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Influence of estimation errors on wayfinding-decisions in unknown street networks – analyzing the least-angle strategy

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Abstract. The least-angle strategy is a common wayfinding method that can be applied in unknown environments if the target direction is known. The strategy is based on the navigator's heuristic to select the street segment at an intersection which is most in line with the target direction. To use this strategy, the navigator needs to know the angles between the target direction and the street segments leading out from the intersection. If the direct view to the target is blocked and the target vector cannot be perceived, the target direction that is needed for the decision process is based on the agent's *believed* position and orientation (estimated through path integration). The agent's believed position and target direction are distorted by human errors in estimation of distances and directions, mainly affecting the path integration process. In this paper we examine how human estimation errors of distance and rotation influence the decision behavior in the wayfinding process in an unknown street environment. To demonstrate the geometrical consequences for a specific test case, we use a simulated software agent which navigates in a simulated street environment.

Key words: decision making, error classification, least-angle strategy, wayfinding

1. Introduction

Humans apply various strategies when navigating to a novel goal in an unknown city environment. Some of the strategies make use of an external map, external sign information in the world, a marked trail, or they are based on communication with other navigators. Another common navigation heuristic is the *least-angle* strategy (Hochmair 2000). It can be applied in an unknown environment if the target (a salient landmark) can be perceived directly from the navigator, at least at the beginning of the navigation process. The strategy consists of choosing the street segment at an intersection that is most in line with the target direction. For the decision process, the target vector plays a major role. If the target cannot be perceived during the navigation process, the navigator's decisions are based on the believed target direction which can lead to false wayfinding decisions. In this paper we describe which human estimation errors play a role in the decision process during wayfinding and how they distort the geometry of the navigator's cognitive map.

At the beginning of the paper we give an overview of previous work in the literature that discusses navigation methods to a novel goal. In the following section we explain the least-angle strategy and a model of the navigator's cognitive map that is used to describe the navigation method. Further we give a classification of human estimation errors and show their influence on the decision behavior in a simulated environment using a simulated software agent.

The goal of the paper is to qualitatively describe relevant human estimation errors for wayfinding in an unknown environment. We presume that no reference direction, e.g., as given by the sun or landmarks, is available during the navigation process. The paper shows how estimation errors deteriorate human decision behavior in the least-angle navigation. Using a simulated agent we demonstrate that in specific situations the least-angle strategy does not result in the shortest path and other sources of external information would be helpful for the navigator to reach his goal faster.

2. Navigating to a novel goal in an unknown environment

Many definitions of the terms *wayfinding* and *navigation* exist in the literature. After Golledge et al. (2000), *wayfinding* involves selecting path segments from an existing network to determine a route between two given points, whereas *navigation* includes the processing of spatial information regarding position between origin and destination summarized as course to be followed. In Allen's opinion (G. Allen, personal communication), the two terms are close in their meaning one to the other: *Navigation*, as given in the first definition of English-language dictionaries, reflects its historical derivation (to control or direct a ship). *Wayfinding* simply expands this theme to direct movement through *any* medium, as in navigating an aircraft. For this similarity of meanings we use the terms navigation and wayfinding as synonyms in this paper.

Allen (1999) categorizes wayfinding tasks into travel with the goal of reaching a familiar destination, exploratory travel with the goal of returning to the point of origin, and travel to a novel destination. Our focus is on the third of Allen's wayfinding tasks: We discuss the errors that occur when a navigator travels to a *novel* destination using the least-angle strategy.

In some navigation strategies that are applied to reach a novel target, the knowledge of the subject's actual position is needed. Loomis et al. (1993)

classify the methods of updating position on the basis of kinematic order into position, velocity, and acceleration. In the position-based navigation, external signals which include signals from visible, audible, or odorous landmarks, indicate the traveler's position. The methods for position-based self-location require several landmarks (trilateration, triangulation) or at least one single landmark (using bearing and distance).

Without external landmarks, the traveler's navigation relies on external or internal signals indicating course and speed. If the emphasis is on velocity signals that are produced by self movement, the process is called path integration or dead reckoning. If the emphasis is on acceleration signals, it is called inertial dead reckoning. To determine the turn since the last known heading, the traveler needs to integrate turn rate over time. Determining displacement is done by integrating velocity. Periodic fixes are necessary to correct any error that may accumulate over time.

The measurement of velocity and turn rate can be based on either external (allothetic) or internal (idiothetic) signals (Mittelstaedt 1985). Allothetic information for locomoting organisms includes optic flow and acoustic flow; idiothetic information includes proprioception from the muscles and joints. Humans tend to rely most heavily on visual or, in cases of visual impairment, on auditory, vestibular, and proprioceptive information (Allen 1999). Path integration allows one to venture into unfamiliar territory for the purpose of seeking a destination and provides the traveler with an ongoing estimate of current position in an unknown environment (Gallistel 1990).

Using a *target vector* (describing bearing and distance) from the actual position or from a landmark to define a target is called *vector encoding* (McNaughton et al. 1991), whereas defining the target through storing the relationships (not necessarily vectorial information) between the target, the position, and at least one landmark is called *relational encoding*. We exclude landmarks from the discussion of the least-angle strategy (except the target) and model the target to be defined through vectorial information from the navigator's position. A target vector can be derived from a map, gained from direct perception of the target or communication with other agents, or learned from previous experience. During the navigation, the course is computed relative to the reference direction (which may be the target vector itself). Direct sensing of a reference point allows updating of the navigator's heading and course.

A further method to reach a novel goal in an unknown environment is following a route description that is given as a sequence of operations (Kuipers 1982). In such case, the target vector does not play a role in decision making. When following given instructions, landmarks help to signal where described actions, particular reorientations, have to be accomplished (Michon and Denis 2001).

3. The least-angle navigation

3.1. *The standard situation*

We imagine the following situation: A traveler walks through an unfamiliar street environment and tries to navigate towards a salient landmark, e.g., a church. His actual position is at an intersection, from which the traveler can perceive the outgoing streets and the landmark. We presume that the navigator has never visited the goal before, and that no other navigators are available that make communication possible.

Figure 1 shows a decision situation in an abstract street network. The target is assumed to be a salient landmark (node T) that can be perceived by the traveler. The traveler's position is at node N2, the edges leading out from N2 are e1 and e2. The traveler must decide whether to select e1 or e2 as the next street. Following the least-angle strategy, the navigator chooses the street segment which has the smallest deviation from the target direction. We assume this heuristic to minimize travel distance to be a common strategy among humans. In the least-angle strategy, the utility of the decision is based on a geometric attribute (i.e., the deviation from the target direction). In general, spatial decisions may also include non-spatial attributes (Timmermans and Golledge 1990), e.g., scenery or safeness of a route, or attractiveness of places (Gärling and Golledge 2000).

In Figure 1, the deviation angels between a street and the target direction are labeled α_1 and α_2. As the navigator directly perceives e1, e2 and T, he can judge whether α_1 or α_2 is the smaller angle. As $\alpha_1 < \alpha_2$ (and therefore, e1 is more in line with the target than e2) the navigator chooses e1 as the next street.

The decision is made in a local reference system, where the angles, α_1 and α_2 indicate a directional difference between the vectors e1, e2 and N2-T. As the least-angle strategy is a rule of thumb, a path chosen by the least-angle method is locally optimal, but does not have to be part of a globally optimal route. In our view, we treat a decision as successful or correct if it provides the locally optimal solution (is most in line with the target direction).

The least-angle navigation, if used in an unknown environment, is an online routing algorithm. Online algorithms model the routing behavior in an unknown network, where information about the network must be obtained through perception. The COMPASS routing algorithm for computer networks (Bose and Morin 1999; Kranakis et al. 1999), which is applicable in plane

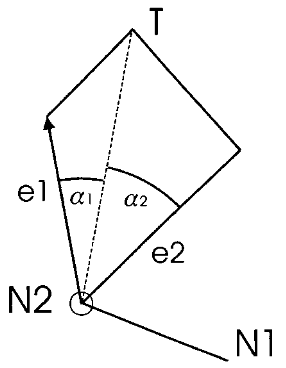

Figure 1. Applying the least-angle strategy at a decision situation.

graphs, always moves a packet to the node that is most "in line" with the target. Thus, the COMPASS algorithm and the least-angle navigation are comparable regarding their decision strategy. In contrast to the COMPASS algorithm, the least-angle strategy describes *human* navigation behavior, and therefore human estimation errors need to be considered in its description. Bose and Morin (1999) describe the functionality of the COMPASS algorithm which depends on the geometry of the graph. For the discussion of human estimation errors with the least-angle strategy, we restrict the discussion to a "standard" situation in a "standard" environment, where the navigator reaches the goal.

3.2. *Using a cognitive map*

We propose that the decision process in the least-angle strategy is based on a mental representation of the environment. Through visual percepts and idiothetic information, the navigator constructs the cognitive map, which contains directions, angles, and vectors. The navigator's errors in perception (i.e., passed distances and performed turns) lead to errors in the cognitive map. For a more abstract elaboration of the task of this work – the discussion of the geometrical nature of errors and their influence on a decision situation – we use the term *agent*, which is a concept from Artificial Intelligence (Wooldridge 1999) and functions as a conceptual paradigm to represent the human navigator.

To model the influence of human estimation errors we use a *two-tiered* reality and beliefs representation, in which reality *(facts)* and the agent's cognition *(beliefs)* are represented separately. Following an AI tradition, the agent's environmental representation is called *belief* to stress the potential differences between reality and the agent's representation (Davis 1990).

If errors in the estimates are too large, the decision can be wrong – the continuous error passing a threshold leads to an error in a discrete direction. Concerning the agent's decision making, we model fact and belief as consisting of the agent's position and heading. Incorrect decisions are caused by two principal errors in the cognitive map:

(1) A *shift* between the agent's real and believed position relative to the target
(2) A *rotation* between the agent's real and believed orientation (heading) relative to the target

A classification of those estimation errors which lead to a distortion of the agent's believed position and heading is given later.

3.3. *A more complex situation: a hidden target*

Compared to Figure 1, a more complex situation in the least-angle navigation is the following: As in the previous situation, the traveler navigates through an unfamiliar environment of narrow streets in a city environment, the traveler's view is limited to the intersecting streets at a crossing and the buildings along the street. Contrary to the previous example, a direct view to the target is given only at the beginning of the navigation process (node N1). At node N2 (the agent's position), we assume that *no* direct view to the target is available (Figure 2). As the agent does not use an external map, he applies path integration to assess his position relative to the starting point, and from which he can deduce position and heading relative to the target. For the navigator, the target position is represented in his egocentric reference system and accessible from his working memory, even when the objects are not visible. Humans seem to do this by constructing a spatial mental framework from extensions of the three body axes and associating objects to the axes (Tversky 2000). Due to estimation errors in path integration, the believed target direction is distorted.

In Figure 2 the believed target direction is directed towards T′. At node N2, the agent determines the smallest α among all outgoing streets for the locally best solution. The correct deviation angles (between the streets and the correct target direction) are labeled α_{i}, the angles in the agent's belief are labeled α'_{i}. In the error free case, α_1 is the minimum angle and e1 is chosen for the next street. Using the believed target direction, α'_2 is the minimum angle, and e2 is falsely chosen. We give a geometric criterion that describes if estimation errors and the resulting distortions in the cognitive map cause an incorrect wayfinding decision or not.

We define α_{c} as the minimum angle between the correct target direction T and the outgoing streets, and α'_{e} as the minimum angle between the estimated target direction T and the outgoing streets. *If the indices c and e are different for the correct and the believed target direction, an incorrect wayfinding*

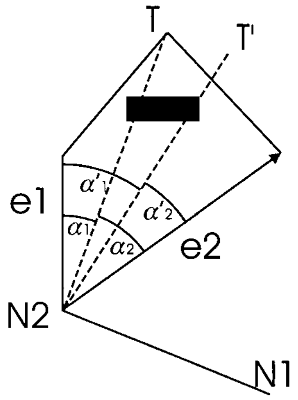

Figure 2. False decision caused by an object blocking the direct view.

decision is made. We can write the criterion as

$$c \neq e \text{ for } \alpha_c = \alpha_{\min}(T), \alpha'_e = \alpha'_{\min}(T') \qquad (1)$$

where *min* means the angle with the smallest *absolute* size. As the indices between the minimum angles in the given situation in Figure 2 are different, an incorrect wayfinding decision is the consequence.

4. Classification of estimation errors

4.1. *Interpreting errors from two perspectives*

Position and heading relative to the target are the parameters that influence the decision. If the navigator perceived the target directly, we assume that the agent estimates the perceived distance to the target correctly, so that the agent's position (relative to the target) and heading are error free in the local reference system. In lieu of the position-target vector, the agent's position can also be given as a fictive vector from a starting point to the actual position.

A distorted position and orientation relative to the origin of travel can be expressed within two reference frames: An allocentric reference frame and the agent's egocentric reference frame. In an allocentric reference frame, the viewpoint is taken from above, and an agent's distorted believed position and heading can be interpreted as path integration error. It emerges from errors in estimating walked distances and turns. The allocentric perspective allows to plot the agent's believed position and heading and compare it with the correct position and heading. The target stays invariant during the navigation process. In special situations, a position error and heading error together can neutralize so that the agent's believed orientation towards the target seems correct.

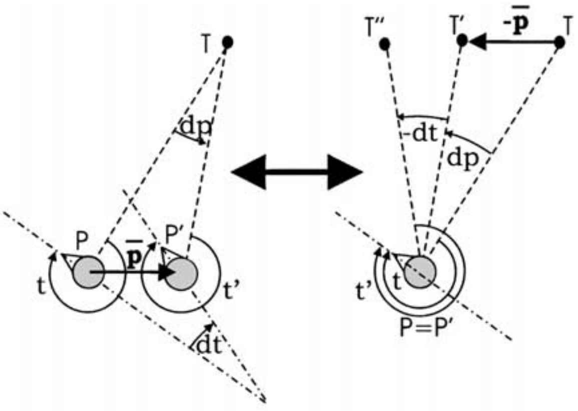

Figure 3. Transformation of correct and believed target vector from allocentric into egocentric reference frame.

In the agent's local reference frame, the error during path integration does not appear as an error in the believed position and heading. Instead, in the navigator's view, the target "moves" from where it should be if the believed position and heading where correct. The path integration error is visualized through a shifted and rotated position of the target in the navigator's egocentric reference frame. The influence of human estimation errors on the target vector can be transformed from one perspective into the other.

Figure 3 shows the same situation in both reference frames. Both views visualize the agent's real and believed position and heading. The left part of Figure 3 visualizes an allocentric view of the situation, whereas the right part visualizes an egocentric view of the same situation. In the left figure, the two errors (position and heading) are labeled as follows: The vector from the agent's correct (P) to the believed (P′) position is called **p**, the angle between the real and believed orientation (t′–t) is called dt. For the transformation of errors from the allocentric to the egocentric reference frame, two steps are to be made:

(1) shift the target position for the vector –**p**
(2) rotate the target for the angle –dt

In the egocentric reference frame, step (1) leads to T′ (right figure). The figure shows that the position error (**p**) operates on the believed target direction like a rotation (dp). After performing step (2), the believed target position in the egocentric reference frame is T″.

In our classification of errors we use the allocentric reference frame. This approach allows to view errors in percepts as errors during the path integration process. This concept seems more natural, as the path integration error is caused by the agent and not by the target.

Table 1. Elements of fact and agent's belief

fact	belief
position	position′
heading	heading′

Table 2. Agent's activities, with their parameters and role for position and orientation

	fact	belief
turn	$\beta \Rightarrow$ t	$\beta \Rightarrow$ t′
move	(d, t) $\Rightarrow$ Pos	(d′, t) $\Rightarrow$ Pos′
move curved	(d, t, c) $\Rightarrow$ Pos, t	(d, t, c′) $\Rightarrow$ Pos′, t′

4.2. *Classification*

In this section we give a classification of human estimation errors that influence decision making during wayfinding. As mentioned before, the agent's believed position and heading are the significant elements for decision making. Table 1 visualizes fact and belief of a navigating agent, split into position and heading. The elements with ′ are part of the agent's belief and are therefore potentially distorted.

The agent performs several actions during his navigation process: turn, move straight, or move along a curve. Each of the activities needs one or several parameters to be completely described. If the parameters are estimated incorrectly by the agent, the action results either in a position error, an orientation error, or both. Table 2 connects the actions with the resulting changes in the cognitive map. The right column of the table shows how incorrect parameters distort the agent's believed position or heading.

The first function describes an agent's *turn* (rotation) that results in a new heading t. Parameter β is the rotation angle. If β' is distorted, the consequence is a distorted heading t′. We call the distortion of β' the *rotation error*.

The second function describes an agent's (straight) *move* which results in a new position. To describe a straight move, the azimuth *t* and the distance *d* are required. A distortion of parameter d′ results in a distorted position. The distortion of d′ is called *distance estimation error*. The navigator's believed (correct or distorted) orientation stays unchanged through a straight move.

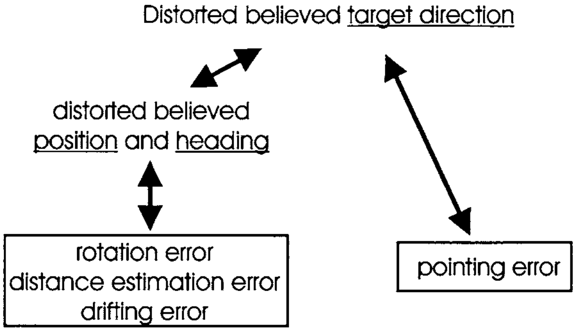

Figure 4. Classification of estimation errors that cause a distorted believed target direction.

The third function is a generalization of the *move* function. It describes the move along a curved line. The parameter *c* stands for the curvature of the street segment that is passed. If $c > 0$ (the agent moves along a curve), then the action results in a changed position and heading, otherwise only the position is changed. If $c' \neq c$, we call this the *drifting error.* A false believed curvature c' can be overloaded with a distorted initial heading t' and a distorted passed distance d'.

The distortion of the parameters in Table 2 (rotation, distance, and curvature) refers to the path integration process. A fourth error that is independent of any of the listed activities is the *pointing error* (ε)*.* It describes the error that occurs when performing a perceptually directed action such as pointing to the target direction. The pointing error occurs if a target is first perceived, and then the direction of the hidden target must be recalled from the working memory and indicated. The pointing error is not cumulative, which means that a pointing error at one position does not influence the decision behavior at the following positions of the wayfinding process.

Figure 4 unifies the errors that cause a distorted believed target direction: Rotation error, distance estimation error and drifting error cause a distorted believed position and heading (see Table 2) which results in a distorted believed target direction in the agent's egocentric reference frame. The pointing error is independent of path integration and deteriorates solely the believed target direction of the navigator.

4.3. *Visualization of estimation errors*

In this section, we visualize each of the four given estimation errors. Several estimation errors can occur simultaneously in navigation situations, e.g., distance estimation error and wrong curvature. For an easier understanding of the figures we are visualizing the consequences for each single error. Believed elements are notated with primed symbols.

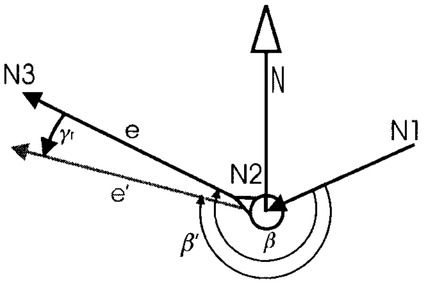

Figure 5. Rotation error caused by distorted estimation of turn at N2.

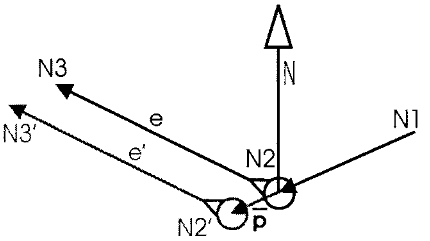

Figure 6. Distance estimation error causing a distorted believed position.

Figure 5 visualizes the consequences of a *rotation error* – a distorted believed heading. The agent moves from node N1 to node N2, and then turns towards node N3 in street e. Through internal signals the agent estimates the rotation angle β' which is distorted by the rotation error γ_r. The distortion results in a false believed heading along e′. A rotation error is cumulative, i.e., rotation errors sum up with each turn.

With a *distance estimation error*, passed distances are perceived either too long or too short. The consequence of a distance estimation error on a straight move is a distorted believed position, as demonstrated in Figure 6. On a curved move, the distance estimation error also results in a distorted believed heading.

Concerning the *drifting error* we look at one specific configuration of the parameters c and c′ in Table 2: If the curvature $c > 0$ and the believed curvature $c' = 0$, the agent walks along a curve but believes to walk straight. This situation may occur if the agent moves a long distance in a large scale space, without any reference direction or landmark, e.g. in the desert when the sky is cloudy.

In the visualization of such a situation in Figure 7, the agent believes to move along e′ to P′ although his real move is along curve e to P. The agent's heading after the move has the azimuth t, his believed heading is t′. The symbol γ_d indicates the angle between believed and real heading, the vector **p**

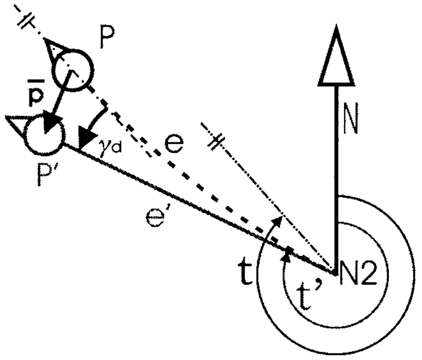

Figure 7. Drifting error causing a false believed heading and position.

visualizes the shift between the agent's believed and real position. The effects of the drifting error – distortion of position and heading – are cumulative.

If environmental constraints prevent the agent from walking along a curved path, e.g., if the agent moves along straight streets in an environment, no drifting error will occur. This is the reason why we set the drifting error (parameterized by the curvature c) of a navigating agent in a city environment to 0.

In Figure 3 we have shown the influence of the position error (dp) and orientation error (dt) on the target direction. The pointing error ε is independent from position and heading. It is added as a rotation to the believed target direction. Figure 8 visualizes the influence of all potential estimation errors on the believed target direction, and in consequence, the influence on the deviation angle α. Line e indicates an outgoing street segment from P. The correct deviation angle between the outgoing street and the correct target T is labeled α. Due the distorted believed target direction of T‴, the distorted deviation angle α' is smaller than α, which may lead to an incorrect wayfinding decision.

The relation between α and α' is

$$\alpha' = \alpha - dp + dt - \varepsilon \tag{2}$$

All of the parameters in Equation 2 *(*$\alpha, dp, dt, \varepsilon$*)* can be positive or negative.

4.4. *Direct perception of the target*

If the target can be directly perceived during the navigation process, the cumulative errors (i.e., orientation error and position error) disappear. Perceiving the target allows the agent to perceive α directly and make a correct decision, whereas a blocked view forces the agent to estimate the deviation angle α. The complete cycle of perception, decision, and action is

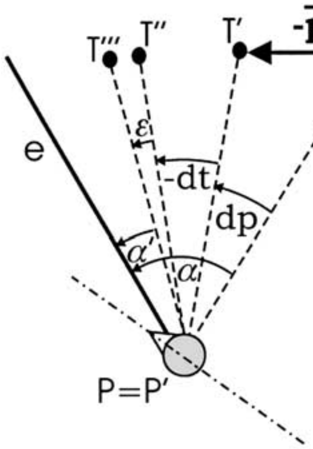

Figure 8. Distortion of deviation angle α through pointing error ε, distorted believed heading (dt), and distorted believed position (dp).

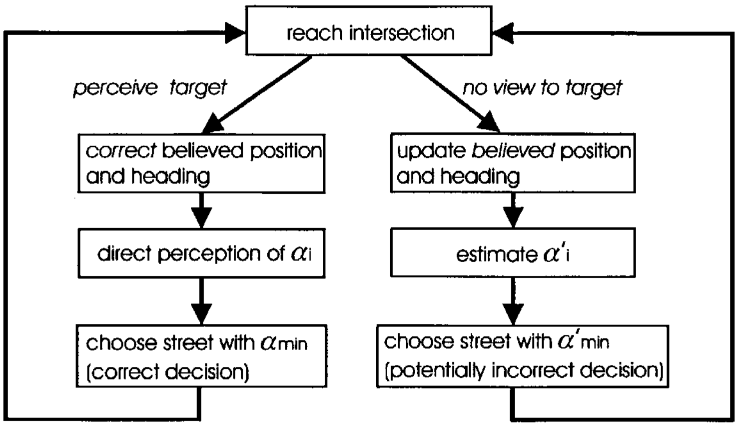

Figure 9. Substeps of navigation process.

visualized in Figure 9. The left branch shows the substeps of the process for a directly perceived target, the right branch for a hidden target.

5. Qualitative assessment of human estimation errors

The scope of this section is to qualitatively describe human estimation errors during navigation. The assessment of tendencies of errors functions as basis for the agent simulation in the next section. As the test situation of the simulation has not been verified with human subjects testing in the real environment so far, we rely on experiments of the literature for the tendencies of errors. Many experiments about triangle completion and path integration have been performed by researchers to determine rotation, distance estimation, and drifting error. Most of these experiments were performed in conditions not adequate to those that are used for the least-angle strategy. An example are experiments that were conducted in open spaces without obstacles, whereas the least-angle strategy is performed in environments where navigation is constrained by buildings. Other experiments about pointing error and triangle completion used blindfolded subjects (e.g., Klatzky et al. 1990; Loomis et al. 1993), of which the results are also not exactly reusable for our simulation.

Other human subjects tests were conducted in geographical spaces, hence, they are basically close to the situation of the least-angle navigation. But these experiments focus on other aspects than geometrical estimation errors of humans. For example, experiments focus on the decision criteria human navigators apply in street networks (Golledge 1995; Bailenson et al. 2000), or determine the role of landmarks for route description (Michon and Denis 2001).

5.1. *Qualitative assessment of the rotation error*

Triangle completion (e.g., Passini et al. 1990) is a common method to test human path integration abilities.

The method can be used to qualitatively determine encoding errors of turns and distances. An extensive series of experiments with a return-to-origin task was realized by Loomis et al. (1993). Testing subjects were led outbound along two legs of a triangle and then attempted to return to the origin. The result showed that small angles between the first and second leg were estimated too large, whereas large angles between the first and second leg were estimated too small. This pattern of systematic errors suggests the presence of systematic errors in the underlying path integration process (Loomis et al. 1993).

Sadalla and Montello (1989) examined the memory for turns of varying angularity encountered during pathway traversal. In their experiment, the relative absence of external cues maximizes the likelihood that subjects rely on egocentric reference systems. The subjects walked eleven 8.3 m pathways, each containing one turn ranging from 15 degrees to 165 degrees from the direction of forward motion. After each pathway, subjects were asked to estimate the angle traversed. The hypothesis of the experiment is that subjects employ a pair of orthogonal reference axes when judging the magnitude of traversed angles. The results of the experiment show that paths containing angles near 0°, 90°, and 180° from the forward motion were the least disorienting and were most accurately remembered. Errors increased as angles diverged from these orthogonal coordinates. Figure 10 visualizes a part of the

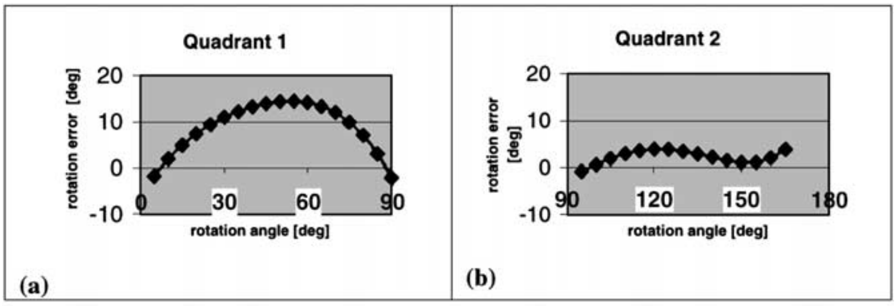

Figure 10. Interpolated curves for the rotation error from 15° to 90° (a) and from 90° to 165° (b), after Sadalla and Montello (1989).

results of the experiment showing the interpolated error curves for quadrant 1 and quadrant 2. Positive values in the figure indicate a distortion of the angle towards a right angle.

The data reveal the tendency to estimate all angles as more like 90° than they actually are: Turns between 0° and 90° are overestimated, while turns between 90°, and 180° are underestimated. This tendency corresponds to the result of Loomis et al. (1993). Due to a range of angles from 15° to 165° we decided to use the data of the experiment of Sadalla and Montello to simulate the rotation error. These data function as an approximation to a rotation error in a real street environment. In a geographical space, additional information gained through visual perception of buildings or reference objects may change these curves.

Other experiments that made use of map-drawing methodologies support the theory that humans distort angles towards 90° (e.g., Lynch 1960; Byrne 1979). Experiments in geographical space (Montello 1991; Golledge et al. 2000) revealed that the estimation of the goal direction towards a hidden goal after traveling a path gets worse with the number of irregular (< > 90°) curves along the path.

5.2. *Assessing the distance estimation error*

Experiments conducted in geographic space revealed a high number of different influences that distort human perception of distance and route-length. The following items list only some of these influences:

- Downs and Stea (1973) found that winding roads are perceived longer than long and continuous roads.
- Distances appear longer in crowded, filled areas than in empty, non crowded areas (Thorndyke and Hayes-Roth 1981).

- Distance estimates are often exaggerated when there are barriers along a route (Kosslyn et al. 1974). Positive effects, on the other side, make distances appear shorter (Golledge and Zannaras 1973).
- Perceived information or features along a route (Sadalla and Staplin 1980) makes distances appear longer.

A specific model may explain the influence of some of these facts: Huttenlocher et al. (1991) propose a model that distinguishes between two different types of encoding spatial relations form perceptual experience: *Fine-grained coding*, which represents a fine-grain stimulus, is treated as unbiased. Contrary to this, *categorical coding* is subject to systematic bias, typically according to prototypes (a central value in the presumed pattern of values across the region it encompasses). If the representation of the perceived information is exact, information at the two levels is perfectly coordinated, whereas with inexact representations, the two levels provide non-redundant information which can be combined in making estimates. Within the estimation process, category information may be used to adjust a recollected fine-grain value in reporting: People may adjust recalled fine-grain values to lie in the range subsumed by a category (the process is called *truncation*). Both, distortion of perceived distances and angles may be explained as a combination of the biased prototype effect and the inexactness of fine-grained coding. Distances may be distorted towards prototype values, e.g., vistas, units of a hierarchically organized space, or boundaries between units of a space. The estimation of rotation angles uses, for example, the front-back and left-right axes as prototypes. The latter supports the theory that subjects regularize perceived angles.

Several indoor experiments were conducted to determine human distance and directional error in the triangle completion, (e.g., Glasauer et al. 1994; Philbeck et al. 1997). Experiments do not show clear tendencies for distance estimation errors, thus, no experimentally verified error can be used for our simulated model. We take a hypothetical error of 10% for our simulated agent, which means that the agent mentally “walks” 10% more in the simulated environment than he actually does.

5.3. *Assessing the pointing error*

In contrast to the return-to-origin task where the subject needs to keep track of the origin with respect to himself, perceptually directed action involves maintaining an internal representation of locations other than the origin (Loomis and Klatzky 1999). In a perceptually directed task, the subject perceives the target, either through vision or sound, and then attempts, with no further perceptual input about the target, to indicate the target location using some motoric response. Many experiments with regard to perceptually directed action have been done. They include visually directed walking (Thomson 1980), visually directed reaching (Foley 1977), and visually directed throwing (Eby and Loomis 1987).

In the literature, two different concepts of the relation between vision and action can be found. The first concept is the idea of *perception-action* coupling (Philbeck et al. 1997), which assumes that certain aspects of the changing visual stimulation are tightly coupled to particular aspects of the action. The other concept hypothesizes that *internal* representations, resulting from sensory, perceptual, and cognitive processing, have a great influence on the explanation of at least some actions.

An experiment of Philbeck et al. (1997) provides evidence that perceived location is an invariant in the control of action by showing that different actions are directed towards a single goal location in space. A pointing error that is found with respect to one specific type of perceptually directed action is therefore representative for those of many other perceptually directed activities. The paths traversed in this experiment show the subjects' tendency to expect the target more right (clockwise) than it is, even when walking on one leg only. This qualitative result is used to define the pointing error (ε) in our simulated agent.

5.4. *Summary*

In the geographic space, many facts influence human estimation of distances, turn angles, and pointing directions. Due to the high number of influences, it is difficult to give correct values for human perception errors during navigation in a street environment without human subjects testing. To design the errors of the simulated agent we take the tendencies of errors determined in experiments of the literature. We assume the following errors to be made in a city navigation, and implement them in the simulated navigating agent:

- *rotation error*: follows Figure 10 (range between –1 and +15 degrees)
- *distance estimation error*: estimate passed distances 10% too long
- *drifting error*: curvature = 0 (no error)
- *pointing error*: 3° (clockwise)

6. Elements in the navigation simulation

In the following sections we describe a simulated agent's decision behavior during the navigation process in a simulated street environment. The virtual test area is a part of the inner city in Vienna. In the simulation, we follow the agent's navigation steps on the way to his target. The decision situations are visualized on photographs (showing the perspective of the navigator) as well as on maps.

We realized the navigation model through algebraic descriptions using classes with functions. The model contains abstractions of the street network and a navigating agent that makes his decisions using the least-angle strategy. Estimation errors of the agent correspond to those defined in the previous section. We implemented the algebraic specifications in the functional programming language Haskell (Thompson 1996). The simulation allows to assess the influence of estimation errors on the chosen route in the given situation. The complete code can be downloaded from *ftp://ftp.geoinfo.tuwien.ac.at/hochmair*.

6.1. *Structure of the navigating agent*

An agent is anything that can be viewed as perceiving its environment through sensors and acting upon the environment through effectors (Russell and Norvig 1995). A software agent, as used in our simulation, "perceives" its environment through encoded input data streams, and gives a result as data streams which describe the result of the decision process and the agent's performed activity. Several types of software agents are introduced in (Russell and Norvig 1995; Wooldridge 1999), ranging from simple reflex agents up to utility-based agents. We use a *utility based agent with state* for our model. This type of agent can store information about himself and the environment in an internal data structure. Further, the agent uses a utility function that allows him to determine the value of all potential activities at a decision point, i.e., an intersection. In our simulated agent, the utility function evaluates the deviation angles α_i between the outgoing streets of an intersection and the target direction. The lower the deviation α for a street segment, the higher is the value of the street for the agent.

The simulated model contains separate representations of the agent's actual position and heading (fact), and the agent's *believed* position and heading (belief). This concept corresponds to the two-tiered reality and beliefs computational model.

6.2. *Environment*

Russell and Norvig (1995) classify environments after their properties. The environment in our approach is *inaccessible* as the agent's sensors cannot permanently perceive all aspects that are relevant for the decision process: The agent does not have knowledge about the complete environment's structure, but perceives only outgoing streets from the actual intersection. Further, the target which is important for the decision process, is not visible from

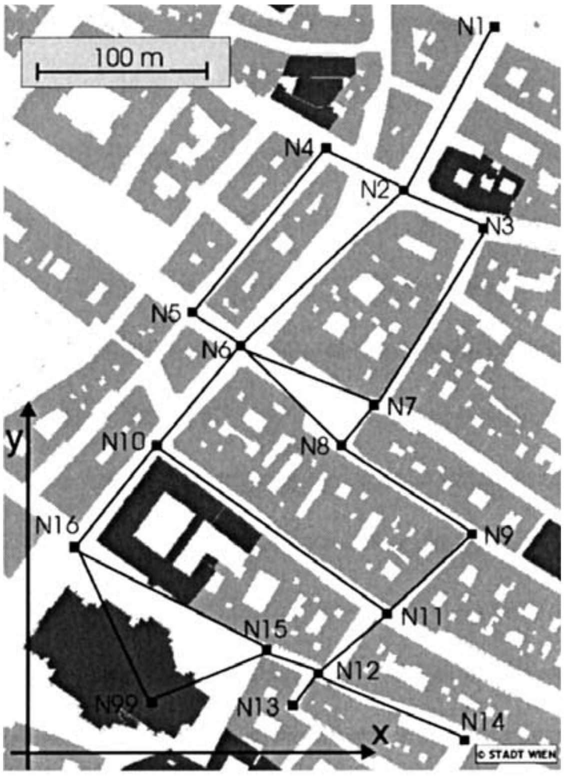

Figure 11. Testing area for the simulated navigation process.

most decision points. The environment is modeled to be *static* throughout the whole navigation process, as the geometry of streets and places does not change during the navigation process.

In the data model of the simulation, intersections of the environment are abstracted as nodes, consisting of an identifier and a two dimensional position; streets are abstracted as edges between two nodes. The complete street network is united in an undirected graph. A map of the testing area is given in Figure 11. The nodes are indicated with labeled dots, the connecting streets are visualized as black lines. The figure is oriented along north-south, a scale bar in the upper left corner gives an impression of the distances. The agent's starting point is N1, the "Schwedenplatz" in the northern part of the inner city; the goal is the steeple of St. Stephen's Cathedral (N99), which is situated in the southern part of the map. A direct view to the target is given at N1, N15, N16, and at the target N99 itself.

In the real world, humans move continuously along a street. During a continuous move, situations may occur, where the agent perceives the target directly, e.g., through a gap in the row of buildings along the street. A direct perception of the target erases the agent's errors of believed heading and position and influences the decision behavior at the next intersection. As in our model the agent's potential positions are discrete on the given nodes, the agent's moves from node to node are implemented as discrete steps, not as a continuous move. Therefore we make the simplification of the real-world case, that direct views to the target are possible at nodes only.

6.3. *Creating the agent*

In this section we build a simulated navigating agent. Among the agent's parts, *belief, next node, goal*, and *history* represent the agent's state.

- *fact*: a tuple consisting of the agent's position and heading. The agent's position is stored as node. Each node consists of an id (*Int*), a coordinate tuple (*Float, Float*) and a boolean variable that says if the target is visible from the node or not (*Bool*). The heading is expressed by a *Float* value and given in radiant.
- *belief*: a tuple consisting of the agent's believed position and heading (same data types as used in *fact*)
- *next node*: the node that the agent plans to visit next. This is the result of the decision process.
- *goal*, represented as node
- *agent's error profile* which describes the agent's quantitative estimation errors. The 4-tuple consists of the factor for the distance estimation error (*Float*), the pointing error in radians (*Float*), a boolean variable that determines if the agent makes a rotation error (following Figure 10) or not (*Bool*), and the curvature of movement (*Float*)
- *history* which includes previously visited nodes; specified as a list of nodes

The agent is designed through the definition of a data type *Agent* (see the code example below), which consists of data types describing the agent's state. The agent is called *tim*. The agent's starting position (*pos*) is at node N1, the heading is set 0 (it could be any other value, too). As the target can be perceived directly from node N1, the agent's believed position (pos') and believed heading ($heading'$) are correct at the beginning of the navigation process. Therefore these parts correspond to the actual position and heading. The next node (*nn*) is not defined at the beginning, therefore set to a dummy node *n0*. After the first decision process, the next node will we updated. For the first test run, we create an error free agent. Therefore, in the 4-tuple of *err*, the distance estimation scale is set 1, the pointing error is set 0, the boolean flag for the rotation error is set False, and the curvature for the drifting error is set 0. The history list (*hi*) of visited nodes contains the start node.

```
data Agent = Agent Fact Belief Next Goal Errors History

tim = Agent (pos, heading) (pos′, heading′) nn goal err hi
 pos = n1               – actual position
 heading = 0            – actual heading
 pos′ = n1              – believed position
 heading′ = 0           – believed heading

 nn = n0                – next node
 goal = n99             – goal node
 err = (1, 0, False, 0) – error profile
 hi = [pos]             – history list
```

We use four functions to describe the agent's activities. The functions are included in the class called *State* (see next code example), where the class signature describes the parameters of the included functions. In the signature, data types and parameters before the last "→" represent the argument types, and the last element represents the type of the result.

The *turn*-function takes a *Graph* (the environmental graph) and the parameterized *agent* as input. Perceiving the adjacent nodes of his actual position, the agent decides for the next node and turns his facing towards the next node. The believed turn can be distorted by a rotation error and result in a distorted believed heading. The result of the function is an agent with the updated real and believed heading.

The *move*-function makes the agent walk to the next node which has been determined in the decision function – the agent walks in the azimuthal direction of his actual heading until he reaches the next node. The function results in an agent with updated position and heading. For a curvature < > 0 the *move* function represents a curved move (3rd function in Table 2).

The *step*-function is a function composition of *turn* and *move*: The agent first turns and then moves.

The *wayFind* function performs a complete navigation process from the start node to the target, using the *step*-function recursively.

```
class Sate agent where

turn : : Graph → agent → agent
move : : agent → agent

step : : Graph → agent → agent
wayFind : : Graph → agent → agent
```

Two conditions are used to stop the recursion in the *wayFind* function: The first condition is identity of the agent's actual position and the goal node (the goal is reached). The other condition is given by a limit of navigation steps.

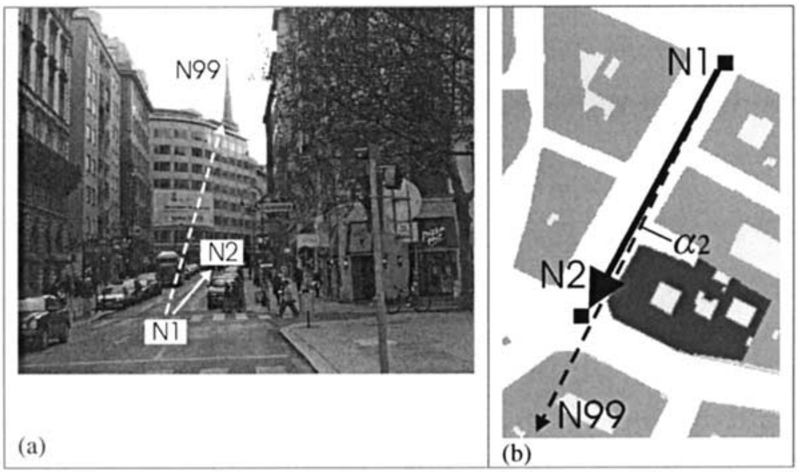

Figure 12. Navigation situation at start node N1. The target can be directly perceived.

This is realized through checking the history list for its number of included elements. If the limit of navigation steps is reached, the agent obviously got caught in a loop of nodes.

7. Simulation

7.1. *Error free agent*

For the first simulation, we follow the route of an error free agent. Figure 12 shows the situation at start node N1 from the agent's view (a) and in a street map (b). The salient landmark St. Stephen's Cathedral (N99) can be seen from this location. As the agent has a direct view to the target, the deviation angle α_2 between the outgoing street segment and the target direction can be determined correctly. As N1 has only one outgoing street segment, α_2 is the smallest deviation angle. N2 is chosen to be the next node.

Applying the *step* function shows the agent's step from N1 to N2. The term *net* in the input line describes the environmental graph of Figure 11. The result of the input shows the agent's state, including *fact* and *belief* of the two-tiered computational model. The first line (fact) shows that the agent has moved to N2 (*pos*). The second line indicates the agent's beliefs. Due to an error free agent, position and believed position, and heading and believed heading are identical. *History* shows the visited nodes N1 and N2.

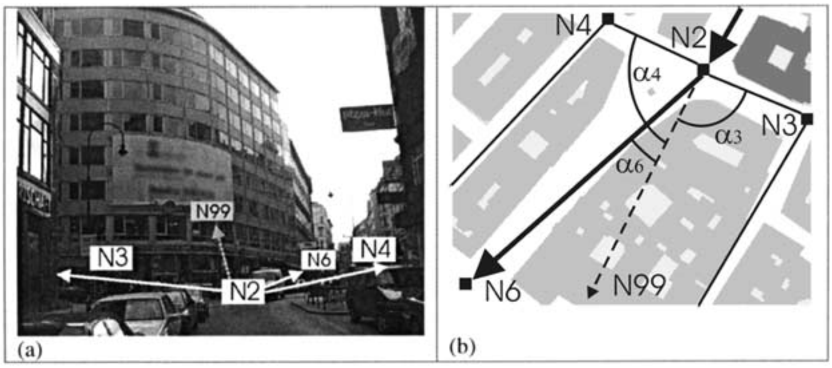

Figure 13. Decision situation at N2, where the target is hidden by an object (a). The agent decides to move to N6 (b).

```
test input > step net tim

FACT pos N2 Pos 224.0 333.0 Heading 3.64522
BELIEFS pos N2 Pos 224.0 333.0 Heading 3.64522
History [N2, N1]
```

At N2 there are three outgoing street segments from the agent's view (Figure 13), one to the left (N3), one to the front (N6), and one to the right (N4). As the goal N99 cannot be perceived directly, the agent determines the deviation angles α_i from his believed position and heading.

As α_6 has the smallest deviation from the target direction to N99 (Figure 13b), the agent decides to choose the street towards N6. We reapply the *step* function on the previously moved agent:

```
test input > step net (step net tim)

FACT pos N6 Pos 129.0 245.0 Heading 3.96522
BELIEFS pos N6 Pos 129.0 245.0 Heading 3.96522
History [N6, N2, N1]
```

The result shows that the agent's actual position is N6 and that his beliefs are correct. The *History*-list shows the visited nodes N6, N2, N1.

This stepwise procedure can be continued until the goal is reached. Another method to simulate the complete navigation process is to apply the *wayFind* function that applies navigation steps recursively.

```
test input> wayFind net tim

FACT pos N99 Pos 74.0 36.0 Heading 2.61767
BELIEFS pos N99 Pos 74.0 36.0 Heading 2.61767
History [N99, N16, N10, N6, N2, N1]
```

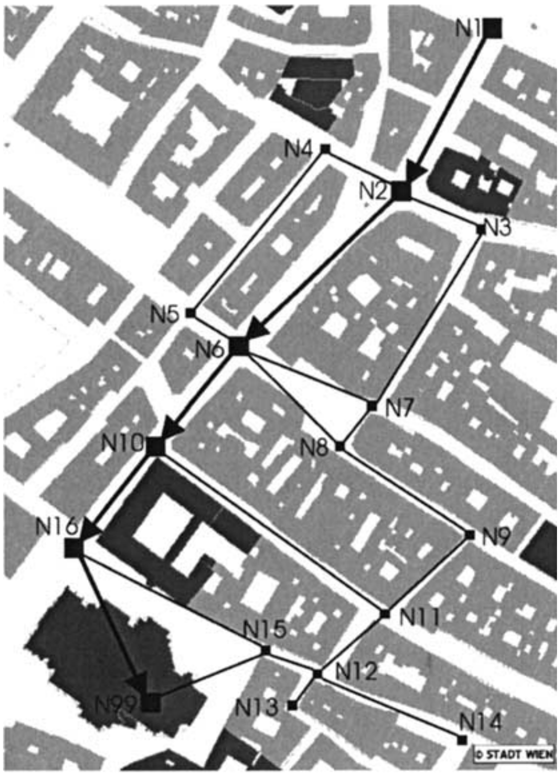

Figure 14. Chosen path of an error free agent.

The result shows that the agent has successfully reached his goal N99. His believed position and heading are correct. The History list shows the agent's traveled path. It is visualized in Figure 14 by arrows.

7.2. *Erroneous agent*

To determine the influence of estimations errors on the wayfinding process in the test environment, we create an erroneous simulated software agent *pete*, that has the following perceptual errors:

- distance estimation factor: 1.1
- pointing error: clockwise 0.05 rad (which corresponds to 3 degrees)
- rotation error: True
- curvature: 0

The errors mean that

- the agent estimates passed distances as 10% too long

- the agent believes mentally updated target positions to be 3 degrees more to the right than they are (in addition to the consequences of rotation and position error)
- the agent's estimations of the turn rate are distorted (following Figure 10)
- the agent believes passed straight street segments to be straight (which is obviously correct)

The agent is created similarly to the error free agent, except for a changed error profile. The navigation process starts at N1. Again, we apply the *step* function to see the first step of the simulated agent:

```
test input > step net pete

FACT pos N2 Pos 224.0 333.0 Heading 3.64522
BELIEFS pos N2 Pos 218.6 323.2 Heading 3.64522
History [N2, N1]
```

The result shows that the agent's believed position (and therefore the agent's egocentric reference frame) is shifted for an amount of about 15m compared to the real position. This can be seen by comparing the coordinate tuples of *pos* in the *FACT*-line with those of *pos* in the *BELIEFS*-line. The position error is caused by the distance estimation error.

In Figure 15, gray elements show the agent's believed position and the connected street elements in an allocentric reference frame. The deviation angles α'_i between outgoing streets and the believed target direction are computed from the believed position and heading. Although slightly changed, α'_6 is the least angle at N2$'$ for the erroneous agent, too. Thus the street segment towards N6 is chosen for further navigation.

```
test input> step net (step net pete)

FACT pos N6 Pos 129.0 245.0 Heading 3.96522
BELIEFS pos N6 Pos 103.567 239.188 Heading 4.0816
History [N6, N2, N1]
```

Reapplying the *step* function, the agent moves to N6. The agent's believed position and heading are distorted (Figure 16). The distorted heading is caused by the rotation error at N2: As the turn rate < 90°, the rotation angle β_2 is overestimated by the agent (compare Figure 10a). The distorted believed position is caused by the rotation error and the distance estimation error. The gap between correct and believed position increases to approximately 25m.

At N6, we reapply the *step* function. The result shows that the agent moves to N10. The position error at N10 has further increased, whereas the difference between real and believed heading has slightly reduced, due to a small (overestimated) left turn at N6.

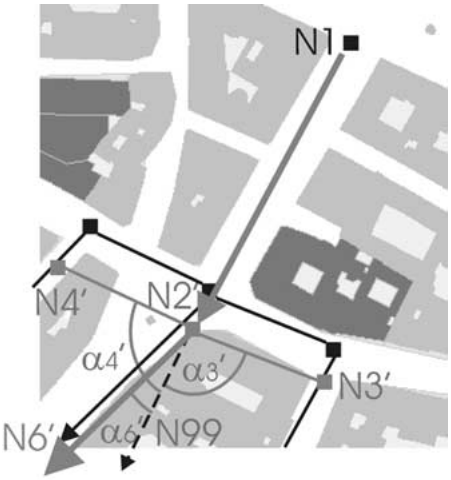

Figure 15. The erroneous agent's believed position at N2 after the first step

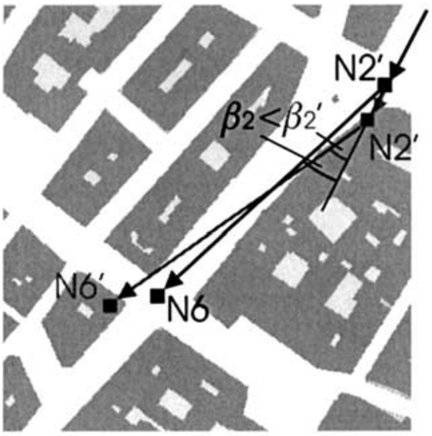

Figure 16. Agent's distorted believed position and orientation at N6.

```
test input> move net (move net (move net pete))

FACT pos N10 Pos 76.0 180.0 Heading 3.82564
BELIEFS pos N10 Pos 38.122 174.165 Heading 3.93022
History [N10, N6, N2, N1]
```

Figure 17 visualizes the decision situation at N10 from a bird's eye view. There are two outgoing streets, one leads to N16, the other one to N11. Corresponding to Figure 8, Figure 18 shows the influence of the pointing error and the agent's distorted believed position (N10′) and heading, on the agent's believed target direction. Position error **p** causes a rotation of the

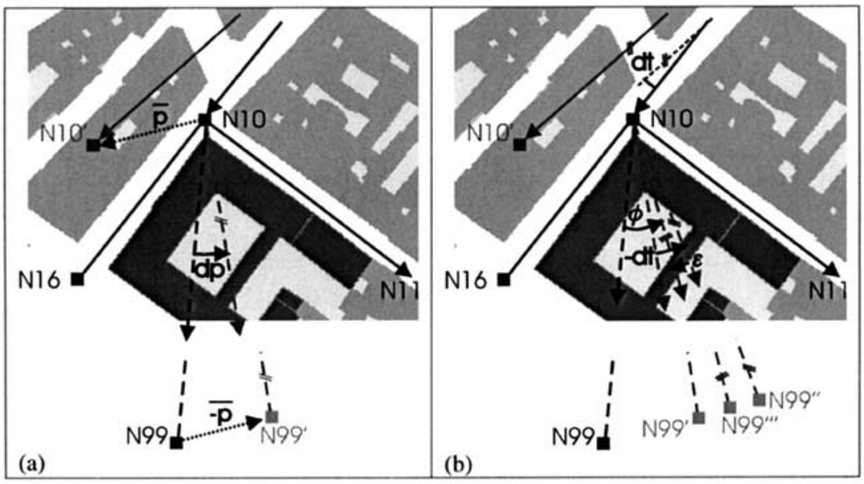

Figure 17. Distortion of the believed target direction: through distorted believed position (a), and distorted believed orientation, and pointing error (b).

target for the angle dp to N99′ (Figure 17a). Further, the believed target vector direction is rotated for the negative orientation error (–dt) to N99″ (Figure 17b). An additional pointing error ε in clockwise direction moves the target direction towards N99‴. The resulting overall error between the correct and the believed target direction is labeled ϕ.

In Figure 18, the deviation angles between the correct target direction and the outgoing streets are labeled α_{16} and α_{11}, the believed angles are labeled α'_{16} and α'_{11}. The figure shows that $\alpha_{16} < \alpha_{11}$, therefore the error free agent decides to take the street to N16. In contrast to this, $\alpha'_{11} < \alpha'_{16}$, – the order of the believed deviation angles changes. The erroneous agent makes an incorrect decision and decides to move to N11.

To see the consequence of this decision for the complete navigation process, we apply the *wayFind* function on the erroneous agent at the starting node N1. The complete navigation path is visualized in Figure 19.

```
test input> wayFind net pete

FACT pos N99 Pos 74.0 36.0 Heading 4.36842
BELIEFS pos N99 Pos 74.0 36.0 Heading 4.36842
History [N99, N15, N12, N11, N10, N6, N2, N1]
```

Again, the agent reaches the goal N99. The estimation errors and the locally incorrect decision at N10 cause a longer path than the one an error free agent takes (compare Figure 14). As the target is reached (and therefore visible), the orientation error amounts to 0, and the believed position is correct. The example shows that estimation errors may result in a changed route but do not

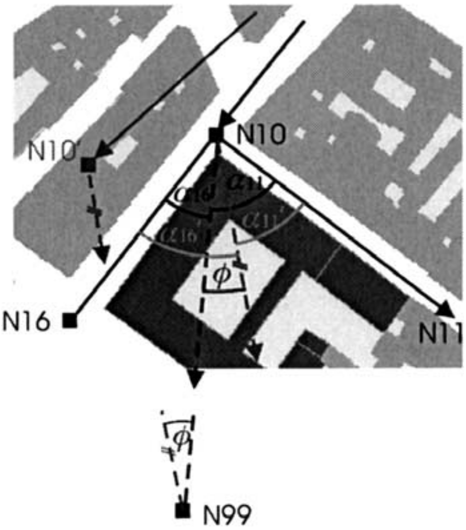

Figure 18. Incorrect decision making due a false believed target direction.

necessarily lead to getting lost. The consequences depend on the configuration of the environment graph and the size of the agent's subjective estimation errors.

8. Discussion

We gave a classification of human estimation errors that play a role during path integration in an unknown environment. We showed how distortion of perceptual information influences the navigators believed position, heading, and target direction. The size of the angles between the (believed) target direction and the outgoing streets from an intersection are the input for the decision function of the least-angle strategy. Besides error discussion, we gave a geometric criterion in Equation 1 that is used to determine if a local wayfinding decision has been made correctly or not.

We discussed distance estimation error, rotation estimation error, drifting error, and pointing error. The first three errors, if combined, result in a distorted belief of the agent's position and heading. The pointing error distorts the believed target direction only.

To demonstrate the role of estimation errors for the least-angle strategy in a city environment, we simulated the navigation process with a navigating agent in a simulated environment, implemented in Haskell. We used a two-tiered conceptual model for the simulated agent, to show the potential

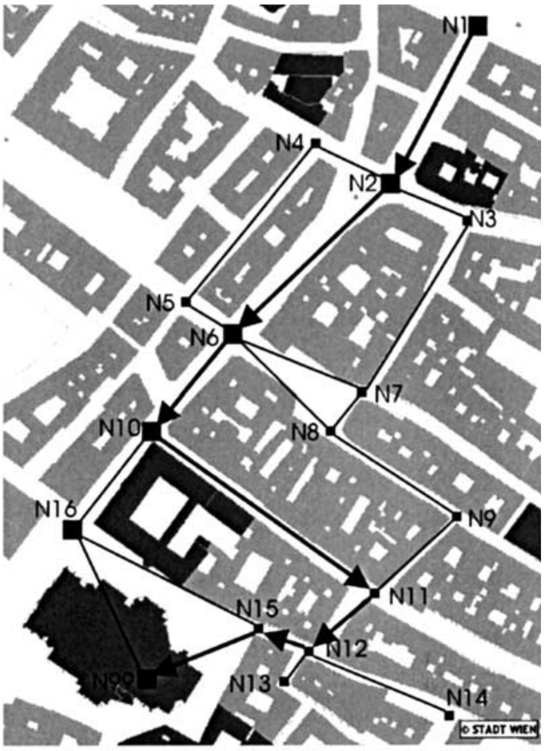

Figure 19. Chosen path of an erroneous agent

differences between the actual agent's position and heading in the environment (*fact*), and the agent's believed position and heading (*belief*). The aim of the simulation is to show the interaction of various errors during the navigation process. The error parameters for the simulated agent are based on results of various experiments reported in the literature. As conditions in the experiments are different than those of a built-up street environment, we do not claim the error parameters to exactly represent human error behavior.

The classification of estimation errors can be applied to any navigation process relying on path integration. The least-angle strategy is only one of them. What the paper revealed is the fact that literature, as far as we can say, lacks a distinct quantitative description of those human perception errors that play a role in the path integration process in a built-up street environment. Compared to laboratory results, additional facts play a role when estimating self-movement in a street environment, e.g., optical flow perceived from buildings, light-shadow-effects, solar direction, additional landmarks, or steepness and width of the street. To reliably clarify the influence of various parameters on the path integration process, human subjects testing in a city environment is needed. A more precise description of human estimation errors (with standard deviations and statistical metadata) could be used to assess the influence of estimation errors on a wayfinding decision more detailed through application of the law of error propagation.

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