UCS-NT: AN UNBIASED COMPRESSIVE SENSING FRAMEWORK FOR NETWORK TOMOGRAPHY

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ABSTRACT

This paper addresses the problem of recovering sparse link vectors with network topological constraints that is motivated by network inference and tomography applications. We propose a novel framework called UCS-NT in the context of compressive sensing for sparse recovery in networks. In order to efficiently recover sparse specification of link vectors, we construct a feasible measurement matrix using this framework through connected paths. It is theoretically shown that, only $O(k\log(n))$ path measurements are sufficient for uniquely recovering any k-sparse link vector. Moreover, extensive simulations demonstrate that this framework would converge to an accurate solution for a wide class of networks.

***Index Terms*—** Compressive Sensing, Network Monitoring, Sparse Recovery, Network Tomography.

1. INTRODUCTION

The monitoring of link features such as delay and packet loss rate, is an important task in network management. For example, inferring the network utilization and performance specifications are attained via fault diagnosis and congestion detection. However, direct measurement of each link in the network can be operationally difficult and costly (mostly requiring the cooperation of middle network nodes). In some applications, this collaboration is impossible due to protocol or topological constraints. Hence, the topic of inferring network internal characteristics from indirect end-to-end (aggregate) measurements, called *Network Tomography*, becomes more significant [1–7]. Often, it is desired to exploit the status of each individual link with a total number of aggregate measurements much smaller than the number of links in a network. This is conceivable if we have prior knowledge about some properties of links (*i.e.*, sparsity) in the network. For instance, we know that the number of congested links in networks is much smaller than the set of all links.

In this paper, we introduce a general framework called "UCS-NT" (Unbiased Compressive Sensing for Network Tomography) in the context of Compressive Sensing (CS). CS [8–12] is a new research domain in signal processing which tries to recover sparse signals from a smaller number of measurements or incomplete observations. Although, most existing works in CS rely on the assumption that any subset of values can be aggregated together [9], [11], this assumption is not necessarily true in the network monitoring problems where only links that induce a path or connected sub-graph can be aggregated together in the same measurement. There have only been a few recent works considering network topological constraints in order to design a feasible measurement matrix over networks (graphs) using compressive sensing [13–17]. In general, those approaches are either deterministic or random. To the best of our knowledge, there are only two distinct random methods in recent literature. In [16] the gossip algorithm which lacks a clear discussion about the sufficient number of measurements is presented, and in [17], additive measurements by using Random Walks (RW) are adopted. However, In the latter method, node selection in each measurement has a significant linear bias towards high-degree nodes. Therefore, this method may be inapplicable in many complex networks.

In this paper, we provide the first results on unbiased compressive sensing for network inference by extracting the sparse specifications in networks. Our specific contributions in this work are summarized as follows:
(1) We provide a stochastic measurement construction over network. The required number of UCS-NT measurements to recover any k-sparse link vector is $O(k\log(n))$ (section 4.2).
(2) We theoretically prove the null-space property for the constructed measurement matrix from the UCS-NT. This condition guarantees the correctness of matrix (section 4.1).
(3) To the best of our knowledge, this is the first paper that provides an unbiased compressive sensing framework for network inference. We evaluate its performance both theoretically and experimentally (sections 3 and 5).

2. MODEL AND PROBLEM FORMULATION

We consider a network, expressed by an undirected graph $\mathcal{G} = (V(\mathcal{G}), E(\mathcal{G}))$, where $V(\mathcal{G}) = \{\mathrm{v}_1, \mathrm{v}_2, ..., \mathrm{v}_n\}$ denotes the set of nodes (vertices) with cardinality $|V(\mathcal{G})| = n$, and $E(\mathcal{G}) = \{e_1, e_2, ..., e_N\}$ is the set of links (edges) with cardinality $|E(\mathcal{G})| = N$. Nodes communicate only over these links. Suppose every link i has a real value x_i, and vector $x = (x_i, i = 1, 2, ..., N)$ is associated with $E(\mathcal{G})$. x is a k-sparse vector if $\|x\|_0 = k$ where $\|.\|_0$ denotes the number of non-zero elements of x. Suppose that we have m end-to-end measurements over the network ($m \ll N$). We would like to identify certain links (*i.e.*, congested links with large delays) from these measurements. Remember that the delay over each measurement is the sum of delays over each link on the connected path (route).

Let x be an $N \times 1$ non-negative vector whose p-th entry is the delay over link p, and $y \in \mathcal{R}^m$ denotes the vector of m measurements whose q-th entry represents the total delay of a connected path in a network. Let $\mathcal{A}$ be an $m \times N$ measurement matrix with its i-th row corresponds to the i-th measurement. $\mathcal{A}_{ij} = 1$ $(i = 1, ..., m, j = 1, ..., N)$ if and only if the i-th measurement includes link j and zero otherwise. We can write in the compact form $y = \mathcal{A}x$. The measurement matrix $\mathcal{A}$ can identify all k-sparse vectors if and only if it satisfies the null-space property which $\mathcal{A}x_1 \neq \mathcal{A}x_2$ for every two different (at most) k-sparse vectors x_1 and x_2 [15]. It is important that sparse recovery over networks using compressive sensing has a closely related field called graph constrained group testing [18–22]. Note that compressive sensing can perform better than group testing based on the required number of measurements [17]. Hence, we have used CS throughout this paper. In addition, CS may abstractly model complex systems even when the measurements from certain elements are not available. Therefore, our approach can be potentially used in other applications besides network tomography.

3. THE PROPOSED FRAMEWORK: UCS-NT

In this section, we propose an Unbiased Compressive Sensing framework for Network Tomography (UCS-NT) which is an efficient sparse recovery algorithm to recover any k-sparse link vector in a sufficiently connected network. In this approach, we construct a random measurement matrix $\mathcal{A}$ to infer the link parameters (such as delay) inside a network through end-to-end probing between nodes along some random routes. The constructed measurement matrix $\mathcal{A}$ from the UCS-NT holds the following properties, and we will theoretically prove the last two, in the next section. These conditions are: (1) Each measurement is feasible in the sense that the links of the same measurement correspond to a connected path. This condition emphasizes on sparse recovery with network topological constraints. (2) The constructed measurement matrix $\mathcal{A}$ satisfies the null-space property for the uniqueness of the sparse solution to the recovered vector. (3) The generated measurement matrix $\mathcal{A}$ will be able to recover any k-sparse link vector using only $O(k \log(n))$ path measurements. In the proposed method, in order to construct each row of the measurement matrix $\mathcal{A}$, the following steps are iteratively performed:
(i) A start node is selected relative to $P(v)$ as the current node. (ii) The probabilities of current node and its neighbors are calculated. (iii) Then, the next node is selected under various conditions. More details can be seen in Algorithm 1.

Algorithm 1 Proposed Framework: UCS-NT

Input: graph $\mathcal{G}(V, E), m, t$
1: m: number of measurements
2: t: number of measurement steps
3: $P(Cn) = \text{NULL}$ /*Prob. of Current-node*/
4: **for** $i = 1 \rightarrow m$ **do**
5: **Foreach** node $v \in V(\mathcal{G})$ **do**
6: $P(v) = \frac{1}{|V(\mathcal{G})|-1} \times (1 - \frac{deg(v)}{2|E(\mathcal{G})|})$
7: **end for**
8: Cn = Select a node relative to $P(v)$
9: **for** $j = 1 \rightarrow t$ **do**
10: **Foreach** neighbors of Current-node ($\mathbb{N}_{Cn}$) **do**
11: $P(w) = \frac{1}{deg(Cn)} \times min(1, \frac{deg(Cn)}{deg(w)})$
12: **end for**
13: **if** $P(Cn) = \text{NULL}$ **then**
14: $P(Cn) = 1 - \sum_{w \in \mathbb{N}_{Cn}} P_w^{UCSNT}$
15: **end if**
16: **if** $\exists w; P(w) \geq P(v = Cn)$ **then**
17: Find all the w with this property
18: Next-node = Select randomly one of these w
19: **else if** $\forall w \in \mathbb{N}_{Cn}; P(w) < P(v = Cn)$ **then**
20: Next-node = Select w relative to $P(w)$
21: **else**
22: Next-node = Trace back to the previous node
23: **end if**
24: Remove the link between Cn and Next-node
25: Current-node = Next-node
26: **end for**
27: **end for**
Output: feasible measurement matrix $\mathcal{A}$

Here, $P(w)$ is the probability of moving from current node to node w where link $(Cn, w) \in E(\mathcal{G})$; $\forall \{Cn, w\} \in V(\mathcal{G})$. In the proposed method, we can avoid biasedness towards high-degree nodes by selecting a good start node in line (6), and also assigning different probabilities to the neighbors of current node in line (11) of Algorithm 1. Moreover, the basic idea is inspired by the Metropolis-Hasting MCMC technique [23; 24], which is unbiased towards high-degree nodes [25]. As a result, we have a uniform stationary distribution $\pi_v^{UCSNT} = \frac{1}{|V(\mathcal{G})|}$ that leads us to construct an efficient measurement matrix for unbiased compressive sensing. Note that these measurements (walks) through the connected paths, according to the assumptions in section 2, leads to feasibility of the measurement matrix (the first condition).

3.1. Analysing the node degree bias

A basic, but very important property of networks (graphs) is their node degree distribution p_d, *i.e.*, the fraction of nodes with degree equal to d, for all $d \geq 0$ [25]. In this part, we analyse the observed node degree bias for measurements (walks), in particular for Random Walk (RW) used in [17] and the UCS-NT used in this paper, over the graph $\mathcal{G}$.

We have summarized some of the notations in Table 1.

Table 1: Summary of Key Notations.

$deg(v)$	degree of node v
$p_d = \frac{1}{\lvert V \rvert} \sum_{v \in V} 1_{\{deg(v)=d\}}$	degree distribution in graph $\mathcal{G}$
$\langle d \rangle = \langle p_d \rangle = \sum_d d\, p_d$	average node degree in graph $\mathcal{G}$
q_d	expected observed degree distribution
$\langle q_d \rangle = \sum_d d\, q_d$	expected observed average node degree

I) *Random Walk (RW)*: Random walks have been widely studied; see [26] for an excellent survey. RW is also used in [17] to generate a measurement matrix in order to recover any k-sparse link vector. In RW for any given connected and aperiodic graph, the probability of being at a particular node v converges to the stationary distribution $\pi_v^{RW} = \frac{deg(v)}{2|E(\mathcal{G})|}$. Therefore, the expected observed degree distribution q_d^{RW} is:

$$q_d^{RW} = \sum_v \pi_v^{RW} \times 1_{\{deg(v)=d\}} = \frac{d}{2|E|} p_d |V| = \frac{d\, p_d}{\langle d \rangle} \quad (1)$$

Consequently, the expected observed average node degree is:

$$\langle q_d^{RW} \rangle = \sum_d d\, q_d^{RW} = \frac{\sum_d d^2\, p_d}{\langle d \rangle} = \frac{\langle d^2 \rangle}{\langle d \rangle} \quad (2)$$

where $\langle d^2 \rangle$ denotes the average squared node degree in $\mathcal{G}$. According to Eq. (2), we can easily see the biasedness of RW towards higher degree nodes, because $\langle q_d^{RW} \rangle > \langle p_d \rangle$.

II) *UCS-NT (UCSNT)*: As we mentioned at the end of section 3, the transition matrix $P_{v,w}^{UCSNT}$ resulted from Algorithm 1 leads to a uniform stationary distribution $\pi_v^{UCSNT} = \frac{1}{|V(\mathcal{G})|}$, and consequently:

$$q_d^{UCSNT} = \sum_v \pi_v^{UCSNT} \times 1_{\{deg(v)=d\}} = p_d \quad (3)$$

$$\langle q_d^{UCSNT} \rangle = \sum_d d\, q_d^{UCSNT} = \sum_d d\, p_d = \langle d \rangle \quad (4)$$

Based on Eq. (4), UCS-NT estimates the true mean and it is unbiased, because it satisfies $\langle q_d^{UCSNT} \rangle = \langle p_d \rangle$. As it is shown, RW is clearly biased towards high-degree nodes, and it may be inapplicable to be used in measurement matrix construction for inferencing networks with diverse degree distributions, ranging from constant-degree (*e.g.*, in regular graphs), a distribution concentrated around the average value (*e.g.*, in Erdös-Rényi random graphs, or in well-balanced peer-to-peer networks), to heavily right-skewed distributions (as the case in World Wide Web, unstructured P2P, Internet at the IP and Autonomous System level, and Online Social Networks). Because in these networks, the congested links are mostly located on the links pointing to the lower degree nodes. On the contrary, the proposed UCS-NT framework is the proper approach to solve the aforementioned problem. Therefore, we offer the UCS-NT framework for analysing all kinds of complex networks.

4. THEORETICAL ANALYSIS

In this section, we focus on the two other mentioned properties in section 3: The null-space property for the UCS-NT and the required number of measurements for sparse recovery.

4.1. Correctness Condition for UCS-NT

Theorem 1. *Let* $\mathrm{y}_{m\times 1} = \mathcal{A}_{m\times N}\, \mathrm{x}_{N\times 1}$. *Suppose that for every subset of columns (say R) of the measurement matrix $\mathcal{A}$ from the UCS-NT with $|R| \le r$ (i.e. the number of columns is no more than r), the corresponding sub-matrix A_R includes at least one row with a single non-zero element. Then the unique recovery of any link vector* x *with regarding to* $\mathrm{y} = \mathcal{A}\mathrm{x}$ *is conceivable, if* $k < \frac{r+1}{2}$.

Proof. See [27]. □

4.2. minimum sufficient path measurements in UCS-NT

In this part, we indicate how many path measurements are sufficient to recover any k-sparse link vector. First, we theoretically analyse the special class of networks called uniform graphs and later in section 5, we will experimentally investigate some other networks such as Erdös-Rényi (The simplest variety of random graphs), Watts-Strogatz and $\mathcal{G}^{(4)}$ ("Small world" graphs with high clustering and low path lengths) and Complete Graph. The undirected and sufficiently connected graph $\mathcal{G}$ is named a (D, c) uniform graph where c is a constant, and $D < deg(v) < cD$ for all $v \in V$. Suppose that a walk based on UCS-NT framework over the network has a stationary distribution μ. The δ-mixing time of $\mathcal{G}$ is defined as the smallest t' such that a UCS-NT walk of length t' starting at any arbitrary node i ends up having a distribution μ' such that $\|\mu - \mu'\|_\infty \le \delta$ where $\|.\|_\infty$ denotes the supremum norm. For $\delta = \frac{1}{(2cn)^2}$, we define $T(n)$ as the δ-mixing time.

In order to ensure the unique recovery of any k-sparse link vector with minimum number of UCS-NT measurements, from [19], we have the following theorem,

Theorem 2. *Consider a degree* $D_0 = O(c^2 k T^2(n))$, *where* $D \ge D_0$ *and* $t = O(\frac{nD}{c^3 k T(n)})$ *such that t is the length of a walk, then the following holds. Let Φ be a link set with $(k-1)$ links in the graph $\mathcal{G}$, and let ℓ be a link not belonging to the set Φ. Then define* $\pi_{\ell,\Phi} = \Omega(\frac{1}{c^4 k T^2(n)})$, *where $\pi_{\ell,\Phi}$ denotes the probability that a walk passes through link ℓ, but misses all the links from the link set Φ.*

Suppose we construct m independent measurements (walks) via UCS-NT framework satisfying the network topological constraints. Consider an arbitrary link set $\Phi' \subseteq E(\mathcal{G})$ with cardinality $|\Phi'| = k'$ where $1 \le k' \le k$. We define $\pi_{\Phi'}$ as the probability that a UCS-NT walk visits one and only one element from the set Φ'. Consider that $\pi_{\ell,\Phi}$ holds for any link ℓ and any link set Φ, with cardinality no bigger than k. Therefore, as defined in Theorem 2, $\pi_{\Phi'} = \Omega(\frac{1}{c^4 k T^2(n)}) \times k'$, since the events of having a single non-zero element can be divided into k' disjoint events where each of them is the event of having a single non-zero elements in one of the corresponding columns in Φ'. Then the probability that there doesn't exist any UCS-NT measurement (walk) with a single non-zero element in the k' columns corresponding to the links from Φ', can be represented by $\mathrm{P} = (1 - \pi_{\Phi'})^m$.

For the reason that there are $\binom{|E|}{k'}$ combinations of choosing the k' links, the probability of existing one link set Φ' of $|\Phi'| = k'$ without any single non-zero element row is:

$$\mathrm{P}_{k',k} \leq \binom{|E|}{k'}(1 - \pi_{\Phi'})^m \quad (5)$$

$$\leq \binom{n^2}{k'}(1 - \Omega(\frac{k'}{c^4 k T^2(n)}))^m \quad (6)$$

$$\leq e^{k'(1+\log(\frac{n^2}{k'}))+m\log(1-\Omega(\frac{k'}{c^4kT^2(n)}))} \quad (7)$$

We want $\mathrm{P}_{k',k}$ to be smaller than 1, hence, according to Eq. (7), we have $e^{k'(1+\log(\frac{n^2}{k'}))+m\log(1-\Omega(\frac{k'}{c^4kT^2(n)}))} < 1$. Thus as long as $m > \max_{1\leq k'\leq k}\left(-\frac{k'(1+\log(\frac{n^2}{k'}))}{\log(1-\Omega(\frac{k'}{c^4kT^2(n)}))}\right)$, with probability $1 - o(1)$, the measurement matrix $\mathcal{A}$ constructed from UCS-NT framework guarantees recovering up to $\frac{k}{2}$-sparse link vectors according to Theorem 2. In fact, $m = O(c^4T^2(n)k\log(n))$ path measurements suffices to recover any k-sparse link vector, where c is a constant and $T(n)$ is the $\delta = \frac{1}{(2cn)^2}$ mixing time of $\mathcal{G}$. $T(n)$ would be small enough with increasing the number of nodes n. Therefore, minimum measurements needed in the network topological constrained problem is $O(k\log(n))$ via UCS-NT framework in compressive sensing while it is $O(k^2\log(n))$ by group testing [19]. Please refer to [27] for more precision analysis.

5. EXPERIMENTAL RESULTS

In this section, we provide numerical simulation results representing the performance of the proposed framework. We consider five different synthetic graphs: Two random graphs derived from the Erdös-Rényi model with link existence probabilities 0.2 and 0.5 (Erdos0.2, Erdos0.5) containing respectively 100 and 70 nodes; one $\mathcal{G}^{(4)}$ graph (G4) with 500 nodes where each node is connected to its four closest neighbors; one Complete Graph (CG) with 50 nodes, and one realization of Watts-Strogatz model (Watts) with 500 nodes with Number of neighbors $m = 4$, and rewiring probability $p = 0.01$.

In all of the test cases, we compare the UCS-NT with the work in [17] which we call RW in short. This work is state-of-the-art CS-based method for network tomography. The positions of sparsity (congested links) in these methods are assumed fix and the network traffic does not change during the time. Thus, they could not work over real-time communication, and extensions to such networks will be of future work.

Experiment 1 (Recovery error): Fig. 1 shows the recovery percentage of ℓ_1-minimization for different number of measurements. In this experiment, the sparsity of link vector is around 7%. As it is shown, the improvement of our framework is more than 61% for G4, and 12% for Erdos0.2, compared to RW. According to Algorithm 1, each measurement traverses many more links than RW and a greater coverage of network information is achieved with less number of measurements. The improvement percentages for other graphs are around 6% for CG, 4% in Erdos0.5 and 3% for Watts.

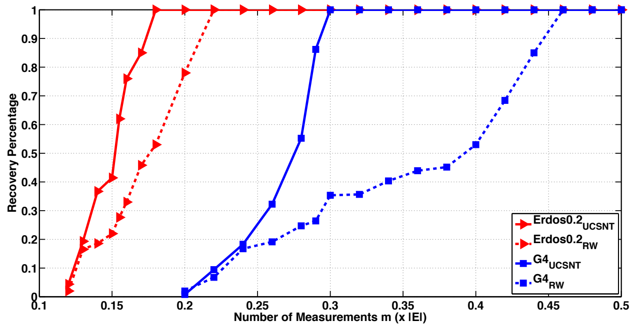

Fig. 1: Recovery Percentage for Erdos0.2 and $\mathcal{G}^{(4)}$

Experiment 2 (Minimum measurements): In Fig. 2, we plot the relationship between the sparsity of link vector and the minimum number of measurements for a recovery percentage greater than 0.9. As it is clear, our work showed a significant improvement in Erdos0.2 (17%) and Watts (15%). These improvements are inspired by increasing the network information coverage with less number of measurements. The improvements in other graphs are around 3% for CG, 4% for Erdos0.5, and 13% for G4.

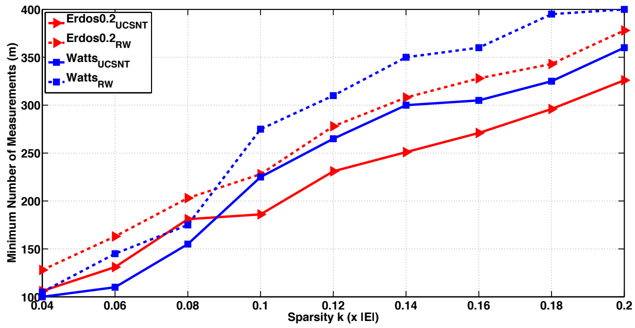

Fig. 2: Minimum Measurements for Erdos0.2 and Watts

6. CONCLUSION

In this paper, we introduced a general framework called UCS-NT in the context of compressive sensing for network tomography. We used this framework to construct a feasible measurement matrix under network topological constraints. By theoretical analysis, we showed that $O(k\log(n))$ UCS-NT measurements are sufficient to uniquely recover any k-sparse link vector. Also, we demonstrated that this framework would be an accurate solution through extensive simulations.

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