Mr Madoff's Amazing Returns: An analysis of the Split Strike Conversion Strategy

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Abstract

It is now known that the very impressive investment returns generated by Bernie Madoff were based on a sophisticated Ponzi scheme. Madoff claimed to use a split strike conversion strategy. This strategy consists of a long equity position plus a long put and a short call. In this paper we examine Madoff's returns and compare his investment performance with what could have been obtained using the split strike conversion strategy based on the historical data. We also analyze the split strike strategy in general and derive expressions for the expected return standard deviation and Sharpe ratio of this strategy.

Keywords: Madoff, split strike conversion strategy, performance measurement.

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1 Introduction

Preliminary work in progress, Please do not quote

In December 2008 the investment operation of Bernie Madoff was exposed as a giant Ponzi scheme. Madoff had attracted a wide following because he delivered consistently high returns with very low volatility over a long period. He claimed to use a split strike conversion strategy to obtain these low risk returns. This strategy involves taking a long position in equities together with a short call and a long put on the index to lower the volatility of the position. We know now that these returns were fictitious. The Madoff affair raises the obvious questions as to why it was not discovered earlier and why investors and regulators missed the various red flags. A number of these points are discussed in Markopolos.(2009).

This paper discusses certain aspects of the split strike strategy and analyzes the Madoff performance. We analyze the Fairfield Sentry Ltd hedge fund which was one of Madoff's feeder funds to represent the Madofff strategy. Fairfield Sentry describes its strategy as follows.

> The Fund seels to obtain capital appreciation of its assets principally through the utilization of a nontraditional options strategy described as a split strike conversion to which the Fund allocates the predominant portion of its assets. The investment strategy has defined risk and reward parameters. The establishment of a typical position entails (i) the purchase of a group or basket of securities that are intended to highly correlate to the S&P 100 Index, (ii) the purchase of out-of-the- money S&P100 Index put options with a notional value approximately equal to the market value of the basket of equity securities and (iii) the sale of out-of-the- money S&P 100 Index call options with a notional value approximately equal to the market value of the basket of equity securities. The basket typically

consists of 40-50 stocks in the S&P 100 Index. The primary purpose of the long put options is to limit the market risk of the stock basket at the strike price of the long puts. The primary purpose of the short call options is to largely finance the cost of the put hedge and increase the stand-still rate of return. The "split strike conversion" strategy is implemented by Bernie L. Madoff Investment Securities LLC("BLM"), a broker dealer registered with the Securities and Exchange Commission through accounts maintained by the Fund in that firm. The services of BLM and its personnel are essential to the continued operation of the Fund and its profitability.

In the next section we analyze the performance of Fairfield Sentry for the period December 1990 to October 2008. The most dramatic aspect of the performance is the very low volatility of the returns. This in turn leads to an unusually high Sharpe ratio. We compare these returns with what could have been obtained by following the strategy in real time using the actual historical returns and find that while the expected return is in the ball park the volatility of the strategy in practice is much higher. In Section Three we analyze some theoretical properties of the distribution of returns of the split strike strategy. In particular we develop closed form expressions for the expected return, standard deviation and Sharpe ratio of this strategy.

2 Analysis of the Empirical Results

In this section we analyze the Fairfield Sentry return performance and contrast the reported returns with those that could be achieved using the split strike conversion strategy . The reported monthly returns for the period Dec 1990 to October 2008 are given in Table 1. These returns are amazingly consistent with an exceptionally low volatility. The monthly

volatility is seventy one basis points corresponding to an annual volatility of 2.45%. The average monthly return for the strategy 84.00 basis points corresponding to an annual average return of 10.59%. Investors clearly put a very high value on this combination of high returns and low volatility given the popularity of Madoff's funds.

If these returns were in fact achievable they would dominate those obtained from investing directly in the Index. Investing in the Index offers comparable returns with much higher volatility. Figure One compares the Fairfield Sentry (FS) performance with the strategy of investing directly in the S&P 500 with dividends reinvested over the period December 1990 to October 2008. It shows how an initial investment of one hundred would have grown under both assumptions. One hundred invested in the FS fund would have accumulated to 603.8 by October 2008 whereas one hundred invested in the S & P would have accumulated to 433.03 by October 2008 reflecting a lower growth rate. The expected annual return from investing in the S&P is 9.64% with a standard deviation of 14.28% over the 17 years and eleven months period. We note that we assume no expenses in the S& P investment and it is not clear if the Fairfield returns are net[1] of expenes.

We note that the growth of the investment in Fairfield Sentry is approximately linear. We explore this issue in Figure Two. The top most green line represents a linear increase of 2.343 per month starting at 100 in December 1990 to October 2008. The bottom most green line assumes that a constant monthly compound[2] growth rate of 84 basis points per year. This assumes that the fund increases at a constant compound rate of return each month. We note that the actual performance of the FS Fund lies inside these two green lines.

[1]It is conventional for hedge fund returns to be quoted net of expenses. These expenses include a fee on the total assets plus an incentive fee based on the performance of the fund.

[2]This rate is actually .008398 and it corresponds to a geometric growth rate per month. This we have $(1.008398)^{215} = 6.0377$.

Table 1: Fairfield Sentry monthly returns from December 1990 to October 2008.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2008	0.63	0.06	0.18	0.93	0.81	-0.06	0.72	0.71	0.50	-0.06	na	na
2007	0.29	-0.11	1.64	0.98	0.81	0.34	0.17	0.31	0.97	0.46	1.04	0.23
2006	0.70	0.20	1.31	0.94	0.70	0.51	1.06	0.77	0.68	0.42	0.86	0.86
2005	0.51	0.37	0.85	0.14	0.63	0.46	0.13	0.16	0.89	1.61	0.75	0.54
2004	0.88	0.44	-0.01	0.37	0.59	1.21	0.02	1.26	0.46	0.03	0.79	0.24
2003	-0.35	-0.05	1.85	0.03	0.9	0.93	1.37	0.16	0.86	1.26	-0.14	0.25
2002	-0.04	0.53	0.39	1.09	2.05	0.19	3.29	-0.13	0.06	0.66	0.09	0.00
2001	2.14	0.08	1.07	1.26	0.26	0.17	0.38	0.94	0.66	1.22	1.14	0.12
2000	2.14	0.13	1.77	0.27	1.30	0.73	0.58	1.26	0.18	0.86	0.62	0.36
1999	1.99	0.11	2.22	0.29	1.45	1.70	0.36	0.87	0.66	1.05	1.54	0.32
1998	0.85	1.23	1.68	0.36	1.69	1.22	0.76	0.21	0.98	1.86	0.78	0.26
1997	2.38	0.67	0.80	1.10	0.57	1.28	0.68	0.28	2.32	0.49	1.49	0.36
1996	1.42	0.66	1.16	0.57	1.34	0.15	1.86	0.20	1.16	1.03	1.51	0.41
1995	0.85	0.69	0.78	1.62	1.65	0.43	1.02	-0.24	1.63	1.53	0.44	1.03
1994	2.11	-0.44	1.45	1.75	0.44	0.23	1.71	0.35	0.75	1.81	-0.64	0.60
1993	-0.09	1.86	1.79	-0.01	1.65	0.79	0.02	1.71	0.28	1.71	0.19	0.39
1992	0.42	2.72	0.94	2.79	-0.27	1.22	-0.09	0.85	0.33	1.33	1.35	1.36
1991	3.01	1.40	0.52	1.32	1.82	0.30	1.98	1.00	0.73	2.75	0.01	1.56
1990												2.77

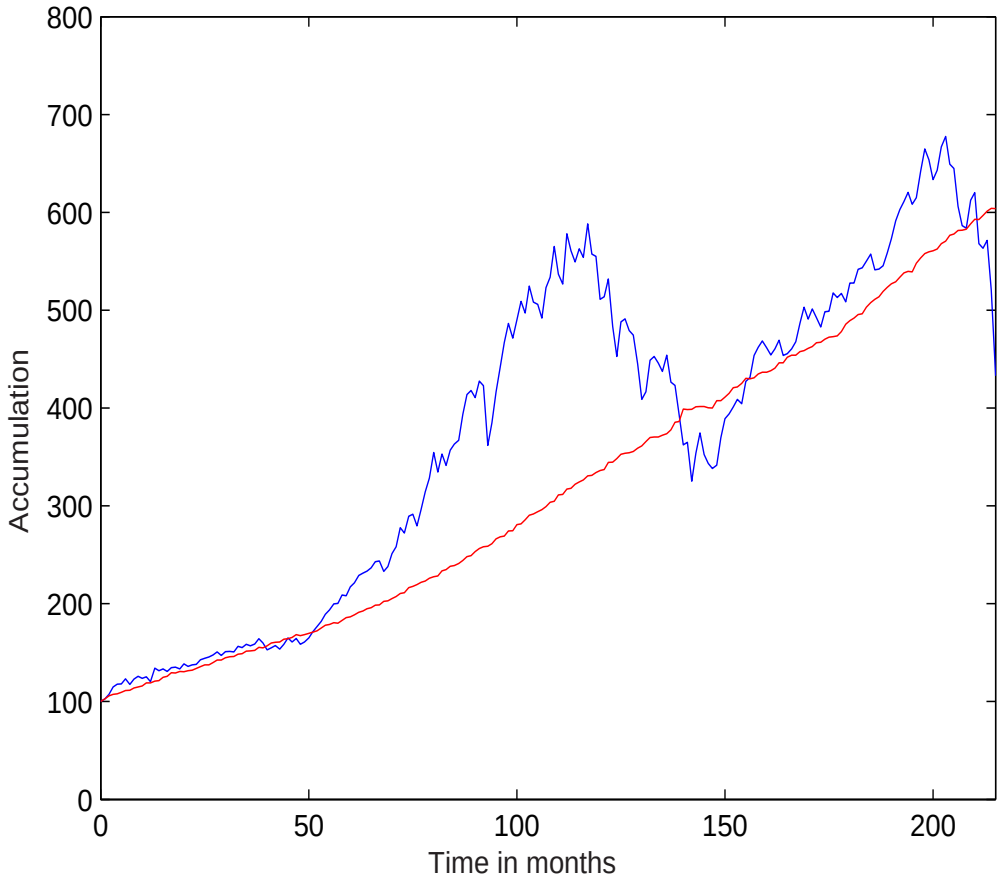

Figure 1: Accumulated investment proceeds under Fairfield Sentry returns given in Table 1 and investment in the S&P 500 with dividends reinvested. The Fairfield Sentry performance is in red and the S& P 500 accumulation is in blue. It is assumed that 100 is invested in each strategy in December 1990 and accumulated until October 2008.

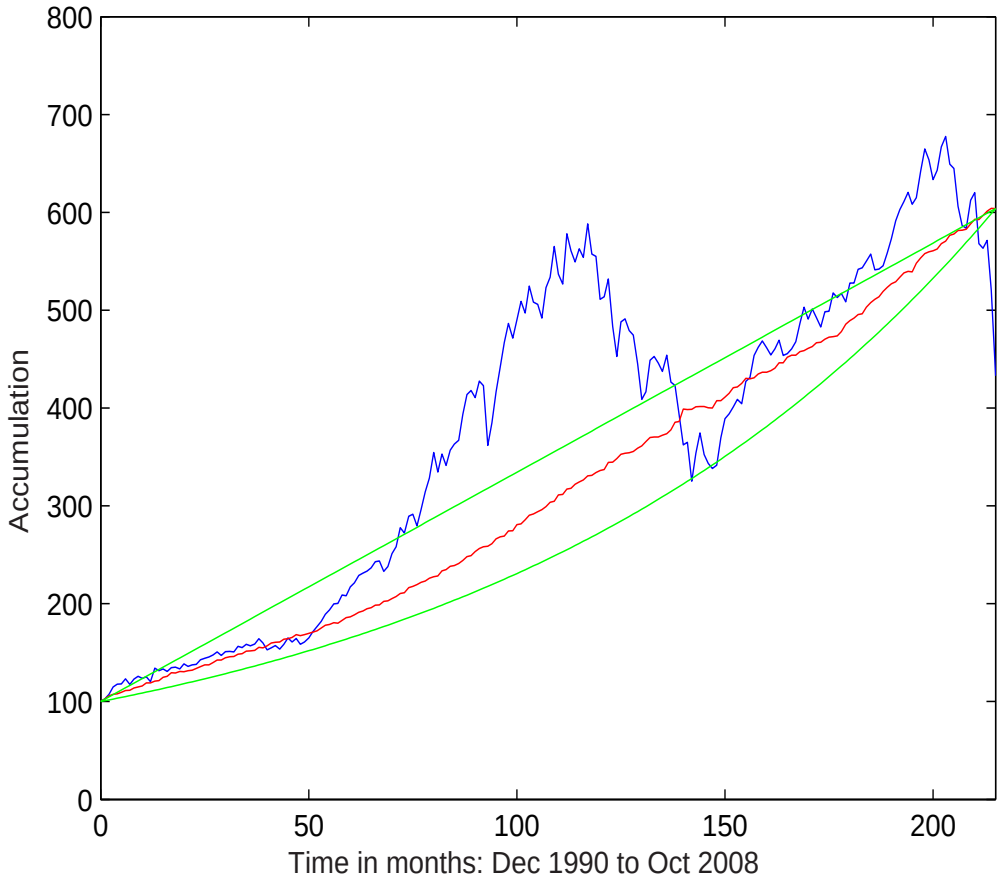

Figure 2: Accumulated investment proceeds under Fairfield Sentry returns given in Table 1 and investment in the S&P 500 with dividends reinvested. The Fairfield Sentry performance is in red and the S& P 500 accumulation is in blue. The top green line shows how the initial 100 accumulates to 603.78 in October 2008 if the dollar increase is constant and equal to 2.343 per month. The bottom green line shows how the initial 100 in December 1990 accumulates to 603.78 in October 2008 under a constant monthly compound growth rate.

Table 2: Summary performance statistics for four strategies for the period December 1990 to October 2008.

Strategy	Invest in S&P	Split strike no volatility skew	Split Strike with volatility skew	Fairfield Sentry
Average return monthly	0.77	0.92	0.63	0.84
Average return annual	9.64	11.68	7.89	10.59
St Deviation(monthly)	4.12	3.09	3.15	0.71
St Deviation(annual)	14.28	10.72	10.91	2.45
Sharpe Ratio monthly	.105	.180	.094	.712
Sharpe Ratio annual	.363	.656	.326	2.47
Max monthly return	11.44	5.37	4.94	3.29
Min monthly return	-16.79	-4.92	-5.63	-0.64
Percent positive	64.65	64.65	64.19	92.33
Correlation with $S\&P$	1.00	0.9480	0.9514	0.3197

Given the consistently high returns and the incredibly low volatility of the FS returns it is of interest to examine what sort of returns could be expected under a split strike strategy during this period. To do so we assume that a hypothetical investor takes a long position in the S&P 500 Index starting in December 1990. At the same time he buys a put option on the Index and sells short a call option on the Index. We assume that the strike price of the put is 5% below the initial spot price of the Index and we assume that the strike price of the call is 5% above the initial spot price of the Index. Both options are assumed to be European and have a one month maturity. Typically the call price will exceed the put price and the proceeds are invested for one month at the risk free rate. At the end of the month the option positions are settled in cash. If there is a loss under the option strategy it is

financed in the first place from the risk free investment of the net option premiums and if that is not enough by selling enough shares of the Index.

It is assumed that all available monies are invested in the Index at the start of the second month and the same option strategy is implemented. This procedure is repeated every month for the 215 month period. We ignore transaction costs and any price impact of the trades. In order to price the options we use the Black Scholes formula. As a proxy for the implied volatility we use we use the VIX to price both[3] the call and put options. The average level of the VIX over this period is 19.24% . We use the prevailing one month US T bill rates to proxy the risk free rates in pricing the options .

The results are summarized in Figure 3. This graph shows that the split strike strategy appears to do quite well as compared to direct investing in the Index. It has a higher expected return than he FS strategy (11.68% as against 10.59%). However the returns are more correlated withe S&P index and have much higher volatility than the FS strategy. As shown in Table 3 the expected return for the split strike strategy is 11.68% per annum with a standard deviation of 10,72% leading to a annual Sharpe ration of 0.656. While this Sharpe ratio is much less than the FS Sharpe ratio of 2.45 it is almost double the Sharpe ratio of investing in the Index over this period. Our later analysis in Section Three will show that the split strike conversion strategy produces a lower Sharpe ratio than direct investing in the Index so this result is surprising.

The option prices that were used to construct Figure 3 assume that both the call and put options were priced using the prevailing value of the VIX. It is well known that there is a volatility skew whereby the implied volatility at a fixed maturity is a decreasing function of the strike price. For example Zhang and Shu find that over the five year year period from 1995 to 1999 the average implied volatility at the money for short term options is 19%

[3]Later we incorporate a volatility skew into the price calculations.

whereas the average implied volatility for out of the money calls is 17% and the average implied volatility for out of the money puts is 21%. They define short term options to be options with less than 45 days to maturity and they define the moneyness m of an option as

$$m = \frac{S}{K} - 1,$$

where S is the spot price and K is the strike price of the option. An extract from Zhang and Shu's results is given in Table 3.

Table 3: Average Implied Volatility of S&P 500 Index Options for period Jan 1995 to December 1999.

Moneyness	Average Implied Volatility for short term options
Out of the Money $-.05 < m < -.032$	17%
At the Money $-.02 < m, -.02$	19%
In the Money $.02 < m < .05$	21

In order to capture the option skew we assume that the out of the money call options in our simulation strategy are priced using 90 percent of the prevailing VIX level. We assume that the out of the money put options in our simulation strategy are priced using 110 percent of the prevailing VIX level. These adjustments are in line with the empirical results of Zhang and Shu. With this change the performance of the split strike strategy is less impressive and its Sharpe ratio is similar to that obtained by investing in the market. We display the results

of the split strike strategy when we include tuis allowance for the volatility skew in Figure 4. The black line denotes the accumulation under the split strike strategy with this adjustment.

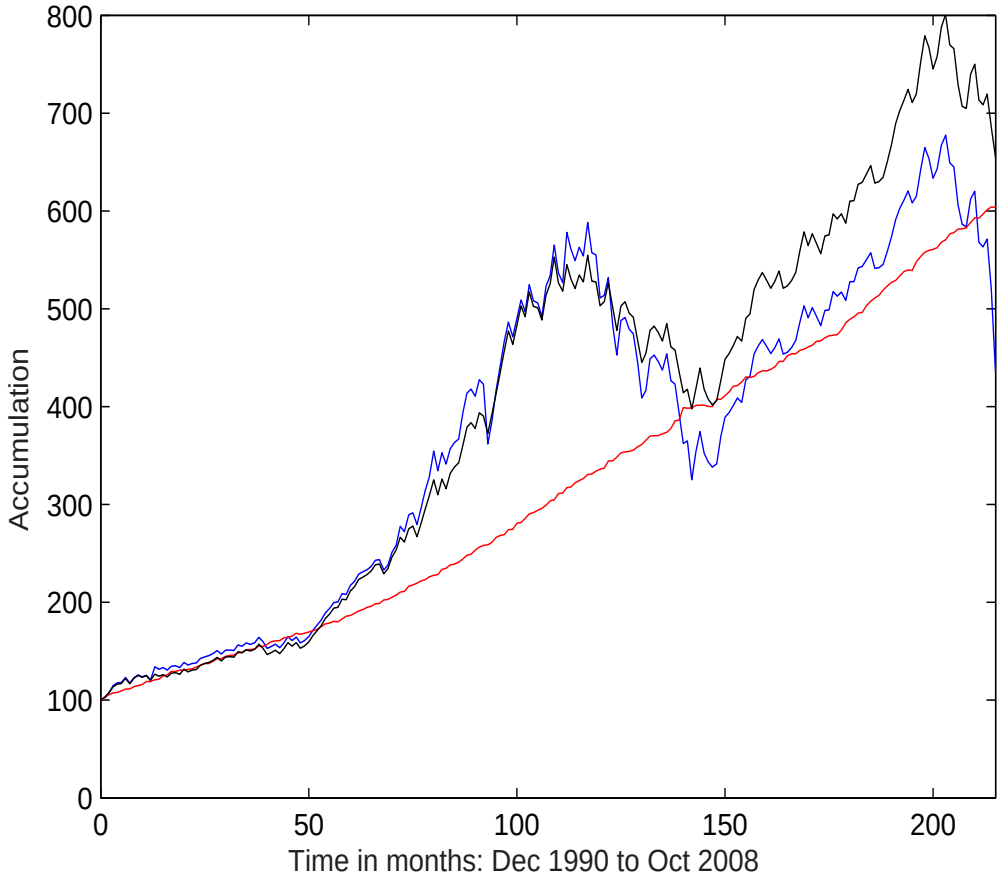

Figure 3: Accumulated investment proceeds under the Fairfield Sentry returns given in Table 1 and investment in the S&P 500 with dividends reinvested. The Fairfield Sentry performance is in red and the S& P 500 accumulation is in blue. The black line shows the results of investing in the split strike strategy when the options are priced using using the prevailing value of the VIX.

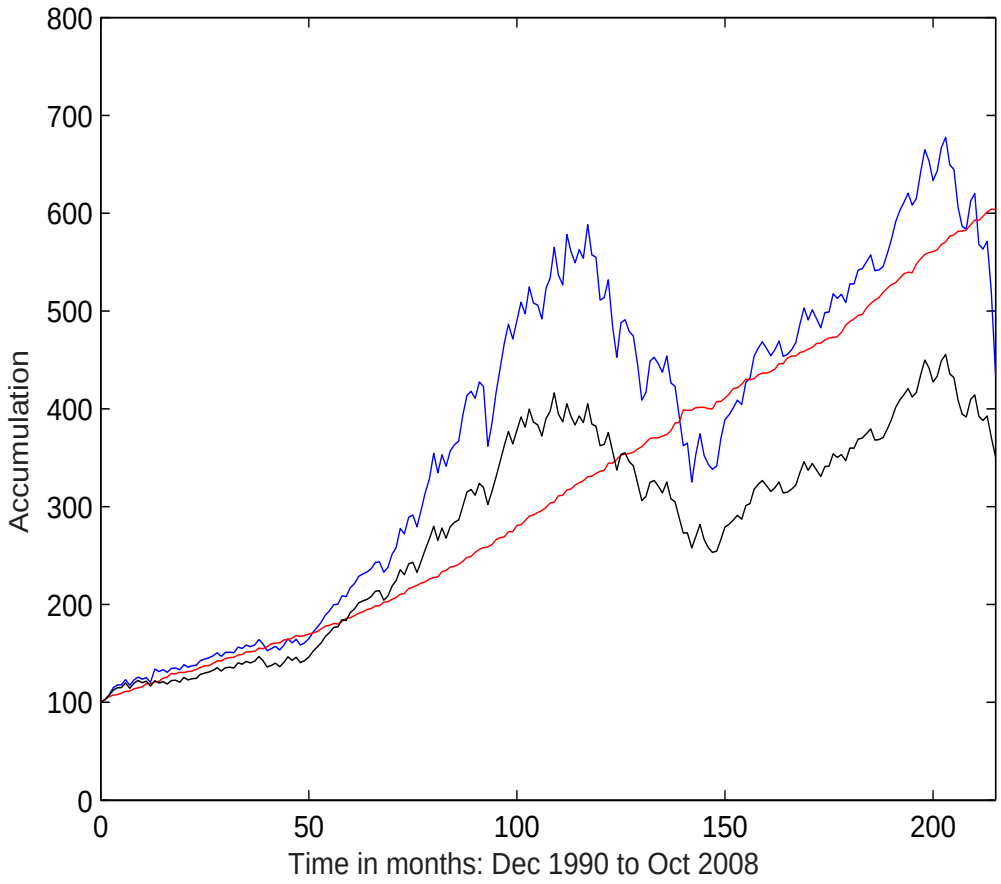

Figure 4: Accumulated investment proceeds under the Fairfield Sentry returns given in Table 1 and investment in the S&P 500 with dividends reinvested. The Fairfield Sentry performance is in red and the S& P 500 accumulation is in blue. The black line shows the results of investing in the split strike strategy when the options are priced using using a volatility skew assumption where an at the money money option is priced using the prevailing value of the VIX. The puts are priced at the prevailing value of the VIX multiplied by 1.1. The calls are priced at the prevailing value of the VIX multiplied by 0.90.

Manipulation Proof Performance Measures

Recently Goetzmann, Ingersoll and Speigel (2007) (GIS)have developed a manipulation free portfolio performance measure. GIS demonstrate that their measure is robust to various manipulation strategies. Even though their measure was not designed to detect outright fraud it can provide valuable insights on the nature of the split strike strategy and the Fairfield Sentry returns.

The formula for the GIS measure, $\hat{\Theta}$ for a series of N monthly returns is defined as follows

$$\hat{\Theta} = \frac{1}{(1-\rho)h}\, ln\left(\frac{1}{N}\sum_{i=1}^{N}\left[\frac{1+r_{pi}}{1+r_{fi}}\right]^{1-\rho}\right) \tag{1}$$

where

- r_{pi} is the rate of return on the portfolio for month i ,
- r_{fi} is the risk free rate for month i ,
- h is the time interval in years . Here $h = \frac{1}{12}$.
- ρ corresponds to the relative risk aversion of the investor.

GIS note that this measure, $\hat{\Theta}$ has an intuitive economic interpretation. It measures the portfolios implied excess return after adjusting for risk. Thus the for the risk free portfolio $\hat{\Theta} = 0$. If one had a portfolio that earned exactly fifty basis points above the risk free rate every month with no variation this portfolio would have $\hat{\Theta} = .06$. If the portfolio is risky then $\hat{\Theta}$ decreases as the investor becomes more risk averse. Table 4 shows the values of the GIS measure for direct investment in the S& P, the split strike strategy (incorporating the volatility skew) and the Fairfield Sentry returns for different levels of ρ.

Table 4: Values of $\hat{\Theta}$ corresponding to different levels of ρ for three investment strategies

Value of ρ	Invest in S&P	Split Strike with volatility skew	Fairfield Sentry
2	0.0309	0.0237	0.0597
3	0.0201	0.0177	0.0594
4	0.0090	0.0117	0.0592
5	-0.0025	0.0056	0.0589
10	-0.0664	-0.0254	0.0575

Table 4 shows that the Fairfield Sentry portfolio outperforms the other strategies based on this measure. If the investor becomes more risk averse the value of $\hat{\Theta}$ declines rapidly for the the investment in the $S\&P$ and the split strike strategy. However $\hat{\Theta}$ hardly changes at all for the Fairfield Sentry returns. The Fairfield Sentry se returns correspond to an extra six percent per year above the risk free rate for all investors even the even the most risk averse. Mr Madoff's returns were ingeniously designed to appeal to even the most risk averse investors.

3 Theoretical Analysis of Split Strike Strategy

To be added

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Other references to be added