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# Integrating Subjective Knowledge Bases through an Extended Belief Game Model

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**Abstract.** Belief merging is concerned with the integration of several (mutually inconsistent) belief bases such that a coherent belief base is developed. Various belief merging models have been developed to address this problem. These models often consist of two key functions, namely: negotiation, and weakening. A negotiation function finds the weakest belief bases among the available belief bases, and then the selected belief bases concede based on a weakening function. This process is iteratively repeated until a consistent belief base is developed. In this paper, we extend the current belief merging models by introducing the extended belief game model. The extended belief game model operates over a subjective belief profile, which consists of belief bases with subjectively annotated formulae. The subjective information attached to each formula enables the proposed model to prioritize the formulae in the merging process. We provide several instantiations of the model by introducing suitable functions. Furthermore, We formally investigate how the outcome of a collaborative modeling process can be obtained as a solution of the extended belief game model.

**Keywords:** Belief Merging, Collaborative Modeling, Formal Conflict Resolution, Social Contraction, Negotiation

## 1. Introduction

Belief merging deals with the issue of acquiring a consistent belief base from several (inconsistent) belief bases, i.e. given a set of information sources that may contain mutual inconsistencies, how can a coherent set of information be developed which is a fair representative of the content conveyed by all the information sources [27]. Several researchers have approached this issue as a multi-stage game [27, 13, 39]. The game is a competition between the information sources where the weakest information sources are considered as the losers of the game and should make appropriate concessions. The basic idea of the merging game is to form some sort of coalition between like-minded information sources, and penalize the information sources that are furthest away from the coalition. The rationality for such an approach under the reliability assumption for the information sources is based on Condorcet's Jury theorem [30]. This theorem argues that in most cases listening to the majority of the members of the jury (coalition) is the most rational decision.

Each stage of the merging game is known as rounds of negotiation or competition. Konieczny's belief negotiation model [27] and Booth's belief negotiation model [13, 12] are among the most popular frameworks for belief merging, which are defined for a pure propositional logic setting. In these models, in each round of negotiation, some of the information sources are selected by the negotiation (choice) function. The selected sources should then weaken their beliefs using a weakening (contraction) function. Focusing on pure propositional logic makes the definition of flexible negotiation and weakening functions more difficult in these frameworks.

Several models have been proposed which exploit priority relationships between the belief bases and their constituting formulae in the process of merge [8, 10, 11]. The priorities provide essential information with regards to the degree of formulae importance. They can be helpful in deciding which formulae need to be discarded in order to restore consistency. Possibilistic logic has been extensively used for this purpose where the formulae in the belief bases are annotated with a weight denoting their degree of necessity. One of the key factors that needs to be incorporated into the priority rankings is the degree of uncertainty of the information sources about their expressed opinions. Possibility theory addresses uncertainty through the necessity and possibility measures. Subjective logic [21], an extension to Dempster-Shafer theory of evidence [15], is also able to explicitly address uncertainty. It defines uncertainty as one of the dimensions of belief in its three dimensional belief structure.

Logic-based merging approaches have been employed in different fields such as multi-agent systems, and distributed computing. Similar to these areas, collaborative modeling is concerned with the employment of information from different sources. Here, the sources of information are human experts that are in most cases cooperating to develop a unique model for a given domain of discourse [48]. The model developed by each expert can be considered as an information source and their integration can be viewed as a belief merging process. The factor distinguishing collaborative modeling from the mentioned fields is that it directly interacts with *human participants*. Humans usually make conception errors due to risk aversion, short-term memory or even framing and perceptual problems [37]. This implies that not all asserted information from the sources are correct or equally reliable. Furthermore, epistemic uncertainty (also known as partial ignorance) is an indispensable element of human judgments that makes them even more susceptible to inaccuracy and imprecision [43]. There is no guarantee that the received information coming from various human sources be consonant or consistent. They may be in many cases arbitrarily (with very few common elements) or disjointedly (no common elements) distributed [43]. Hence collaborative modeling needs to consider the significant role of uncertainty and imprecision in the manipulation of the experts expressions.

In this paper, we propose an extended belief game model, where the significance of each formula is addressed through the framework of Subjective logic using subjective opinions. Each belief source is represented by a Subjective belief base (SBB). The process of integrating various subjective belief sources is accomplished in two steps. In the first step (social contraction), some of the inconsistent belief bases (selected by the choice function) are both syntactically and semantically enhanced (using the enhancement function) to make them mutually coherent. For this step, some choice and enhancement functions are defined based on the subjective opinions of the belief bases. In the second step (combination), the coherent belief bases acquired from the first step are combined using the Subjective consensus operator, which can have a reinforcement effect on the final subjective belief of the formulae.

We will further formally define how the outcome of a collaborative modeling process can be obtained as a solution of the extended belief game model.

The rest of the paper proceeds as follows. We give some preliminaries and an introduction to the belief game model by Konieczny in the next two sections. Section 4 provides an overview of Evidence theory. Consensus formation in evidence theory is introduced in Section 5. In Section 6, our extended belief game model is introduced. Specific choice and enhancement functions for the extended belief game model are given in Section 7. We instantiate the proposed model and discuss its properties in Section 8. The application of the extended belief game model to collaborative modeling and an illustrative example are given in Section 9. Finally some related work are reviewed and the paper is concluded with some concluding remarks in Section 11.

## 2. Preliminaries

Throughout this paper, we let $\mathcal{L}$ be a propositional language over a finite alphabet $\mathcal{P}$ of propositional symbols. $\Omega$ represents the set of possible interpretations. An interpretation is a function from $\mathcal{P}$ to $\{\bot, \top\}$, where $\bot$, and $\top$ denote truth and falsehood, respectively. An interpretation $\omega$ is a model of formula $\phi$, noted as $\omega \models \phi$ which makes the formula true. Furthermore, let $\phi$ be a formula, $mod(\phi)$ is employed to denote the set of models of $\phi$, i.e. $mod(\phi) = \{\omega \in \Omega \mid \omega \models \phi\}$. Classical propositional logic deduction is represented using $\vdash$. Two formula such as $\phi$ and $\varphi$ are equivalent, expressed $\phi \equiv \varphi$, if and only if $\phi \vdash \varphi$ and $\varphi \vdash \phi$. A formula $\phi$ satisfying $mod(\phi) \neq \bot$ is considered to be consistent.

A belief base $\varphi$ is a consistent propositional formula (or, equivalently, a finite consistent set of propositional formulae $\{\phi_1, \phi_2, ..., \phi_n\}$, considered conjunctively: $\phi_1 \wedge \phi_2 \wedge .... \wedge \phi_n$). Let $\varphi_1, ..., \varphi_n$ be $n$ not necessarily different belief bases, we define a belief profile as a multi-set $\Psi$ consisting of those $n$ belief bases, i.e. $\Psi = (\varphi_1, ..., \varphi_n)$. This definition permits two or more different sources to be identical. The conjunction of the belief bases of $\Psi$ is represented as $\bigwedge \Psi = \varphi_1 \wedge ... \wedge \varphi_n$. Belief profile inclusion is denoted as $\sqsubseteq$, and belief profile union is represented by $\sqcup$. The cardinality of a finite belief profile $\Psi$ is noted as $\sharp(\Psi)$.

A belief profile $\Psi$ is consistent if and only if $\bigwedge \Psi$ is consistent. Let $\mathcal{E}$ be the set of all finite non-empty belief profiles. Two belief profiles $\Psi_1$ and $\Psi_2$ are equivalent ($\Psi_1 \equiv \Psi_2$) if and only if there is a bijection $(f)$ between $\Psi_1$ and $\Psi_2$ such that each belief base of $\Psi_1$ is logically equivalent to its image in $\Psi_2$ ($\forall \varphi \in \Psi_1, f(\varphi) \in \Psi_2, \varphi \equiv f(\varphi)$).

## 3. Belief Game Model

Richard Booch has proposed a framework for incrementally merging sources of information that may contain inconsistencies [13]. This framework, Belief Negotiation Model (BNM), provides several rounds of negotiation through which certain information sources make some concessions to reach an agreeable state. Suppose the belief profile $\Psi$ is provided by the information sources. The negotiation process commences in the first round of negotiation by analyzing $\Psi^0$. If $\Psi^0$ is consistent, it is considered as the solution. But if $\Psi^0$ is diluted with inconsistency a round of negotiation between the information sources is performed. The operationalization of this process is performed using two base functions, namely: choice function, and weakening function. In each round, the choice function identifies the losers of the negotiation process. The losers must then make some concessions by conforming to other possibilities through the employment of the weakening functions. BNM possess two important features. First, it does not require all information sources to make concessions in each round. This means that even if two information sources reveal equivalent belief bases, one may need to weaken while the other does not. Second, each information source can possess its own individual weakening (contraction) function. The belief negotiation model has its roots in belief revision, which is defined in terms of two primary operations: belief contraction, and belief expansion [35].

Additionally, the Belief Game Model (BGM) has been proposed by Konieczny [27]. Analogous to BNM, BGM consists of two base functions: choice function, which selects from every belief profile in $\mathcal{E}$, a set of its belief bases, and the weakening function that equally weakens the selected belief bases. As opposed to BNM, BGM incorporates more anonymity by stating that only the contents of a belief base should be considered in the choice function. Furthermore, it weakens all the selected belief bases in a similar manner.

**Definition 1** *A choice function is a function $g : \mathcal{E} \to \mathcal{E}$ such that:*

*(c1)* $g(\Psi) \sqsubseteq \Psi$,

*(c2) If* $\bigwedge \Psi \not\equiv \top$, *then* $\exists \varphi \in g(\Psi)$ *s.t.* $\varphi \not\equiv \top$,

*(c3) If* $\Psi \equiv \Psi'$, *then* $g(\Psi) \equiv g(\Psi')$.

The choice function attempts to find the belief sources that need to be weakened at each round of negotiation. The first condition ensures that the selected belief bases are a subset of the belief profile $\Psi$. The second condition states that at least one non-Tautological belief base should be selected. This condition is necessary so that a BGM has a termination condition. The last condition is irrelevance of syntax. It asserts that the choice of the belief bases to be weakened is only based on their informational value and not on the syntactical representation.

**Definition 2** *A weakening function is a function $\blacktriangledown : \mathcal{L} \to \mathcal{L}$ such that:*

*(w1)* $\varphi \vdash \blacktriangledown(\varphi)$,

*(w2) If* $\varphi \equiv \blacktriangledown(\varphi)$, *then* $\varphi \equiv \top$,

*(w3) If* $\varphi \equiv \varphi'$, *then* $\blacktriangledown(\varphi) \equiv \blacktriangledown(\varphi')$.

The two first conditions of the weakening function ensure that the selected belief bases are replaced by a strictly weaker belief base. The last condition is an irrelevance of syntax, which guarantees that weakening is only dependant on semantical information conveyed by the belief bases.

**Definition 3** *A Belief Game Model is a pair $\mathcal{N} = \langle g, \blacktriangledown \rangle$ where $g$ is a choice function and $\blacktriangledown$ is a weakening function. The solution to a belief profile $\Psi$ for a Belief Game Model $\mathcal{N} = \langle g, \blacktriangledown \rangle$ under the integrity constraint $\mu$, noted $\mathcal{N}_\mu(\Psi)$, is the belief profile $\Psi^\mu_\mathcal{N}$ defined as:*

*(w1)* $\Psi_0 = \Psi$,

*(w2)* $\Psi_{i+1} = \blacktriangledown_{g(\Psi_i)}(\Psi_i)$,

*(w3)* $\Psi^\mu_\mathcal{N}$ *is the first* $\Psi_i$ *that is consistent with* $\mu$.

The solution to the belief profile is the outcome of a structured game on the semantics of the belief sources. At each round of negotiation the weakest bases (losers) are identified. These have to concede by weakening their beliefs. In some scenarios, the result of the integration has to conform to some predefined constraints. The third condition ensures that the result of the belief game model is consistent with the constraints ($\bigwedge \Psi^\mu_\mathcal{N} \wedge \mu$).

## 4. Basics of Evidence Theory

Evidence theory is one of the theoretical models, which is able to numerically quantify the lack of knowledge with regards to a certain phenomenon in an effective manner [44]. It is a potentially useful tool for the evaluation of the reliability of information sources. Dempster-Shafer (DS) theory of evidence is one of the most widely used models that provides means for approximate and collective reasoning under uncertainty [15]. It is basically an extension to probability theory where probabilities are assigned to sets as opposed to singleton elements. The employment of the DS theory requires the definition of the set of all possible states in a given setting, referred to as the frame of discernment represented by $\theta$. The powerset of $\theta$, denoted $2^\theta$, incorporates all possible unions of the sets in $\theta$ that can receive belief mass.

The truthful subsets of the powerset can receive a degree of belief mass; therefore, the belief mass assigned to an atomic set such as $\psi \in 2^\theta$ is taken as the belief that the given set is true. Moreover, the belief mass ascribed to a non-atomic set such as $\psi \in 2^\theta$ is interpreted as the belief that one of the atomic sets in $\psi$ is true, but uncertainty rules out the possibility of pinpointing the exact atomic set.

**Definition 4** *A belief mass assignment is a mapping $m : 2^\theta \rightarrow [0, 1]$ that assigns $m_\theta(\psi)$ to each subset $\psi \in 2^\theta$ such that:*

*(1)* $m_\theta(\psi) \geq 0$,

*(2)* $m_\theta(\varnothing) = 0$,

*(3)* $\sum_{\psi \in 2^\theta} m_\theta(\psi) = 1$.

$m_\theta(\psi)$ is then called the belief mass of $\psi$. A belief mass assignment is called dogmatic if $m_\theta(\theta) = 0$, since all the possible belief masses have been spent on the subsets of $\theta$.

The belief in $\psi$ is interpreted as the absolute faith in the truthfulness of $\psi$, which not only relies on the belief mass assigned to $\psi$ but also to belief masses assigned to subsets of $\psi$.

**Definition 5** *A belief function corresponding with $m_\theta$, a belief mass assignment on $\theta$, is a function $b : 2^\theta \to [0, 1]$ defined as:*

$b(\psi) = \sum_{\varphi \subseteq \psi} m_\theta(\varphi), \quad \varphi, \psi \in 2^\theta.$

Analogously, disbelief is the total belief that a set is not true.

**Definition 6** *A disbelief function corresponding with $m_\theta$, a belief mass assignment on $\theta$, is a function $d : 2^\theta \to [0, 1]$ defined as:*

$d(\psi) = \sum_{\varphi \cap \psi = \varnothing} m_\theta(\varphi), \quad \varphi, \psi \in 2^\theta.$

To address the degree of uncertainty which is inherent in the above definitions, Jøsang provides a complementary definition, uncertainty, which computes the degree of possible confusion in belief assignment [21].

**Definition 7** *An uncertainty function corresponding with $m_\theta$, a belief mass assignment on $\theta$, is a function $u : 2^\theta \to [0, 1]$ defined as:*

$u(\psi) = \sum_{\varphi \cap \psi \neq \varnothing, \varphi \not\subseteq \psi} m_\theta(\varphi), \quad \varphi, \psi \in 2^\theta.$

With the above definitions, Subjective logic [24] extends Dempster-Shafer theory of evidence. A belief expression in Subjective logic is defined as a 3-tuple $\chi_x^A = (b_x^A, d_x^A, u_x^A)$ also known as the opinion of expert $A$ about hypothesis $x$ ($\chi_x^A$). It can be shown with this definition that belief ($b_x^A$), disbelief ($d_x^A$), and uncertainty ($u_x^A$) elements of an opinion should satisfy:

$$b_x^A + d_x^A + u_x^A = 1. \tag{1}$$

The above condition restricts the possible values that can be expressed as an opinion by an expert only to the points placed in the interior surface of an equal-sided triangle. The three constituent elements determine the position of an opinion within the triangular space. Figure 1 shows the three axis that can be used to identify the position of an opinion point in the triangle. In the opinion triangle, the line connecting absolute belief and absolute disbelief corners (right and left corners) is called the probability axis. This is because the removal of uncertainty from Subjective logic will result in a pure probabilistic interpretation of belief (i.e. $b_x^A + d_x^A = 1.$), which respects the additivity condition. The opinions which are situated on this axis are named dogmatic opinions since they do not contain any degree of uncertainty. Among dogmatic beliefs, the two opinions located on the

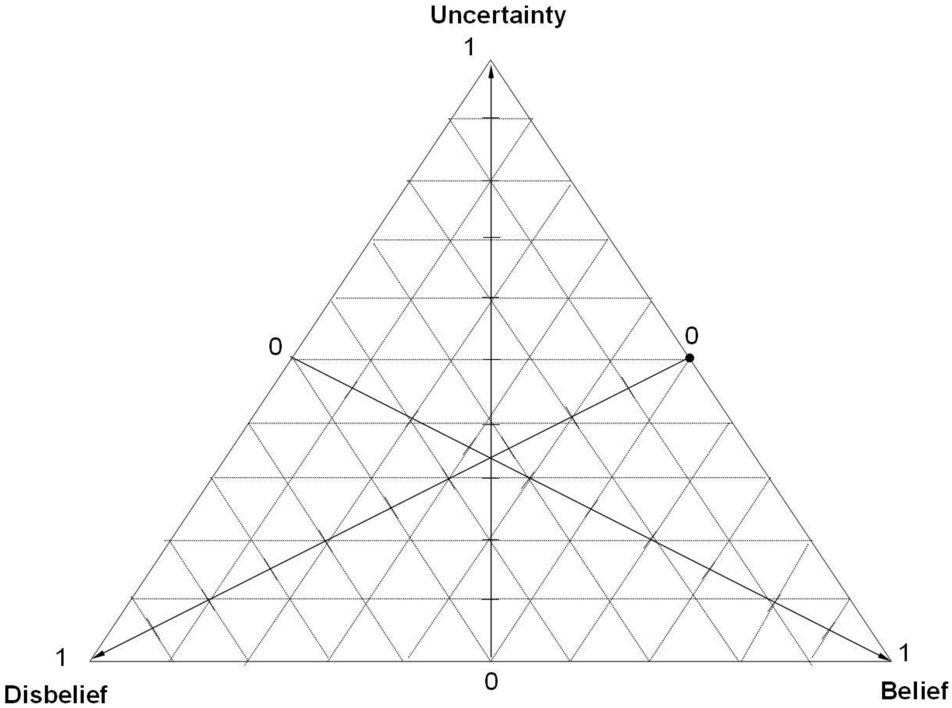


*Figure 1.* The Opinion Space Can be Mapped into the Interior of an Equal-sided Triangle.

extreme ends of the probability axis are called absolute opinions and represent inflexible agreement or disagreement with a given hypothesis.

A Subjective Belief Base (SBB) is a set of formulae annotated with subjective opinions in the form of $B = \{(\phi_i^*, \chi_{\phi_i}^A) : i = \{1, .., n\}$, where $\chi_{\phi_i}^A$ is a subjective opinion such that $\chi_{\phi_i}^A = (b_{\phi_i}^A, d_{\phi_i}^A, d_{\phi_i}^A)$. The classical propositional form of B is represented by $B^*$ $(\{\phi_i | (\phi_i^*, \chi_{\phi_i}^A) \in B\})$. A subjective belief profile $\mathcal{SP}$ consists of multiple SBBs. $\mathcal{SP} = (B_1, ..., B_n)$ is consistent if and only if $B_1^* \cup ... \cup B_n^* \cup \mu$ is consistent. $\mathcal{SE}$, $\mathcal{F}_B$, and $\mathcal{K}$ are employed to denote the set of all finite non-empty subjective belief profiles, the set of all formulae in belief base $B$ and the set of all subjective belief bases, respectively.

**Definition 8** *Let $B$ be a SBB, and $\alpha \in (0, 1]$. The $\alpha$-cut of $B$ is $B_{\succeq\alpha} = \{\phi \in B^* | (\phi, \chi) \in B$ and $(b_\chi \geq \alpha$ or $d_\chi \geq \alpha)\}$.*

Two subjective belief bases are equivalent, shown as $B \equiv_s B'$, if and only if $\forall \chi \in (0, 1]$, $B_{\succeq\chi} \equiv B'_{\succeq\chi}$. Furthermore, two $\mathcal{SE}$s are equivalent, denoted $\mathcal{SE} \equiv_s \mathcal{SE}'$, if and only if there is a bijection between them.

## 5. Semantics of Consensus Formation

The intention of combining information is to meaningfully summarize and simplify a corpus of data elements coming from various sources in order to develop a consensus between the participating sources [22]. There exist several possible techniques for aggregating multiple belief mass assignments in Dempster-Shafer theory of evidence. The base combination rule for multiple mass assignment functions is Dempster's rule of combination, which is a generalization of Bayes' rule [36]. This combination operator ignores the conflicts between the functions and emphasizes on their agreements.

**Definition 9** *Let $m_1$ and $m_2$ be two belief mass assignments defined on a frame of discernments $\theta$ which stem from two distinct sources. Let the combined resulting belief mass assignment from Dempster's rule of combination be $m_{\oplus} = m_1 \oplus m_2$ where $\oplus$ denotes the operator of combination.*

$$m_{\theta}(\beta) = \frac{\sum_{\varphi,\psi\subseteq\theta,\varphi\cap\psi=\beta} m_1(\varphi)m_2(\psi)}{1-\sum_{\varphi,\psi\subseteq\theta,\varphi\cap\psi=\emptyset} m_1(\varphi)m_2(\psi)}, \forall\beta\subseteq\theta, \beta\neq\emptyset$$

*when* $\sum_{\varphi,\psi\subseteq\theta,\varphi\cap\psi=\emptyset} m_1(\varphi)m_2(\psi) \neq 1$.

Dempster's rule redistributes the conflicting masses over the non-conflicting masses and therefore insists on the mutual agreements and removes conflicts [31]. This approach to belief integration has been criticized due to its counter-intuitive results under highly conflicting belief expressions [50]. Several authors have proposed models to overcome this problem. For instance, Smets has proposed the assignment of the conflicting masses to $\phi$. His interpretation of conflicts is that they occur when the hypothesis space is not exhaustive [45]. In a different approach, Yager proposes the assignment of conflict masses to $\theta$, and interprets it as the degree of overall ignorance [49]. Within the framework of Subjective logic, the combination operation is performed through the application of the Consensus operator.

**Definition 10** *Let $\chi_x^A = (b_x^A, d_x^A, u_x^A)$ and $\chi_x^B = (b_x^B, d_x^B, u_x^B)$ be two opinions about a common fact $x$ stated by two different information sources $A$ and $B$, and let $\kappa = u_x^A + u_x^B - u_x^A u_x^B$. When $u_x^B \to 0$, and $u_x^A \to 0$, the relative dogmatism between the two opinions are defined using $\gamma = u_x^B / u_x^A$. Now $\chi_x^{A,B} = (b_x^{A,B}, d_x^{A,B}, u_x^{A,B})$ is a fair representative of both opinions, Consensus operator $\oplus$ outcome, such that:*

| when $\kappa \neq 0$ | when $\kappa = 0$ |
|---|---|
| $b_x^{A,B} = (b_x^A u_x^B + b_x^B u_x^A)/\kappa$ | $b_x^{A,B} = \frac{\gamma b_x^A + b_x^B}{\gamma+1}$ |
| $d_x^{A,B} = (d_x^A u_x^B + d_x^B u_x^A)/\kappa$ | $d_x^{A,B} = \frac{\gamma d_x^A + d_x^B}{\gamma+1}$ |
| $u_x^{A,B} = (u_x^A u_x^B)/\kappa$ | $u_x^{A,B} = 0$ |

The Consensus operator has been shown to have a stable behavior under various conditions and even while merging conflicting dogmatic beliefs [23]. It satisfies two important algebraic properties i.e. commutativity ($A \oplus B = B \oplus A$), and associativity ($A \oplus [B \oplus C] = [A \oplus B] \oplus C$), which are of great significance while merging two peer information sources.

## 6. An Extended Belief Game Model

In this section, we propose an extended belief game model that incorporates features from both BGM [27], and BNM [13]. In the proposed model, the significance of the formulae is specified through subjective belief annotation values. Each source of belief in this model is represented using a SBB. It is assumed in our model that each SBB is internally consistent.

**Definition 11** *A choice function under the integrity constraint $\mu$ is a function $g_\mu : \mathcal{SE} \rightarrow \mathcal{SE}$ such that:*

*(ec1)* $g_\mu(\mathcal{SP}) \sqsubseteq \mathcal{SP}$,

*(ec2) If* $\bigwedge \mathcal{SP} \wedge \mu \not\equiv \top$, $\exists B_i, B_j$ *such that* $B_i^* \wedge B_j^* \wedge \mu \not\equiv \top$, *then* $(B_i$ *or* $B_j) \in g_\mu(\mathcal{SP})$,

*(ec3) If* $\mathcal{SP} \equiv_s \mathcal{SP}'$ *then* $g_\mu(\mathcal{SP}) \equiv_s g_\mu(\mathcal{SP}')$.

Condition $ec1$ is a direct generalization from $c1$ in BGM. The second condition states that the choice function does not select those SBBs which are not inconsistent with any other SBBs. Therefore, it minimally includes any of the two SBBs whose conjunction produces an inconsistent belief base with regards to the integrity constraint $\mu$. $ec2$ is required to ensure that the proposed extended belief game model properly terminates under any setting. It can also be seen in $ec1$ that the choice function is dependent on the syntactical format of the SBBs, since consistency checking is performed on the belief base without attention to the subjective opinion annotation values. Finally, a negotiation function $g$ is syntax-dependant if it satisfies $ec3$.

**Definition 12** *An enhancement function is a function* $\blacktriangledown : \mathcal{K} \times \mathcal{SE} \times \mathcal{SE} \rightarrow \mathcal{K}$ *where for a SBB B, and two subjective belief profiles* $\mathcal{SP}, \mathcal{SP}'$, *if* $\mathcal{SP}' \sqsubseteq \mathcal{SP}$ *and* $B \in \mathcal{SP}'$ *then* $\blacktriangledown_{\mathcal{SP},\mathcal{SP}'}(B)$ *satisfies:*

*(ee1)* $\blacktriangledown_{\mathcal{SP},\mathcal{SP}'}(B) - \blacktriangledown^a_{\mathcal{SP},\mathcal{SP}'}(B) \subseteq B$,

*(ee2) If* $\exists B_i \in \mathcal{SP}$ *such that* $B^* \wedge B_i^* \wedge \mu \not\equiv \top$ *then* $\blacktriangledown^a_{\mathcal{SP},\mathcal{SP}'}(B)^* \wedge B_i^* \equiv \top$,

*(ee3) If* $B = \blacktriangledown_{\mathcal{SP},\mathcal{SP}'}(B)$, *then* ($\nexists B_i \in \mathcal{SP}$ *such that* $B^* \wedge B_i^* \wedge \mu \not\equiv \top$ *and* $\blacktriangledown^a_{\mathcal{SP},\mathcal{SP}'}(B) = \varnothing$).

*where* $\blacktriangledown^a_{\mathcal{SP},\mathcal{SP}'}(B)$ *denotes the justifications on B.*

The enhancement function can be extended to subjective belief profiles by allowing $\blacktriangledown_{\mathcal{SP},\mathcal{SP}'}(\mathcal{SP}) = \{\blacktriangledown_{\mathcal{SP},\mathcal{SP}'}(B) : B \in \mathcal{SP}\}$.

The enhancement function promotes the contents of the information sources. Analogous to the weakening function in BGM, this function weakens the belief bases in order to remove inconsistencies, but additionally allows the information sources to add extra consistent formulae to the belief base to justify or extend their standpoint. $ee1$ ensures that the result of the enhancement function has been weakened except for the additional consistent formulae that have been added by the information source for standpoint justification. $ee2$ states that the additional formulae should be consistent with the formulae in the peer belief base with which the inconsistencies had initially occurred. Furthermore, the result of the enhancement function is only equivalent to the initial subjective belief base if a peer inconsistent belief base does not exist and no further justifications have been added to the belief base ($ee3$).

**Definition 13** *An extended belief game model is a pair $\mathcal{N} = \langle g, \blacktriangledown \rangle$ where $g$ is a choice function and $\blacktriangledown$ is an enhancement function. The final solution to a subjective belief profile $\mathcal{SP}$ for an extended belief game model $\mathcal{N} = \langle g, \blacktriangledown \rangle$, under the integrity constraint $\mu$, denoted $\mathcal{N}_\mu(\mathcal{SP})$, is the subjective belief profile $\mathcal{SP}^\mu_\mathcal{N}$ defined as:*

*(sp1)* $\mathcal{SP}_0 = \mathcal{SP}$,

*(sp2)* $\mathcal{SP}_{i+1} = \blacktriangledown_{\mathcal{SP}_i, g(\mathcal{SP}_i)}(\mathcal{SP}_i)$,

*(sp3)* $\mathcal{SP}^\mu_\mathcal{N}$ *is the first* $\mathcal{SP}_i$ *that is consistent with* $\mu$.

Let $\mathcal{SP} = \{B_1, ..., B_n\}$ be a subjective belief profile. The integration of the subjective belief bases in the extended game model is achieved through several rounds of negotiation. In each round the belief bases which create inconsistencies are weakened and justified through the enhancement function in order to obtain a consistent subjective belief profile ($\mathcal{SP}^\mu_\mathcal{N}$)(social contraction step). Once a consistent $\mathcal{SP}^\mu_\mathcal{N}$ has been created, the SBBs in $\mathcal{SP}^\mu_\mathcal{N}$ are combined using the subjective consensus operator which creates a fair trade-off between the information of different sources (combination step) as a result of which a ultimate subjective belief base is obtained.

## 7. Choice and Enhancement Functions

### *7.1. Choice Function*

The structure of the choice function is based on a fitness measure (analogous to belief entropy functions, its lower values are more desirable) that defines the quality of the subjective belief bases (SBBs) i.e. those belief bases which are less eminent than the others are selected as suitable candidates in the choice function. We define the fitness measure based on intrinsic and extrinsic properties of the belief bases. The intrinsic properties of a belief base address the internal features inferred from the subjective structure of the belief base, while the extrinsic properties relate to the behavior of the belief base with regards to $\mathcal{SP}$.

**Definition 14** *The fitness measure of a subjective belief base $B$ under integrity constraints $\mu$ is a function $fm : \mathcal{K} \times \mathcal{SE} \rightarrow [0, \infty)$. Given two SBBs $B$ and $B' \in \mathcal{SP}$, $B$ is considered more competent than $B'$ if and only if $fm^{\mu}_{\mathcal{SP}}(B) < fm^{\mu}_{\mathcal{SP}}(B')$.*

Now we introduce the building blocks of the fitness measure that can be employed for evaluating the quality of each individual belief base.

*7.1.1. Intrinsic Properties of belief bases in $\mathcal{SP}$* The intrinsic properties of each belief base is closely related with the subjective annotation values of its constituting formulae. These properties represent the strength and validity of the stated formulae, and the fortitude of beliefs of the information source that is providing the belief base. Two major intrinsic features are considered, namely: Ambiguity, and Indecisiveness.

Before introducing these properties, an aggregation function [39] needs to be introduced. The aggregation function is required in order to accumulate the overall value of each property for a subjective belief base.

**Definition 15** *An aggregation function is a total function $f$ that assigns a non-negative integer to every finite tuple of integers and satisfies non-decreasingness (if $x \leq y$ then $f(x_1, ..., x_n) \leq f(x_1, ..., y, ..., x_n)$), minimality ($f(x_1, ..., x_n) = 0 \; iff \; x_1 = ... = x_n = 0$), and identity ($\forall x \in \mathbb{N},\ f(x, ..., x) = x$).*

Now we can define the intrinsic properties. First, the ambiguity property provides the basis to calculate the degree of confusion in the subjective belief base about the exact fraction of belief that should be assigned to each formulae.

**Definition 16** *Let $\phi = (\phi^*, \chi_\phi)$ be a subjective formula in a subjective belief base, and $\epsilon$ be a normalization factor. Ambiguity is a function $\zeta : \mathcal{F} \rightarrow [0, 1]$, defined as:*

$$\zeta(\phi) = -\left(\frac{1}{e^{1-(b_\phi + d_\phi)}} - 1\right)/\epsilon.$$

Ambiguity is similar to the belief entropy metric in the generalized entropy criterion [47]. Furthermore, the indecisiveness property is a measure of the ability of the information source to firmly state a given formula. The further away the degree of belief and disbelief of a given formula are, the stronger and more decisive the formula is.

**Definition 17** *Let $\phi = (\phi^*, \chi_\phi)$ be a subjective formula in a subjective belief base, and $\epsilon'$ be a normalization factor. Indecisiveness is a function $\vartheta : \mathcal{F} \rightarrow [0, 1]$, defined as:*

$$\vartheta(\phi) = \left(\frac{2}{e^{(b_\phi - d_\phi)} + e^{(d_\phi - b_\phi)}} - \frac{2e}{e^2 + 1}\right)/\epsilon'.$$

The intrinsic properties of the formulae can be extended to a subjective belief base $B = \{\phi_1, ..., \phi_n\}$, by allowing $\zeta(B) = f(\zeta(\phi_1), ..., \zeta(\phi_n))$, and $\vartheta(B) = f(\vartheta(\phi_1), ..., \vartheta(\phi_n))$, where $f$ is an aggregation function (e.g. sum). These properties are considered intrinsic since they can be computed within the context of a single belief base. Therefore, the setting of the other belief bases in the subjective belief profile is not important in the behavior of the intrinsic properties.

*7.1.2. Extrinsic Properties of belief bases in $\mathcal{SP}$* These properties show the behavior of the belief bases and their sources in relationship with the others. They are somewhat also dependant on the past behavior of the information sources. These extrinsic properties are only meaningful in comparison and within the context of a specific $\mathcal{SP}$, and not in vacuum. We introduce two extrinsic properties, namely: Conflict and Reliability.

The conflict property defines the degree of inconsistency between the belief of a single belief base towards one formula in comparison with another belief base's opinion on the given formula. Conflict can be taken as the extent of disagreement and divergence of the belief bases.

**Definition 18** *Let $\phi = (\phi^*, \chi_\phi)$, and $\phi' = (\phi'^*, \chi_{\phi'})$ be two subjective formula, and $\epsilon''$ be a normalization factor. Conflict is a function $\delta : \mathcal{F} \times \mathcal{F} \to [0,1]$, defined as:*

$$\delta(\phi, \phi') = \frac{(b_\phi \times d_{\phi'} + b_{\phi'} \times d_\phi)}{2 \times \epsilon''}.$$

Analogous to the intrinsic properties, conflict can be extended to two subjective belief bases $B = \{\phi_1, ..., \phi_n\}$ and $B' = \{\phi'_1, ..., \phi'_n\}$ by letting $\delta(B, B') = f(\delta(\phi_1, \phi'_1), ..., \delta(\phi_n, \phi'_n))$, where $f$ is an aggregation function (e.g. $\sum$, or $max$) and $\phi'_i$ is the image of $\phi_i$. Now based on this extension, we can define the overall degree of conflict of a single belief base i.e. $\delta(B_j) = f\left(\delta(B_j, B_1), ..., \delta(B_j, B_{\sharp(\mathcal{SP})})\right)$. The conflict property of a belief base shows the degree of divergence of a given belief base with regards to the other belief bases in the same belief profile.

The second extrinsic property of a belief base is related to overall behavior of the information source throughout a period of observation called reliability. Under different contexts researchers have employed terms such as trustworthiness [14], reputation [20], confidence [28], and others to refer to the same issue. We formalize the reliability of the source of a belief base as a subjective opinion that represents the degree of belief in the fact that the information source is going to reveal a correct and consistent belief base.

**Definition 19** *Let $\chi_B = (b_B, d_B, u_B)$ be a subjective opinion about the reliability of a given belief base $B$. Reliability is a function $\Re : \mathcal{K} \to [0,1]$ such that:*

$$\Re(B) = \frac{b_B + \tau}{d_B}.$$

*where $\tau$ is a small correction value. It is used to prevent divide by zero when $\Re^{-1}$ is employed.*

*7.1.3. Fitness Measure-based Choice Function* The formalization of the intrinsic and extrinsic properties of the belief bases provide the basis for the characterization of an ordering mechanism based on the fitness measure.

**Definition 20 (Extends Definition 14)** *The fitness measure of a subjective belief base $B$ under integrity constraints $\mu$ is a function $fm : \mathcal{K} \times \mathcal{SE} \to [0, \infty)$ such that:*

$$fm^{\mu}_{\mathcal{SP}}(B) = \left|\overrightarrow{(\zeta(B), \vartheta(B), \delta(B), \Re^{-1}(B))}\right|.$$

**Definition 21** *Let $\mathcal{SP} = \{B_1, ..., B_n\}$ be a subjective belief profile. A fitness measure-based ordering, $\prec_{fm}$, of two SBBs $B_i$, and $B_j \in \mathcal{SP}$ is defined as:*

$$\begin{aligned} B_i \prec_{fm} B_j \quad & \textit{iff } (B_i^* \equiv \top \textit{ and } B_j^* \not\equiv \top) \\ & \textit{or } (B_i^* \not\equiv \top \textit{ and } B_j^* \not\equiv \top \textit{ but } fm^{\mu}_{\mathcal{SP}}(B_i) < fm^{\mu}_{\mathcal{SP}}(B_j)). \end{aligned}$$

THEOREM 22 *Let $B_1 \equiv_s B_1'$, $B_2 \equiv_s B_2'$, $\Re_{B_1} = \Re_{B_1'}$, and $\Re_{B_2} = \Re_{B_2'}$. If $B_1 \prec_{fm} B_2$ then $B_1' \prec_{fm} B_2'$.*

**Proof:** The ordering of subjective belief bases is directly dependent on the fitness measure which is itself reliant on intrinsic and extrinsic properties of the belief bases. So if the properties of two belief bases are equivalent, it can be concluded that $fm^{\mu}_{\mathcal{SP}}(B) = fm^{\mu}_{\mathcal{SP}}(B')$. Based on Definitions 16, and 17, the intrinsic properties of a belief base are focused on the subjective opinions of the formulae and the internal structure of each individual belief base. Since $B_1 \equiv_s B_1'$; therefore, $\zeta(B) = \zeta(B')$ and $\vartheta(B) = \vartheta(B')$. Furthermore, the conflict property considers the subjective opinions of each formula in $B$ and the subjective opinions of its image in $B'$. Based on Definition 8 and its subsequence, $\delta(B) = \delta(B')$. Now, given that $fm^{\mu}_{\mathcal{SP}}(B) = fm^{\mu}_{\mathcal{SP}}(B')$; therefore, if $B_1 \prec_{fm} B_2$ then $B_1' \prec_{fm} B_2'$. ■

Theorem 22 shows that fitness measure-based ordering is syntax-independent.

It would be inferred that $B_j$ is more convergent than $B_i$ within the context of $\mathcal{SP}$ when $B_i \prec_{fm} B_j$. We can now finally define the choice function based on the fitness measure-based ordering.

**Definition 23** *Let $\mathcal{SP} = \{B_1, ..., B_n\}$ be a subjective belief profile. A fitness measure-based choice function under the integrity constraint $\mu$ is defined as follows:*
$B_i \in g^{fm}_{\mu}$ *iff* $\forall B_j \in \mathcal{SP}$ *such that* $B_i^* \wedge B_j^* \wedge \mu \not\equiv \top$, *and* $B_j \prec_{fm} B_i$.

According to this choice function, from the set of mutually inconsistent belief bases the ones that are divergent within the context of $\mathcal{SP}$ are selected. The fitness measure-based choice function $g^{fm}_{\mu}$ should be checked to see wether it satisfies $ec1 - ec3$. Based on Definition 23, $ec1$ is satisfied. $ec2$ is also satisfied from the definition of the choice function and the fitness measure-based ordering. $ec3$ is also satisfied according to the following corollary.

**Corollary 24** *$g^{fm}_{\mu}$ satisfies condition $ec3$ of the extended belief game model choice function, that is, it is syntax-independent.*

The corollary is a straight derivation from Theorem 22. It can be seen that $g^{fm}_{\mu}$ satisfies $ec1 - ec3$ and is therefore a valid choice function.

### 7.2. Enhancement Functions

The subjective opinions attached to each belief base formulae provide a suitable basis for developing attractive enhancement functions. Here, we first introduce an enhancement function that deletes the weakest inconsistent formula, and also provides the possibility for the information source to offer justifications in the form of additional formulae. Second, an enhancement function is developed, which does not require any deletions. It only manipulates the subjective opinions of the belief bases.

**Definition 25** *Let $\mathcal{SP}$ be a subjective belief profile and $(\phi, \chi) \in \cup(\mathcal{SP})$ be a subjective belief formula forming a singleton belief base $C_\phi = \{(\phi, \chi)\}$. $(\phi, \chi)$ is considered to be one of the weakest inconsistent formula of $\mathcal{SP}$ if and only if:*

*(wif1)* $(\phi, \chi)$ *is in conflict in* $\cup(\mathcal{SP})$,

*(wif2)* $\forall(\phi_i, \chi_i) \in \cup(\mathcal{SP})$ *such that* $\phi \wedge \phi_i \wedge \mu \not\equiv \top$ *then* $C_{\phi_i} \prec_{fm} C_\phi$.

**Definition 26** *Let $\mathcal{SP} = \{B_1, .., B_n\}$ be a subjective belief profile, $\mathcal{SP}'$ be a subset of $\mathcal{SP}$, $B \in \mathcal{SP}'$ and $C = \{(\phi, \chi) \in B | (\phi, \chi)$ be one of the weakest inconsistent formula in $\cup(\mathcal{SP})\}$. The weakest inconsistent-based (WI) enhancement function for $B$ is defined as:*

$\blacktriangledown^{WI}_{\mathcal{SP},\mathcal{SP}'} = B \setminus C$ *, if* $C \neq \varnothing$.

For each of the selected belief bases through the choice function, the WI enhancement function deletes those formulae which are among the weakest inconsistent formulae in $\cup\mathcal{SP}$, else the belief base remains intact. We now check that $\blacktriangledown^{WI}$ is a enhancement function. It is clear that *ee*1 is satisfied. *ee*2 is also satisfied since the information source is bound to respect this condition if any justifications are provided. Based on the definition of the weakest inconsistent formula (Definition 25), *ee*3 is satisfied because no weak subjective formula can be found for elimination under such a condition, and therefore, $B = \blacktriangledown^{WI}(B)$.

Let us now define a more flexible enhancement function that increases negotiability between the information sources.

**Definition 27** *Let $\Re_B = (b_{\Re_B}, d_{\Re_B}, u_{\Re_B})$ be the general opinion about the reliability of information source $B$, $\chi^B_p = (b^B_{\chi_p}, d^B_{\chi_p}, u^B_{\chi_p})$ be $B$'s opinion about some proposition $p$. Discounting of $\chi^B_p$ by $\Re_B$, denoted $\chi^B_p \otimes \Re_B$, is defined as follows:*

$$b^{\otimes}_p = b_{\Re_B} \times b^B_{\chi_p}$$
$$d^{\otimes}_p = b_{\Re_B} \times d^B_{\chi_p}$$
$$u^{\otimes}_p = d_{\Re_B} + u_{\Re_B} + b_{\Re_B} u^B_{\chi_p}$$

**Definition 28** *Let $\mathcal{SP} = \{B_1, .., B_n\}$ be a subjective belief profile, $\chi$ represent all possible subjective opinions, $\Re_j$ be the reliability of the information source $j$, and*

*$\mathcal{SP}'$ be a subset of $\mathcal{SP}$. Let $B \in \mathcal{SP}'$, $(\phi_i, \chi_i) \in B$ and $C = \{(\phi', \chi'), ..., (\phi^{(n)}, \chi^{(n)})\}$ be the set of all images of $(\phi_i, \chi_i)$ in other belief bases in $\mathcal{SP}$. The conformance subjective belief for $(\phi_i, \chi_i)$ is a function $\wp : \mathcal{F}_B \rightarrow \chi$ defined as:*

$$\wp^B_{\phi_i} = \oplus_{(\phi_j,\chi_j)\in C} [\otimes(\Re_j, \chi_j)].$$

The conformance subjective belief for $(\phi_i, \chi_i) \in B$, denoted $\wp^B_{\phi_i}$, represents the general consensus about a given formulae. It is produced by the application of the consensus operator on the set of images of $\phi_i$ in peer belief bases discounted by the reliability of the information sources.

**Definition 29** *Let $\mathcal{SP} = \{B_1, .., B_n\}$ be a subjective belief profile, $\mathcal{SP}'$ be a subset of $\mathcal{SP}$, $B \in \mathcal{SP}'$ and $C = \{(\phi, \chi) \in B | (\phi, \chi)$ be one of the weakest inconsistent formula in $\cup(\mathcal{SP})\}$. The conformance belief-based (CB) enhancement function for $B$ is defined as:*

$$\blacktriangledown^{CB}_{\mathcal{SP},\mathcal{SP}'} = \{(\phi_1, \chi'_1), ..., (\phi_n, \chi'_n)\}$$

*where $\chi'_i$ is a half-open interval between the original subjective belief $\chi_i$ and $\wp^B_{\phi_i}$.*

In the CB-based enhancement function, each belief base in $\mathcal{SP}'$ is required to update the subjective belief values of the formulae in its belief base. The effect of the update on the subjective beliefs should remain in a half-open interval recommended by the enhancement function for each individual formula. This way the information sources have more freedom in updating and improving the contents of their belief base. If all belief bases conform to the recommended subjective opinion $\wp^B_{\phi_i}$ consensus is reached and inconsistencies are removed. Furthermore, according to $ee2$, the information sources are able to add justifications as long as new inconsistencies do not arise. Similar to $\blacktriangledown^{WI}$, it is easy to show that $\blacktriangledown^{CB}$ satisfies $ee1 - ee3$, and is therefore, a valid enhancement function.

## 8. Framework Instantiation and Properties

### *8.1. Instantiation*

Different arrangements of the choice functions and the enhancement functions will produce dissimilar extended belief game models, and belief base merging strategies that can produce disparate final outcomes. In the models below, we show and discuss how the different choice and enhancement functions can be integrated and the results that they will each produce.

- $\langle g^{fm,max}, \blacktriangledown^{WI} \rangle$: The choice function uses the maximum aggregation function. In each round of negotiation, those subjective belief bases that have the highest quantities of intrinsic and extrinsic properties assigned to their formulae are selected and the weakest inconsistent formulae are removed from $\cup\mathcal{SP}$.

- $\langle g^{fm,\Sigma},\ \blacktriangledown^{WI}\rangle$: In this setting, those subjective belief bases that have the highest value of intrinsic and extrinsic properties assigned to their formulae are selected by the choice function that benefits from a sum aggregation function. The weakest inconsistent formulae are then removed.
- $\langle g^{fm,max},\ \blacktriangledown^{CB}\rangle$: In this case, the selected belief bases (i.e. those belief bases that consist of formulae that have the highest quantity of intrinsic and extrinsic properties) through the choice function are required to update their formulae belief based on a recommended belief interval.
- $\langle g^{fm,\Sigma},\ \blacktriangledown^{CB}\rangle$: In each round of negotiation, the choice function selects those subjective belief bases that have formulae which possess the highest value of intrinsic and extrinsic properties. Here, the enhancement process is similar to $\langle g^{fm,max},\ \blacktriangledown^{CB}\rangle$.

By comparing these four possible settings, it can be seen that those compositions that benefit from conformance belief-based enhancement function have the lowest *information loss* with regards to each individual belief base. This is because in the enhancement process, conformance belief-based enhancement function does not directly delete any formula, and only requires the information sources to amend their subjective opinions for each formula. Although this is a positive feature in many cases and provides the possibility for formal negotiation between the information sources by belief adjustment, it is not so much desirable in situations where fast consensus achievement and simple computation is sought. For such conditions, the employment of the weakest inconsistent-based enhancement function yields faster results; however, the accuracy of its results is arguable.

### 8.2. *Logical Properties*

The merging operators derived from the extended belief game model possesses several interesting logical properties. Here, we describe these properties and show how they are satisfied by the variants of the extended belief game model.

**Definition 30** *Let $\Delta$ be a subjective merging operator, and $\mathcal{SP}, \mathcal{SP}_1$, and $\mathcal{SP}_2$ be subjective belief profiles, and $\mu$ be integrity constraints. A merging operators derived from the extended belief game model should satisfy the following logical properties:*

*(lp1)* $\Delta_\mu(\mathcal{SP}) \wedge \mu$ *is consistent,*

*(lp2)* *Let* $\mathcal{SP} = \{B_1, ..., B_n\}$*. If* $B_1 \cup ... \cup B_n$ *is consistent, then* $[\Delta_\mu(\mathcal{SP})]^* \equiv [B_1 \cup ... \cup B_n]^*$ *and* $\forall\phi$*, if* $\exists i$ *such that* $(\phi, \chi) \in B_i$ *then there exists* $\chi'$ *such that* $(b_\chi \leq b_{\chi'}$ *or* $d_\chi \leq d_{\chi'})$ *and* $(\phi, \chi') \in \Delta_\mu(\mathcal{SP})$*,*

*(lp3)* *if* $\mathcal{SP}_\infty \equiv_s \mathcal{SP}_\in$ *then* $\Delta_\mu(\mathcal{SP}_\infty) \equiv_s \Delta_\mu(\mathcal{SP}_\in)$*,*

*(lp4)* *Let* $\mathcal{SP} = \{B_1, ..., B_n\}$ *and* $\bigcup \mathcal{SP} = B_1 \cup ... \cup B_n$*. Let* $(\phi, \chi) \in \bigcup(\mathcal{SP})$*. If* $(\phi, \chi)$ *is not in conflict in* $\bigcup \mathcal{SP}$*, then* $(\phi, \chi') \in \Delta_\mu(SP)$ *where* $(b_\chi \leq b_{\chi'}$ *or* $d_\chi \leq d_{\chi'})$*.*

The first property ($lp1$) states that the final subjective belief base should be consistent with regards to the integrity constraints. $lp2$ asserts that when the original information bases are consistent and there is no conflict between them, the merging operator should reinstate all the original information along with a reinforced effect on their subjective opinions. $lp3$ is the principle of irrelevance of syntax. Finally, $lp4$ emphasizes that all non-conflicting information of the knowledge bases should be present after merge.

THEOREM 31 *Let $\oplus$ be the consensus operator, and let $\mathcal{N} = \langle g^{d,f}, \blacktriangledown \rangle$, where $d = fm$, $f = min$ or $f = \sum$, and $\blacktriangledown = \blacktriangledown^{WI}$. Then the operator $\Delta_{\mathcal{N},\oplus}$ satisfies properties $(lp1) - (lp4)$.*

**Proof:** —

($lp1$) The first property is satisfied by definition (see Definition 13), since the outcome of the extended belief game process is $\bigwedge \mathcal{SP}^{\mu}_{\mathcal{N}} \wedge \mu$.

($lp2$) Based on Definition 13, if $\mathcal{SP}$ is consistent, then $\mathcal{SP}_{\mathcal{N}} = \mathcal{SP}$; therefore, $\Delta_{\mathcal{N},\oplus}(\mathcal{SP})$ is equal to the effect of the consensus operator on the subjective opinions of the conjunction of all belief bases. Furthermore, the consensus operator has a reinforcement effect on consistent subjective opinions.Hence, we have if $\exists i$ such that $(\phi, \chi) \in B_i$ then there exists $\chi'$ such that $(b_\chi \leq b_{\chi'}$ *or* $d_\chi \leq d_{\chi'})$ and $(\phi, \chi') \in \Delta_\mu(\mathcal{SP})$, and $lp2$ is satisfied.

($lp3$) Let $\mathcal{SP}_1 \equiv_s \mathcal{SP}_2$ where $\mathcal{SP}_i = \{B^i_1, ..., B^i_n\}$. Suppose $\forall i$ , $B^1_i \equiv B^2_i$. According Corollary 24, the choice function $g^{fm}_\mu$ is syntax-independent, and also $\blacktriangledown^{WI}(B^j_i) \equiv_s \blacktriangledown^{WI}(B^k_i)$. The consensus operator is also clearly syntax-independent. Therefore, $\Delta_{\mathcal{N},\oplus}$ satisfies $lp3$.

($lp4$) Since the choice function selects the weakest inconsistent formulae, and consequently they are removed from the belief bases; hence, the non-conflicting formulae will certainly exist in the final outcome. The reinforcement effect is also satisfied by the consensus operator. It can be seen that $\Delta_{\mathcal{N},\oplus}$ satisfies $lp4$.

■

THEOREM 32 *Let $\oplus$ be the consensus operator, and let $\mathcal{N} = \langle g^{d,f}, \blacktriangledown \rangle$, where $d = fm$, $f = min$ or $f = \sum$, and $\blacktriangledown = \blacktriangledown^{CB}$. Then the operator $\Delta_{\mathcal{N},\oplus}$ satisfies properties $(lp1) - (lp4)$.*

**Proof:** The proof of Theorem 32 is easy to show, similar to Theorem 31. ■

## 9. Framework Application and Examples

### *9.1. Collaborative Modeling*

Collaborative modeling is a process through which multiple experts interact in order to develop a unique model (usually based on a given a meta-modeling formalism) for a given domain of discourse. In most cases in this process, each of the experts focuses on a special aspect of the problem domain and devises a separate model. The individually developed models should be finally integrated to form a unique model of the problem at hand. One of the greatest challenges in this process is the proper integration of the individual models. This is because many of the individual models have overlapping areas of concern. The specifications provided for the overlapping segments of each model may be inconsistent, and therefore, require meticulous methods for addressing their integration.

According to the definitions in [46], the inconsistencies between software models can be formally defined based on the overlap relationships between the interpretations assigned to domain models by the experts.

**Definition 33** *The interpretation of a domain model DM represented through a set of interrelated model elements E is a pair (U,I) where U is the domain of interpretation of the model, and I is a total morphism that maps* $\forall e_i \in E$ *onto a relation* $R \subseteq U^n$ *called the extension of* $e$*.*

An interpretation of a domain model relates each model element onto its corresponding concept in the domain of discourse; therefore, an interpretation conveys how an expert views the developed model with regards to the given domain.

**Definition 34** *Let* $e_i$ *and* $e_j$ *be two model elements of domain models* $DM_i$*, and* $DM_j$*, and let* $T_{i,A} = (U_{i,A}, I_{i,A})$*, and* $T_{i,B} = (U_{i,B}, I_{i,B})$ *be interpretations of* $DM_i$ *and* $DM_j$ *assigned by experts A and B, respectively. The overlap relationships can be defined as follows:*

- *(no overlap)* $I_{i,A}(e_i) \neq \varnothing$, $I_{j,B}(e_j) \neq \varnothing$, *and* $I_{i,A}(e_i) \cap I_{j,B}(e_j) = \varnothing$,
- *(totally overlap)* $I_{i,A}(e_i) \neq \varnothing$, $I_{j,B}(e_j) \neq \varnothing$, *and* $I_{i,A}(e_i) = I_{j,B}(e_j)$,
- *(inclusively overlap)* $I_{i,A}(e_i) \neq \varnothing$, $I_{j,B}(e_j) \neq \varnothing$, *and* $I_{i,A}(e_i) \subseteq I_{j,B}(e_j)$,
- *(partially overlap)* $I_{i,A}(e_i) \neq \varnothing$, $I_{j,B}(e_j) \neq \varnothing$, *and* $I_{i,A}(e_i) \cap I_{j,B}(e_j) \neq \varnothing$, $I_{i,A}(e_i) - I_{j,B}(e_j) \neq \varnothing, I_{j,B}(e_j) - I_{i,A}(e_i) \neq \varnothing$.

Based on these definitions, an inconsistency between domain models can be defined in terms of overlaps.

**Definition 35** *Let* $DM = \{DM_1, ..., DM_n\}$ *be a set of domain models, and* $O = \{(DM_i, DM_j) | DM_i \in DM,\ DM_j \in DM\}$ *be the overlap relationship between them. Furthermore, let CR be a set of consistency rules instructed by the meta-modeling*

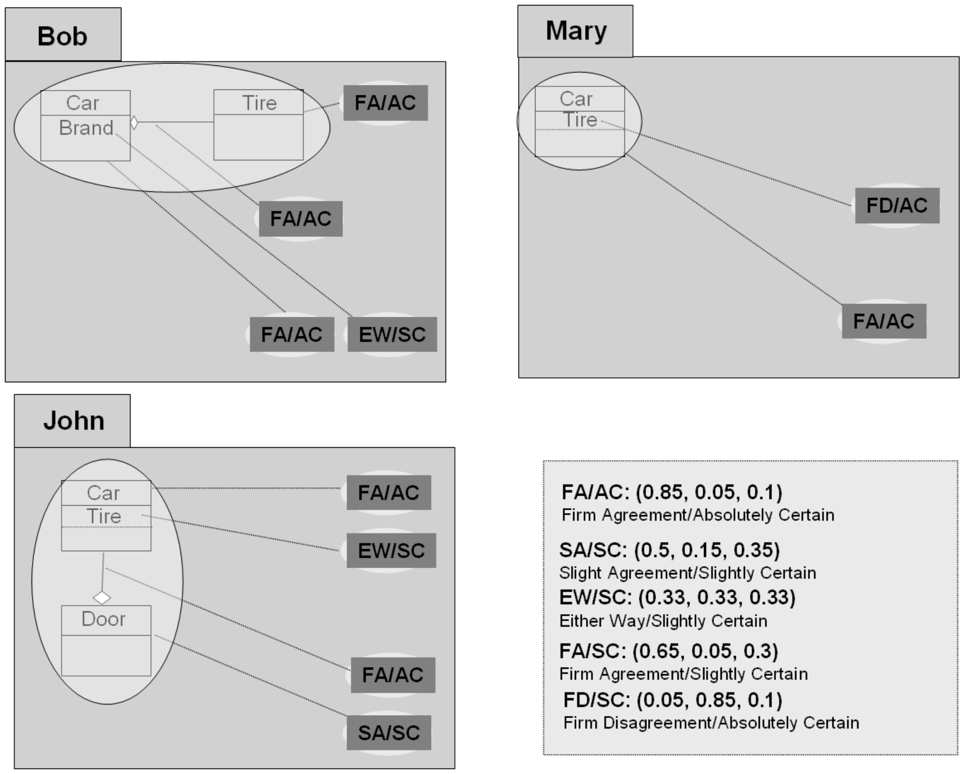


*Figure 2.* Small Portions of Three Models Developed by Different Experts.

*language and domain restrictions. $DM$ is said to be inconsistent if $CR$ is not satisfied under merge strategy $\Delta$.*

It is now possible to show that a collaborative modeling process can be supported by an extended belief game model. The domain models in a collaborative modeling process represent the subjective belief bases, and the experts symbolize the information sources in an extended belief game model.

**Definition 36** *A collaborative modeling process is an extension to the extended belief game model, $\mathcal{M} = \langle g, \blacktriangledown \rangle$, where $g$ is a choice function and $\blacktriangledown$ is an enhancement function. The final unique model developed from a set of domain models $\mathcal{DM}$ developed by various experts under the consistency rules $CR$, denoted $\mathcal{M}_{\mathcal{CR}}(\mathcal{DM})$, is the domain model $\mathcal{DM}_{\mathcal{M}}^{\mathcal{CR}}$ defined as:*

*(cm1)* $\mathcal{DM}_0 = \mathcal{DM}$,

*(cm2)* $\mathcal{DM}_{i+1} = \blacktriangledown_{\mathcal{DM}_i, g(\mathcal{DM}_i)}(\mathcal{DM}_i)$,

*(cm3)* $\mathcal{DM}_{\mathcal{M}}^{\mathcal{CR}}$ *is the first* $\mathcal{DM}_i$ *that is consistent with* $CR$.

### *9.2. Illustrative Example*

In this section, we develop an example to show the behavior of two extended belief game model based merge methods. We show and compare the outcome of merge according to $\langle g^{fm,max},\ \blacktriangledown^{WI}\rangle$ and $\langle g^{fm,max},\ \blacktriangledown^{CB}\rangle$.

**Example 37** *Three experts: Bob, John, and Mary are collaboratively designing the exterior and interior of a Car. Each of them designs a model from his/her own perspective and according to his/her area of expertise. We focus on a small segment of the models highlighted in Figure 2. The models are summarized as subjective propositions in a subjective belief profile* $\mathcal{SP} = \{\mathcal{M}_{Bob},\ \mathcal{M}_{John},\ \mathcal{M}_{Mary}\}$, *where:*

$\mathcal{M}_{Bob} = \{(p, [FA/AC]), (q, [EW/SC]), (r, [FA/AC]), (t, [FA/AC]^c), (s, [FA/AC])\}$,
$\mathcal{M}_{John} = \{(p, [FA/AC]), (t, [FA/AC]), (u, [SA/SC]), (v, [FA/AC])\}$,
$\mathcal{M}_{Mary} = \{(p, [FA/AC]), (t, [FD/AC])\}$,

*p: Car is a class, q: Brand is an attribute of Car, r: Tire is a class, s: A Car can have multiple Tires, t: Tire is an attribute of Car, u: Door is a class, v: A Car can have multiple Doors.*
*Without loss of accuracy, let us assume that all three experts are equally reliable ($\Re_i = (1, 0, 0)$); and hence ignore its effect. In order to merge the developed models, a two step game should be performed. In the first step the weakest model should be selected as the loser of the game, and subsequently, in the second step the loser has to make some concessions through the enhancement function; therefore, we initially calculate the intrinsic and extrinsic properties of all models and compute the fitness measure, in order to find the weakest model through the fitness measure-based choice function. The values of the properties of the models in the first round of negotiation are shown in Table 1.*

*Table 1.* First Round Property Values for Each of the Models

| | $\mathcal{M}_{Bob}$ | $\mathcal{M}_{John}$ | $\mathcal{M}_{Mary}$ |
|---|---|---|---|
| Ambiguity | 0.28 | 0.29 | 0.09 |
| Indecisiveness | 0.35 | 0.35 | 0.1 |
| Conflict | 0.77 | 0.77 | 0.77 |
| $fm$ | 0.88 | 0.89 | 0.78 |

*A fitness measure-based ordering can be inferred from Table 1 that is $\mathcal{M}_{Mary} \prec_{fm} \mathcal{M}_{Bob} \prec_{fm} \mathcal{M}_{John}$. Furthermore, $\mathcal{M}_{John}$ is inconsistent with both $\mathcal{M}_{Mary}$ and $\mathcal{M}_{Bob}$ on the same formulae; therefore, $\mathcal{M}_{John}$ is selected as the loser of this round by the fitness measure-based choice function ($g^{fm,max}$). The behavior of the enhancement functions in this step are different. The WI-enhancement function selects the weakest inconsistent formulae and removes them from the model. The* *weakest inconsistent formula in* $\mathcal{M}_{John}$ *is t which will be removed from the model. This removal will make all three models compatible and they can be merged easily.*

*In the CB-enhancement function, the function recommends suitable opinion values that can make the experts beliefs converge. The upper value for the intervals that are proposed by the CB-enhancement function* ($\wp^{B}_{\phi_i}$) *for all formula are: for* $p = (0.89, 0.05, 0.06)$*, for* $t = (0.05, 0.89, 0.06)$*, for* $u = (0.5, 0.15, 0.35)$*, and for* $v = (0.85, 0.05, 0.1)$ *which clearly have a reinforcement effect. It is up to the John to decide about the final value that it wants to select from within this range (e.g. for t the range is from* $(0.85, 0.05, 0.1)$ *to* $(0.05, 0.89, 0.06)$*). Lets suppose that John selects the proposed values of the CB enhancement function, so inconsistencies are removed and integration can easily be performed, but if he makes other decisions other rounds of negotiation are required.*

$$\begin{aligned}\mathcal{M}_{\oplus \mathcal{WI}} = \{&(p, [(0.91, 0.06, 0.03)]), (q, [(0.33, 0.33, 0.33)]), (r, [(0.85, 0.05, 0.1)]),\\ &(t, [(0.05, 0.89, 0.6)]), (s, [(0.85, 0.05, 0.1)]), (u, [(0.5, 0.15, 0.35)]),\\ &(v, [(0.85, 0.05, 0.1)])\},\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{\oplus \mathcal{CB}} = \{&(p, [(0.92, 0.05, 0.03)]), (q, [(0.33, 0.33, 0.33)]), (r, [(0.85, 0.05, 0.1)]),\\ &(t, [(0.05, 0.92, 0.3)]), (s, [(0.85, 0.05, 0.1)]), (u, [(0.5, 0.15, 0.35)]),\\ &(v, [(0.85, 0.05, 0.1)])\}^1.\end{aligned}$$

*As it can be seen, the WI-enhancement function is faster in making a final decision but has a high information loss; whereas, CB-enhancement function provides more flexibility for negotiation but is obviously slower. The CB-enhancement function is most suitable for cases where a correct belief base is singled out due to the negligence or lack of knowledge of the other information sources. It allows the proper justification of the belief bases standpoints by having more patience in merge.*

## 10. Related Work

We review some of the related literature from two perspective. First, we introduce those work that address belief merging, and second, we provide an overview of some of the work in collaborative modeling that are most relevant to the theme of this paper.

### *10.1. Belief Merging*

The belief game model [27] and the belief negotiation model [13] are two seminal work in the field of belief merging. Although these models are structurally quite similar, they have some fundamental differences. One of the major differences is that BGM focuses on the logical content of the belief bases in making the choice for contraction, while BNM considers each source as a candidate. Therefore, BNM may weaken one of the two identical belief bases, and leave the other intact; whereas in BGM, these two belief bases are always dealt with similarly. This feature of BGM adds more anonymity to the process, since individual characteristics of the belief bases are not important. Furthermore, BNM provides the possibility to consider the history of the rounds of negotiation while making a decision in the choice function. For instance, it can make the condition that each belief base cannot be weakened two times in a row. However in BGM, the choice decisions are more Markovian meaning that each round of negotiation is totaly independent of the previous rounds.

More recently, a class of general belief merge operators called $DA^2$ have been proposed that encode many previous merging operators (both model-based and syntax-based) as its special cases [26]. The general $DA^2$ framework is defined using a distance function $d$ and two aggregation function $\oplus$ and $\odot$. The major shortcoming of the mentioned models is that they function over pure propositional logic belief bases, which makes the definition of choice functions complicated. It is therefore important to consider the role of formulae priorities while merging belief bases.

Some researchers have defined formulae priorities through the employment of the necessity degree of each formula in possibility theory [34]. There are two approaches to possibilistic belief base merging. In the first approach, researchers believe that inconsistency is totally undesirable and should be removed after merging [10, 9]. The second approach claims that inconsistencies are unavoidable and they can exist after merging [1, 7, 11]. Qi et al. [39] propose a prioritized belief negotiation model that merges information sources represented by possibilistic belief bases. They introduce several negotiation and weakening functions that manipulate the necessity degrees of the formulae and employ the conjunctive operator (that can have a reinforcement effect) over the obtained consistent belief bases for combination.

The split-combination (S-C) merging operators [41] are another set of operators that effectively employ possibilistic information to merge belief bases. The basic idea of the S-C approach is that it initially splits the belief profile into two sub-profiles based on a splitting method ($\mathcal{B} = \langle \mathcal{C}, \mathcal{D} \rangle$). Then the belief bases in each belief profile are merged separately resulting in two belief bases C and D. The final outcome of the S-C merging approach is $B_{S-C}$=C∪D. The authors have also proposed two splitting methods, namely: *upper-free-degree based* splitting and the *free-formula based* splitting. In a different approach, Amgoud and Prade [2] propose an argumentation framework for merging conflicting prioritized belief bases. In this model, the preference orderings between the arguments makes it possible to distinguish different types of relations between the arguments. A set of acceptable arguments can be developed from the framework, which can be considered as the required results. Other merging models based on possibilistic theory can be found in the related literature [29, 40, 33, 32, 38].

### *10.2. Collaborative Modeling*

A survey conducted in the United States revealed that seventy percent of the people involved in the modeling and design divisions of their company believe that sequential engineering products are insufficient and that they will need collaborative design software [18]. Despite the growing need for collaborative modeling tools, only recently has this domain gained attention. There are two major models of collaboration, namely: artifact-neutral and model-oriented. The former is focused around providing suitable tools for enhancing communication and cooperation between the participating experts; while the latter is concerned with the improvement of the quality of the final product by focusing on the model bing developed as the subject of collaboration.

In a model-oriented collaboration setting, the experts are cooperating to develop a unique model based on the structure of a given meta-model. The integration of the single models of each expert yields the final unique product. In the integration process any inconsistencies need to be resolved and a fair representative of all the models should be developed. Many models, algorithms and tools have been developed to find the differences, and discrepancies between various kinds of models, and to integrate them into a unique representation. The Epsilon Merging Language is a rule-based language that provides tools for merging models designed using diverse meta-models [17]. It provides its users with the possibility of defining pluggable algorithms called strategies. JDiff is a tool that compares two object-oriented programs and identifies differences and correspondences [3]. For this purpose, JDiff reduces this problem to a hammock matching problem in a graph. UMLDiff is a model specific differencing tool that operates on two UML class models reverse engineered from Java code, and produces a structural change tree depicting the additions, removals, refactorings, and etc [19].

Furthermore, [42] introduces a distributed consistency checking tool that given a set of models and the mappings between them, it first merges the models and then verifies the derived product against consistency constraints. Three types of consistency checks that are allowed are compatibility, multiplicity, and reachability. $\chi$bel is another inconsistency management framework that provides means for reasoning over multiple inconsistent state machine models [16]. The models are annotated with Quasi-Boolean logic values and a model checker called $\chi$chek performs an evaluation over the defined properties. Moreover, AGORA is a goal-oriented discrepancy identification framework [25]. The framework computes two metrics, vertical and diagonal variances, that allow the identification of cross-perspective inconsistency. Finally, the Integration Suite [6, 4] is a collaborative conceptual modeling framework that employs belief theory to capture the uncertainty in the expressions of multiple experts. Uncertainty is employed in this framework to formally guide the participating modelers towards consensus. A survey of other work on collaborative modeling can be found in [48].

## 11. Concluding Remarks

In this paper, we have proposed an extended belief game model which extends Konieczny's belief game model [27]. The proposed model functions over a subjective belief profile which consists of different belief bases annotated with subjective opinions. Our model is different from BGM in several aspects. First, our model considers extrinsic and intrinsic properties of the belief bases in order to make a final decision regarding the belief bases that are selected by the choice function;

therefore, similar to BNM our model does not perform similarly over content-wise identical belief bases. Second, Konieczny argues that social pressure is a permissible influence on the information sources [27]. This may lead to cases where a belief base is singled out and forced to conform with the general opinion although it possess the correct information. To overcome this issue, our extended model consists of an enhancement function that provides more space for negotiation by permitting the addition of justifications in the enhancement function (Definition 12). The flexible definition of the enhancement function provides more *room to maneuver* for those belief bases that have to concede in order for them to justify their stand point. Third, we consider the reliability of the information sources expressing the content of a belief base as one of its extrinsic properties. The reliability value is a static subjective belief about the trustworthiness of the information source. Our previous experiments show that in many cases the reliability value that dynamically changes as a result of the rounds of negotiation is not very much different from the initial reliability value that was ascribed to a belief base [5]; therefore, reliability is a static subjective opinion in our model.

We have also introduced several choice and enhancement functions. The CB enhancement function is demonstrates enduring behavior, while the WI enhancement function shows to be more impatient. This is because the CB function tolerates inconsistencies and requires partially conformance to its proposed recommendations, but the WI function immediately removes the weakest inconsistent formula from the belief base. The functions introduced in this paper are shown to satisfy the *deadlock avoidance* condition. Moreover, the CB function exhibits greater respect for *minimality of change* by tolerating inconsistencies to a great extent. It has also been formally shown that a collaborative modeling process can be supported by an extended belief game model if the inconsistent model elements are appropriately annotated with subjective opinions by the modeling experts.

## Notes

1. Although $t$ exists in the final belief base, the belief base is still consistent because $t$ possess a definite disbelief mass, which is interpreted as $\neg t$.

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