/3$ random bits. The blinding security of the scheme at this weakest point is therefore about 62 bits[4]. This is mitigated because even knowing x_2 leaves an equation in two unknowns, so the overall blinding is still higher (and bigger with the curve order: $fuzzbits \approx (curvebits - reservedbits)/2) - valuebits)$, though the number of bits in x_2 may roughly suggest the number of bits in $fuzzedValue$.

The sender (Prover) must prove to the network participant (Verifier), in zero knowledge, that each x_k^2 really is the square of x_k, and that x_k are both sufficiently small that the sum of x_k^2 will not overflow.

[1] Three squares can be used, which need not reveal a Δ, but the storage and computation requirements are 50% higher than the two squares approach.

[2] To save space, $\sum_{j=0}^{outputs} \Delta_j$ is revealed as an unsigned integer at the transaction level.

[3] A better algorithm[39] for finding sums of squares may solve this. The sender can use a different method to get x_1 and x_2, without any change to the verifier logic.

[4] x_1 and x_2 can, of course, be permuted with a probability of 1/2 for an extra bit.

The following commitments to x_k are made:

$$E_1 = x_1 * G \qquad E_2 = x_2 * G$$

$$F_1 = x_1 * E_1 \qquad F_2 = x_2 * E_2$$

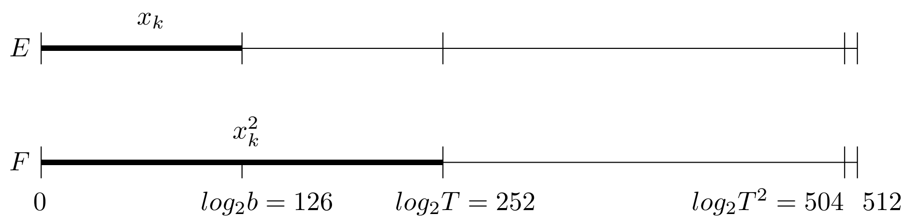
It follows that the Verifier can check the commitment equality without knowing x_k, so *commitment* does not need to be published:

$$commitment = F_1 + F_2 + \Delta$$

3.5.2 Proof of Square and Small Magnitude

The Proof of Equality of Two Discrete Logarithms[37] is adapted to the Elliptic Curve setting. Prover and Verifier agree on an elliptic curve C over a field F_n, generator $G \in E/R_n$, a hash function $Hash$ and security parameter t. Prover and Verifier both know a point commitment to x, $E = x * G$. The prover also knows that $x < b$ where b is a public integer. The security parameter[5] implies expansion of $T = 2^t b - 1$ between what is required to construct the proof, and what the Verifier can be convinced of. It is expected that $T < \sqrt{N}/(maxOutputs * 2)$, to prevent overflow in the transaction.

The bit representations of x_k and x_k^2 do not use up the whole curve, to allow the proof to be constructed:



Protocol $PKSqSmall(G, x, E : E = x * G \wedge F : F = x * E \wedge x \in [-T, T])$

1. Prover chooses random $r \in [0, T]$ and then computes $U = r * G$ and $V = r * E$.
2. Then she computes $c = Hash(U|V) \mod 2^t$.
3. Finally she computes $m = r + c * x$.
4. If $m > c * b$ and $m < T$, she sends[6] (m, U, V) to Verifier, otherwise she starts again the protocol.
5. Verifier checks all of the following are true, which convinces Verifier that $x \in [-T, T]$ and $F = E * x = G * x^2$:
   - $c * b < m < T$
   - $m * G = U + E * c$
   - $m * E = V + F * c$

[5] The Schnorr soundness security parameter t determines the probability of a cheating prover succeeding as 2^{-t}.

[6] The original protocol also sent c, but this is not necessary because the Verifier already receives U, V and can compute $c = Hash(U|V)$

3.6 Fuzz Balancing

If the transaction has more outputs than inputs, the sum of the input fuzz alone would not be sufficient to generate good output fuzz for each output. This is solved by allowing negligible input coin value to drain into the public fee. Likewise, if there is excess fuzz, it is allowed to drain into the output sum. The fuzz bits can be thought of as blinding sub-satoshis. Inputs are outputs to the next transaction, so there is a fixed bound on the possible value within the system, which verified to be unchanged by the transaction:

$$\sum_{i=1}^{inputs} vinCommit_i + coinbase * G = \sum_{j=1}^{outputs} (F_{1j} + F_{2j}) + (fee + \sum_{j=1}^{outputs} \Delta_j)) * G$$

Individual senders may choose to weaken the blinding by setting the fuzz to zero to gain a satoshi. This is economically insignificant and does not reveal any more than if the sender were to reveal the value itself. The sender of the following transaction may re-initialise the random fuzz bits normally.

Coinbase transactions may choose any publicly verifiable deterministic constant to initialise the fuzz (zero results in a round number for satoshi quantity). The Coinbase total can be randomly partitioned between multiple outputs by the same algorithm (such as the one included in Appendix A), to improve privacy of the coinbase outputs:

$$cbvalue_{0,\ldots,outputs} = Distribute(outputs, coinbase, valuebits)$$

4 Transaction Output Format

In addition to signature and script, and instead of the publicly visible value, the enhanced transaction output contains several elements to prove the validity of the transaction to the network:

- $voutCommit_j$: public commitments to the output value, including the fuzz bits, represented as points:

$$voutCommit_j = E_{1j}, E_{2j}, F_{1j}, F_{2j}$$

- $voutProof_j$: a compact NIZKP proof of unsigned smallness on the committed values:

$$voutProof_j = m_j, U_j, V_j$$

- $voutCrypt_j$: a one-time encryption of the value as performed by the sender by Elliptic Curve Diffie-Hellman exchange using $value_j$ as the private key for the source transaction, and the receiver's public view key:

$$commonSecret_j = value_j * voutViewPubKey_j$$

$$voutCrypt_j = Hash(commonSecret_j) \oplus value_j$$

  The receiver can later obtain the same shared secret from the transaction by using her private view key:

$$commonSecret_j = voutViewPrivKey_j * voutCommit_j$$

The encrypted information is not necessarily limited to the only the value, and can include arbitrary messages.

4.1 Script and Multi-signature Transactions

The coin scripting language is not affected, except for the verify opcodes, which accept the extra view key. The view key can be ignored for the purpose of script evaluation. For N-of-M transactions, the view keys can be shared as necessary.

4.2 Coinbase

If coinbase subsidy could be both randomised similar to Luckycoin[40] (and earlier version of Dogecoin[41]), and hidden while proved in a narrow range, this could provide extra initial privacy for the miners. This is considered too expensive to implement. The coinbase is instead constrained to be spent into a minimum of 3 outputs. This ensures that a miner's payee will not be able to determine the exact amounts sent to other payees from the single transaction output.

4.3 Sender and Receiver Responsibilities

Sender and receiver must not disclose the view key, amount and fuzz bits used in each transaction. It is up to the sender of a transaction to guarantee its secrecy by generating good randomness for the fuzz bits of each output. Once the details of a transaction are made public, it is likely that they can not be hidden again.

4.4 Transaction Size

For an output, the value hiding enhancement adds four commitments of 66 bytes each, two smallness square proofs of 198 bytes each, and an encrypted value of 64 bytes. In net terms, about 700 bytes are added. While this grows quickly with the number of outputs, only one commitment (66 bytes) needs to be kept in each unspent transaction output. At the user level, value hiding can encourage fewer transaction outputs to maintain the same level of privacy.

Since the introduction of multi-signature addresses, the average Bitcoin transaction size has risen to about 600 bytes. For a typical two-input, two-output transaction, the hiding overhead is then about 1.4KB (230%).

These numbers are a guide, and are not absolute. There is an adjustable tradeoff between the bit-security preventing forgeability, as defined by security parameter t; and the bit-security of value hiding, due to the security parameter $fuzzbits$. There are also adjustable tradeoffs between the security bit-levels, commitment size, curve order and $maxOutputs$. Special cases such as single-input single-output transactions do not require expensive proofs.

Note that ultra-small value transactions ("dust") cannot be rejected, because their value is not known. To prove that each output is big enough to be economically important, would further increase the transaction size and verification time. Instead, a positive non-zero transaction fee can be mandated by the miners to protect the ledger from abuse (note that as of version 0.1, Bitcoin encourages non-zero fees). Beyond the scope of this paper, dust can be also be mitigated by changing transaction incentives to reduce the size of the unspent output set.

4.5 Computational Load

The value verification requires, for every transaction input and output, two ECC additions to verify the sum. For every output, the verification of the proof also requires five ECC multiplications and two ECC additions. Public key recovery from the signature presents additional costs.

The performance impact of decrypting the encrypted value is small, and this need only performed for transactions in the local wallet. Constructing a transaction requires two ECC multiplications for every output. Construcing an output proof requires four (or probabilistically slightly more) ECC multiplications and one ECC addition. Transaction construction is a much less common task than transaction verification.

5 Implications

5.1 Features

Audit is still possible through equivalence proofs, or trivially by sharing a view key. Transaction value statistics are not available in, though the total number of coins created is known, because the coinbase values are public. The enhancement does not provide support for the creation of Coloured Coins[43], without revealing values. In addition to standing on its own, the enhancement can be implemented as a sidechain[44] or integrated into the Bitcoin (or Monero) protocol as a hard fork with a new transaction version. Spent transaction pruning[45] is still possible with the enhancement.

Of course, the enhancement does not preclude a user from using additional address hiding protocols such as mixing, though the linkability of both the view key and the spend key must be considered. A hidden satoshi will mix as well as a hidden coin, so this enhancement further improves CoinJoin. Hidden values deprecate privacy preserving strategies such as merge avoidance[42]. This scheme, of course, does not depend on a tiered network architecture, but can be integrated into Dash[16]. A user need not mix coins of related denominations, she can simply spend to multiple addresses.

This method does not hide all aspects of a transaction like like ZeroCash, but by hiding the amount, it does hide the most important aspect, while avoiding the complexity of zkSNARKS and additional initialisation functions.

If almost all inputs or outputs in any transaction are revealed, this will also reveal the remaining input (or output). This effect can propagate to related transactions if they are themselves almost fully revealed.

5.2 Social

The enhancement makes Bitcoin more like cash. Users of cash tend not to discover the value of other user's transactions. Without this enhancement, Bitcoin nodes discover the value of each unrelated transaction on the global network.

With cash transactions, if a party chooses to publicly disclose a transaction amount, their claim would initially stand unconfirmed. They would need additional evidence, such as a confirmation from the counterparty (or intermediary) to try to back up the claim. With Bitcoin, such disclosure is immediately and forever provable on the blockchain. This permanent and undeniable record should discourage the use of this technology for nerfarious purposes.

Properly kept, crypto-currencies such as Bitcoin are the most difficult asset class to take from an owner without consent. This process reduces to probabilistic rubber hose cryptanalysis in extremis, which may only be feasible on a small scale. Unfortunately, not all owners can be assumed to make sufficient effort to protect their coins, and the public visibility of Bitcoin values make the owners a target[46]. This enhancement to Bitcoin hides value to discourage the selection and prioritisation of targets for the application of cryptanalysis.

6 Conclusion

It has been shown to be not only possible, but very practical, to enhance a crypto-currency like Bitcoin to hide transaction values from public view, while maintaining the integrity of each transaction, with only a small increase in computational and storage requirements over the non-hiding protocols. Zero-knowledge proofs are performed on the elliptic curve by the sender of every transaction, these convince all honest nodes that the sum of the transaction outputs equals the sum of the transaction inputs, and that overflow did not occur. Elliptic point commitments include a random fuzz component to deflect brute force attacks, and this component is uniformly rebalanced in the outputs of each transaction. This design is significantly simpler to implement than other methods proposed thus far. Some implications of the technology are discussed.

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A The Distribute algorithm

Algorithm 1 To partition a *total* into j random parts while preserving the sum

procedure DISTRIBUTE($j, total, bits$)
    $Assert(0 < j < total)$ ▷ input validation
    $Assert(j * 2^{bits-2} < total < j * 2^{bits})$
    $mask = 2^{bits} - 1$
    for $x = 1$ to j **do**
        $\mathrm{ret}_j \leftarrow 0$ ▷ initialise return values
    end for
    while $total > 1$ **do** ▷ work until *total* is fully disbursed
        $\mathrm{fences}_0 \leftarrow 0$
        $\mathrm{fences}_{j+1} \leftarrow total$
        for $x = 1$ to j **do** ▷ random without replacement
            $\mathrm{fences}_x \leftarrow Random(U(1, total) \cap \mathrm{fences}_{0,\ldots,x-1})$
        end for
        $\mathrm{fences}_{0,\ldots,j+1} \leftarrow Sort(\mathrm{fences}_{0,\ldots,j+1})$
        for $x = 0$ to j **do** ▷ partition the total into *delta*s
            delta $\leftarrow \mathrm{fences}_{x+1}$ - fences_x
            $\mathrm{ret}_x \leftarrow \mathrm{ret}_x$ + delta
            $total \leftarrow total$ - delta
            while $\mathrm{ret}_x > mask$ **do** ▷ carry *delta* overflow
                $\mathrm{ret}_x \leftarrow \mathrm{ret}_x$ - $mask$
                $total \leftarrow total + mask$
            end while
        end for
    end while
    return $\mathrm{ret}_{0,\ldots,j}$ ▷ implicitly, $total \equiv \sum ret_{0,\ldots,j}$
end procedure