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Reference-Dependent Risk Attitudes

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Abstract

Kőszegi and Rabin (forthcoming) develop a model building from prospect theory that a) combines the reference-dependent "gain-loss utility" with standard "consumption utility," b) bases the reference point on beliefs about outcomes rather than on the status quo, and, to incorporate probabilistic beliefs, c) allows for stochastic reference points. In this paper we develop an extension of that model, and apply both the original and the extension to study the taste for insurance and other preferences over monetary risk. The model predicts that the environment—through its effect on expectations—heavily influences risk attitudes toward modest-scale risk. When exposure to potential modest-scale changes in wealth is a surprise, the model corresponds to a form of classical prospect theory, and hence predicts a distaste for insuring surprise losses. When potential losses are anticipated, planned spending on insurance does not generate sensations of loss while uncertain bad outcomes do, leading to a very high willingness to pay for insurance. When there are risks a person cannot or does not want to avoid, insuring additional risks does not eliminate the possibility of painful loss, so her willingness to pay for insurance is reduced. The model also allows for consumption utility to dominate attitudes towards large-scale risks, predicting a general taste for insurance against large-scale risks independently of the environment.

Keywords: Loss Aversion, Personal Equilibrium, Prospect Theory, Reference-Dependent Preferences, Risk Attitudes

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1 Introduction

How a person assesses an economic outcome is often influenced by how it compares to a reference level. Two regularities have been emphasized by Kahneman and Tversky's (1979) prospect theory and the literature building from it: a diminishing sensitivity to changes as an outcome moves further from the reference level, and a significantly greater aversion to losses than appreciation of gains. These two regularities generate risk attitudes different from those implied by the classical diminishing-marginal-utility-of-wealth model: diminishing sensitivity implies risk lovingness in the loss domain, and loss aversion implies a non-trivial dislike of modest-scale risks.[1]

Yet many observed risk behaviors do not seem consistent with prospect theory applied, as is conventional, with the reference level as the status quo. While in laboratory experiments many people display risk lovingness in the loss domain, consumers exhibit strong risk *aversion* in the loss domain when purchasing extended warranties on household items or choosing low deductibles on car and homeowners insurance. Although most people reject a favorable modest-stakes gamble when it is offered unexpectedly, subjects are much closer to risk neutral when selling a lottery they are endowed with.[2] Other behavior—most notably attitudes toward large-scale losses—seems to accord more closely to the classical model than to prospect theory.

This paper applies the model from Kőszegi and Rabin (forthcoming) and an extension to study behavior in the face of monetary risk, unifying in a single theory the above and other behaviors that seem inconsistent when viewed through the lens of other models. Our model a) combines the reference-dependent "gain-loss utility" with standard "consumption utility," b) bases the reference point on beliefs about outcomes rather than on the status quo, and, to incorporate probabilistic

[1]The calibrational inconsistency of modest-scale risk aversion with the classical diminishing-marginal-utility-of-wealth model has been pointed out by many researchers and formalized by Rabin (2000). For instance, a person with $1 million in lifetime wealth who obeys this model and who rejects a fifty-fifty lose $500 or gain $550 gamble would also turn down an equal-probability bet of losing $4,000 or gaining $100,000,000,000,000. While nobody would turn down the large-scale bet, most people would turn down the $500 gamble. Barberis, Huang, and Thaler (2003), for example, find that the majority of MBA students, financial advisors, and even very rich investors (with median financial wealth over $10 million) reject this bet.

[2]See Knetsch and Sinden (1984) and Birnbaum, Coffey, Mellers, and Weiss (1992).

beliefs, c) allows for stochastic reference points. Because of feature (a), our theory is consistent with aversion to all large-scale risk. Because of feature (b), it predicts both risk lovingness in response to *surprise* modest-scale losses, and—since anticipated premium payments do not generate sensations of loss while bad outcomes in uncertain situations do—first-order risk aversion in response to *expected* risks. Because of features (b) and (c), our theory predicts that being endowed with some risk decreases aversion to additional risk. And because it predicts that risk attitudes will be highly scale and context-dependent, it explains why estimates of risk aversion based on the classical model are wildly variable. It also accommodates and clarifies phenomena, such as the disposition effect, that are often associated with prospect theory, but are in fact inconsistent with classical versions of it.

In Section 2, we specify a person's utility as $u(c|r) \equiv m(c)+\mu(m(c)-m(r))$, where c is the wealth level and r is the "reference" wealth level. The reference-independent "consumption utility," $m(c)$, corresponds to the classical notion of outcome-based utility. "Gain-loss utility," $\mu(m(c) - m(r))$, depends on the difference between the consumption utility of the outcome and of the reference level. The shape of μ captures loss aversion and diminishing sensitivity.

Rather than the more common assumption of status quo, we assume that a person's reference point is the *beliefs* she held in the recent past about outcomes. An employee who had expected a \$50,000 salary will assess a salary of \$40,000 as a loss, and a taxpayer who had expected to pay \$30,000 in taxes will treat a \$20,000 tax bill as a gain. Because a person may be uncertain about outcomes, our theory allows for the reference point to be a distribution $G(\cdot)$. In comparing c to $G(\cdot)$, the decisionmaker evaluates $u(c|r)$ for all r possible under $G(\cdot)$, and takes an average of these "mixed feelings." A stochastic wealth outcome is evaluated according to its expected utility given the reference point.

While our complete model incorporates a theory of how expectations are determined, some key intuitions can be seen from how behavior depends on expectations given exogenously. To illustrate implications for modest-scale risk, where consumption utility is approximately linear, consider a person's decision of whether to pay \$55 to insure a 50% chance of losing \$100. If she had expected to retain the status quo of \$0, our model makes the same prediction as prospect theory: because

of diminishing sensitivity, she does not wish to insure the risk. If she had expected to pay \$55 for insurance, taking the gamble exposes her to a fifty-fifty chance of losing \$45 or gaining \$55. With a conventional estimate of two-to-one loss aversion, she dislikes this gamble and buys the insurance. If she had expected to be exposed to the 50% chance of losing \$100, she would assess paying \$55 partially as a loss of \$55 and partially as a gain of \$45. Because insurance therefore does not prevent sensations of loss, she does not buy it.

To complete our model, we assume that expectations fully reflect the true probability distribution of outcomes generated by a person's choice, and define solution concepts for two prototypical timelines of decisions. When a person makes a decision shortly before outcomes occur, at that moment she takes the reference point as fixed. For these situations, we define in Section 3 an "unacclimating personal equilibrium" (UPE) as behavior where the stochastic outcome generated by utility-maximizing choices conditional on expectations coincides with expectations. We refer to the person's favorite UPE as her "preferred personal equilibrium" (PPE).

Our two scenarios above showing that the decisionmaker insures the potential \$100 loss if and only if she expects to do so means that both insuring and not insuring are UPE. And because risky expectations mean that more outcomes generate unpleasant sensations of loss, they tend to decrease expected utility, so that the PPE is to insure. This strong taste for insurance reflects the fact that while a bad outcome of an *uncertain* lottery is evaluated as a loss relative to better possible outcomes, a fully expected premium payment is not evaluated as a loss. Hence, we formally capture the psychological intuition of previous researchers (e.g. Novemsky and Kahneman 2005) that money given up in regular purchases is not a loss.

While our model predicts a strong preference for insuring expected risks, it also implies that because risky expectations make it impossible to avoid sensations of loss, risks a person cannot or does not want to avoid decrease her aversion to further risk.

In Section 4, we consider a person's behavior when she makes a committed decision long before outcomes occur, and hence affects the reference point by her choice. For these situations, we define a "choice-acclimating personal equilibrium" (CPE) as a decision that maximizes expected utility, given that it determines both the reference lottery and the outcome lottery. Like PPE,

CPE predicts that the decisionmaker strongly prefers to insure expected risks, and has a decreased aversion to additional risk when there is risk she cannot or does not want to avoid. But there are also important differences between CPE and PPE risk attitudes. Suppose the cost of insuring the fifty-fifty chance of losing \$100 is not \$55, but \$75. With two-to-one loss aversion, the unique UPE (and PPE) is then to choose the lottery. Yet because the chance of not having to pay \$100 means a sense of loss if she does have to pay, with the realization that she would take it the lottery yields the decisionmaker lower expected utility than insurance. This means both that the availability of risky options can decrease welfare in PPE, and that—with the ability to avoid such options—risk aversion is greater in CPE.

The above sensitivity of behavior to the situation applies only to modest-scale choices, where risk attitudes are necessarily dominated by the gain-loss component of preferences. In Section 5, we investigate attitudes towards large-scale risk, where consumption utility cannot be assumed to be linear. We show that under reasonable conditions, the reference point has only a minor impact on expected utility from a very large gamble, so that independent of the environment a person may exhibit risk aversion reflecting diminishing marginal utility of wealth.

Beyond helping to explain in a unified framework seemingly contradictory behaviors, the endogenous specification of the reference point helps make our model readily portable to many settings. To facilitate applications, in fact, in Appendix A we present an array of risk-characterization concepts and results. Yet our theory is far from incorporating all important issues related to reference-dependent risky choice. In Section 6, we conclude the paper by discussing some of the shortcomings of our model, emphasizing especially its failure to capture important components of bounded rationality in reference-dependent risk attitudes.

2 Reference-Dependent Utility

In this section we present the one-dimensional version of the utility function in Kőszegi and Rabin (forthcoming), and derive some results on how behavior depends on expectations about outcomes. As with all models of risky choice (outside of full-fledged life-cycle consumption models in macroe-

conomics), our theory takes as a primitive the choice set of gambles that a person is assumed to focus on, in isolation from other risks and choices she faces. We return in the conclusion to a discussion of how our assumptions about focusing and bracketing might affect the positive and normative implications of our model.

For a riskless wealth outcome $c \in \mathbb{R}$ and riskless reference level of wealth $r \in \mathbb{R}$, utility is given by $u(c|r) \equiv m(c) + \mu(m(c) - m(r))$.[3] The term $m(c)$ is intrinsic "consumption utility" usually assumed relevant in economics, and also utility when wealth is deterministic and equal to the reference point. The term $\mu(m(c) - m(r))$ is the reference-dependent "gain-loss utility." While it surely exaggerates the tight connection between the two components, our model assumes that how a person feels about gaining or losing relative to a reference point depends solely on the changes in consumption utility associated with such gains or losses.

Since we assume below that the reference point is beliefs about outcomes, we allow for the reference point to be a probability measure G over $\mathbb{R}$:

$$U(c|G) = \int u(c|r)dG(r). \tag{1}$$

This formulation captures the notion that the evaluation of a wealth outcome is based on comparing it to all possibilities in the support of the reference lottery. For example, if the reference lottery is a gamble between \$0 and \$100, an outcome of \$50 evokes a mixture of two feelings, a gain relative to \$0 and a loss relative to \$100.

An alternative to our mixed-feelings formulation, pursued by Gul (1991) and Shalev (2000) discussed below, is to collapse the reference lottery into some type of certainty equivalent.[4] With such a specification, two reference lotteries that have the same certainty equivalent generate the same risk preferences. This is inconsistent with our theory's—we suspect broadly correct—prediction

[3] Both because it is likely that many of the intuitions would go through in a more fully elaborated model, and because it seems that people often experience sensations of gain and loss directly from wealth changes, this paper studies monetary wealth rather than consumption.

[4] There is some suggestive evidence of mixed feelings when there are multiple counterfactuals relative to which outcomes can be evaluated. For instance, Larsen, McGraw, Mellers, and Cacioppo (2004) find that when a subject receives \$5 from a lottery that could have paid \$5 or \$9, she has both positive and negative emotions—presumably from winning \$5 and not winning \$9, respectively. Losing \$5 when losing \$9 is possible evokes similar mixed feelings.

that a person is more inclined to accept a risk if she has rather than has not been expecting to face that risk.

More than saying a person separately compares an outcome to all components of the reference point, our formulation of $u(c|r)$ below implies that losses relative to a stochastic reference point count more than gains, so that the \$50 above yields negative gain-loss utility. An alternative specification is one where the relief of avoiding the loss outweighs the disappointment of not getting the gain. This alternative seems difficult to reconcile with loss aversion relative to riskless reference points. It would also seem to imply that—in the hope of creating pleasant reliefs—people will seek to endow themselves with risks.

Our utility function is also closely related to Sugden (2003), where outcome lotteries are compared to reference lotteries state by state. Sugden's (2003) formulation captures a form of state-contingent regret missing from our model, but—contrary to strong intuition as well as evidence in Mellers, Schwartz, and Ritov (1999)—implies that when wealth is drawn from the reference lottery, a person would never experience sensations of gain or loss.

When c is drawn according to the probability measure F, utility is given by

$$U(F|G) = \iint u(c|r)dG(r)dF(c). \tag{2}$$

For simplicity and contrary to Kahneman and Tversky (1979) and its extensions, we assume that preferences are linear in probabilities. Although non-linear probability weighting is an important determinant of risky choice, it seemingly interacts little with the results we stress.

We assume μ satisfies the following properties:

A0. $\mu(x)$ is continuous for all x, twice differentiable for $x \neq 0$, and $\mu(0) = 0$.

A1. $\mu(x)$ is strictly increasing.

A2. If $y > x \geq 0$, then $\mu(y) + \mu(-y) < \mu(x) + \mu(-x)$.

A3. $\mu''(x) \leq 0$ for $x > 0$ and $\mu''(x) \geq 0$ for $x < 0$.

A4. $\frac{\mu'_-(0)}{\mu'_+(0)} \equiv \lambda > 1$, where $\mu'_+(0) \equiv \lim_{x\to 0} \mu'(|x|)$ and $\mu'_-(0) \equiv \lim_{x\to 0} \mu'(-|x|)$.

Properties A0-A4, first stated by Bowman, Minehart, and Rabin (1999), correspond to Kahneman and Tversky's (1979) explicit or implicit assumptions about their "value function" defined on

$c - r$. Loss aversion is captured by A2 for large stakes and A4 for small stakes, and diminishing sensitivity is captured by A3. While the inequalities in A3 are most realistically considered strict, to characterize the implications of reference dependence with loss aversion but without diminishing sensitivity as a force on behavior, we define a subcase of A3:

A3′. For all $x \neq 0, \mu''(x) = 0$.

When we apply A3′ below, we will parameterize μ as $\mu'_+(0) = \eta$ and $\mu'_-(0) = \lambda\eta > \eta$, so that η can be interpreted as the weight on gain-loss utility.

To determine behavior, we need to combine the utility function introduced above with a theory of reference-point determination. We assume that a person's reference point is not the status quo, but her *rational expectations* about the relevant outcome held between the time she first focused on the outcome and shortly before it occurs.[5] For example, if an employee had been expecting a salary of \$100,000, she would assess a salary of \$90,000 not as a large gain relative to her status quo wealth, but as a loss relative to her expectations of wealth. Some evidence indicates that expectations are important in determining sensations of gain and loss, but our primary motivation for this assumption is that it helps unify and reconcile in an intuitive way existing discussions.[6]

For the remainder of this section and in the next two sections, we investigate the decision-maker's attitudes toward modest-scale risk, such as \$100 or \$1,000, where $m(\cdot)$ can be taken to be approximately linear—and where we therefore derive formal results under the assumption that $m(c) = c$.[7] In Section 6, we return to an exploration of large-scale risk, where risk preferences can

[5]Our theory posits that preferences depend on lagged expectations, rather than expectations contemporaneous with the time of consumption. This does not assume that beliefs are slow to adjust to new information or that people are unaware of the choices that they have just made—but that preferences do not instantaneously change when beliefs do. When somebody finds out 5 minutes ahead of time that she will for sure not receive a long-expected \$100, she would presumably immediately adjust her expectations to the new situation, but she will still 5 minutes later assess not getting the money as a loss.

[6]For intuition, see our discussion of disposition effects below, as well as further discussions in Kőszegi and Rabin (forthcoming). For more direct evidence of expectations-based counterfactuals affecting reactions to outcomes, see for instance Mellers, Schwartz, and Ritov (1999), Breiter, Aharon, Kahneman, Dale, and Shizgal (2001), and Medvec, Madey, and Gilovich (1995).

[7]Even for a person who has only \$1 million in lifetime wealth and a very high consumption-utility coefficient

be substantially influenced by diminishing marginal utility of wealth. Here we first analyze the decisionmaker's risk-taking behavior when the reference point is fixed, considering both deterministic and stochastic reference points. The analysis provides foundations for the results in Sections 3 and 4, which are derived under our complete model that incorporates a theory of expectations formation.

Proposition 2 of Kőszegi and Rabin (forthcoming) shows that when the decisionmaker expects to keep the status quo, her behavior is identical to that predicted by prospect theory modified to assume that decisions weights are probabilities. More generally, our model predicts that for deterministic expectations the decisionmaker's behavior satisfies the properties of prospect theory around the expected wealth level. Suppose, for instance, that a shopper had long been expecting to pay $100 for groceries, and is now given a fifty-fifty chance of paying double or nothing. Prospect theory based on the status quo says that this choice is driven by risk lovingness in the loss domain, so that the person accepts the gamble; our theory says that for gambles around expected wealth, loss aversion applies, so she rejects the gamble.

These results can be used to interpret the nature of "disposition effects" found by Odean (1999) for stocks and Genesove and Mayer (2001) for houses—whereby people appear disproportionately reluctant to sell an asset for less than they paid. Although not derived from classical prospect theory, the intuition commonly invoked for the disposition effect is that because the purchase price operates as a reference point for the selling price, people will be risk-seeking in waiting for a price to recover before selling. Our model predicts that the expected resale price governs selling behavior. While probably unrealistic in saying that the purchase price per se plays no role, this prediction clearly helps in determining when and how the disposition effect matters at all. Because home and stock owners usually expect to make money, they will be risk loving when these investments unexpectedly lose money. But when items—such as cars, appliances, or furniture—are known to lose their value over time, people will be unbothered by selling them below the purchase price. And if a person expects an investment—such as a house in a booming market or inventory a merchant

of relative risk aversion of 10, winning or losing $1,000 (a difference of $2,000 in wealth) only changes marginal consumption utility by 1.8 percent.

expects to resell at a large margin—to make a large positive return, she will even be reluctant to sell at prices insufficiently above the purchase price.

While our model replicates prospect theory around a riskless reference point, there are two senses in which the expectation of risk increases the decisionmaker's willingness to take on additional risks. To state these results, we use $H + H'$ to denote the distribution of the sum of independent draws from the distributions H and H'. (Thus, $(H + H')(z) = \int H(z - s)dH'(s)$.) When it creates no confusion, a real number will denote both a deterministic wealth level and the lottery that assigns probability 1 to that amount of wealth. Proposition 1 says that under $A3'$, a person is more willing to accept a given lottery if her reference point is risky rather than riskless:

Proposition 1 *Suppose $m(\cdot)$ is linear and μ satisfies A3$'$. For any lotteries G and F and constant k, if $U(k + F|k) \geq U(k|k)$, then $U(G + F|G) \geq U(G|G)$.*

When F is added to a riskless reference point k, the positive outcomes of F are assessed as pure gains, while the negative outcomes of F are assessed as pure losses. When F is instead added to a lottery G, positive outcomes of F partially eliminate losses suffered due to G, and are thus evaluated more favorably than pure gains; and negative outcomes of F partially eliminate gains from G, and are hence evaluated less unfavorably than pure losses. For both of these reasons, the decisionmaker is more willing to accept F. The mean of G and the value of k play no role in this result because with a linear $m(\cdot)$ risk attitudes do not depend on wealth.

The second sense in which expecting risk decreases risk aversion is that a person is approximately risk neutral in accepting an additional lottery if it is "small" relative to the reference lottery:

Proposition 2 *Suppose $m(\cdot)$ is linear. For any lottery F with positive expected value:*

1. There exist $A, \epsilon > 0$ such that for any lottery G where $Prob_G[r \in (k - A, k + A)] < \epsilon$ for all constants k, $U(G + F|G) > U(G|G)$.

2. For any continuously distributed lottery G, there is a $\bar{t} > 0$ such that for any $t \in (0, \bar{t}]$, $U(G + t \cdot F|G) > U(G|G)$.

In evaluating an extra lottery that is small relative to the reference lottery G, neither loss aversion nor diminishing sensitivity plays a major role. On the one hand, it is unlikely that an

outcome resulting from the extra lottery turns a gain or loss from G into the opposite, weakening the force of loss aversion. On the same hand, there is little diminishing sensitivity over the range of the extra lottery, weakening this force as well.

Because this will be key for a number of later results, some care must be taken in understanding the logic and implications of Propositions 1 and 2. While a greater expectation of risk makes the person more risk neutral, this very much does not say that she is unbothered by the risk she faces. In fact, the same force that decreases risk aversion also decreases expected utility: when the reference point is stochastic, the decisionmaker cannot avoid the possibility of utility-decreasing losses by avoiding additional risk.

3 UPE and PPE Risk Attitudes

In the next two sections, we consider the decisionmaker's behavior when her expectations are endogenously determined by the environment she faces. From the psychological hypothesis that the reference point is equal to rational expectations from the recent past, we develop different reduced-form models of behavior depending on when the decisionmaker makes (commits to) her choice. In this section, we analyze her behavior in one extreme possibility, when she anticipates the choices she faces but cannot commit to a choice until shortly before the outcome. This assumption applies, for example, to insurance choices on short-term rentals such as cars or skis.

Suppose the decisionmaker has probabilistic beliefs over possible compact choice sets described by $\{D_1, q_1; D_2, q_2; \ldots; D_L, q_L\}$, where choice set $D_l \subset \Delta(\mathbb{R})$ occurs with probability q_l. For simplicity, when we refer to a single choice set D, we mean $L = 1, q_1 = 1$, and $D_1 = D$. Since the person makes the decision shortly before the resulting outcome, the beliefs determining the reference point are past and hence unchangable at the time of choice. This means that she maximizes utility taking the reference point as given, so that she could rationally expect a choice if and only if she is willing to make it after having expected to do so.

Definition 1 *A selection $\{F_l \in D_l\}_{l=1,\ldots,L}$ is an unacclimating personal equilibrium (UPE) if for*

all l *and* $F_l' \in D_l$, $U\left(F_l \left| \sum F_l q_l \right.\right) \geq U\left(F_l' \left| \sum F_l q_l \right.\right)$.[8]

If the person expects to choose F_l from choice set D_l, then given her expectations over possible choice sets she expects the distribution of outcomes $\sum F_l q_l$. Definition 1 says that with those expectations as her reference point, she should indeed be willing to choose F_l from choice set D_l.

UPE is closely related to the notion of "loss-aversion equilibrium" that Shalev (2000) defined for multiplayer games as a Nash equilibrium fixing each player's reference point, where the reference point is equal to the player's (implicitly defined) reference-dependent expected utility. Although Shalev does not himself pursue this direction, reformulating his notion of loss-aversion equilibrium using our utility function and applying it to individual decisionmaking corresponds to UPE.

In the decision between a fifty-fifty chance of paying \$100 and buying insurance for \$55, when is choosing the lottery a UPE? If the lottery is the reference point, the following inequality indicates when it is preferred to paying \$55:

$$\left[\frac{1}{2}(-100) + \frac{1}{2}0\right] + \left[\frac{1}{4}\mu(100) + \frac{1}{4}\mu(-100)\right] \geq [-55] + \left[\frac{1}{2}\mu(45) + \frac{1}{2}\mu(-55)\right]. \tag{3}$$

There can often be multiple UPE in a given situation, and generically a decisionmaker gets different expected utilities from different UPE. To the extent that she can choose any plan so long as she will follow it through, she will choose her *preferred personal equilibrium* (PPE):

Definition 2 *A selection* $\{F_l \in D_l\}_{l=1,\ldots,L}$ *is a* <u>*preferred personal equilibrium*</u> *(PPE) if it is a UPE, and* $U\left(\sum F_l q_l \left| \sum F_l q_l \right.\right) \geq U\left(\sum F_l' q_l \left| \sum F_l' q_l \right.\right)$ *for all UPE selections* $\{F_l' \in D_l\}_{l=1,\ldots,L}$.

A major feature of UPE and its refinement PPE is the constraint that choice must be optimal *given* expectations at the time. This means that the decisionmaker does not internalize the effect of her choice on expectations, so that she often does not maximize ex-ante expected utility among the choices available to her. A person may prefer following through on a plan not to take a gamble to following through on taking the gamble, but nonetheless UPE and PPE predict she will take the gamble if after the fact she finds it attractive.

[8] Because each of our solution concepts are examples of personal equilibrium as first defined in Kőszegi (2003), Theorem 1 of that paper implies that when $\sum D_l q_l$ is convex and compact, all three types of equilibria exist.

To begin our analysis of UPE and PPE behavior, we note that the results in Section 2 on fixed expectations can be thought of as applying UPE or PPE to situations where a person's eventual choice set D is a "surprise": if she had been expecting to face choice set D' with near certainty and D with very small probability, and to choose $F \in D'$ (perhaps because $D' = \{F\}$), her reference point is "close to" F independently of D or what she had been expecting to choose from D.

Such surprise situations describe many real-life environments—including the ones in which disposition effects have been observed—as well as most experimental settings. From the perspective of our model, this means that most experiments, by virtue of fixing the reference point at a clear level, are on the one hand ideal for identifying people's value functions, but are on the other hand misleading because in situations that do not come as a surprise, people are likely to have different—and endogenous—expectations.

To illustrate risk attitudes in this and especially later situations, we carry through our model's implications in a parameterized example with $m(c) = 10,000 \ln(c)$, and $\mu(x) = \sqrt{x}$ for $x \geq 0$ and $\mu(x) = -3\sqrt{-x}$ for $x \leq 0$.[9] Table 1 summarizes some of the implications for attitudes toward a fifty-fifty gamble that pays either \$1,000,100 or \$999,900. Rows correspond to environments the gamble might be evaluated in, with the first seven rows considering surprise situations with various expecations. The first column identifies the gamble's certainty equivalent for each environment. The second column gives the coefficient of relative risk aversion $\tilde{\rho}$ that would be inferred from the gamble's certainty equivalent if one assumed reference-independent CRRA utility and a wealth of \$1 million. In response to the surprise gamble, the decisionmaker displays extreme risk lovingness if the gamble is a small loss relative to expectations, and extreme risk aversion when it is a small gain or involves both a loss and a gain. While loss aversion is a stronger force than is diminishing sensitivity, the decisionmaker displays more risk aversion for gains than for mixed gambles because in the latter case she evaluates a payment for insurance as a loss. But in contrast to such strong

[9]Note that, unless A3$'$ holds, our model is not invariant to affine transformations of $m(\cdot)$, so that the appropriate specification of $m(\cdot)$ and $\mu(\cdot)$ involves a substantive assumption about their relative scaling. This scaling amounts to an assumption about the speed of diminishing sensitivity in gain-loss utility. Indeed, the choice to specifiy our example with $m(c) = 10,000 \ln(c)$ is to get both gain-loss utility to dominate for small stakes and consumption utility to dominate for large stakes.

non-neutrality to risk, when the surprise gamble far from the decisionmaker's prior expectations, she is virtually risk neutral in evaluating it. All said, the calculations of $\tilde{\rho}$ reported in Tables 1-3 are indicative of our interpretation of the literature estimating risk attitudes with the mis-specified model that assumes risk aversion is independent of gains and losses. The range of implied $\tilde{\rho}$ from -1,700 to 23,000 for small-scale \$100 risk and of -14 to 15 for modest-scale \$10,000 correspond to the wild range of estimates of $\tilde{\rho}$ observed in the literature. The fact that for huge-scale risk $\tilde{\rho}$ hovers very close to 1 corresponds in turn to the far greater stability of estimates in the literature for huge-scale risks.

To study decisions for choice situations that are anticipated, we first establish that a person has a strong preference to insure risks—she is first-order risk averse—and then show that expecting risk to start with decreases her aversion to additional risk.

Proposition 3 *Suppose $m(\cdot)$ is linear. For any $c \in \mathbb{R}$ and mean-zero lottery $F \neq 0$ with bounded support, there exist $\bar{k}, \bar{t} > 0$ such that for any positive $t < \bar{t}, k < \bar{k}$, the unique PPE in the choice set $\{c, c + t(F + k)\}$ is to choose c.*

Proposition 3 says that a person will always turn down better-than-fair bets that are sufficiently small and insufficiently attractive.[10] The intuition is related to two observations in the previous section. First, choosing c is a UPE: when the reference point is c, the decisionmaker's reaction to modest-scale risks accords to prospect theory around c, so that due to loss aversion she turns down a gamble slightly more favorable than c. Second, c yields higher expected utility than the gamble—and hence is a PPE—because anticipated risk makes the decisionmaker unhappy: she prefers a riskless *reference point* to a risky one, and once the reference point is riskless, our previous point says that she prefers a riskless *outcome* to a risky one as well.

"Status-quo prospect theory" says that diminishing sensitivity governs attitudes toward insuring losses, so that except for a willingness to pay to avoid small-probability losses because of non-linear probability weighting, people will dislike insurance. We reverse this prediction by capturing an intuition regarding the difference between "costs" and "losses" that has been around in the

[10] Proposition 10 in Appendix A identifies a precise condition on the attractiveness of a vanishingly small lottery that determines whether the decisionmaker accepts the lottery.

literature (e.g. Kahneman and Tversky 1984, Novemsky and Kahneman 2005), but has not been precisely formulated or formalized. In our model, an expected premium payment is not evaluated as a loss, but a bad realization of a stochastic lottery is. Because loss aversion therefore plays a central role in the decision of whether to insure, first-order risk aversion results.

Row 8 of Table 1 quantifies the implications of this point and the limit result in Proposition 3 for our parameterized example. The decisionmaker's certainty equivalent is lower than for any of the surprise situations, and is in particular much lower than when she expected \$1,000,000 and was surprised by the gamble.

We conclude with results on two ways in which expecting risk to start with decreases aversion to additional risk. Although our formal results identify the effect of facing a lottery G the decisionmaker *cannot* avoid, this unavoidability can be the reduced-form representation of a situation where she can, but in equilibrium *does not* avoid G. First, the decisionmaker is more likely to accept a given lottery if she is rather than is not already facing:

Proposition 4 *Suppose $m(\cdot)$ is linear and μ satisfies A3$'$. For any lotteries G and F and constant k, if choosing G is a UPE with the choice set $\{G, G+F\}$, then choosing k is a PPE with the choice set $\{k, k+F\}$.*

And as a corollary to Proposition 2, a person prefers not to buy actuarially unfair insurance for risks that are small relative to the risk she is already expecting:

Proposition 5 *Suppose $m(\cdot)$ is linear. For any lottery F with positive expected value:*

1. There exist $A, \epsilon > 0$ such that for any lottery G where $Prob_G(r \in [k-A, k+A]) < \epsilon$ for all constants k, the unique UPE for the decision set $\{G, G+F\}$ is to choose $G+F$.

2. For any continuously distributed lottery G, there is a $\bar{t} > 0$ such that for any $t \in (0, \bar{t}]$, the unique UPE for the decision set $\{G, G+t \cdot F\}$ is to choose $G + t \cdot F$.

Rows 9, 10, and 11 of Table 1 illustrate these propositions for our parameterized example, showing that the decisionmaker approaches risk neutrality for relatively modest amounts of background risk. Even an unavoidable risk on the order of \$100 decreases the premium she is willing to pay from \$48 to \$9, and unavoidable risk on the order of \$1,000 makes her virtually risk neutral.

4 CPE Risk Attitudes

We now analyze risk preferences regarding outcomes that are resolved long after all decisions are committed to, a situation that applies to most small and modest-scale insurance choices. In this case, the expectations relative to which outcomes are evaluated are formed after—and therefore incorporate the implications of—the decision.

Definition 3 *A selection* $\{F_l \in D_l\}_{l=1,\ldots,L}$ *is a* <u>*choice-acclimating personal equilibrium*</u> *(CPE) if* $U\left(\sum F_l q_l \,|\sum F_l q_l\right) \geq U\left(\sum F'_l q_l \,|\sum F'_l q_l\right)$ *for all selections* $\{F'_l \in D_l\}_{l=1,\ldots,L}$.

If the decisionmaker makes a set of choices $F_l \in D_l$, these choices will determine her reference point by the time the relevant wealth outcome occurs. Thus, when evaluating her resulting expected utility, both the reference and outcome lotteries are equal to $\sum F_l q_l$. Our notion of CPE is related to Gul's (1991) model of "disappointment aversion," where outcomes are also evaluated relative to a reference lottery that is identical to the chosen lottery. As we have mentioned, however, in Gul's formulation evaluation is relative to the reference lottery's certainty equivalent.

Returning to our example of choosing between a 50% chance of losing \$100 and insuring this risk for \$55, selecting the lottery is a CPE if

$$\left[\frac{1}{2}(-100) + \frac{1}{2}0\right] + \left[\frac{1}{4}\mu(100) + \frac{1}{4}\mu(-100)\right] \geq [-55] + [0]. \tag{4}$$

The difference between our concepts is in the right-hand sides of Inequalities (3) and (4), which capture the decisionmaker's expected utilities when deviating from the purported UPE and CPE, respectively. In UPE, the reference point does not adjust to the deviation, so paying \$55 is assessed partly as a loss of \$55 and partly as a gain of \$45. In CPE, the reference point adjusts to the choice of paying \$55, so that there is no sensation of gain or loss once that outcome occurs.

As with PPE, except in knife-edge cases there will be a unique CPE. But unlike in PPE, where the decisionmaker can choose her favorite plan only from among UPE plans, in CPE she can commit to her overall favorite lottery. Hence, there cannot be a divergence between behavior and welfare.

It bears emphasizing that UPE/PPE and CPE are not different theories of reference-dependent preferences. Indeed, the decisionmaker's expected utility from choosing the lottery (the left-hand

side of Inequality (3) or (4)) does not depend on whether the choice is determined by UPE or CPE. Rather, the two concepts are motivated by the same theory of preference as manifested differently depending on whether the person can commit to her choice ahead of time.

We note first that Propositions 3 through 5 derived above for PPE also apply to CPE.[11] Because the decisionmaker dislikes anticipated risk, she is first-order risk averse. Furthermore, expecting risk reduces her aversion to additional risk. This latter result follows partly from the force behind our analogous results above, that when expecting uncertainty to start with, avoiding further risk does not eliminate sensations of loss. When a person chooses both her reference and outcome lotteries, there is an additional, parallel force acting in the same direction: when the outcome lottery is uncertain to start with, avoiding the expectation of further risk cannot eliminate sensations of loss.

Despite these similarities, CPE is consistent with risk aversion that is qualitatively different not only from standard expected-utility-over-wealth models and prospect theory, but also from the predictions of UPE, PPE, or any model where the reference point is taken as given at the moment of choice. Strikingly, the motive to avoid risky expectations can lead the decisionmaker choose a stochastically dominated option. To illustrate this possibility, consider a lottery F that yields $g > 0$ with probability $p \geq 0$ and zero with probability $1 - p$, and μ satisfies A3$'$. Then

$$U(F|F) = [pg + (1-p)0] + [p(1-p)\mu(g) + p(1-p)\mu(-g)] = pg[1 - (1-p)\eta(\lambda - 1)].$$

If $\eta(\lambda - 1) > 1$, the decisionmaker chooses $p = 0$ over a small $p > 0$. Intuitively, raising expectations of getting g makes an outcome of no gain feel more painful. To avoid such disappointments, the person gives up the fragile hope of making gains. In fact, if gain-loss utility is sufficiently important, reducing exposure to sensations of loss is the decisionmaker's central concern:

Proposition 6 *Suppose $m(\cdot)$ is linear and the decisionmaker faces the finite choice set D containing the deterministic outcome c and no greater deterministic outcomes. For any given $\mu_0(\cdot)$, there is an $\bar{\eta}$ such that if $\mu(\cdot) = \eta\mu_0(\cdot)$ with $\eta > \bar{\eta}$, the unique CPE is to choose c.*

More than a theoretical possibility illustrated by our example above and implied by Proposition 6, CPE predicts the rejection of uncertain gains for calibrationally plausible specifications of the

[11] The proofs of Propositions 3 through 5 in Appendix B establish the statements for CPE as well.

utility function. In fact, $\eta(\lambda - 1) > 1$ whenever *observed* loss aversion is at least two-to-one—whenever overall sensitivity to losses, $1 + \eta\lambda$, is at least twice as high as overall sensitivity to gains, $1 + \eta$.

While the tendency to choose a stochastically dominated lottery may seem counterintuitive, it is consistent with the flavor of some discussions in the psychology literature on welfare and risk-taking. Frederick and Loewenstein (1999) discuss, for example, how a prisoner may be made worse off by a small chance of being released, because that makes the outcome of remaining in prison much more difficult to bear. In the domain being examined in this paper, we also feel our result that the decisionmaker chooses a stochastically dominated lottery captures in extreme form the strong risk aversion consumers display when purchasing insurance for long-term modest-scale losses, choosing low deductibles on existing insurance, and selecting expensive fixed-fee contracts.

There are ways in which our result must be qualified, however. Importantly, the preference for dominated lotteries clearly arises only when such lotteries reduce exposure to gain-loss sensations. In addition, diminishing sensitivity can substantially reduce a person's dislike of risk for modest stakes. As g increases, the sensation of loss from comparing nothing to g increases slower and slower, whereas consumption utility increases linearly. Hence, it becomes more and more attractive to take the chance at getting g.

Beyond illustrating a unique property of CPE, the rejection of a probabilistic gain is also an example of a more general difference between the CPE and PPE: there are risky options that a person takes in PPE that she would not take in CPE because they do not maximize her ex-ante expected utility among available options.[12] Intuitively, in PPE the decisionmaker realizes that she will take a lottery that is attractive *fixing expectations*, and because she incorporates the possibility of good outcomes into her reference point, her sense of loss from low outcomes is increased.

[12]Suppose, for instance, that A3$'$ holds and the person has two-to-one loss aversion ($(1 + \eta\lambda)/(1 + \eta) = 2$), and consider the choice set consisting of a riskless \$0 and a fifty-fifty lose \$10 or gain \$21 gamble. If the person expected \$0, the prospect of \$21 would be sufficiently attractive for her to take the lottery, and this preference would be even stronger if she expected the lottery. Therefore, the unique UPE, and hence also the PPE, is to take the lottery. It is easy to check, however, that the lottery yields lower expected utility.

While a person therefore often takes more risk in PPE than in CPE, the following proposition shows that under A3$'$, the converse cannot happen. Hence, except for complications caused by diminishing sensitivity, we predict people will be more risk averse when decisions are committed to well in advance than when uncommitted.

Proposition 7 *Suppose $m(\cdot)$ is linear, A3$'$ holds, and for different lotteries $F, F' \in D$, F' is a mean-preserving spread of $F + k$ for some constant k. If F is a PPE, F' is not a CPE.*

Both the similarities and differences between PPE and CPE are nicely illustrated by the example in Table 1. Comparing rows 12 through 15 with rows 8 through 11, it is clear that with any given amount of background risk, CPE choices are more risk averse than PPE choices. Nevertheless, CPE behavior also approaches risk neutrality with even moderate amounts of background risk.

5 Immodest Risk

We now illustrate some of our model's implications for attitudes towards large-scale risk, showing that for such stakes $\mu(\cdot)$ can become largely irrelevant in determining risk preferences. This means that our model can reconcile prospect-theoretic behavior in modest-stakes gambles with classical predictions for larger stakes.

Proposition 8 shows that when diminishing sensitivity is a significant-enough feature of gain-loss utility, the expected utility from very risky outcomes is little influenced by the reference point:

Proposition 8 *Suppose $m(\cdot)$ has full range and $\lim_{x\to\infty} \mu'(x) = \lim_{x\to-\infty} \mu'(x) = 0$. For any $r, r' \in \mathbb{R}$, $r > r'$ and $\epsilon > 0$, there is a $\delta > 0$ such that if F is continuously distributed with density less than δ everywhere, then $U(F|r') - U(F|r) < \epsilon$.*

If F is a very risky gamble, most of its outcomes are far from both r and r'. Since sensitivity of gain-loss utility to changes approaches zero far from the reference point, it makes little difference whether these outcomes are being compared to r or r'.

Beyond this analytical result, our model's predictions for large-scale risks can be illustrated by applying the parameterized version we have considered above to larger stakes. Table 2 and 3 perform this exercise for two gambles. Table 2 considers the decisionmaker's attitudes toward a fifty-fifty \$990,000 or \$1,010,000 gamble in various surprise, PPE, and CPE situations, and Table 3 considers a fifty-fifty \$500,000 or \$1,500,000 gamble. Recall that for the \$100 gamble considered in Table 1, the decisionmaker is extremely risk-loving for surprise losses, and her behavior is extremely sensitive to the circumstances in which she is facing that gamble. While Table 2 illustrates that she exhibits a similar qualitative pattern for \$10,000 stakes, the sensitivity of her behavior is significantly reduced. And for the \$500,000 gamble, her risk attitudes are close to what they would be without gain-loss utility, being largely determined by $m(\cdot)$ independently of the environment.

6 Caveats, Discussion, and Conclusion

We have developed a theory of reference-dependent preferences that can help explain, under a single umbrella, risk attitudes that would look inconsistent when interpreted through other models. From the perspective of our model, therefore, it is no surprise that different measurements of risk aversion based on the neoclassical reference-independent paradigm yield very different conclusions. Our model not only implies that such measures will vary across contexts and scales, but, as reflected in the great variation in the inferred coefficients of relative risk aversion in Tables 1 to 3, predicts *how*: when measured on very large-stakes data, single-digit coefficients will be found; when measured on modest-stakes data, triple-digit coefficients will be found; when measured on small-stakes data, coefficients too embarrassingly large to report will be found.[13]

[13]The deductible choices of American homeowners analyzed by Sydnor (2005) and the behavior of Paraguayan farmers analyzed by Schechter (2005) imply coefficients of relative risk aversion in the triple digits. Mehra and Prescott (1985) estimate that to explain the historical equity premium, investors have to have a coefficient of relative risk aversion well in the double digits. Estimating an unemployment model with consumption and search-effort choices using data on unemployment durations, Chetty (2003) estimates a coefficient of relative risk aversion of around 7. Based on hypothetical choices between large gambles on lifetime wealth, Barsky, Juster, Kimball, and Shapiro (1997) estimate an average coefficient of relative risk averison of around 5. Chetty (2005) shows that existing evidence on

But the reduced-form model of this paper bypasses several complications that must be addressed for a full understanding of reference-dependent risk preferences. The greatest weakness of our theory is that it takes as one of its primitives the set of decisions and risks a person is considering, as distinct from all the decisions and risks she is facing. Indeed, Proposition 2 and 5 can be interpreted as saying that if people incorporated all present and future risks into their expectations—a substantial amount of risk for most people—reference dependence would essentially not affect risk attitudes. Although psychological evidence indicates that people often "narrowly bracket"—they isolate individual decisions and risks from others, even when they would benefit substantially from thinking more broadly—relatively little is known about the extent, patterns, and effects of such bracketing phenomena. Our model, as well as any plausible model of risk preferences, is far too sensitive to such bracketing to be complacent about its status as a model of rational utility maximization.[14]

Another major issue with our model concerns welfare. Since the hedonic effect of choice includes both gain-loss sensations and consumption utility, using our utility function as a first-pass welfare measure is realistic. For two specific reasons, however, we are more ambivalent about our model's welfare conclusions than about its behavioral conclusions. First, people underestimate how quickly the reference point will adjust to a choice, and hence put too much weight on gain-loss sensations when making decisions.[15] Second, people's narrow focus on individual decisions and risks may lead them to care too much about a gain or loss that is likely to be eliminated soon by an offsetting loss or gain.

For both of these reasons, the appropriate welfare measure is likely to be closer to consumption

the income elasticity of labor supply comfortably bounds the coefficient of relative risk aversion from above by 2. Finally, many papers that report small coefficients of risk aversion even for small stakes do so by dint of using the same terminology to describe mathematically different measures; by variously defining wealth as monthly income or potential income over the course of a one-hour experiment rather than lifetime wealth, these papers report figures that are, by arithmetic fiat, orders of magnitude different than would be found with a consistent measure.

[14]For papers highlighting the role of bracketing in this and other domains, see Kahneman and Lovallo (1993), Benartzi and Thaler (1995), Read, Loewenstein, and Rabin (1999), and Thaler (2000).

[15]This relates to what Kahneman (2003) refers to as the "transition heuristic," and what Loewenstein, O'Donoghue, and Rabin (2003) refer to as "projection bias."

utility than we assume in this paper. This could significantly alter some of our welfare conclusions. For example, although we showed above that the availability of a lottery can lower welfare defined to include gain-loss disutility, the same analysis implies that this would not be possible for welfare based solely on consumption utility.

The underestimation of changes in the reference point also has behavioral implications. If a person underestimates the effect of changes in her expectations on her preferences, she may not appreciate fully how she can rid herself of risky reference points by changing her expectations. Because our predictions of first-order risk aversion are driven partly by a person's desire to avoid risky expectations, this underappreciation can reduce risk aversion.

Our model also glosses over a set of issues related to what expected and realized outcomes she pays attention to. In contrast to our formal model, different outcomes resulting from the same choice often differ in salience. As noted in Sydnor (2005), for instance, having to pay for repairing an uninsured house is a very salient loss, but *not* having to pay rarely results in a strong sensation of gain. A person who focuses more on such losses than what is deemed in our model as "gains" presumably has an even stronger taste for insurance than our model predicts.

Finally, our model ignores a source of utility—anticipatory emotions—that seems important in many risky situations. For instance, an investor's anxiety about funding her child's education is likely to affect both her welfare and many of her financial decisions. Insofar as anticipatory feelings are about future consumption and gain-loss utilities, our qualitative results would not be affected by adding them to the model. Quantitatively, however, anticipatory emotions can affect the degree of risk aversion; Caplin and Leahy (2001), for instance, show that the attempt to avoid anxiety about uncertain outcomes can increase a person's preference for riskless options.

Table 1 — Attitudes Toward a Fifty-Fifty \$999,900/\$1,000,100 Gamble		
	Certainty Equivalent	Inferred $\widetilde{\rho}$
Surprise, expected \$500,000	$\$1,000,000 - \epsilon$	$1 + \epsilon$
Surprise, expected \$990,000	$\$1,000,000 - \epsilon$	3.4
Surprise, expected \$999,900	\$999,981	3,936
Surprise, expected \$1,000,000	\$999,991	1,844
Surprise, expected \$1,000,100	\$1,000,033	-7,171
Surprise, expected \$1,010,000	$\$1,000,000 + \epsilon$	-5.6
Surprise, expected \$1,500,000	$\$1,000,000 - \epsilon$	.97
PPE, no background risk	\$999,952	11,389
PPE, U[-100,+100] background risk	\$999,991	1,717
PPE, U[-1,000,+1,000] background risk	$\$1,000,000 - \epsilon$	111
PPE, U[-10,000,+10,000] background risk	$\$1,000,000 - \epsilon$	6.0
CPE, no background risk	\$999,929	23,348
CPE, U[-100,+100] background risk	\$999,971	6,041
CPE, U[-1,000,+1,000] background risk	\$999,998	368
CPE, U[-10,000,+10,000] background risk	$\$1,000,000 - \epsilon$	14.4

Note: the uniform distributions are discrete uniforms with atoms on multiples of \$50.

Table 2 — Attitudes Toward a Fifty-Fifty \$990,000/\$1,010,000 Gamble			
		Certainty Equivalent	Inferred $\widetilde{\rho}$
Surprise, expected \$500,000		$\$1,000,000-\epsilon$	$1+\epsilon$
Surprise, expected \$990,000		$\$999,671$	6.5
Surprise, expected \$999,900		\$999,531	9.4
Surprise, expected \$1,000,000		\$999,568	8.6
Surprise, expected \$1,000,100		\$999,603	7.9
Surprise, expected \$1,010,000		\$1,000,712	-14.3
Surprise, expected \$1,500,000		\$999,951	.97
PPE, no background risk		\$999,243	15.2
PPE, U[-1,000,+1,000] background risk		\$999,364	12.8
PPE, U[-10,000,+10,000] background risk		$\$999,761$	4.7
CPE, no background risk		\$999,243	15.2
CPE, U[-1,000,+1,000] background risk		\$999,364	12.8
CPE, U[-10,000,+10,000] background risk		$\$999,638$	7.2

Note: the uniform distributions are discrete uniforms with atoms on multiples of \$50.

Table 3 — Attitudes Toward a Fifty-Fifty \$500,000/\$1,500,000 Gamble			
		Certainty Equivalent	Inferred $\widetilde{\rho}$
Surprise, expected \$500,000		$\$864,160$	$1+\epsilon$
Surprise, expected \$990,000		$\$867,523$	.99
Surprise, expected \$999,900		$\$867,752$	.99
Surprise, expected \$1,000,000		\$867,754	.99
Surprise, expected \$1,000,100		$\$867,756$	.99
Surprise, expected \$1,010,000		\$867,972	.99
Surprise, expected \$1,500,000		$\$871,571$	.96
PPE, no background risk		\$861,498	1.04
PPE, U[-1,000,+1,000] background risk		\$861,606	1.03
PPE, U[-10,000,+10,000] background risk		$\$861,824$	1.03
CPE, no background risk		\$861,498	1.04
CPE, U[-1,000,+1,000] background risk		\$861,606	1.03
CPE, U[-10,000,+10,000] background risk		$\$861,824$	1.03

Note: the uniform distributions are discrete uniforms with atoms on multiples of \$50.

Appendix A

In this appendix we present an array of concepts and results that may be of practical use in applying our model, but which are not key to any of the main results of the paper. We begin with identifying a condition such that with the reference point being the status quo, the decisionmaker rejects all fair gambles.

Proposition 9 *Suppose $m(\cdot)$ is linear and the reference point is \$0. The decisionmaker rejects all fair gambles if and only if* $\lim_{x\to\infty} \mu'(-x) \geq \mu'_{+}(0)$.

Assumption A3 allows the decisionmaker's risk lovingness in losses to be much stronger than her risk aversion in gains, which may even lead her to accept unfair gambles given a reference point of \$0. Proposition 9 says that when $m(\cdot)$ is linear, a necessary and sufficient additional condition to rule out such possibilities is that sensitivity to losses is everywhere greater than sensitivity to gains. When $m(\cdot)$ is concave, this condition is of course sufficient, but not necessary.

For our results on PPE and CPE behavior, we introduce three definitions characterizing the riskiness of lotteries. Our first definition is the conventional one of second-order stochastic dominance, but allows comparisons of lotteries with different means:

Definition 4 *A lottery F is <u>less risky</u> than the lottery F' if F' is a mean-preserving spread of $F + k$ for some constant $k \in \mathbb{R}$.*

Because it turns out to be an especially pertinent measure of riskiness in some special cases of our model, we also introduce a more specific concept that is (to our knowledge) undefined and unexplored in the literature on risk preferences:

Definition 5 *For a lottery $F \in \Delta(\mathbb{R})$, the <u>average self-distance</u> of F is*

$$S(F) \equiv \iint |x - y| dF(x) dF(y).$$

The average self-distance of a lottery is the average distance between two independent draws from the lottery. A lower self-distance is a necessary but not sufficient condition for one lottery to be unambiguously less risky than another:

Lemma 1 *If F is less risky than F', then F has lower average self-distance than F'.*

Finally, we introduce a measure for how a lottery's possible gains compare to its possible losses.

Definition 6 *For a lottery F, let $F_+ = E_F[\max\{x, 0\}]$ and $F_- = E_F[\max\{-x, 0\}]$. The* <u>*favorability*</u> *of F is defined as $\Phi(F) \equiv 1$ if $F_+ = F_- = 0$, and $\Phi(F) \equiv F_+/F_-$ otherwise.*

The favorability of a lottery is the ratio of the average gain of the lottery (relative to zero) and the average loss. Using the above concepts, Proposition 10 precisely identifies the extent of the decisionmaker's first-order risk aversion, generalizing Arrow's theorem to reference-dependent risky choice. This extends the limit result of Proposition 3 in the text, which said that small bets that are insufficiently better-than-fair will be rejected.

Proposition 10 *Suppose $c \in \mathbb{R}$, and F is a lottery with bounded support.*

1. If $\Phi(F) < (1 + \mu'_-(0))/(1 + \mu'_+(0))$, then there exists a $\bar{t} > 0$ such that for any positive $t < \bar{t}$, the unique PPE in the choice set $\{c, c + t \cdot F\}$ is to choose c. If $\Phi(F) > (1 + \mu'_-(0))/(1 + \mu'_+(0))$, then there exists a $\bar{t} > 0$ such that for any positive $t < \bar{t}$, the unique PPE in the choice set $\{c, c + t \cdot F\}$ is to choose $c + t \cdot F$.

2. If $2E[F] < (\mu'_-(0) - \mu'_+(0))S[F]$, then there exists a $\bar{t} > 0$ such that for any positive $t < \bar{t}$, the unique CPE in the choice set $\{c, c + t \cdot F\}$ is to choose c. If $2E[F] > (\mu'_-(0) - \mu'_+(0))S[F]$, then there exists a $\bar{t} > 0$ such that for any positive $t < \bar{t}$, the unique CPE in the choice set $\{c, c + t \cdot F\}$ is to choose $c + t \cdot F$.

Part 1 says that when applying PPE, small bets will be accepted if and only if their favorability is greater than the "coefficient of loss aversion" associated with $u(c|r)$—which is the ratio of the kink in $u(c|r)$ at $c = r$. Part 2 says that when applying CPE, a small bet will be accepted if and only if twice its expected value is greater than the product of its average self-distance and the difference in the decisionmaker's sensitivity to small losses and gains.

Proposition 11 shows that when consumption utility is linear and A3′ holds, we can characterize a person's CPE attitude toward a lottery purely in terms the lottery's mean and average self-distance:

Proposition 11 *Suppose $m(c) = c$ and $\mu(\cdot)$ meets A3′. Then,*

I. For any lottery F,

$$U(F|F) = E[F] - \frac{1}{2}\eta(\lambda - 1)S[F].$$

II. For a choice set D, define $y(D) = \{F \in D|$ for all $F' \in D$, either (i) $S[F] < S[F']$, or (ii) $S[F] = S[F']$ and $E[F] \geq E[F']\}$. For a sufficiently high η, $y(D)$ is the set of CPE.

Part II represents a lexicographic ranking of lotteries by their lowest average self-distance, and then the highest mean. There will generally be a unique lottery with minimal average self-distance, in which case that lottery is chosen when gain-loss utility is very important even if it is stochastically dominated by other options.

The role of average self-distance in determining a person's CPE choices is best seen with a simple but striking observation. In a personal equilibrium (of any sort), every possible sensation of gain—say from comparing an outcome x to a counterfactual $y < x$—is matched by an equally likely and equally large loss—from comparing y to x. Because the losses are more heavily felt, on net gain-loss utility will be proportional to the negative of the average of these distances.

Finally, we show two properties of risky choice in our model that it shares with the standard model. Although expected risk decreases aversion to risk under either of our solution concepts, without diminishing sensitivity it never eliminates the risk aversion completely:

Proposition 12 *Suppose A3′ holds and F second-order stochastically dominates $F' \neq F$. Then both the unique PPE and the unique CPE from the choice set $\{F, F'\}$ is to choose F.*

We also note that if F first-order stochastically dominates F', then for any given reference lottery the decisionmaker prefers F to F'. Thus, choosing F' cannot be a UPE. While mathematically trivial, this result is of interest because it contrasts with some of our results for CPE.

Proposition 13 *Suppose the decisionmaker faces the choice set D. If $F \in D$ first-order stochastically dominates F', then F' is not a UPE.*[16]

Appendix B

Proof of Proposition 1. Let $F_+ = E_F[\max\{x, 0\}]$ and $F_- = E_F[\max\{-x, 0\}]$. That is, F_+ and F_- are the expectations of the positive and negative parts of F, respectively.

[16] In fact, this result relies solely on Assumption A1, guaranteeing that $u(c|r)$ is increasing in c, and on no other feature of μ.

Clearly, $U(k+F|k) \geq U(k|k)$ if and only if $(1+\eta)F_+ \geq (1+\eta\lambda)F_-$. Now $U(G+F|G) \geq U(G|G)$ is equivalent to

$$\iiint \left[c + c' + \mu(c + c' - r)\right] dG(r)dG(c)dF(c') \geq \iint \left[c + \mu(c - r)\right] dG(r)dG(c)$$

or

$$\iiint \left[c' + \mu(c + c' - r) - \mu(c - r)\right] dG(r)dG(c)dF(c') \geq 0.$$

Notice that for any $c' \geq 0$, $\mu(c+c'-r)-\mu(c-r) \geq \eta c'$, and for any $c' \leq 0$, $\mu(c+c'-r)-\mu(c-r) \geq \eta\lambda c'$. Hence

$$\begin{aligned} & \iiint \left[c' + \mu(c + c' - r) - \mu(c - r)\right] dG(r)dG(c)dF(c') \\ \geq \; & \iiint \left[c' + \eta \max\{c', 0\} + \eta\lambda \min\{c', 0\}\right] dG(r)dG(c)dF(c') \geq (1+\eta)F_+ - (1+\eta\lambda)F_- \geq 0. \end{aligned}$$

This completes the proof.

Proof of Proposition 2. Let F' be the mean-zero lottery that satisfies $F' = F + k$ for a constant k.

1. We prove that for any $\epsilon_1 > 0$, there are $A, \epsilon > 0$ such that if $Prob_G(r \in [-A, A]) < \epsilon$, then for any $a \in \mathbb{R}$,

$$\iint \left(\mu(a + c - r) - \mu(a - r)\right) dF'(c)dG(r) > -\epsilon_1. \tag{5}$$

This is sufficient because it implies that $U(G + F'|G) - U(G|G) > -\epsilon_1$, and hence $U(G + F|G) - U(G|G) \geq U(G + F'|G) - U(G|G) + k > k - \epsilon_1 > 0$ for ϵ_1 to be sufficiently small.

Since μ is differentiable other than at zero and is concave in gains and convex in losses, both $\lim_{x\to\infty} \mu'(x)$ and $\lim_{x\to-\infty} \mu'(x)$ exist. This implies that for any $\epsilon_2 > 0$, there is an A such that

$$h(b) \equiv \int \left(\mu(b + c) - \mu(b)\right) dF'(c) > -\epsilon_2$$

for any $|b| > A$. h is also bounded; let its bound be M. Notice that

$$\iint \left(\mu(a + c - r) - \mu(a - r)\right) dF(c)dG(r) = \int h(a - r)dG(r).$$

Since $Prob_G[a-r \in [-A,A]] < \epsilon$ and $h(\cdot) > -\epsilon_2$ outside this range, the above integral is greater than $-\epsilon M - \epsilon_2$. Therefore, we can choose A, ϵ, and ϵ_2 such that if G satisfies the conditions of the proposition, the above integral is greater than $-\epsilon_1$.

2. Whenever $G(\cdot)$ is a continuous distribution, the function

$$h(\cdot) \equiv \int u(\cdot|r)dG(r)$$

is differentiable everywhere. Hence, for any $c \in \mathbb{R}$,

$$\lim_{t\to 0} \frac{\int [h(c+t\cdot c') - h(c)]\, dF'(c')}{t} = 0$$

everywhere. Hence

$$\lim_{t\to 0} \frac{U(G+t\cdot F'|G) - U(G|G)}{t} = \iint \left[\lim_{t\to 0} \frac{\int [h(c+t\cdot c') - h(c)]\, dF'(c')}{t}\right] dG(c) = 0.$$

This implies that

$$\lim_{t\to 0} \frac{U(G+t\cdot F|G) - U(G|G)}{t} > 0,$$

completing the proof.

Proof of Proposition 3. Let $G = F+k$. As in the proof of Proposition 1, let $G_+ = E_G[\max\{x,0\}]$ and $G_- = E_G[\max\{-x,0\}]$. We prove that if $G_+/G_- < (1+\mu'_-(0))/(1+\mu'_+(0))$, then there is a $\bar{t}$ satisfying the statement of the proposition. This is sufficient because for $k=0$, $G_+/G_- = 1$.

We have

$$\begin{aligned}\lim_{t\to 0} \frac{U(c+tG|c) - U(c|c)}{t} &= \frac{\int [\max\{0,tc'\} + \mu(\max\{0,tc'\})]\, dG(c') + \int [\min\{0,tc'\} + \mu(\min\{0,tc'\})]\, dG(c')}{t} \\ &= (1+\mu'_+(0))G_+ - (1+\mu'_-(0))G_- < 0.\end{aligned}$$

Hence, there is a $\bar{t}$ such that for $t < \bar{t}$, choosing c is a UPE in the choice set $\{c, c+tG\}$.

Also:

$$
\begin{aligned}
\lim_{t\to 0}\frac{U(c+tG|c+tG)-U(c|c)}{t} &= \int c'dG(c') + \frac{\iint \mu(t(c'-r))dG(c')dG(r)}{t} \\
&= \int c'dG(c') - \frac{1}{2}(\mu'_-(0)-\mu'_+(0))A(G) \\
&\leq G_+ + G_- - (\mu'_-(0)-\mu'_+(0))(G_+ + G_-) \\
&< (1+\mu'_+(0))G_+ - (1+\mu'_-(0))G_- < 0,
\end{aligned}
$$

where the second-to-last inequality is true because $G_+ \geq G_-$. This establishes that c is both a PPE and CPE.

Proof of Proposition 4. By Proposition 1, it is immediate that choosing k is a UPE. Hence, it is sufficient to prove that k is preferred to any $F \in D$.

That k is a UPE implies that $U(k|k) \geq U(F|k)$. Hence, there is a $j \geq 0$ such that for $F' = F+j$, $U(k|k) = U(F'|k)$. We prove that $U(k|k) > U(F'|F')$ (which is clearly sufficient since $U(F'|F') \geq U(F|F)$). The condition that $U(F'|k) = U(k|k)$ can be written as

$$
\int cdF'(c) + \int \mu(c-k)dF'(c) = k. \tag{6}
$$

Clearly, we must have $k < \int cdF'(c)$. Otherwise, by the concavity of μ, we would have $\int \mu(c-k)dF'(c) < 0$, a contradiction.

We want to prove that

$$
\int cdF'(c) + \iint \mu(c-r)dF'(r)dF'(c) < k.
$$

Given Equation 6, this is equivalent to

$$
\iint \mu(c-r)dF'(r)dF'(c) < \int \mu(c-k)dF'(c)
$$

Now since the mean of F' is strictly greater than k, there is a $j > 0$ such that the mean of $F' - j$ is k. Notice that

$$
\iint \mu(c-r)dF'(r)dF'(c) < \iint \mu(c-r)d(F'-j)(r)dF'(c),
$$

which by the concavity of μ is less than or equal to

$$\int \mu(c-k)dF'(c).$$

This completes the proof.

We now prove the same statement for CPE. We want to prove that if $U(k+F|k+F) \geq U(k|k)$, then $U(G+F|G+F) \geq U(G|G)$. Since k just shifts both sides of the first inequality by a constant, it is sufficient to prove for $k=0$. We will prove that $U(F|F) \geq U(G+F|G+F) - U(G|G)$.

Notice that the two sides are equal in consumption utility (and equal to the expectation of F). Hence, we prove the inequality for the gain-loss-utility component. We take advantage of a geometric analogy: for any distribution H, the negative of the gain-loss utility part of $U(H|H)$ is proportional to the average self-distance of H. Therefore the above statement is equivalent to the following: when the distribution F is added to the distribution G, the increase in the average self-distance is lower than the average self-distance of F. To show this, consider any two realizations a and b of G, and any two realizations x and y of F. By the triangle inequality,

$$|(a+x)-(b+y)| \leq |a-b| + |x-y|$$

or

$$|(a+x)-(b+y)| - |a-b| \leq |x-y|,$$

completing the proof.

Proof of Proposition 5. As in the proof of Proposition 2, define F' as the mean-zero lottery such that $F' = F + k$ for a constant k.

1. As an obvious implication of Proposition 2, there are $A, \epsilon > 0$ such that the unique UPE, and hence also the PPE, is to choose $G + F$.

The proof for CPE is only slightly more complicated. By the proof of Proposition 2, for any $\epsilon_1 > 0$ there are $A, \epsilon > 0$ such that if $Prob_G(r \in [-A, A]) < \epsilon$, then $U(G+F'|G) - U(G|G) > -\epsilon_1$. Applying a similar argument, there are $A, \epsilon > 0$ such that if $Prob_G(r \in [-A, A]) < \epsilon$, $U(G +$

$F'|G+F') - U(G+F'|G) > -\epsilon_1$. Hence, there are $A, \epsilon > 0$ such that if $Prob_G(r \in [-A, A]) < \epsilon$, $U(G+F|G+F) - U(G|G) = U(G+F'|G+F') - U(G|G) + k > k - 2\epsilon_1 > 0$ for a sufficiently small ϵ_1.

2. By Proposition 2, there is a $\bar{t} > 0$ such that for $t < \bar{t}$, the unique UPE, and hence also the PPE, in the choice set $\{G, G+tF\}$ is to choose $G+tF$.

We now prove for CPE. By the same argument as in the proof of Proposition 2,

$$\lim_{t\to 0} \frac{U(G+t\cdot F'|G+t\cdot F') - U(G|G)}{t} = 0,$$

so that

$$\lim_{t\to 0} \frac{U(G+t\cdot F|G+t\cdot F) - U(G|G)}{t} = k > 0.$$

Proof of Proposition 6.

For any lottery F,

$$U(F|F) = e(F) + \eta \iint (\bar{\mu}(c'-r)dF(r)dF(c') = e(F) + \frac{1}{2}\eta \underbrace{\iint (\bar{\mu}(|c'-r|) + \bar{\mu}(-|c'-r|))dF(r)dF(c')}_{\equiv -n(F)}. \tag{7}$$

By A2, $n(F) > 0$ for any non-deterministic lottery F. Let $x = \min_{F\in D, F\neq c} n(F)$ and $y = \max_{F\in D, F\neq c} e(F)$. If $\eta > \frac{2(y-c)}{x}$, the unique CPE is to choose c.

Proof of Proposition 7. We prove that if F is a UPE, then $U(F|F) > U(F'|F')$, so that F' is not a CPE. If $F' = F + k$ for some k, then the result is immediate, since in that case we would have to have $k < 0$.

That F is a UPE when means that $U(F|F) \geq U(F'|F)$. Hence, there is a constant $k' \geq 0$ such that for $F'' = F' + k'$, we have $U(F|F) = U(F''|F)$. We prove that $U(F|F) > U(F''|F'')$, which is sufficient because $U(F''|F'') \geq U(F'|F')$.

The condition that $U(F''|F) = U(F|F)$ can be written as

$$\int cdF''(c) + \iint \mu(c-r)dF(r)dF''(c) = \int cdF(c) + \iint \mu(c-r)dF(r)dF(c). \tag{8}$$

Clearly, we must have $\int cdF(c) < \int cdF''(c)$. Otherwise, because μ is strictly increasing and concave and F'' is riskier than F, we would have $\iint \mu(c-r)dF(r)dF''(c) < \iint \mu(c-r)dF(r)dF(c)$, a contradiction.

We want to prove that

$$\int cdF''(c) + \iint \mu(c-r)dF''(r)dF''(c) \leq \int cdF(c) + \iint \mu(c-r)dF(r)dF(c).$$

Given Equation 8, this is equivalent to

$$\iint \mu(c-r)dF''(r)dF''(c) \leq \iint \mu(c-r)dF(r)dF''(c)$$

Now since the mean of F'' is greater than that of F, there is a $k'' > 0$ such that $F'' - k''$ and F have the same mean. Notice that

$$\iint \mu(c-r)dF''(r)dF''(c) < \iint \mu(c-r)d(F''-k'')(r)dF''(c),$$

which in turn is less than or equal to

$$\iint \mu(c-r)dF(r)dF''(c)$$

because $F'' - k''$ is a mean-preserving spread of F and μ is concave. This completes the proof.

Proof of Proposition 8. Let $M = -\mu(-(m(r) - m(r')))$. By properties A2 and A3 of $\mu(\cdot)$, $\mu(x + m(r) - m(r')) - \mu(x) \leq M$ for any $x \in \mathbb{R}$.

Since $\lim_{x\to\infty} \mu'(x) = \lim_{x\to-\infty} \mu'(x) = 0$, for any $\epsilon_1 > 0$ there is an A such that if $|x| > A$, then $\mu'(x) < \epsilon_1$. Furthermore, since $m(\cdot)$ has full range, for all $\epsilon_2 > 0$ there is a $\delta > 0$ such that if the density of F is less than δ everywhere, $Prob_F[m(c) \in [m(r') - A, m(r) + A]] < \epsilon_2$. Denote the

interval $[m(r') - A, m(r) + A]$ by B. Under these conditions,

$$
\begin{aligned}
U(F|r') - U(F|r) &= \int [\mu(m(c) - m(r')) - \mu(m(c) - m(r)]dF(c) \\
&= \int_{m(c)\in B} [\mu(m(c) - m(r')) - \mu(m(c) - m(r)]dF(c) + \int_{m(c)\notin B} [\mu(m(c) - m(r')) - \mu(m(c) - m(r)]dF(c) \\
&\leq \epsilon_2 M + \epsilon_1(m(r) - m(r')),
\end{aligned}
$$

which is less than ϵ for appropriately chosen ϵ_1, ϵ_2.

Proof of Proposition 9. Obvious.

Proof of Lemma 1. Since constant shifts in a distribution clearly leave the average self-distance unchanged, we can assume without loss of generality that F and F' have the same mean. Then, using that the absolute-value function is convex,

$$
\iint |x-y| dF(x)dF(y) \leq \iint |x-y| dF'(x)dF(y) = \iint |x-y| dF(y)dF'(x) \leq \iint |x-y| dF'(y)dF'(x).
$$

Proof of Proposition 10. Obvious from the proof of Proposition 3.

Proof of Proposition 11. Part I follows from Equation 7. Part II is trivial from Part I.

Proof of Proposition 12. Notice that under A3$'$, the utility function $U(\cdot|F) = \int u(\cdot|r)dF(r)$ is weakly concave. Hence $U(F|F) \geq U(F'|F)$, so that choosing F is a UPE.

We now prove that $U(F|F) > U(F'|F')$, both completing the proof that F is a PPE and proving

that it is a CPE. Since the expected consumption utilities cancel, this inequality is equivalent to

$$\iint \mu(c-r)dF(c)dF(r) > \iint \mu(c-r)dF'(c)dF'(r),$$

or $a(F) < a(F')$. To show this, note that by the convexity of the absolute-value function,

$$\iint |c-r|dF(c)dF(r) \leq \iint |c-r|dF'(c)dF(r) = \iint |c-r|dF(r)dF'(c).$$

Using the same thing, that $F' \neq F$, and that the support of F' contains both negative a positive numbers, the above is strictly less than

$$\iint |c-r|dF'(r)dF'(c) = \iint |c-r|dF'(c)dF'(r).$$

Proof of Proposition 13. For any reference lottery G, the function

$$U(\cdot|G) = \int u(\cdot|r)dG(r)$$

is strictly increasing. Hence, for any two distributions F, F' such that F first-order stochastically dominates F', $U(F|G) > U(F'|G)$. This implies that F' cannot be a UPE when F is available.

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