GPU accelerated maximum cardinality matching algorithms for bipartite graphs

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Abstract. We design, implement, and evaluate GPU-based algorithms for the maximum cardinality matching problem in bipartite graphs. Such algorithms have a variety of applications in computer science, scientific computing, bioinformatics, and other areas. To the best of our knowledge, ours is the first study which focuses on the GPU implementation of the maximum cardinality matching algorithms. We compare the proposed algorithms with serial and multicore implementations from the literature on a large set of real-life problems where in majority of the cases one of our GPU-accelerated algorithms is demonstrated to be faster than both the sequential and multicore implementations.

Keywords: GPU, maximum cardinality matchings, bipartite graphs, breadth-first search

1 Introduction

Bipartite graph matching is one of the fundamental problems in graph theory and combinatorial optimization. The problem asks for a maximum set of vertex disjoint edges in a given bipartite graph. It has many applications in a variety of fields such as image processing [18], chemical structure analysis [16], and bioinformatics [2] (see also another two discussed by Burkard et al. [4, Section 3.8]). Our motivating application lies in solving sparse linear systems of equations, as algorithms for computing a maximum cardinality bipartite matching are run routinely in the related solvers. In this setting, bipartite matching algorithms are used to see if the associated coefficient matrix is reducible; if so, substantial savings in computational requirements can be achieved [7, Chapter 6].

Achieving good parallel performance on graph algorithms is challenging: they are memory bounded; there are poor localities of the memory accesses; and the dependencies among the computations are irregular. Algorithms for the matching problem are no exception. There have been recent studies that aim to improve the performance of matching algorithms on multicore and manycore architectures. For example, Vasconcelos and Rosenhahn [19] propose a GPU implementation of

an algorithm for the maximum weighted matching problem on bipartite graphs. Fagginger Auer and Bisseling [10] study an implementation of a greedy graph matching on GPU. Halappanavar et al. [13] also study approximate matching on GPU. Çatalyürek et al. [5] propose different greedy graph matching algorithms for multicore architectures. Azad et al. [1] introduce several multicore implementations of maximum cardinality matching algorithms on bipartite graphs.

We propose GPU implementations of two maximum cardinality matching algorithms. We analyze their performance and employ further improvements. We thoroughly evaluate their performance with a rigorous set of experiments on many bipartite graphs from different applications. The experimental results conclude that one of the proposed GPU-based implementation is faster than its existing multicore counterparts.

The rest of this paper is organized as follows. The background material, some related work, and a summary of contributions are presented in Section 2. Section 3 describes the proposed GPU algorithms. The comparison of the proposed GPU-based implementations with the existing sequential and multicore implementations is given in Section 4. Section 5 concludes the paper.

2 Background and contributions

A bipartite graph $G = (V_1 \cup V_2, E)$ consists of a set of vertices $V_1 \cup V_2$ where $V_1 \cap V_2 = \emptyset$, and a set of edges E such that for each edge, one of the endpoints is in V_1 and other is in V_2. Since our motivation lies in the sparse matrix domain, we will refer to the vertices in the two classes as row and column vertices.

A matching $\mathcal{M}$ in a graph G is a subset of edges E where a vertex in $V_1 \cup V_2$ is in at most one edge in $\mathcal{M}$. Given a matching $\mathcal{M}$, a vertex v is said to be matched by $\mathcal{M}$ if v is in an edge of $\mathcal{M}$, otherwise v is called unmatched. The cardinality of a matching $\mathcal{M}$, denoted by $|\mathcal{M}|$, is the number of edges in $\mathcal{M}$. A matching $\mathcal{M}$ is called maximum, if no other matching $\mathcal{M}'$ with $|\mathcal{M}'| > |\mathcal{M}|$ exists. For a matching $\mathcal{M}$, a path $\mathcal{P}$ in G is called an $\mathcal{M}$-alternating if its edges are alternately in $\mathcal{M}$ and not in $\mathcal{M}$. An $\mathcal{M}$-alternating path $\mathcal{P}$ is called M-augmenting if the start and end vertices of $\mathcal{P}$ are both unmatched.

There are three main classes of algorithms for finding the maximum cardinality matchings in bipartite graphs. The first class of algorithms is based on augmenting paths (see a detailed summary by Duff et al. [8]). Push-relabel-based algorithms form a second class [12]. A third class, pseudoflow algorithms, is based on a more recent work [14]. There are $\mathcal{O}(\sqrt{n}\tau)$ algorithms in the first two classes (e.g., Hopcroft-Karp algorithm [15] and a variant of the push relabel algorithm [11]), where n is the number of vertices and τ is the number of edges in the given bipartite graph. This is asymptotically best bound for practical algorithms. Most of the other known algorithms in the first two classes and the ones in the third class have the runtime complexity of $\mathcal{O}(n\tau)$. These three classes of algorithms are described and compared in a recent study [17]. It has been demonstrated experimentally that the champions of the first two families are comparable in performance and better than that of the third family. Since

we investigate GPU acceleration of augmenting-path-based algorithms, a brief description of them is given below (the reader is invited to two recent papers [8, 17] and the original resources cited in those papers for other algorithms).

Augmenting-path-based algorithms follow the same common pattern: given an initial matching $\mathcal{M}$ (possibly empty), they search for an $\mathcal{M}$-augmenting path $\mathcal{P}$. If none exists, then $\mathcal{M}$ is maximum by a theorem of Berge [3]. Otherwise, $\mathcal{P}$ is used to increase the cardinality of $\mathcal{M}$ by setting $\mathcal{M} = \mathcal{M} \oplus E(\mathcal{P})$ where $E(\mathcal{P})$ is the edge set of the path $\mathcal{P}$, and $\mathcal{M} \oplus E(\mathcal{P}) = (\mathcal{M} \cup E(\mathcal{P})) \setminus (\mathcal{M} \cap E(\mathcal{P}))$ is the symmetric difference. This inverts the membership in $\mathcal{M}$ for all edges of $\mathcal{P}$. Since both the first and the last edges of $\mathcal{P}$ were unmatched in $\mathcal{M}$, we have $|\mathcal{M} \oplus E(\mathcal{P})| = |\mathcal{M}| + 1$. The augmenting-path-based algorithms differ in the way the augmenting paths are found and the associated augmentations are realized. They mainly use either breadth-first-search (BFS), or depth-first-search (DFS), or a combination of them to locate and perform the augmenting paths.

Multicore counterparts of a number of augmenting-path based algorithms are proposed in a recent work [1]. The parallelization of these algorithms is achieved by using atomic operations at BFS and/or DFS steps of the algorithm. Although atomic operations might not harm the performance on a multicore machine, they might cause a significant performance degradation on a GPU due to the possible serialization of very large number of concurrent thread executions.

As a reasonably efficient DFS is not feasible with GPUs, we accelerate two BFS-based algorithms, called HK [15] and HKDW [9]. HK has the best known worst-case runtime complexity of $O(\sqrt{n}\tau)$ for a bipartite graph with n vertices and τ edges. HKDW is a variant of HK and incorporates techniques to improve the practical runtime while having the same time complexity. Both of these algorithms use BFS to locate the shortest augmenting paths from unmatched columns, and then use DFS-based searches restricted to a certain part of the input graph to augment along a maximal set of disjoint augmenting paths. HKDW performs another set of DFS-based searches to augment using the remaining unmatched rows. As is clear, the DFS-based searches will be a big obstacle to achieve efficiency. In order to overcome this hurdle, we propose a scheme which alternates the edges of a number of augmenting paths with a parallel scheme that resembles to a breadth expansion in BFS. The proposed scheme offers a high degree of parallelism but does not guarantee a maximal set of augmentations, potentially increasing the worst case time complexity to $O(n\tau)$ on a sequential machine. In other words, we trade theoretical worst case time complexity with a higher degree of parallelism to achieve better practical runtime with a GPU.

3 Methods

We propose two GPU-based algorithms which find the augmenting paths via BFS, speculatively perform some of them, and fix any inconsistencies that can be resulting from speculative augmentations. The proposed algorithms exploit the GPU's vast number of thread parallelisms by assigning each vertex to a thread. Then, the threads concurrently perform independent operations for vertices in

each kernel call, even though actual work is done for a portion of the vertices. Therefore, the GPU algorithm differs from a multi-core algorithm in which a shared data structure is used with atomic operations.

The overall structure of the first GPU-based algorithm is given in Algorithm 1, APsB. It largely follows the common structure of most of the existing sequential algorithms and corresponds to HK. It performs a combined BFS starting from all unmatched columns to find unmatched rows, thus locating augmenting paths. Some of those augmentations are then realized using a function called ALTERNATE (will be described later). The parallelism is exploited inside the INITBFSARRAY, BFS, ALTERNATE, and FIXMATCHING functions. Algorithm 1 is given the adjacency list of the bipartite graph with its number of rows and columns. Any prior matching is given in $rmatch$ and $cmatch$ arrays as follows: $rmatch[r] = c$ and $cmatch[c] = r$, if the row r is matched to the column c; $rmatch[r] = -1$, if r is unmatched; $cmatch[c] = -1$, if c is unmatched.

Algorithm 1: SHORTEST AUGMENTING PATHS (APSB)

```
   Data: cxadj, cadj, nc, nr, rmatch, cmatch
 1 augmenting_path_found ← true;
 2 while augmenting_path_found do
 3 |   bfs_level ← L0;
 4 |   InitBfsArray(bfs_array, cmatch, L0);
 5 |   vertex_inserted ← true;
 6 |   while vertex_inserted do
 7 |   |   predecessor ←Bfs(bfs_level, bfs_array, cxadj, cadj, nc, rmatch,
 8 |   |                    vertex_inserted, augmenting_path_found);
 9 |   |   if augmenting_path_found then
10 |   |   |   break;
11 |   |   bfs_level ← bfs_level + 1;
12 |   ⟨cmatch, rmatch⟩ ← Alternate (cmatch, rmatch, nc, predecessor);
13 |   ⟨cmatch, rmatch⟩ ← FixMatching (cmatch, rmatch);
```

The outer loop of Algorithm 1 iterates until no more augmenting paths are found, thereby guaranteeing a maximum matching. The inner loop is responsible from completing the breadth-first-search of the augmenting paths. A single iteration of this loop corresponds to a level of BFS. The inner loop iterates until all shortest augmenting paths are found. Then, the edges in these shortest augmenting paths are alternated inside ALTERNATE function. Unlike the sequential HK algorithm, APsB does not find a maximal set of augmenting paths.

By removing the lines 9 and 10 of Algorithm 1, another matching algorithm is obtained. This method will continue with the BFSs until all possible unmatched rows are found; it can be therefore considered as the GPU implementation of the HKDW algorithm. This variant is called APFB.

We propose two implementations of the BFS kernel. Algorithm 2 is the first one. The BFS kernel is responsible from a single level BFS expansion. That is, it

Algorithm 2: BFS Kernel Function-1 (GPUBFS)

```
   Data: bfs_level, bfs_array, cxadj, cadj, nc, rmatch,
         vertex_inserted, augmenting_path_found
 1 process_cnt ← getProcessCount(nc);
 2 for i from 0 to process_cnt − 1 do
 3     col_vertex ← i × tot_thread_num + tid;
 4     if bfs_array[col_vertex] = bfs_level then
 5         for j from cxadj[col_vertex] to cxadj[col_vertex + 1] do
 6             neighbor_row ← cadj[j];
 7             col_match ← rmatch[neighbor_row];
 8             if col_match > −1 then
 9                 if bfs_array[col_match] = L0 −1 then
10                     vertex_inserted ← true;
11                     bfs_array[col_match] ← bfs_level + 1;
12                     predeccesor[neighbor_row] ← col_vertex;
13             else
14                 if col_match=−1 then
15                     rmatch[neighbor_row] ← −2;
16                     predeccesor[neighbor_row] ← col_vertex;
17                     augmenting_path_found ← true;
```

takes the set of vertices at a BFS level and adds the union of the unvisited neighbors of those vertices as the next level of vertices. Initially, the input bfs_array is filled with $bfs\_array[c] = L_0 - 1$ if $cmatch[c] > -1$ and $bfs\_array[c] = L_0$ if $cmatch[c] = -1$ by a simple InitBfsArray kernel (L_0 denotes BFS start level).

The GPU threads partition the column vertices in a single dimension. Each thread with id tid is assigned a number of columns which is obtained via the following function:

$$getProcessCount(nc) = \begin{cases} \lceil \frac{nc}{tot\_thread\_num} \rceil & \text{if } tid < nc \bmod tot\_thread\_num, \\ \lfloor \frac{nc}{tot\_thread\_num} \rfloor & \text{otherwise.} \end{cases}$$

Once the number of columns are obtained, the threads traverse their first assigned column vertex. The indices of the columns assigned to a thread differ by tot_thread_num to allow coalesced global memory accesses. Threads traverse the neighboring row vertices of the current column, if its BFS level is equal to the current bfs_level. If a thread encounters a matched row during the traversal, its matching column is retrieved. If the column is not traversed yet, its bfs_level is marked on bfs_array. On the other hand, when a thread encounters an unmatched row, an augmenting path is found. In this case, the match of the neighbor row is set to -2, and this information is used by Alternate later.

Algorithm 3 gives the description of the Alternate function. This kernel alternates the matched edges with the unmatched edges of the augmenting paths found; some of those paths end up being augmenting ones and some are only partially alternated. Here, each thread is assigned a number of rows. Since $rmatch$ of an unmatched row (that is also an endpoint of an augmenting path) has been

Algorithm 3: ALTERNATE

```
   Data: cmatch, rmatch, nc, nr, predecessor
 1 process_vcnt ← getProcessCount(nr);
 2 for i from 0 to process_vcnt − 1 do
 3     row_vertex ← i × tot_thread_num + tid;
 4     if rmatch[row_vertex] = −2 then
 5         while row_vertex ≠ −1 do
 6             matched_col ← predecessor[row_vertex];
 7             matched_row ← cmatch[matched_col] ;
 8             if predecessor[matched_row] = matched_col then
 9                 break;
10             cmatch[matched_col] ← row_vertex;
11             rmatch[row_vertex] ← matched_col;
12             row_vertex ← matched_row;
```

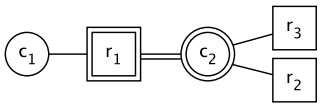


Fig. 1. Vertices r_1 and c_2 are matched; others are not. Two augmenting paths starting from c_1 are possible.

set to -2 in the BFS kernel, only the threads whose row vertices' matches are -2 start ALTERNATE. Since there might be several augmenting paths for an unmatched column, race conditions while writing on *cmatch* and *rmatch* arrays are possible. Such a race condition might cause infinite loops (inner while loop) or inconsistencies, if care is not taken. We prevent these by checking the predecessor of a matched row (line-8). For example, in Fig. 1, two different augmenting paths that end with r_2 and r_3 are found for c_1. If the thread of r_2 starts before the thread of r_3 in ALTERNATE, the match of c_2 will be updated to r_2 (line-10). Then, r_3's thread will read *matched_row* of c_2 as r_2 (line-7). This would cause an infinite loop without the check at line-8. Inconsistencies may occur when the threads of r_2 and r_3 are in the same warp. In this case, the if-check will not hold for both threads, and their row vertices will be written on *cmatch* (line-10). Since only one thread will be successful at writing, this will cause an inconsistency. Such inconsistencies are fixed by FIXMATCHING kernel which implements: $rmatch[r] \leftarrow -1$ for any r satisfying $cmatch[rmatch[r]] \neq r$.

Algorithm 4 gives the description of a slightly different BFS kernel function. This function takes a *root* array as an extra argument. Initially, the root array is filled with $root[c] = 0$ if $cmatch[c] > -1$, and $root[c] = c$ if $cmatch[c] = -1$. This array holds the root (as the index of the column vertex) of an augmenting path, and this information is transferred down during BFS. Whenever an augmenting path is found, the entry in bfs_array for the root of the augmenting path is set to $L_0 - 2$. This information is used at the beginning of the BFS kernel. No more BFS

traversals is done if an augmenting path is found for the root of the traversed column vertex. Therefore, while the method increases the global memory accesses by introducing an extra array, it provides an early exit mechanism for BFS.

Algorithm 4: BFS Kernel Function-2 (GPUBFS-WR)

```
   Data: bfs_level, bfs_array, cxadj, cadj, nc, rmatch, root
         vertex_inserted, augmenting_path_found
 1 process_cnt ← getProcessCount(nc);
 2 for i from 0 to process_cnt − 1 do
 3     col_vertex ← i × tot_thread_num + tid;
 4     if bfs_array[col_vertex] = bfs_level then
 5         myRoot ← root[col_vertex];
 6         if bfs_array[myRoot] < L0 − 1 then
 7             continue;
 8         for j from cxadj[col_vertex] to cxadj[col_vertex + 1] do
 9             neighbor_row ← cadj[j];
10             col_match ← rmatch[neighbor_row];
11             if col_match > −1 then
12                 if bfs_array[col_match] = L0 −1 then
13                     vertex_inserted ← true;
14                     bfs_array[col_match] ← bfs_level + 1;
15                     root[col_match] ← myRoot;
16                     predeccesor[neighbor_row] ← col_vertex;
17             else
18                 if col_match=−1 then
19                     bfs_array[myRoot] ← L0 − 2;
20                     rmatch[neighbor_row] ← −2;
21                     predeccesor[neighbor_row] ← col_vertex;
22                     augmenting_path_found ← true;
```

We further improve GPUBFS-WR by making use of the arrays *root* and bfs_array. BFS kernels might find several rows to match with the same unmatched column, and set $rmatch[\cdot]$ to -2 for each. These cause Alternate to start from several rows that can be matched with the same unmatched column. Therefore, it may perform unnecessary alternations, until these augmenting paths intersect. Conflicts may occur at these intersection points (which are then resolved with FixMatching function). By choosing L_0 as 2, we can limit the range of the values that bfs_array takes to positive numbers. Therefore, by setting the bfs_array to $-(neighbor\_row)$ at line 19 of Algorithm 4, we can provide more information to the Alternate function. With this, Alternate can determine the beginning and the end of an augmenting path, and it can alternate only among the correct augmenting paths. APsB-GPUBFS-WR (and Alternate function used together) includes these improvements. However, they are not included in APFB-GPUBFS-WR since they do not improve its performance.

4 Experiments

The proposed implementations are compared against the sequential HK and PFP implementations [8], and against the multicore implementations P-PFP, P-DBFS, and P-HK obtained from [1]. The CPU implementations are tested on a computer with 2.27GHz dual quad-core Intel Xeon CPUs with 2-way hyper-threading and 48GB main memory. The algorithms are implemented in C++ and OpenMP. The GPU implementations are tested on NVIDIA Tesla C2050 with usable 2.6GB of global memory. C2050 is equipped with 14 multiprocessors each containing 32 CUDA cores, totaling 448 CUDA cores. The implementations are compiled with gcc-4.4.4, cuda-4.2.9 and -O2 optimization flag. For the multicore algorithms, 8 threads are used. A standard heuristic (called the cheap matching, see [8]) is used to initialize all algorithms. We compare the runtime of the matching algorithms after this common initialization. The execution times of the GPU algorithms exclude memory copy time. But including memory copy time decreases the reported mean speedups across all data set by at most 6%.

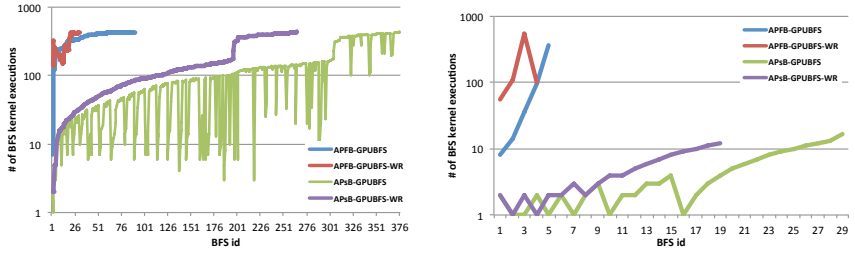
The two main algorithms APFB and APsB can use different BFS kernel functions (GPUBFS and GPUBFS-WR). Moreover, these algorithms can have two versions (i) CT: uses a constant number of threads with fixed number of grid and block size (256×256) and assigns multiple vertices to each thread; (ii) MT: tries to assign one vertex to each thread. The number of threads used in the second version is chosen as MT = $\min(nc, \#threads)$ where nc is the number of columns, and $\#threads$ is the maximum number of threads of the architecture. Therefore, we have eight GPU-based algorithms.

We used 70 different matrices from variety of classes at UFL matrix collection [6]. We also permuted the matrices randomly by rows and columns and included them as a second set (labeled RCP). These permutations usually render the problems harder for the augmenting-path-based algorithms [8]. For both sets, we report the performance for a smaller subset which contains those matrices in which at least one of the sequential algorithms took more than one second. We call these sets O_S1 (28 matrices) and RCP_S1 (50 matrices). We also have another two subsets called O_Hardest20 and RCP_Hardest20 that contain the set of 20 matrices on which the sequential algorithms required the longest runtime.

Table 1. Geometric mean of the runtime (in seconds) of the GPU algorithms on different sets of instances.

	APFB				APsB			
	GPUBFS		GPUBFS-WR		GPUBFS		GPUBFS-WR	
	MT	CT	MT	CT	MT	CT	MT	CT
O_S1	2.96	1.89	2.12	**1.34**	3.68	2.88	2.98	2.27
O_Hardest20	4.28	2.70	3.21	**1.93**	5.23	4.14	4.20	3.13
RCP_S1	3.66	3.24	1.13	**1.05**	3.52	3.33	2.22	2.14
RCP_Hardest20	7.27	5.79	3.37	**2.85**	12.06	10.75	8.17	7.41

Table 1 compares the proposed GPU implementations on different sets. As we see from the table, using a constant number of threads (CT) always increases

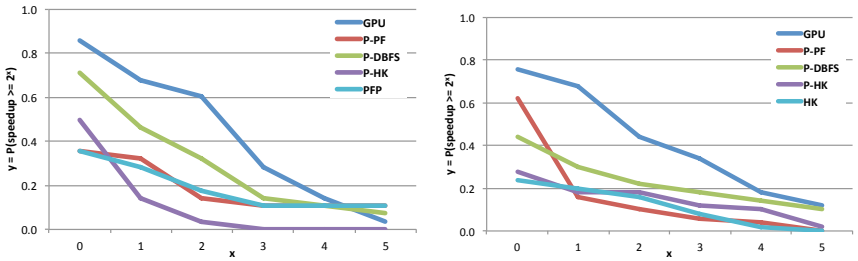


(a) Hamrle3 (b) Delanuay_n23

Fig. 2. The BFS ids and the number of kernel executions for each BFS in APsB and APFB variants for two graphs. The x axis shows the id of the **while** iteration at line 2 of APsB. The y axis shows the number of the **while** iterations at line 6 of APsB.

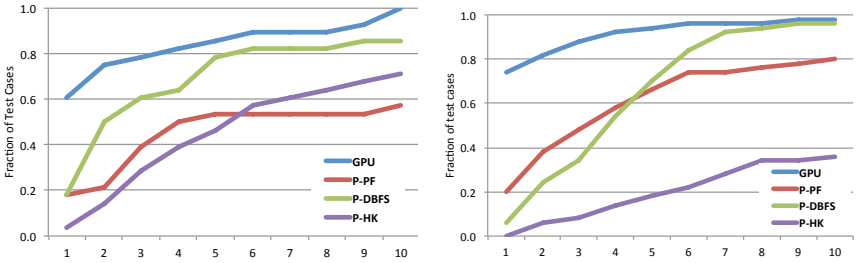
the performance of an algorithm, since it increases the granularity of the work performed by each thread. GPUBFS-WR is always faster than GPUBFS. This is due to the unnecessary BFSs in the GPUBFS algorithm. GPUBFS cannot determine whether an augmenting path has already been found for an unmatched column, therefore it will continue to explore. This unnecessary BFSs not only increase the time, but also reduce the likelihood of finding an augmenting path for other unmatched columns. Moreover, the ALTERNATE scheme turns out to be more suitable for APFB than APsB, in which case it can augment along more paths (there is a larger set of possibilities). For example, Figs. 2(a) and 2(b) show the number of BFS iterations and the number of BFS levels in each iteration for, respectively, Hamrle3 and Delanuay_n23. As clearly seen from the figures, APFB variants converge in smaller number of iterations than APsB variants; and for most of the graphs, the total number of BFS kernel calls are less for APFB (as in Fig. 2(a)). However, for a small set of graphs, although the augmenting path exploration of APsB converges in larger number of iterations, the numbers of the BFS levels in the iterations are much less than APFB (as in Fig. 2(b)). Unlike the general case, APsB outperforms APFB in such cases. Since APFB using GPUBFS-WR and CT is almost always the best algorithm, we only compare the performance of this algorithm with other implementations in the following.

Figures 3(a) and 3(b) give the log-scaled speedup profiles of the best GPU and multicore algorithms on the original and permuted graphs. The speedups are calculated with respect to the fastest of the sequential algorithms PFP and HK (on the original graphs HK was faster; on the permuted ones PFP was faster). A point (x, y) in the plots corresponds to the probability of obtaining at least 2^x speedup is y. As the plots show, the GPU algorithm has the best overall speedup. It is faster than the sequential HK algorithm for 86% of the original graphs, while it is faster than PFP on 76% of the permuted graphs. P-DBFS obtains the best performance among the multicore algorithms. However, its performance degrades on permuted graphs. Although P-PFP is more robust than P-DBFS to permutations, its overall performance is inferior to that of P-DBFS. P-HK is outperformed by the other algorithms in both sets.



(a) Original graphs (b) Permuted graphs

Fig. 3. Log-scaled speedup profiles.



(a) Original graphs (b) Permuted graphs

Fig. 4. Performance profiles.

Figures 4(a) and 4(b) show the performance profiles of the GPU and multicore algorithms. A point (x, y) in the plots means that with y probability, the corresponding algorithm obtains a performance that is at most x times worse than the best runtime. The plots clearly mark the GPU algorithm as the fastest in most cases, especially for the original graphs and for $x \leq 7$ for the permuted ones. In particular, the GPU algorithm obtains the best performance in 61% of the original graphs, while this ratio increases to 74% for the permuted ones.

Figure 5 gives the overall speedups. The proposed GPU algorithm obtains average speedup values of at least 3.61 and 3.54 on, respectively, original and permuted graphs. The speedups increase for the hardest instances, where the GPU algorithm achieves 3.96 and 9.29 speedup, respectively, on original and permuted graphs. Moreover, the runtimes obtained by the GPU algorithm are robust among the repeated executions on a graph instance. We calculated the standard deviation of the execution times for different repetitions. For the graphs in O_S1 set, the ratios of the standard deviations to the average time are observed to be less than 10%, 18%, and 47% for 20, 5, and 3 graphs, respectively.

Table 2 gives the actual runtime for O_Hardest20 set for the best GPU and multicore algorithms, together with the sequential algorithms. The compete set of runtimes can be found at `http://bmi.osu.edu/hpc/software/matchmaker2/maxCardMatch.html`. As table shows, except six instances among

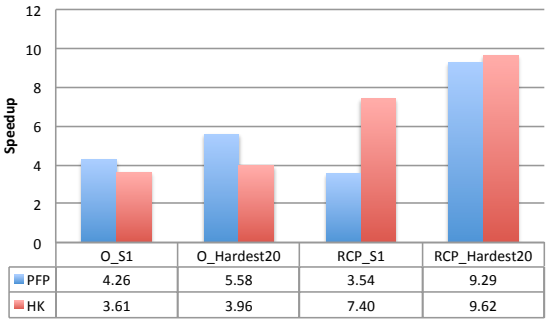


Fig. 5. Overall speedup of the proposed GPU algorithm w.r.t. PFP (left bars) and HK (right bars) algorithms.

the original graphs and another two among the permuted graphs, the GPU algorithm is faster than the best sequential algorithm. It is also faster than the multicore ones in all, except five original graphs.

Table 2. The actual runtime of each algorithm for the O_Hardest20 set.

	Original graphs				Permuted graphs			
Matrix name	GPU	P-DBFS	PFP	HK	GPU	P-DBFS	PFP	HK
roadNet-CA	**0.34**	0.53	0.95	2.48	**0.39**	1.88	3.05	4.89
delaunay_n23	**0.96**	1.26	2.68	1.11	**0.90**	5.56	3.27	14.34
coPapersDBLP	**0.42**	6.27	3.11	1.62	0.38	1.25	**0.29**	1.26
kron_g500-logn21	**0.99**	1.50	5.37	4.73	**3.71**	4.01	64.29	16.08
amazon-2008	**0.11**	0.18	6.11	1.85	**0.41**	1.37	61.32	4.69
delaunay_n24	**1.98**	2.41	6.43	2.22	**1.86**	12.84	6.92	35.24
as-Skitter	**0.49**	1.89	7.79	3.56	**3.27**	5.74	472.63	29.63
amazon0505	**0.18**	22.70	9.05	1.87	**0.24**	15.23	17.59	2.23
wikipedia-20070206	**1.09**	5.24	11.98	6.52	**1.05**	5.99	9.74	5.73
Hamrle3	1.36	2.70	**0.04**	12.61	**3.85**	7.39	37.71	57.00
hugetrace-00020	**7.90**	393.13	15.95	15.02	**1.52**	9.97	8.68	38.27
hugebubbles-00000	13.16	**3.55**	19.81	5.56	**1.80**	10.91	10.03	38.97
wb-edu	33.82	8.61	**3.38**	20.35	17.43	20.10	**9.49**	51.14
rgg_n_2_24_s0	3.68	2.25	25.40	**0.12**	**2.20**	12.50	5.72	31.78
patents	0.88	**0.84**	92.03	16.18	**0.91**	0.97	101.76	18.30
italy_osm	5.86	1.20	**1.02**	122.00	**0.70**	3.97	6.24	18.34
soc-LiveJournal1	**3.32**	14.35	243.91	21.16	**3.73**	7.14	343.94	20.71
ljournal-2008	**2.37**	10.30	360.31	17.66	**6.90**	7.58	176.69	23.45
europe_osm	57.53	**11.21**	14.15	1911.56	**7.21**	37.93	68.18	197.03
com-livejournal	**4.58**	22.46	2879.36	34.28	**5.88**	17.19	165.32	29.40

5 Concluding remarks

We proposed a parallel GPU implementation of a BFS-based maximum cardinality matching algorithm for bipartite graphs. We compared the performance of the proposed implementation against sequential and multicore algorithms on various datasets. The experiments showed that the GPU implementation is faster than

the existing multicore implementations. The speedups achieved with respect to well-known sequential implementations varied from 0.03 to 629.19, averaging 9.29 w.r.t. the fastest sequential algorithm on a set of 20 hardest problems. A GPU is a restricted memory device. Although, an out-of-core or distributed-memory type algorithm is amenable when the graph does not fit into the device, a direct implementation of these algorithms will surely not be efficient. We plan to investigate matching algorithms for extreme-scale bipartite graphs on GPUs.

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