

Actuator and sensor selection for an active vehicle suspension aimed at robust performance

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A recently presented method for actuator and sensor selection for linear control systems is applied and evaluated for an active vehicle suspension control problem. The aim is to eliminate the actuator/sensor combinations for which no controller exists that achieves a specified level of robust performance. Complete controller synthesis is avoided by using necessary conditions for robust performance. Due to this, it cannot be guaranteed that a stabilizing and robustly performing controller exists for the combinations satisfying the conditions. This is a major shortcoming of the method. Depending on the application and implementation, the computation time consumed by the selection procedure can be large.

1. Introduction

Prior to controller synthesis, an appropriate number, place, and type of actuators and sensors should be decided on. This process will be called Input-Output (IO) selection. The system outputs are composed of measured variables y and controlled variables; the latter are defined prior to IO selection and are not necessarily equivalent to y . In the context of IO selection, 'outputs' is used to refer to the measurements only. By analogy, 'inputs' refers to the control actions u only and not to exogenous variables, such as disturbances, measurement noise, and reference signals. Compared to other steps in control system design, such as modelling, identification, and controller construction, IO selection has received relatively little attention. Nevertheless, it is of crucial importance. In the first place, the employed combination of actuators and sensors (the IO set) may place fundamental limitations on the achievable performance (Boyd and Barratt 1991). For instance, a particular IO set may introduce zeros in the right half-plane, which impose restrictions on the achievable bandwidth, regardless of the controller type that is used (Freudenberg and Looze 1985). Secondly, the IO set affects hardware costs, maintenance effort, reliability, and the complexity of a control system (Nett 1989).

If there are only a few possible IO sets, or if the desired IO set is clear from the requirements, there is no direct need for specialized IO selection methods. However, if there are a large number of candidate IO sets, a systematic IO selection method could be useful to ensure that favourable candidates are not overlooked. Of course, such a method should complement (not replace) the designer's experience and physical insight in the system. The number of candidate IO sets grows extremely rapidly

Received 24 February 1997. Revised 2 October 1997.

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with the number of candidate inputs and outputs. Suppose there are N_u candidate inputs and N_y candidate outputs, making up the ‘full IO set’. During IO selection, a subset of n_u inputs and n_y outputs (an $n_u \times n_y$ IO set’) must be selected from the full IO set. The number of distinct $n_u \times n_y$ sets is given by

$$\binom{N_u}{n_u} \binom{N_y}{n_y}, \quad \text{with} \quad \binom{I}{i} := \frac{I!}{i!(I-i)!}$$

The total number of distinct subsystems is $(2^{N_u} - 1)(2^{N_y} - 1)$. For $N_u = N_y = 10$, this yields 1 046 529 distinct candidate IO sets. If the quality of a single IO set could be assessed in 30 s, it would still take one year to assess all candidates. Hence, an IO selection method should be *efficient*, in the sense that a large number of candidate IO sets can be evaluated quickly and easily. It should also be *effective*, i.e. it should be able to eliminate ‘nonviable’ candidates and to keep ‘viable’ ones. Also, viability should be addressed rigorously (see the discussion below). Though controller design and closed-loop evaluation for each IO set might be the most effective approach, it is only feasible for a small number of alternatives.

Various IO selection methods are mentioned by Cao (1995) and Van de Wal and De Jager (1995). Three limitations are commonly encountered. First, IO selection is often restricted to systems with an equal number of inputs and outputs. Second, the controlled and measured variables are not always treated separately; instead, it is frequently assumed that controlled variables can either be measured directly, or can be suitably represented by measured variables. Third, if employed at all, quantitative performance specifications and uncertainty characterizations are usually restricted to a particular frequency or frequency range. To remedy these shortcomings, the standard formulation for robust control problems can be employed (see, e.g. Zhou *et al.* 1996). In this respect, the goal for IO selection in this paper is *to find all IO sets for which a desired level of Robust Performance (RP) can be achieved*. Thus, with these IO sets it must be possible to construct a stabilizing controller that meets the performance specifications in the face of a class of uncertainties. Such IO sets will be termed *viable*. It should be emphasized that the discussion in the sequel is restricted to linear control systems.

Only a few IO selection methods accounting for robustness can be found in the literature. Those by Braatz *et al.* (1996) (RP) and Rivera (1989) (robust stability) are only applicable for an equal number of inputs and outputs, the performance specifications are imposed on the measurements (via the notion of sensitivity in the conventional feedback configuration (see, e.g. Zhou *et al.* 1996, Chapter 5), and application is restricted to the steady-state (integral control is required). The IO selection method by Banerjee and Arkun (1995) provides a necessary condition for combined nominal performance and robust stability against unstructured uncertainties. It exhibits the first two limitations, but it partially resolves the third one by considering the frequency region below cross-over. The method by Van de Wal and De Jager (1996) resolves all three shortcomings, but is only useful for nominal performance, or robust stability against separate (unstructured) uncertainties. Since these two properties are necessary for RP, the method may be useful for screening candidate IO sets prior to RP considerations. Essentially, the IO selection method checks a necessary and sufficient condition for the existence of a controller meeting an \mathcal{H}_∞ norm specification for the closed-loop. Finally, Lee *et al.* (1995) proposed an IO selection method that aims at RP and resolves the three limitations

mentioned above. The approach is based on necessary conditions for the existence of a controller achieving a desired RP level. The necessity is a result of dropping the stabilizing property of the controller. Consequently, IO sets may be accepted for which a stabilizing controller achieving RP cannot be constructed, i.e. nonviable IO sets may be accepted.

The main contribution of this paper is to investigate the practical usefulness of the method proposed by Lee *et al.* (1995). For this evaluation, an active suspension control problem for a tractor–semitrailer combination is used, providing a set-up for RP controller design for active suspensions. Contrary to the distillation column example in Lee *et al.* (1995), dynamic effects (instead of steady-state effects only) and joint input and output selection (instead of output selection only) are considered in the IO selection. The effectiveness of the IO selection method is assessed and the efficiency is compared with that of μ -synthesis.

The paper is organized as follows. Section 2 discusses the RP concept. Section 3 summarizes the IO selection conditions. After the control problem has been (quantitatively) formulated in section 4, the IO selection method is applied and evaluated in section 5. Finally, conclusions are drawn and some suggestions for future research are given in section 6.

2. Robust performance

The development of the IO selection method in Lee *et al.* (1995) is based on the structured singular value (abbreviated to ‘ μ ’), the relevant issues of which are mentioned in section 2.1. The use of μ to address RP in control system analysis is briefly explained in section 2.2.

2.1. The structured singular value

The structured singular value is defined as a matrix function operating on a complex matrix $M \in \mathbb{C}^{g \times f}$. Its value does not only depend on M , but also on an underlying structure Δ of block-diagonal matrices, defined as follows:

$$\Delta = \left\{ \text{diag}(\delta_1 I_{r_1}, \dots, \delta_k I_{r_k}, \Delta_1, \dots, \Delta_l) : \delta_i \in \mathbb{C}; \Delta_i \in \mathbb{C}^{s_i \times t_i} \right\}$$

$$\sum_{i=1}^k r_i + \sum_{i=1}^l s_i = f; \sum_{i=1}^k r_i + \sum_{i=1}^l t_i = g \tag{1}$$

with I_{r_i} the $r_i \times r_i$ identity matrix. Δ is composed of repeated scalar blocks $\delta_i I_{r_i}$, and full blocks Δ_i . Only complex blocks in Δ are considered, since the IO selection method is not able to deal with real uncertainties. Contrary to the treatment by Lee *et al.* (1995), the Δ_i blocks are not necessarily square. Now, μ is defined as (Packard and Doyle 1993):

$$\mu_{\Delta}(M) := \frac{1}{\min_{\Delta \in \Delta} (\bar{\sigma}(\Delta) : \det(I - M\Delta) = 0)} \tag{2}$$

and $\mu_{\Delta}(M) := 0$ if no $\Delta \in \Delta$ makes $I - M\Delta$ singular. Because μ cannot be computed exactly in an efficient way (Toker and Özbay 1995), upper and lower bounds are used. The latter will not be given attention, since the development of the IO selection theory is based on the upper bound $\bar{\mu}$:

$$\mu_{\Delta}(M) \leq \inf_{D \in \mathcal{D}} (D_z M D_w^{-1}) := \bar{\mu}_{\Delta}(M) \tag{3}$$

Here, $D \in \mathbf{D}$ is shorthand for $D_z \in \mathbf{D}_z, D_w \in \mathbf{D}_w$ and D_z and D_w are the so-called D -scales from the sets:

$$\begin{aligned} \mathbf{D}_z &:= \left\{ \text{diag}(D_1, \dots, D_k, d_1 I_{l_1}, \dots, d_l I_{l_l}) : D_i \in \mathbb{C}^{r_i \times r_i}, D_i = D_i^* > 0; d_i \in \mathbb{R}, d_i > 0 \right\} \\ \mathbf{D}_w &:= \left\{ \text{diag}(D_1, \dots, D_k, d_1 I_{s_1}, \dots, d_l I_{s_l}) : D_i \in \mathbb{C}^{r_i \times r_i}, D_i = D_i^* > 0; d_i \in \mathbb{R}, d_i > 0 \right\} \end{aligned} \quad (4)$$

where $\{\cdot\}^*$ denotes the complex conjugate transpose. D_z and D_w differ in dimensions, but share the same variables D_i and d_i . Each full block Δ_i in Δ is accompanied by diagonal scaling matrices $d_i I_{l_i}$ and $d_i I_{s_i}$, while each repeated block $\delta_i I_{r_i}$ in Δ is accompanied by a full scaling matrix D_i . Without loss of generality, the D -scales are normalized with respect to the last diagonal matrix, so $d_l = 1$. Computation of the μ upper bound is a convex problem and, in general, the bound is close to the exact value (Packard and Doyle 1993). Therefore, the upper bound is often useful in practice.

2.2. Robust performance test condition

While in the previous section μ was discussed for constant matrices, it is now treated in the context of dynamic control systems and Transfer Function Matrices (TFMs), which are also complex matrices if they are considered at one frequency.

In the standard set-up for finite-dimensional, linear, time-invariant control systems of figure 1, the following signals play a role: the manipulated variables $u \in \mathbb{R}^{n_u}$ (inputs); the measured variables $y \in \mathbb{R}^{n_y}$ (outputs); the controlled variables \bar{z}^* , which are defined so they are zero ideally; the exogenous variables \bar{w}^* , such as reference signals, disturbances, and sensor noise; the signals \bar{p} and \bar{q} related with model uncertainties (see discussion below). Finally,

$$\bar{w} := \begin{pmatrix} \bar{p} \\ \bar{w}^* \end{pmatrix} \in \mathbb{R}^{n_w}, \quad w := \begin{pmatrix} p \\ w^* \end{pmatrix} \in \mathbb{R}^{n_w}$$

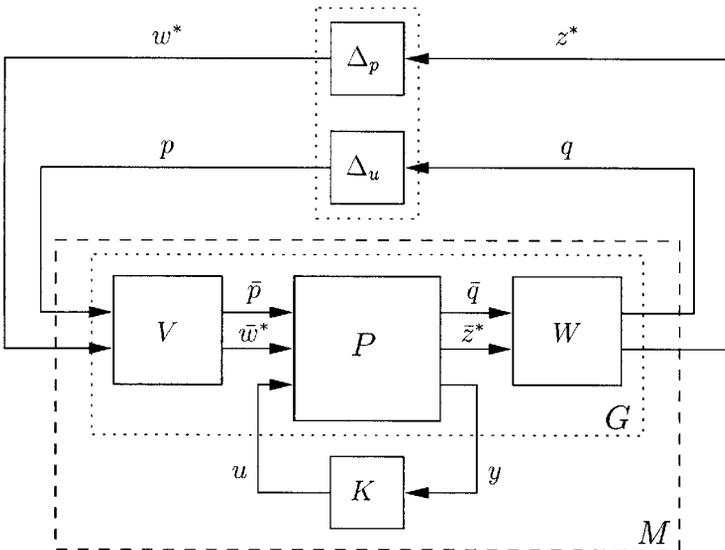


Figure 1. Standard control system set-up.

with w a scaled version of the physical, exogenous variables \bar{w} and

$$\bar{z} := \begin{pmatrix} \bar{q} \\ \bar{z}^* \end{pmatrix} \in \mathbb{R}^{n_z}, \quad z := \begin{pmatrix} q \\ z^* \end{pmatrix} \in \mathbb{R}^{n_z}$$

with z a weighted version of the physical, controlled variables \bar{z} .

The set-up in figure 1 incorporates the blocks Δ_u and Δ_p . The possibly structured block Δ_u accounts for the plant uncertainties. The corresponding entries in the (assumed diagonal) TFMs V and W are used to model these uncertainties, such that Δ_u in figure 1 is a scaled representation of the actual uncertainties. In this paper, controller design and IO selection deal with dynamic uncertainties. This implies a conservative treatment of real parametric uncertainties, which are instead covered by the larger class of dynamic uncertainties. In the context of control systems, the uncertainty structure Δ_u corresponding to Δ_u takes the same form as (1) for the constant matrix case, but with the diagonal blocks replaced by proper, real-rational, and stable TFMs, i.e. $\Delta_u \in \mathcal{RH}_\infty$. The unstructured, fictitious block Δ_p accounts for performance specifications. As will become clear below, it is introduced to arrive at the RP condition. The corresponding entries in V can be used to characterize the magnitude and frequency-dependence of the components in \bar{w} , while the entries in W can be used to indicate which components and frequencies in \bar{z} must be suppressed (for more detail see, e.g. Balas *et al.* 1995, Chapter 3). In analogy to the uncertainty block Δ_u , the performance block Δ_p is assumed in \mathcal{RH}_∞ . Together with Δ_u it forms the augmented block $\Delta := \text{diag}(\Delta_u, \Delta_p)$.

So, the generalized plant G in figure 1 incorporates nominal plant data P and design filters V and W and is partitioned as:

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \quad (5)$$

In figure 1, K denotes the controller. To guarantee closed-loop internal stability, it is assumed that G and K are stabilizable and detectable (Zhou *et al.* 1996, Chapter 16). The generalized closed-loop M is given by:

$$M(G, K) = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21} \quad (6)$$

The proposed IO selection method aims at achieving RP, i.e. at guaranteeing performance for a particular class of uncertainties. The required performance level is quantified by the \mathcal{H}_∞ norm of the uncertain closed-loop TFM between w^* and z^* , denoted as \tilde{M} . The following provides a necessary and sufficient condition for an RP level γ (Zhou *et al.* 1996, Chapter 11):

Robust performance: Assume M is stable and let $\gamma > 0$. For all $\Delta_u \in \Delta_u$ with $\|\Delta_u\|_\infty \leq 1/\gamma$, the uncertain closed-loop \tilde{M} is stable and $\|\tilde{M}\|_\infty < \gamma$ (i.e. RP is achieved) if and only if:

$$\|M\|_\mu := \sup_\omega \mu_\Delta(M(j\omega)) < \gamma \quad (7)$$

So, for a specified frequency grid, μ can be used for control system analysis. In analogy to the constant matrix case in section 2.1, the frequency-dependent μ in (7) is replaced by its upper bound $\bar{\mu}$. The stronger inequality:

$$\|M\|_{\bar{\mu}} := \sup_\omega \bar{\mu}_\Delta(M(j\omega)) = \sup_\omega \inf_{D \in \mathcal{D}} \bar{\sigma}(D_z(\omega)M(j\omega)D_w^{-1}(\omega)) < \gamma \quad (8)$$

is used to test the RP property of a given closed-loop M and serves as the starting point for the development of the IO selection method in Lee *et al.* (1995). The use of the μ upper bound, instead of the exact value, is acceptable for three reasons. As we already remarked in section 2.1, this bound is generally close to the exact μ . Moreover, μ is used as a robust stability/performance measure and it would be more dangerous to underestimate μ than to overestimate it. Also, robustness against particular time-varying uncertainties is covered by the upper bound (Poolla and Tikku 1995).

3. Input-output selection theory

This section provides the conditions for IO selection as documented in Lee *et al.* (1995). The complete derivation is not repeated, but only the major steps are mentioned and the resulting tools are discussed. As remarked before, Δ is not restricted to be built up of square blocks as in Lee *et al.* (1995). The formulas for this more general case are straightforwardly obtained by using the same derivation as Lee *et al.* (1995).

The basic idea for the IO selection method is to check whether or not $\min_{K \in \mathcal{K}_S} \|M(G, K)\|_\mu < \gamma$ is possible, with \mathcal{K}_S the set of all proper, real-rational, and stabilizing controllers (actually, $\gamma = 1$ in Lee *et al.* (1995)). As we announced in section 2.1, this inequality is replaced by:

$$\min_{K \in \mathcal{K}_S} \sup_{\omega} \inf_{D \in \mathcal{D}} \bar{\sigma}(D_z M(G, K) D_w^{-1}) < \gamma \quad (9)$$

yielding a sufficient (and generally almost necessary) condition for existence of a robustly performing controller. The Youla controller parameterization (see, e.g. Maciejowski 1989, Chapter 6) is invoked to create an expression for the closed-loop M that is affine in the new controller parameter Q , whereas the expression (6) is not affine in K . Inequality (9) is then replaced by its equivalent:

$$\min_{Q \in \mathcal{R} \mathcal{H}_\infty} \sup_{\omega} \inf_{D \in \mathcal{D}} \bar{\sigma}(D_z M(N, Q) D_w^{-1}) < \gamma \quad (10)$$

where N is a modified version of G . Note that the requirement of K being stabilizing (K itself is not necessarily stable) is replaced by the equivalent requirement of Q itself being stable ($Q \in \mathcal{R} \mathcal{H}_\infty$).

To arrive at the controller-independent IO selection conditions of Lee *et al.* (1995), the requirement $Q \in \mathcal{R} \mathcal{H}_\infty$ is replaced by $Q \in \mathcal{R} \mathcal{M}$. Here, $\mathcal{R} \mathcal{M}$ is the set of all proper, real-rational TFMs, which do not need to be stable ($\mathcal{R} \mathcal{H}_\infty \subset \mathcal{R} \mathcal{M}$). Hence, the requirement that the controller must be stabilizing is dropped. This gives rise to the major shortcoming of the IO selection method and its necessary character. Lee *et al.* (1995) claim that dropping the stability requirement on Q is mathematically equivalent to dropping the causality requirement on Q . In this context, we refer to Zhou *et al.* (1996, Section 4.3), where a TFM which is analytic and bounded in the open left half-plane (i.e. a TFM in $\mathcal{R} \mathcal{H}_\infty$) is defined as anti-stable or anti-causal.

Lee *et al.* (1995) expect that the consequences of dropping the stabilizing property are 'only significant around cross-over'. This can be made plausible for single input single output (SISO) systems in the conventional feedback configuration. To specify performance, magnitude bounds are imposed on the open-loop transfer function $L(s)$. Typically, the joint performance and stability requirements are especially difficult to meet in the cross-over region, which includes the frequency where

$|L(j\omega)| = 1$. To guarantee stability, the phase of $L(s)$ must obey certain requirements. From Bode's gain-phase relationships it is known that this puts restrictions on the allowable roll-off rate of $|L(j\omega)|$ (Maciejowski 1989). Hence, dropping the stabilizing property could also be interpreted as dropping the requirements on the phase of $L(s)$ for a given magnitude. In that case, magnitude bounds on $L(s)$ can always be met (provided these are the only remaining requirements and the bounds are physically meaningful) and a possible source of ineffectiveness for the IO selection method is introduced.

Without going into further detail, the following condition for IO selection results:

IO selection: *A necessary condition for the existence of a proper, real-rational, stabilizing controller achieving the μ upper bound specification $\|M\|_{\mu} < \gamma$ is the existence of an $X_z \in \mathbf{D}_z$ and an $X_w \in \mathbf{D}_w$ such that for all frequencies ω*

$$\hat{G}_{21\perp}(j\omega) \{G_{11}^*(j\omega)X_z(j\omega)G_{11}(j\omega) - \gamma^2 X_w(j\omega)\} \hat{G}_{21\perp}^*(j\omega) < 0 \quad (\text{output selection}) \quad (11)$$

and

$$\hat{G}_{12\perp}^*(j\omega) \{G_{11}(j\omega)X_w^{-1}(j\omega)G_{11}^*(j\omega) - \gamma^2 X_z^{-1}(j\omega)\} \hat{G}_{12\perp}(j\omega) < 0 \quad (\text{input selection}) \quad (12)$$

In the above, $\hat{G}_{12} := G_{12}(G_{12}^*G_{12})^{-1/2}$, $\hat{G}_{21} := (G_{21}G_{21}^*)^{-1/2}G_{21}$, and $\{\cdot\}_{\perp}$ denotes the orthogonal complement such that:

$$\begin{bmatrix} \hat{G}_{21} & \hat{G}_{21\perp} \end{bmatrix} \in \mathbb{C}^{n_w \times n_w} \quad \text{and} \quad \begin{bmatrix} \hat{G}_{12} \\ \hat{G}_{12\perp} \end{bmatrix} \in \mathbb{C}^{n_z \times n_z}$$

are both unitary matrices for all ω (a square matrix Γ is unitary if $\Gamma^* \Gamma = I$). Like the D -scales in (4), the frequency-dependent X_z and X_w share the same variables and only differ in size. In Lee *et al.* (1995), $X_z = X_w = X$, due to the restriction to square blocks in Δ . \hat{G}_{12} exists if and only if G_{12} has full column rank for all ω , while \hat{G}_{21} exists if and only if G_{21} has full row rank for all ω . This will be assumed from now on, which implies $n_z \geq n_u$ and $n_w \geq n_y$.

A few comments can be made with respect to the IO selection conditions. First, the individual conditions (11) and (12) are each convex feasibility problems in the form of Linear Matrix Inequalities (LMIs). However, they must be checked jointly, which is a nonconvex feasibility problem and therefore more difficult to solve. According to Lee *et al.* (1995), the joint check can only be solved straightforwardly if Δ consists of two full blocks, e.g. for a RP problem with Δ_p and a single full block Δ_u . In that case, X_z and X_w consist of two real diagonal blocks (see (4)), the second of which is an identity matrix. As a result, each condition can be solved for one scalar: (11) for α_1 associated with X_z and X_w ; (12) for α_2 corresponding to X_z^{-1} and X_w^{-1} . Next, we can check whether or not the solution intervals of α_1 ($\alpha_1 \in [\underline{\alpha}_1, \bar{\alpha}_1]$) and $1/\alpha_2$ ($\alpha_2 \in [\underline{\alpha}_2, \bar{\alpha}_2]$) intersect, i.e. if (11) and (12) are jointly met. An alternative, possibly more efficient, approach is to determine the solution interval of one LMI and use this as an additional constraint when checking the feasibility of the other.

Second, note that (11) only depends on G_{11} and G_{21} . This implies that (11) does not depend on the considered input set, provided there are no uncertainties in Δ_u that are directly linked to the inputs u , e.g. multiplicative or additive input uncertainties. Therefore, this condition will be referred to as the 'output selection' condition. By analogy, the 'input selection' condition (12) does not depend on the output set if there are no uncertainties directly linked to y . Hence, under these

restrictions to the uncertainty model, one possible approach to IO selection is to check (11) for all candidate output sets and (12) for all candidate input sets. This approach is not restricted to two full blocks in Δ . However, compared to the joint test, additional nonviable IO sets may be accepted, since no check is made to see whether the combination of accepted input and output sets can yield the desired result.

Third, the input selection condition (12) drops out if z and u have the same dimension $n_z = n_u$ (\hat{G}_{12} is nonsingular and square and hence $\hat{G}_{12\perp}$ is empty). In this case, the input set is considered perfect by the IO selection method. By analogy, the output selection condition (11) drops out if \hat{w} and y have the same dimension $n_w = n_y$ (\hat{G}_{21} is nonsingular and square, so $\hat{G}_{21\perp}$ is empty) and the output set is regarded perfect. Finally, all IO sets with $n_z = n_u$ and $n_w = n_y$ will pass the necessary conditions for IO selection, but the sufficient and almost necessary condition for RP (9) is not guaranteed to be met. In practice (and for the application studied here), actuator weights and sensor noise are usually included in z and w , respectively. So, usually $n_z > n_u$ and $n_w > n_y$ and neither (12) nor (11) drops out.

4. Control problem formulation

The IO selection conditions (11) and (12) are used to select sensors and actuators for an active suspension applied to the six Degrees of Freedom (DOF) model of the tractor–semitrailer combination in figure 2. This is a modified version of the ten DOF model of De Jager (1995), for which a MATLAB file generating the state-space description can be requested. A choice must be made among three candidate actuators (u_1, u_2, u_3), which are placed between the axles and the tractor and semi-trailer chassis. Nine candidate sensors measure the suspension deflections (y_1, y_2, y_3),

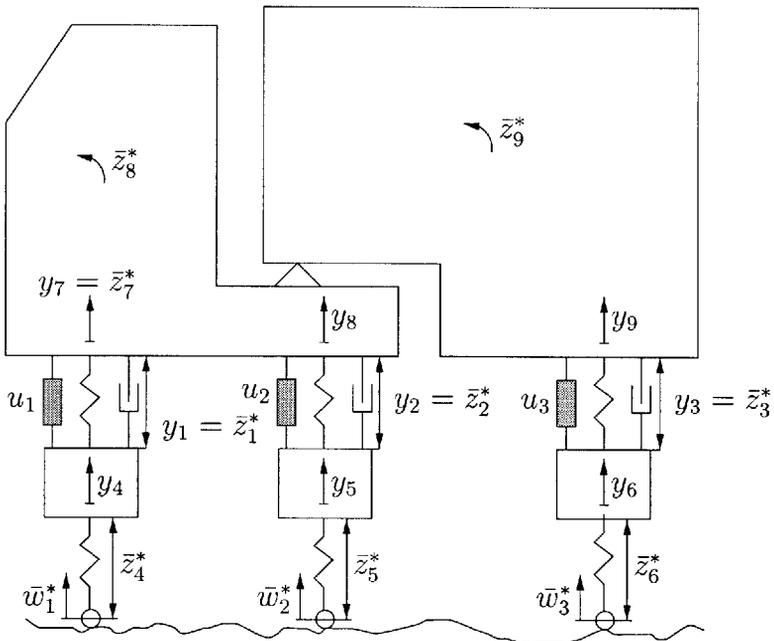


Figure 2. The six DOF tractor–semitrailer model.

the axle accelerations (y_4, y_5, y_6) , and the vertical tractor and semitrailer accelerations (y_7, y_8, y_9) . Though additional candidate inputs (e.g. tyre-actuating systems, see Al-Sulaiman and Zaman (1994)) and outputs (e.g. velocity and absolute displacement sensors) could be proposed, this study is restricted to the ones above. Combining the suggested inputs and outputs yields 3577 candidate IO sets, among which are the 3×9 full IO set and 27 1×1 IO sets.

4.1. Performance specifications

The exogenous input \bar{w}^* contains the excitation by the road surface, i.e. the road surface heights \bar{w}_1^*, \bar{w}_2^* , and \bar{w}_3^* , and measurement noises $\bar{w}_4^*, \dots, \bar{w}_{12}^*$. The road surface height was assumed to be lowpass-filtered, white noise and the corresponding diagonal entries in V were chosen as:

$$V_{1,2,3} = \frac{v_0}{s/\omega_0 + 1} \tag{13}$$

(see also figure 3). For a realistic motorway and a forward vehicle speed of 25 m s^{-1} , $\omega_0 = 2\pi \cdot 0.25 \text{ rad s}^{-1}$ and $v_0 = 8 \times 10^{-3}$ are representative choices (Braun and Hellenbroich 1991).

The measurements were assumed to be disturbed with zero-mean noises. For this purpose, the manufacturers' data of particular displacement and acceleration sensors were consulted, along with experimental data from tests with the vehicle. The suspension deflections y_1, y_2 , and y_3 can be measured with an accuracy of 10^{-3} m . The noise for the acceleration sensors was assumed to stem from two sources. First, the measurements are disturbed with noise with a RMS level of $2.5 \times 10^{-2} \text{ m s}^{-2}$. Second, the acceleration sensors are sensitive to accelerations in the transverse direction, i.e. 1.3% of the transverse accelerations is passed through to y_4, \dots, y_9 as if they were vertical accelerations. A RMS level of $2.5 \times 10^{-2} \text{ m s}^{-2}$ was used as a suitable representation of a worst-case situation. The RMS levels associated with the two acceleration noise sources are simply added to give V_7, \dots, V_{12} . Based on these numbers,

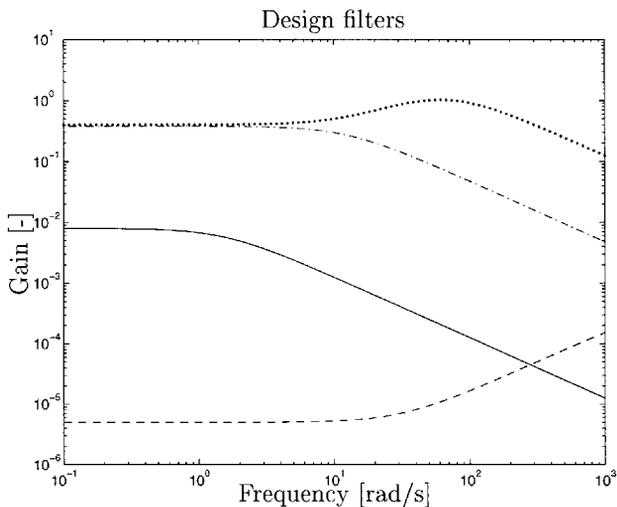


Figure 3. Dynamic design filters: $V_{1,2,3}$ (—), W_7 ($\cdots\cdots$), W_8 (- · - ·), and $W_{10,11,12}$ (- -).

the following constant entries in V were chosen, guaranteeing G_{21} to have full row rank for all α

$$\begin{aligned} V_i &= 1.0 \cdot 10^{-3}, \quad i = 4, 5, 6 && \text{(suspension reflections)} \\ V_i &= 5.0 \cdot 10^{-2}, \quad i = 7, \dots, 12 && \text{(axle and chassis accelerations)} \end{aligned} \quad (14)$$

Four main design goals are distinguished in \bar{z}^* (De Jager 1995). The first and second are to limit the suspension deflections $\bar{z}_{1,2,3}^*$ (due to space limitations) and the tyre deflections $\bar{z}_{4,5,6}^*$ (for good handling and minimum road surface damage). For these design goals, one is actually interested in restricting the \mathcal{Q}_1 norm of the TFM between the road surfaces $\bar{w}_{1,2,3}^*$ and the deflections $\bar{z}_{1,\dots,6}^*$. However, it is not obvious how this is best transformed into the control problem setting considered here, which in fact only accounts for objectives based on the \mathcal{H}_∞ system norm. A multi-objective problem formulation would be more appropriate (De Jager 1995). As an approximation, constant weights were chosen, the numerical values of which will be chosen later:

$$\begin{aligned} W_1 &= \rho_{1f}, \quad W_2 = \rho_{1r}, \quad W_3 = \rho_{1t} && \text{(suspension deflections)} \\ W_4 &= \rho_{2f}, \quad W_5 = \rho_{2r}, \quad W_6 = \rho_{2t} && \text{(tyre deflections)} \end{aligned} \quad (15)$$

The subscripts f, r, and t indicate the tractor front suspension, tractor rear suspension, and semitrailer suspension.

The third design goal is to limit the tractor's accelerations to guarantee good driver comfort and to limit the semitrailer's acceleration to avoid cargo damage. The weighting filters for the tractor's vertical acceleration \bar{z}_7^* and rotational acceleration \bar{z}_8^* are based on plots of human sensitivity provided by Corbridge and Griffin (1986). First, the sensitivity contour for \bar{z}_7^* is approximated by the magnitude of:

$$W_7 = \rho_3 \omega_1^2 \frac{s/\omega_2 + w_{10}}{(s + \omega_1)^2} \quad (16)$$

with $\rho_3 = 1$, $w_{10} = 0.4$, $\omega_1 = 2\pi \times 10 \text{ rad s}^{-1}$, and $\omega_2 = 2\pi \times 5 \text{ rad s}^{-1}$. Second, since the driver's horizontal acceleration can be approximated by a constant multiplied by the rotational acceleration \bar{z}_8^* , the weight for \bar{z}_8^* is based on the horizontal sensitivity, which is approximated by the magnitude of:

$$W_8 = \rho_4 \frac{w_{20}}{s/\omega_3 + 1} \quad (17)$$

with $w_{20} = 1$, $\omega_3 = 2\pi \times 2 \text{ rad s}^{-1}$, and ρ_4 a constant to be determined. The acceleration weights (16) and (17) are depicted in figure 3. The choice of the weight W_9 for the semitrailer's rotational acceleration strongly depends on the cargo in question. Here, it was taken as a constant to be determined: $W_9 = \rho_5$. Limiting the accelerations' power spectra due to the stochastic road surface modelled by (13) actually involves limiting the \mathcal{H}_2 norm of the corresponding TFM (De Jager 1995). With the weights $W_{7,8,9}$, an attempt is made to approximate this goal in the \mathcal{H}_∞ norm setting.

The fourth and final control objective is to limit the controller outputs (actuator weights, $\bar{z}_{10}^* = u_1$, $\bar{z}_{11}^* = u_2$, $\bar{z}_{12}^* = u_3$). Since the actuator bandwidths are limited, high-frequency inputs cannot be realized, so high-frequency controller outputs should be avoided. This is accounted for by the following bi-proper

weighting filter (see also figure 3), which guarantees that G_{12} has full column rank for all ω

$$W_{10,11,12} = \rho_6 \frac{s/\omega_4 + 1}{s/\omega_5 + 1} \quad (18)$$

Here, $\rho_6 = 5 \times 10^{-6}$ and $\omega_4 = 2\pi \times 5 \text{ rad s}^{-1}$ (the actuator bandwidth is assumed to be 5 Hz). Though $\omega_5 = \infty$ is a natural choice, a finite value is needed for a state-space realization of G . To give the actuator weights a ‘nonproper character’ in the frequency range of interest, $\omega_5 = 100 \times \omega_4 \text{ rad s}^{-1}$ was chosen.

The ‘ ρ -parameters’ $\rho_{1_{i,x,l}}$, $\rho_{2_{i,x,l}}$, and ρ_4, ρ_5 in W could be chosen so they express the relative importance of the corresponding control objectives, or so subsequent controller design yields acceptable closed-loop performance. Here, the ρ -parameters were obtained by iterative \mathcal{H}_∞ optimization and closed-loop simulations for the nominal model and the full IO set ($\rho_3 = 1$ and $\rho_6 = 5 \times 10^{-6}$ were fixed). Though the suspension and tyre deflection limits are normally not exceeded for stochastic road surfaces, this may happen for deterministic ones. To account for this to some extent, rounded pulses as a class of deterministic road surfaces (De Jager 1995) were used for the simulations. The characteristic height and length of each rounded pulse (classified from ‘tiny’ to ‘huge’) is such that at least one of the suspension tyre deflections hits its limits for the passively suspended system. The iterative process was continued until the closed-loop responses were acceptable; only for ‘large’ and ‘huge’ rounded pulses did the front suspension deflection of the controlled system slightly exceed its limits. No attempt was made to improve the response further, since this is beyond the scope of this paper. The following values finally resulted:

$$\begin{aligned} \rho_{1_r} &= 1.28, & \rho_{1_t} &= 10.11, & \rho_{1_l} &= 0.68 \\ \rho_{2_r} &= 4.50, & \rho_{2_t} &= 136.16, & \rho_{2_l} &= 23.34 \\ \rho_4 &= 0.38, & \rho_5 &= 0.30 \end{aligned} \quad (19)$$

With the chosen design filters V and W , an \mathcal{H}_∞ optimization for the full IO set yields $\|M\|_\infty = 0.10$. It should be noted that the influence of sensor noise and actuator weights is rather small: modifying $V_i, i = 4, \dots, 12$ simultaneously by a multiplicative factor $10^{\pm 1}$ leaves $\|M\|_\infty$ almost unchanged, while multiplying the actuator weighting parameter ρ_6 by a factor $10^{\pm 1}$ gives $\|M\|_\infty = 0.10 \pm 0.01$.

4.2. Uncertainty model

In the six DOF tractor–semitrailer model, various uncertainties play a role. Due to possibly large variations in the cargo, important parametric uncertainties are related to the semitrailer mass \tilde{M}_t and semitrailer inertia \tilde{J}_t . It was assumed that the location of the semitrailer’s centre of mass is invariant and that \tilde{J}_t increases proportionally with \tilde{M}_t . With M_t and J_t the nominal values of \tilde{M}_t and \tilde{J}_t the relations $\tilde{M}_t = M_t(1 + \tau\delta)$ and $\tilde{J}_t = J_t(1 + \tau\delta)$ with $\delta \in \mathbb{R}, |\delta| \leq 1$ were used for the uncertainty modelling. Pulling δ out of the uncertain plant gives rise to a repeated, real block $\Delta_u = \delta I_3$ in the set-up of figure 1. The corresponding scaling filters were chosen as $V_p = \sqrt{\tau} I_3$ and $W_q = \sqrt{\tau} I_3$, which form the upper left blocks in V and W . It was assumed that \tilde{M}_t and \tilde{J}_t vary between the values for an empty and fully loaded semitrailer. The means of these extreme values were taken as the nominal values M_t and J_t , giving $\tau = 0.90$.

Two remarks can be made with respect to the uncertainty model. First, the actual uncertainties are real parametric ones, but the IO selection method can only treat them as dynamic uncertainties, see section 2.2. The controller design in this paper is also restricted to complex blocks in Δ , though μ -analysis (Young *et al.* 1995) and μ -synthesis (Young 1996) have been ‘solved’ for mixed real and complex blocks.

Second, to evaluate the IO selection method it should be investigated whether conditions (11) and (12) jointly eliminate more IO sets than they do individually. Recall that the joint check requires Δ_u to be a full block. To study the effect of the assumption that Δ_u is full, μ -synthesis was performed by the D - K iteration approach (see, e.g. Zhou *et al.* 1996, Chapter 11), which continues till $\|M\|_{\bar{\mu}}$ decreases no longer. This ‘optimal μ -synthesis’ was performed for the full IO set and two distinct uncertainty blocks: (1) for a 3×3 repeated, complex Δ_{μ} ; and (2) for a 3×3 full, complex Δ_u . Both approaches gave the same value $\|M\|_{\bar{\mu}} = 0.90$. For comparison, the open-loop μ -value for a repeated, complex block is $\|G\|_{\bar{\mu}} = 1.51$ and for a full, complex block $\|G\|_{\bar{\mu}} = 1.72$. The value $\|M\|_{\bar{\mu}} = 0.90$ is due to the semitrailer’s mass and inertia being zero for $\delta = -1/0.90$. In that case the two $j\omega$ -axis poles of the semitrailer’s axle/tyre-dynamics cannot be stabilized by control. Note, however, that uncertainties $|\delta| = 1/0.90$ are outside the considered class $|\delta| \leq 1$. Since μ -syntheses for the repeated and full Δ_u yielded the same $\|M\|_{\bar{\mu}}$, the remainder of this paper will treat Δ_u as a full, complex block. In general, this may introduce conservatism into the IO selection (and the controller design), possibly leading to the faulty rejection of IO sets. So, the IO selection method is not suitable if the actual uncertainty model is not sufficiently accurately described by the more restrictive model required by the IO selection method.

5. Input–output selection application

Two major practical aspects of the IO selection method are investigated: effectiveness and efficiency. For ten typical IO sets, section 5.1 assesses the effectiveness by comparing the results of optimal μ -synthesis and the IO selection conditions for decreasing γ -values. The selection from all possible IO sets together with efficiency and some additional effectiveness results are discussed in section 5.2.

5.1. Results for typical IO sets

Since the stabilizing property of the controller is dropped, there may be a difference between the lowest achievable $\|M\|_{\bar{\mu}}$ with a stabilizing controller and the γ for which the IO set is first eliminated by the test conditions. To investigate this source of ineffectiveness and to get insight into the preferable position and type of actuators and sensors, table 1 was composed as follows. First, for ten typical IO sets, optimal μ -syntheses were performed, yielding $\|M\|_{\bar{\mu}}$. For the involved D - K iteration, a frequency grid of 51 logarithmically spaced points between 10^{-1} and 10^3 rad s $^{-1}$ was used. All computations were done with the MATLAB μ -analysis and Synthesis Toolbox, Version 3.0 (Balas *et al.* 1995). Second, for the same ten IO sets, the output and input selection conditions (11) and (12) were checked for distinct γ -values. Starting with $\gamma = 2$, a bisection algorithm was used to find the smallest γ for which the selection condition passes. Like in the \mathcal{H}_{∞} optimization part of the D - K iteration, $\epsilon_{01} = 10^{-2}$ was used as the termination criterion for the bisection. For the joint input and output selection conditions, γ_{opt}^* in table 1 corresponds to the

	IO set	Characteristic	$\ M\ _{\bar{\mu}}$	γ_{opt}^*	γ_{opt}	Failing condition	
1	y_1, \dots, y_9	$u_1 u_2 u_3$	Full IO set	0.90	0.39	0.17	Input
2	y_1, \dots, y_9	u_1	Front tractor actuator	1.60	1.60	1.60	Input
3	y_1, \dots, y_9	u_2	Rear tractor actuator	1.57	1.57	1.57	Input
4	y_1, \dots, y_9	u_3	Semitrailer actuator	1.15	1.16	1.16	Input
5	$y_1 y_2 y_3$	$u_1 u_2 u_3$	Suspension deflections	0.94	0.91	0.91	Output
6	$y_4 y_5 y_6$	$u_1 u_2 u_3$	Axle accelerations	0.91	0.79	0.38	Output
7	$y_7 y_8 y_9$	$u_1 u_2 u_3$	Chassis accelerations	0.90	0.41	0.17	Input
8	$y_1 y_4 y_7$	$u_1 u_2 u_3$	Front tractor sensors	1.34	1.33	1.33	Output
9	$y_2 y_5 y_8$	$u_1 u_2 u_3$	rear tractor sensors	1.57	1.57	1.57	Output
10	$y_3 y_6 y_9$	$u_1 u_2 u_3$	Semitrailer sensors	1.31	1.31	1.31	Output

Table 1. Results of μ -synthesis and IO selection conditions for ten typical IO sets.

smallest γ for which the IO set was accepted. By analogy, for the separate conditions, γ_{opt} denotes the smallest γ for which the IO set was accepted and the condition that failed first is explicitly indicated. The IO selection was based on the same frequency grid as μ -synthesis, starting with $10^{-1} \text{ rad s}^{-1}$ and working upwards until a ‘frequency fails’, or $\omega = 10^3 \text{ rad s}^{-1}$ is reached. For each frequency, the feasibility of the LMIs (11) and (12) was checked with the MATLAB package LMITOOL by El Ghaoui *et al.* (1995), which acts as an interface for the Semidefinite Programming package SP developed by Vandenberghe and Boyd (1994). In the case of the joint test, the solution interval $\alpha_2 \in [\underline{\alpha}_2, \bar{\alpha}_2]$ associated with the input selection LMI (12) was determined via a minimization and a maximization, subject to feasibility of the LMI. Given $\underline{\alpha}_2$ and $\bar{\alpha}_2$, the requirement $1/\bar{\alpha}_2 \leq \alpha_1 \leq 1/\underline{\alpha}_2$ was imposed as an additional constraint in checking the feasibility of the output selection LMI (11), see section 3.

To start with, the preference of actuators and sensors for RP is studied on the basis of the optimal $\|M\|_{\bar{\mu}}$ -values. None of the IO sets 2, 3, and 4 is viable for $\gamma = 1$, i.e. none of them achieves RP. Obviously, all other IO sets based on a single actuator are also nonviable, since eliminating sensors will never improve the best achievable control. In the case of a single input, the actuator u_3 at the semitrailer is preferable for RP. However, in the absence of uncertainties (nominal performance), u_1 and u_2 are equally good ($\|M\|_{\infty} = 0.11$) and slightly better than u_3 ($\|M\|_{\infty} = 0.12$). Studying $\|M\|_{\bar{\mu}}$ for IO sets 5, 6, and 7, it turns out that these IO sets are viable for $\gamma = 1$ and that they are approximately as good as the full IO set. There is no clear preference for suspension deflection measurements, axle accelerations, or chassis accelerations. IO sets 8, 9, and 10 are nonviable for $\gamma = 1$, but a comparison shows that sensors mounted on the semitrailer are preferred. In the case of nominal performance, the sensors mounted on the tractor are preferred to those on the semitrailer. Finally, noting that IO sets 5, 6, and 7 give better results than 8, 9, and 10, this suggests that viable IO sets should employ a ‘mixed geometric placement’ of sensors.

Let us take a look at the differences between the results with optimal μ -synthesis and with the IO selection conditions with decreasing γ . On the one hand, for IO sets 2, 3, 4, 8, 9, and 10 the differences between $\|M\|_{\bar{\mu}}$ and γ_{opt}^* for the joint test are within the bisection tolerance $\epsilon_{\text{tol}} = 10^{-2}$. Note that these IO sets are nonviable for $\gamma = 1$. On the other hand, $\|M\|_{\bar{\mu}} - \gamma_{\text{opt}}^*$ is relatively large for the viable IO sets 1, 5, 6, and 7. This illustrates the earlier remarks that the IO selection conditions may be

ineffective, since IO sets may incorrectly be accepted. Although this is not the case for the typical IO sets and test level $\gamma = 1$, IO sets may incorrectly be accepted for other γ -values. For instance, IO sets 1, 6, and 7 are incorrectly accepted for $\gamma = 0.8$. It was also observed (results not listed) that $\|M\|_{\bar{\mu}} - \gamma_{\text{opt}}^*$ decreases for the viable IO sets if y -noise or u -weights increase. As illustrated by the results in table 1 for the nonviable IO sets, this does not imply that $\|M\|_{\bar{\mu}} - \gamma_{\text{opt}}^*$ is always large if the influence of u -weights or y -noise is small, which is the case for the problem in question (see section 4.1).

Table 1 also shows that only for IO sets 1, 6, and 7 there is a difference between γ_{opt}^* (joint check) and γ_{opt} (separate check). Recall from section 3 that output selection based on condition (11) implicitly assumes that the input set is perfect. By analogy, input selection based on (12) assumes the output set to be perfect. As a result, $\gamma_{\text{opt}} \leq \gamma_{\text{opt}}^*$ and additional IO sets might be eliminated for the joint test. Apparently, this source of ineffectiveness only plays a role for IO sets 1, 6, and 7, for which the combination of imperfect output set and input set places additional restrictions on the achievable RP level. For the separate tests, it is explicitly indicated whether the input or output selection condition fails. For the full IO set, the input selection condition (12) fails first and so if inputs are eliminated (12) should again be the first to fail. This is indeed supported by the results for IO sets 2, 3, and 4. IO sets 5, ..., 10 use less sensors and hence the output selection condition may now break down first. This is true, except for IO set 7, for which the input selection condition still fails first. According to the IO selection conditions, control does not suffer from eliminating y_1, \dots, y_6 , resulting in the same γ_{opt} -values for IO set 7 and the full IO set (the γ_{opt}^* -values are also very close). This statement is supported by comparing the $\|M\|_{\bar{\mu}}$ -values for IO sets 1 and 7.

5.2. IO selection results

The results of IO selection for the RP level $\gamma = 1$ are discussed. For the nine candidate sensors and three candidate actuators, there are 511 candidate output sets, seven candidate input sets, and 3577 candidate IO sets. The following three-step strategy was implemented. In the first step, the candidate output sets are subjected to the output selection condition (11) for the first frequency. Starting with the full output set, subsets are tested, but nonviable output sets and their subsets are directly eliminated. For the output sets passing the first frequency, this procedure is repeated for the next frequency. The second step uses a similar procedure for input selection based on (12). The third step involves the joint test for all remaining output sets and input sets. Note that determining the solution interval $\alpha_i \in [\underline{\alpha}_i, \bar{\alpha}_i]$, $i = 1$ or 2 , involves a minimization and a maximization, subject to feasibility of a LMI (see section 3). Generally, these optimizations take more time than a feasibility check and therefore, depending on the number of remaining candidate output and input sets, the solution intervals are computed for either the output selection LMI (11), or the input selection LMI (12). Next, all remaining IO sets are generated and the feasibility of the other LMI is checked, subject to the corresponding solution interval constraint. Performing the joint test, subsets of nonviable IO sets are again directly eliminated. Efficiency is the reason to perform the separate tests in steps one and two prior to the joint test in step three: the joint test generally takes more time, so the computation time can be reduced significantly if the number of joint tests is smaller.

The procedure described above will be called the ‘LMI-based IO selection’. The results will be compared with those from ‘ μ -based IO selection’. In that approach, μ -syntheses are performed to check for the existence of a stabilizing controller achieving the RP level $\gamma = 1$. A similar three-step strategy was used as for the LMI-based IO selection. First, the candidate output sets are investigated, using the full input set. Second, the candidate input sets are investigated, using the full output set. Third, the IO sets generated from the accepted candidate output and input sets are checked. In each step, all subsets of nonviable candidates are eliminated directly. The D - K iteration started with an \mathcal{H}_∞ optimization, as if Δ were unstructured. The D - K iteration employed the same frequency grid as the LMI-based IO selection and it used third-order approximations for each diagonal entry of the D -scales. The iteration was terminated in two cases: if $\|M\|_{\bar{\mu}} < 1$ (set is viable), or if $\|M\|_{\bar{\mu}}$ did not reduce more than $\epsilon_{01} = 10^{-2}$ in a single D - K iteration step (set is nonviable). One of these criteria was always met within four iteration steps.

In the following, the results of each IO selection step are discussed. Besides the CPU times (Silicon Graphics, Indy, 200 MHz, R4400SC) and the number of tests performed in each step, table 2 shows how many candidate IO sets remain after each step. Checking the output selection condition in the first step, the LMI-based IO selection accepted 455 (out of the 511) candidate output sets. Performing the test for increasing frequencies, the 56 nonviable output sets first dropped out for ω between 8 and 44 rad s^{-1} . Testing the nine nonviable output sets with a single sensor for the complete frequency grid, it appeared that the frequency ranges for which the output selection LMIs are infeasible fit well with the frequency ranges for which $\mu_{\Delta}(M)$ from optimal μ -syntheses with the full input set is larger than one; the ‘LMI-ranges’ are always included in the ‘ μ -ranges’. This statement also holds for nonviable output sets with two or three sensors that are all mounted on the tractor’s front or rear, or on the semitrailer, but it does not hold for other nonviable output sets. The 18 smallest viable output sets use two measurements (in the IO selection context, ‘smallest’ refers to the least number of sensors or actuators): one of the sensors mounted on the semitrailer, together with one of the tractor sensors. The output selection part of the μ -based IO selection accepted 441 output sets, while 14 output sets (using only tractor sensors) were eliminated in addition to the LMI-based approach. The 18 smallest viable output sets are the same as for the LMI-based approach.

In the second step, the same three (out of seven) candidate input sets were termed viable by the LMI-based and the μ -based IO selection: $u_1u_2u_3$, u_1u_3 , and u_2u_3 . So, to meet the RP requirement, the semitrailer actuator u_3 together with one of the tractor

Step	LMI-based IO selection			μ -based IO selection		
	Remaining possibly viable IO sets	Tests	CPU (s)	Remaining possibly viable IO sets	Tests	CPU (s)
1	3185	459	4.49×10^3	3087	443	1.56×10^4
2	1365	5	4.55×10^1	1323	5	3.46×10^2
3	1356	1358	1.39×10^4	1225	1231	4.67×10^4
Total:		1822	1.84×10^4		1679	6.27×10^4

Table 2. Results for LMI-based and μ -based IO selection.

actuators must be used. Table 1 already indicated the importance of the semitrailer actuator (and sensors) for RP. In the LMI-based IO selection, the four nonviable input sets first dropped out for $\omega = 8.32 \text{ rad s}^{-1}$ or $\omega = 10.00 \text{ rad s}^{-1}$. The frequencies for which the input selection LMIs are infeasible correspond to the frequencies where $\mu_{\Delta}(M)$ from optimal μ -syntheses (with the full output set) is larger than one.

These are the results for the third step. As discussed above, for both IO selection approaches the number of viable input sets is considerably smaller than the number of viable output sets. Hence, for the LMI-based IO selection, the solution intervals $\alpha_2 \in [\underline{\alpha}_2, \bar{\alpha}_2]$ were determined for the input selection LMI. The 1365 candidate IO sets remaining after the second step were subjected to the joint test and 1356 IO sets were ultimately accepted. So, accounting for the coupling of imperfect input sets and output sets eliminated only nine additional IO sets. The 33 smallest viable IO sets use two actuators and two sensors: either $u_1 u_3$ or $u_2 u_3$ as the input set and any of the semitrailer sensors together with any of the tractor sensors as the output set. The only 2×2 nonviable IO sets are $y_3 y_4 / u_1 u_3$, $y_4 y_6 / u_1 u_3$, and $y_4 y_9 / u_1 u_3$ that use the front tractor actuator and front axle acceleration. The third step of the μ -based IO selection eliminated 98 IO sets of the 1323 candidates remaining after the second step. So 1225 IO sets were ultimately termed viable. Compared to the LMI-based IO selection, the μ -based IO selection thus eliminated 131 extra IO sets (3.7% of the original number of 3577 candidates). The 23 smallest (2×2) IO sets all use one tractor sensor and actuator, together with the semitrailer actuator u_3 and one semitrailer sensor. Performing optimal μ -syntheses for these 23 IO sets, $y_8 y_9 / u_2 u_3$ yielded the smallest value $\|M\|_{\mu} = 0.90$. Also, the μ -values for the IO sets with the front tractor actuator u_1 are just below one ($0.99 < \|M\|_{\mu} < 1.00$). If desired, the number of accepted IO sets can be further reduced by testing for smaller γ -values.

In table 2, CPU times are listed for the two IO selection approaches. With the current implementation, the CPU time for the IO selection based on suboptimal μ -synthesis was 3.4 times larger than for the LMI-based IO selection. It should be emphasized that the differences in the CPU times in table 2 for the two approaches strongly depend on the particular application and on implementation aspects. The application affects the CPU time, e.g. by the convergence in the μ -synthesis and the first encountered frequency for which nonviable candidates drop out. The implementation affects the CPU time, e.g. by the definition of the frequency grid (for both IO selection approaches), by the choice of the D -scale order and the tolerance for \mathcal{H}_{∞} optimization (for the μ -based IO selection), and by the sequence in which the frequencies are checked and the choice of the initial estimates for X_z and X_w (for the LMI-based IO selection). Also, the 'number of operations' in the third step of the IO selection differs for the LMI-based and μ -based approaches, since the latter approach does not need to determine solution intervals for α_2 . Clearly, the CPU times listed in table 2 should be taken with a grain of salt.

6. Conclusions and further directions

The IO selection method proposed by Lee *et al.* (1995) was used for an active suspension control problem. The method relies on necessary conditions, since the stabilizing requirement of the controller is dropped. For the application, the IO selection based on suboptimal μ -synthesis (aimed at constructing a stabilizing controller) indeed eliminated additional, nonviable IO sets. Other possible sources of ineffectiveness are the following. First, control problems with a single, repeated Δ_u ,

or with multiple (repeated or full) blocks in Δ_u involve solving a nonconvex optimization, for which there is currently no efficient solution. To partially resolve this, all possible combinations of Δ_p and full blocks in Δ_u could be studied, but an IO set accepted by this strategy may not pass for the original Δ_u . A structured Δ_u could also be replaced by a single, full, complex block, but then IO sets may incorrectly be rejected. Second, the IO selection cannot deal with real parametric uncertainties, but treating them as dynamic uncertainties (as was done for the application) may introduce conservatism and incorrectly reject IO sets. Third, critical frequencies may be overlooked in the specified grid, so nonviable IO sets may be accepted.

The IO selection method was ineffective for the example considered, since 131 (9.7%) of the accepted IO sets were nonviable. For some typical IO sets, it was shown that the number of incorrectly accepted IO sets may depend on the required RP level γ . Moreover, previous research has made clear that the effectiveness is considerably affected by the choice of the design filters, possibly leading to poor IO selection results. At present, it is unclear to which type of problems the IO selection method can be applied (more) effectively. Once we have gained more insight into this, the effectiveness of the IO selection method may be predictable to some extent and, if so, the approach would become more attractive for practical use. For the example investigated, the IO selection method was not significantly more efficient than (the effective approach of) μ -synthesis. In general, the required computation time for the IO selection is hard to predict beforehand, since this is affected by many aspects. Nevertheless, the IO selection can be useful, especially if only a small number of important frequencies is tested, e.g. only $\omega = 0$, as in Lee *et al.* (1995).

Two IO selection methods that are more efficient are discussed in Van de Wal and De Jager (1997*b*, 'Method 1') and Van de Wal and De Jager (1997*a*, 'Method 2'). Both methods are motivated by the following notion. If the structure in the combined uncertainty/performance block can effectively be reduced to an unstructured one, the efficient IO selection method proposed by Van de Wal and De Jager (1996) can be used again. The first step in both methods is to perform optimal μ -synthesis for the full IO set, yielding the closed-loop M^* . The second step is different. Method 1 generates minimum-phase $\mathcal{R} \mathcal{H}_\infty$ approximations $\hat{D}_z(s)$ and $\hat{D}_w(s)$ for the D -scales corresponding to M^* and Δ and extends the plants for the candidate IO sets with these approximations. Method 2 could be interpreted as a 'complementary' approach. Based on the μ definition (2), the smallest uncertainty Δ_u which violates RP is computed for M^* and Δ over the frequency range. By replacing this uncertainty with a suitable $\mathcal{R} \mathcal{H}_\infty$ TFM and absorbing it into the plants for all IO sets, the (structured) uncertainty block Δ_p is eliminated and an unstructured performance block remains. In the end, both IO selection methods boil down to checking \mathcal{H}_∞ controller existence conditions. Method 1 relies on sufficiency and IO sets may incorrectly be rejected, while Method 2 relies on necessity and IO sets may incorrectly be accepted.

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