A BUNDLING ARGUMENT FOR NARROW BANKING*

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Abstract

This study demonstrates a substantial inefficiency in the banking industry resulting from allowing banks to maintain less than a one-hundred percent reserve requirement, thereby exposing depositors to unwanted risks such as investment risks as well as bank runs and bank crises (e.g., the crisis of Scandinavian banks during the early 1990s, and the S&L in the US during the 1980s). This distortion occurs since depositors with uncertain liquidity needs are unable to find riskless no-return banks who will store their money and perform other basic services (ATM, electronic transfers, checkbooks, and bill payment) by charging nominal fees without exposing depositors to any risk. Thus, it is demonstrated that a substantial welfare loss occurs when profit-maximizing banks bundle deposits accounts with risk taking. We further demonstrate that with the rapid development of securities markets a narrow-banking policy will expand consumers' saving/investment opportunities despite the lending restrictions imposed on the banks. Finally, we demonstrate that deposit insurance policies do not eliminate this inefficiency.

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1. Introduction

In recent years, we have witnessed a rapid worldwide consolidation process of the banking industry. The securitization of markets has broken the traditional link between taking deposits and making loans, thereby challenging the domain of old-fashioned banking, which was based on collection of savings in order to provide capital for entrepreneurial projects. Nowadays large corporations have direct access to international capital markets at terms which outperform banks. Thus, banks have to operate in a world where markets for financial services – banking, brokerage as well as insurance – have become increasingly competitive. This development has serious consequences from the point of view of traditional banking, since, as Kay (1998) recently wrote,

"The rational for the traditional association of functions that we call a bank has simply disappeared, and most of these specific functions – retail marketing of financial services, financial advice to companies, monitoring the creditworthiness of large companies – are better done by some specialist institution that is not necessarily a bank."

Also, with the vast development of money market funds, insurance investment, and investment via brokerage firms in the US, Europe and Asia, and with the fast globalization of investment opportunities, consumers do not suffer from lack of investment opportunities and startup firms can raise capital in a wide variety of markets without approaching conventional banks. In fact, given the large number of money market funds and financial instruments existing today, consumers can always pick such a ratio of risk and return that exactly match their degree of risk aversion.

Many observers question whether the regulatory system has been able to keep pace with the rapid changes in the financial industry as outlined above. Banks presently make use of their retail deposit base as collateral for risky investment activities in securities markets such as trading, market making and placing. However, risky activities take place in a regulatory environment where these risks are effectively underwritten by the governments, and ultimately by the taxpayers, and this applies in particular with respect to the banks which are large enough to enjoy full public protection under the prevailing "too-big-to-fail"-doctrine [see, for example, Feldman and Rolnick (1998)]. With banks extending their domain to the full range of risky wholesale market activities these regulations, primarily designed to protect small and less sophisticated depositors as well as to prevent bank panics, will effectively also serve as a public subsidy for the risk taking by banks, in particular by those large-sized banks which qualify for "too-big-to-fail"-protection. Consequently, the regulatory system serves as a subsidy towards banks which bundle basic retail banking services with unrelated risk taking in the securities markets. Of course, public support of such bundling imposes large distortions on the economy.

Keeping pace with the quick changes in the financial landscape creates substantial challenges not only for regulators but also for academic research on financial intermediation. A theory of banking should fundamentally be able to characterize those specific intermediation functions which a bank can perform more efficiently than an insurance company, a mutual fund or the capital market in general and a well developed theory should also be able to explain the underlying mechanisms. Fundamental issues like these have been analyzed in different contexts by Diamond (1984, 1997) as well as Diamond and Rajan (1998). The present paper sheds some light on the fundamental issue of financial markets and banks as competing mechanisms for providing investors with liquidity, i.e. access to their capital on short notice. Our analysis makes clear that there might be substantial social costs associated with the promotion of a banking system which essentially bundles liquidity provision and risky investment activities, at least in a world where nothing prevents the consumers from pursuing the investments within the framework of well-developed and diversified investment markets.

The existing literature views the depository institutions as "pools of liquidity" providing households with insurance against idiosyncratic shocks that affect their consumption needs. In the influential model by Diamond and Dybvig (1983) banks provide liquidity to depositors who are, ex ante, uncertain about their intertemporal preferences over consumption sequences [see also Bryant (1980) and Villamil (1991)]. In their model demand deposits are needed because liquidity shocks are not publicly observed and therefore cannot be insured. They demonstrate how deposit contracts offer insurance to households and how such contacts can offer a Pareto efficient allocation of risk. However, as Diamond and Dybvig show, there exists a second, inefficient Nash equilibrium where the interaction between pessimistic depositor expectations generates bank runs. As Diamond and Dybvig prove, deposit insurance systems can eliminate such inefficient Nash equilibria. Jacklin (1987) offers an extension of the Diamond-Dybvig model by introducing a market for outside investment opportunities open to consumers as well as banks. However, his paper does not the effects of bank runs on depositors' welfare. Moreover, due to Jacklin's assumption of a continuum of depositors, in his model banks can avoid facing runs with probability 100% by maintaining a reserve ratio (at least) equal to the probability of a liquidity shock of a representative depositor.

Despite the indisputable insurance benefits, empirical observations as well as theoretical research convincingly demonstrate how federal deposit insurance will encourage banks to engage in excessive risk taking [see, for example, Cooper and Ross (1998)]. Therefore, researchers have also systematically investigated mechanisms other than deposit insurance as instruments for reducing the instability of the banking system. In line with Freixas and Rochet (1997), the most natural of the conceivable mechanisms is *narrow banking*, where banks are required to back demand deposits entirely by safe and liquid short-term assets.¹

Wallace (1996), using a dynamic model, argues against narrow banking by showing that it is antagonistic to the efficient provision of liquidity insurance. In this paper, we show that Wallace's result may fail to hold when consumers have access to investment opportunities *outside* the banking industry. In our analysis we argue that since most developed economies possess an almost complete spectrum of liquid investment markets outside the banking industry, there are no social gains associated with allowing banks to bundle deposits with risk, that is, no social gains are generated by allowing ordinary banks to integrate the activities of

¹In this paper we associate narrow banking with maintaining one-hundred percent reserves, however, in some literature narrow banking is defined as a policy which prohibits banks from taking risks (thus, allowing banks to invest some of deposited funds in government bonds).

deposit-taking and risky lending.² The reason is that in the presence of outside investment funds exhibiting any arbitrary degree of diversification available to consumers at transaction costs similar to those offered by the banking industry, the welfare of consumers might be better served within an institutional framework where they are offered narrow banking, i.e. banking channels which are separated from risk. Furthermore, our analysis shows that, even when ignoring the generated moral hazard problems while taking into account the direct costs of funding, the introduction of deposit insurance will always be welfare reducing under a wide-banking policy. Furthermore, we establish that deposit insurance systems cannot substitute for narrow-banking policies in any welfare-improving way.

We conclude our literature review with a discussion of a recent paper by Kashyap, Rajan, and Stein (1999) which, *contrary* to the message of the present paper, attempts to justify wide banking. They ask exactly the opposite question to what we ask: What ties together the traditional commercial banking activities of deposit-taking and lending? In other words, they ask: "why is it important that one institution carry out both functions under the same roof?" whereas we attempt to demonstrate an argument why the two functions ought to be performed by different institutions, or more mildly, at least be carried out in different floors of the same institution. Kashyap et al. narrow down their search for a justification for wide banking to asking whether or not there is a real economic synergy between deposit-taking and lending. As they argue, without such synergies, the current distortions in the regulatory environment lead to

"...that deposit insurance has encouraged an artificial gluing together of the two activities, as banks attempt to maximize the value of the insurance put option by engaging in risky lending. Under this view, one would naturally tend to be sympathetic to "narrow banking" proposals, which effectively call for the breaking up of banks into separate lending and deposit-taking operations..."

²Unless someone has a reason to believe that ordinary banks are able to assess the riskiness of investment projects at a much higher precision than alternative financial market players such as investment banks and brokerage firms.

Indeed, the purpose of the present paper is to formalize this idea without introducing adhoc assumptions regarding the existence and nature of any real economic synergies between deposit-taking and lending. As Kashyap et al. demonstrate, introducing strong synergies will justify wide banking. However, a strong correlation between deposit withdrawals and the degree of nonpledgeable returns on illiquid loan-financed projects will reduce the likelihood of having these synergies. We believe that such a correlation attributes a major cause for the emergence of banking crises. Such a view would be in line with the predictions of the credit cycle models, for example Kiyotaki and Moore (1997), which show how the interaction between debt and collateral values can amplify fluctuations facing an economy. Also, it would find support in Lamont's (1995) model emphasizing the macroeconomic costs of debt overhang. In a general equilibrium context, Holmström and Tirole (1998) have developed the implications arising from limited pledgeability of the returns of illiquid projects for asset pricing, thereby offering a theory of how agency costs should be reflected in asset pricing models, which incorporate liquidity premia.

In our model, a narrow-banking policy generates a higher social welfare than the commonly used wide-banking policy because under wide banking consumers are unable to find riskless banks to deposit their money, thereby forcing them to sacrifice liquidity. Thus, we show that the separation of banking from investment actually increases the variety of investment means consumers can utilize to intertemporally transfer money across periods as it adds a riskless money storage technology which is missing from banks who are governed by the incentives created by available opportunities of bundling deposit-taking and lending.

Consider an economy where consumers face an intertemporal indivisibility constraint such that they can spend their money on either first-period or second-period consumption, depending on a random consumption (liquidity) need which is independently distributed across consumers. The assumed consumption indivisibility represents a simple way of capturing the welfare consequences of a bank run in the first period. If a consumer faces a (random) liquidity need in the first period (say, an opportunity or a need to buy a house or a car, or simply high medical and education expenses), and if the bank is out of reserves, then the consumer is unable to fulfill his consumption need, and his consumption is deferred to the second period (forced saving). Had we assumed that consumption is divisible, a bank run would not necessarily constitute an ex-ante utility loss as consumers will be able to reduce only part of their first-period consumption in a case of a bank run. Hence, what we mean by a bank run is that consumers are unable to obtain their entire deposits for the use of large purchases, such as a house.

With wide banking, the bank lends out some of the deposited money it has, so consumers face a risk of not being able to fulfill their liquidity need in the first period. With narrow banking, consumers can secure their consumption needs as the bank maintains 100% reserves. In addition, if consumers have access to other types of savings (say, investment projects financed by mutual funds), a consumer who wants to sacrifice liquidity and save for a higher expected return can simply do it outside the banking system. However, if the consumer would like to secure his stochastic liquidity needs, wide-banking policy does not provide the consumer with any means of depositing money with no risk. Altogether, a narrow banking policy has the advantage of maintaining investment opportunities outside the banking sector, while providing consumers with an additional instrument in the form of safe deposits to secure their liquidity needs as well as other banking services. The market failure associated with wide banking stems from the fact that consumers are unable to find banks who will store their money and provide other banking services without risk at all. Thus, the distortion here stems from the *bundling* of deposit accounts with risk. Further, we show that the introduction of deposit insurance can be welfare reducing since a tax burden associated with financing a bank failure may increase the number of consumers who are unable to fulfill a liquidity need.

The suggestion for narrow banking is not new. Following Henry Simons and Lloyd Mints, Friedman (1960, Ch.3) lists what he calls the defects of the present fractional reserve banking system. Similar to our results, Friedman envisions that under one-hundred reserve requirement there will be two different types of institutions. One that stores deposits and provide banking services for nominal fees, and one that does the intermediation between lenders and (risky) borrowers. On pages 69–70 Friedman states:

"The effect of this proposal would be to require our present commercial banks to divide themselves into two separate institutions. One would be a pure depository institution, a literal warehouse of money. ...The other institution that would be formed would be an investment trust or brokerage firm."

In this paper, we demonstrate that in the presence of highly developed securities markets, the separation of investment from banks resulting from a narrow banking policy would actually expand consumers' saving/investment opportunities by adding a risk-free zero-return saving opportunities to the wide spectrum of risky investment opportunities.

Our analysis proceeds as follows. The basic model is set in Section 2. Section 3 solves for the profit-maximizing reserve ratio maintained by the commercial bank as a function of the minimum legal reserve requirement set by the central bank; and defines a menu of two available policy options. The first of these policy options, the wide-banking policy, is analyzed in Section 4. The other policy option, that of the narrow-banking policy, is investigated in Section 5. Section 6 consists of a comparison of the welfare implications of wide banking and narrow banking. In Section 7 we introduce deposit insurance and analyze its implication for the comparison between a wide-banking and a narrow-banking policy. Finally, Section 8 offers some concluding comments.

2. The Model

Consider an economy with two consumers (depositors), one commercial bank (bank in what follows) and a regulating authority (central bank in what follows).

2.1 Storage of money technology

The bank has access to a safe zero-return technology which we refer to as the storage technology. This means that the bank has the ability to costlessly store money and such storage does not yield any return. In contrast, consumers do not have access to this zero-return technology. The interpretation of the storage technology is simple. Consumers need a commercial bank because a bank has the ability to store money with significantly lower risks of theft and robbery compared with consumers. For this reason, consumers need the bank in order to make a deposit since storing money at home without risk is prohibitively costly to individual consumers. Of course, consumers need the bank for other services as well (payment services, transfers etc.), which are also implicit in the storage technology assumed above.

2.2 Investment project

All agents (the consumers as well as the bank) have access to a risky investment project. The investment decisions are made in period t = 0, and the return on the investment is realized at period t = 2, that is, in the second period after the investment was made. Thus, the investment project cannot be liquidated at t = 1. This assumption reflects the tradeoff the bank (and consumers via the bank) face between a zero-return risk-free perfectly liquid storage of money, and the illiquidity associated with the positive expected return available through the risky investment.

The investment project yields an uncertain gross return x (per unit invested) given by

$$x = \begin{cases} R & \text{with probability } \alpha \\ 0 & \text{with probability } 1 - \alpha \end{cases}, \quad \text{where} \quad 0 < \alpha \le 1 \quad \text{and} \quad \alpha R > 1.$$
 (1)

Thus, the risky investment project dominates the money storage technology in terms of its expected net return.

We view this risky investment project as an aggregate of economy-wide real investment opportunities available on an equal basis to banks and consumers in this economy. We assume the consumers to have access to this investment project either directly or via mutual funds (which are not explicitly modeled in the present paper). This investment project represents the only instrument available to the consumers or to the bank for generating a return on their money and one could think of this as the return on the fully diversified market portfolio. However, the investment is illiquid in nature and requires a sacrifice of one period of liquidity.³

2.3 Timing

The decisions by the agents in this economy take place in accordance with the sequence described by the five stages below.

- **Stage** -2: The central bank sets a minimum reserve requirement, ρ^{\min} , $(0 \le \rho^{\min} \le 1)$.
- Stage -1: The commercial bank sets its own profit-maximizing reserve ratio, ρ , ($\rho^{\min} \leq \rho \leq 1$).
- Stage 0 (t = 0): Each consumer allocates his endowment of \$1 either to the investment project (1), or makes a deposit with the bank for the full amount. Then, the bank stores a fraction of ρ as cash and allocates a fraction of 1ρ of the deposits to the risky investment project
- Stage 1 (t = 1): Each consumer realizes a liquidity need with probability θ . Each consumer who realizes a liquidity need attempts to withdraw \$1 from his demand deposit (if he has a bank account). The bank must pay back its liabilities on demand deposits provided that it has a sufficient amount of liquidity available.
- Stage 2 (t = 2): The investment return is realized.
 - Return on the bank's investment: The bank returns the remaining balance on demand deposits to depositors. Depositors who receive money from the bank consume their entire endowment of \$1. If the investment project fails and does not yield any return, depositors are not able to transform demand deposits into consumption.

³We are well aware of the fact that, in reality, most mutual funds offer some liquidity. However, this liquidity depends on the funds' performance, and therefore cannot be perfect. Including liquidity in this model may reduce the parameter range in which narrow banking yields higher social welfare than wide banking, but will not eliminate this possibility due to the fact that liquidity will not be possible if the fund underperforms. For this reason, and in order to simplify the model, we assume that the investment project is illiquid in its first period.

Return on consumers' investment: If, instead of depositing with the bank, a consumer allocates his entire endowment to the investment project in t = 0, the consumer gets to consume an amount of R with probability α and 0 otherwise, as described in (1).

The above description of timing provides a general idea of how this economy operates. In the subsections below we elaborate on the precise decision-making processes of consumers and the bank.

2.4 The bank

The bank does not pay interest on demand deposits. In period t = 0 a profit-maximizing bank accepts deposits by consumers. Depending on its selected reserve ratio, the bank allocates the fraction of $1 - \rho$ of consumers' funds to the investment project defined by (1). The deposits that are not invested by the bank remain stored as cash in the bank and do not bear any risk and do not yield any return to the bank. Clearly, if the bank does not lend out any amount to the investment project, it will end up with normal (zero) profits. Therefore, in stage -1 the bank would attempt to maximize its investment in the project (choosing minimal ρ) subject to the mandated reserve requirement *and* subject to consumers' willingness to make deposits in a more risky bank.

Finally, note that in the event that the bank lends out money to the investment project and the investment project fails (x = 0 with probability $1 - \alpha$), the bank is bankrupt and will not be able to pay back its liabilities on demand deposits. Thus, consumers who deposit with the bank face an increase in risk when the bank increases the amount it lends out (a decrease in ρ).

2.5 Consumers

The two consumers are identical ex-ante. Initially, at t = 0 each consumer is endowed with \$1. Each consumer must decide whether to invest the entire amount in the investment project described by (1), thereby sacrificing liquidity in period t = 1, or whether to deposit the entire amount in the bank, thereby maintaining the option of being able to meet potential liquidity needs at t = 1.

We assume that the consumers face an intertemporal indivisibility constraint with respect to their consumption possibilities. Each consumer is allowed to consume all his endowment plus returns (if any) exclusively either in period 1 or period 2. Let c_t denote a consumer's consumption level in period t, t = 1, 2. We normalize the price level to equal unity so that consumption and units of money are expressed in the same units. In order to capture the importance of liquidity to consumers, we view the consumption good as a major indivisible expense such as purchasing an apartment, a car, or a major medical or educational service. Thus, we assume that a consumer will need at least \$1 to purchase this consumption good. Formally,

Assumption 1

Let m_t denote the period t amount of cash money withdrawn by a consumer. Then,

$$c_t = \begin{cases} m_t & \text{if } m_t \ge 1\\ 0 & \text{if } m_t < 1. \end{cases}$$

Therefore, in the present model, the consumption bundles available to the consumer are: $(c_1, c_2) = (1, 0)$, or $(c_1, c_2) = (0, m_2)$, if $m_2 \ge 1$ and (0, 0) if $m_2 < 1$.

In period t = 1, each consumer will face a liquidity shock with probability θ ($0 \le \theta \le 1$). More precisely, at t = 1 each consumer will confront a consumption need with probability θ . We assume that such a consumption need can be realized only if the consumer can collect his deposit in the bank, provided, of course, that the consumer has made such a deposit and that the bank has a sufficient amount of liquidity to cover for such a withdrawal. With probability $1 - \theta$ each consumer will not utilize any consumption opportunity in t = 1, and will therefore delay consumption to period t = 2.

Let β , $(0 \le \beta \le 1)$ denote the time discount factor. We define a consumer's utility by

$$U = \begin{cases} c_1 & \text{when experiencing } t = 1 \text{ liquidity shock} \\ \beta c_2 & \text{otherwise} \end{cases}$$
(2)

where c_t satisfies Assumption 1. The utility function (2) implies that a consumption need may be realized in t = 1. If it is not realized, then the consumption need is realized in t = 2. In either case, consumption is utilized provided that the consumer can generate a sufficient amount of cash as described in Assumption 1.

2.6 Liquidation policy

It is important to define the policy maintained by the commercial bank for liquidating deposits in events when consumers, wishing to withdraw from their demand deposits, find the bank lacking the sufficient funds.

When only one consumer makes a deposit, no particular policy needs to be assumed. If the bank maintains strictly less than 100% reserves, Assumption 1 implies that the consumer is unable to fulfill his liquidity need in the event of realizing a liquidity shock, and therefore such a consumer will not withdraw in t = 1.

When two consumers make a deposit, a bank run may occur in periods t = 1 and t = 2in the case where both wish to withdraw their entire deposits at the same time. If the bank maintains less than \$1 of funds, Assumption 1 implies that no consumer is able to fulfill the liquidity need and will therefore not withdraw. Clearly if the bank maintains \$2 of funds, it will be able to satisfy all liquidity need. Thus, the only case where we need to make an explicit assumption concerning the commercial bank's liquidation rule is when the bank maintains at least \$1 of funds but strictly less than \$2 of funds.

Assumption 2

Suppose that the bank maintains funds to satisfy the liquidity needs of one and only one consumer. In an event that both consumers wish to withdraw their deposits (in any given period), the bank allocates the funds on a first-come-first-served basis. Formally, with probability 1/2 each consumer is able to withdraw his deposit.

3. Reserves

In this section we analyze how the reserve ratio chosen by the commercial bank affects consumers' decisions and utility. Subsection 3.1 demonstrates the effects of the reserve ratio using only extreme reserve ratios (maintaining no reserves and 100% reserves). Subsection 3.2

generalizes the analysis to all possible reserve ratios set by the commercial bank. Subsection 3.3 concludes with solving for the equilibrium reserve ratio chosen by the commercial bank as a function of the minimum reserve requirement set by the central bank.

3.1 Extreme reserve ratios

It is instructive to first look at two extreme reserve ratios set by the bank: $\rho = 0$ (no reserves) and $\rho = 1$ (100% reserves). Intermediate values of the reserve ratio, analyzed in subsection 3.2 below, are somewhat more difficult to analyze as consumers' consumption will also depend on the number of consumers who realize a liquidity need in t = 1.

3.1.1 No reserves $(\rho = 0)$

In this case, a consumer who makes a deposit will never be able to satisfy a liquidity need in an event of a consumption opportunity. Hence, consumption is possible only in t = 2provided that the bank is not bankrupt. Hence, $c_1 = 0$ and $Ec_2 = \alpha$, where "E" is the expectation operator. Consequently, in view of (2), the consumer's expected utility is

$$EU = \alpha U(0, 1) + (1 - \alpha)U(0, 0) = \beta \alpha.$$

This is lower than the expected utility, $\beta \alpha R$, which is obtained if the consumer does not make a deposit with the bank and invests directly in the investment project (1). Thus, under $\rho = 0$ the consumer will never choose to make a deposit.

3.1.2 Full reserves $(\rho = 1)$

With 100% reserves, the consumer is able to consume his entire endowment whenever he realizes a consumption need. Hence,

$$U = \begin{cases} 1 & \text{if the liquidity need is realized in } t = 1\\ \beta & \text{otherwise.} \end{cases}$$

Thus, in this case the consumer's expected utility is $EU = \theta + \beta(1 - \theta)$.

3.2 Extension: Arbitrary reserve ratios

Demand deposits may bear two types of risks: (i) the risk of a bank run, when two consumers realize a liquidity need (probability θ^2), and (ii) the risk of an investment failure of the bank (probability $1 - \alpha$). We define a bank run as a situation where a consumer, facing a liquidity shock, is unable to withdraw his \$1 from his bank account. Bank runs can occur only in period t = 1 when consumers face a realization of their liquidity needs.

The likelihood of having a bank run in t = 1 depends on three elements: (i) the number of depositors (zero, one or two), (ii) the realizations of the liquidity shocks, and (iii) the bank's reserves ratio, ρ . Table 1 lists the circumstances under which a bank run occurs in t = 1.

# Depositors	Reserve Ratio	# Shocks	Prob.	t = 1 Run?	Ec_1
2	$\rho < 1/2$	2	θ^2	Yes	0
2	$1/2 \le \rho < 1$	2	θ^2	Yes	1/2
2	$\rho = 1$	2	θ^2	No	1
2	$\rho < 1/2$	1	$\theta(1-\theta)$	Yes	0
2	$\rho \ge 1/2$	1	$\theta(1-\theta)$	No	1
1	$\rho < 1$	1	θ	Yes	0
1	$\rho = 1$	1	θ	No	1

Table 1: Probability of a bank run and expected t = 1 consumption of a consumer facing a liquidity need in t = 1.

Table 1 demonstrates the consequences of different reserve ratios on the possibility of a bank run and the expected t = 1 consumption of a depositor facing a liquidity shock in t = 1. When two depositors experience a liquidity need, a bank run occurs unless the bank maintains 100% reserves (i.e., $\rho = 1$). When the bank maintains at least 50% reserves but less than 100%, only one consumer can fulfill his consumption need by withdrawing \$1 from his demand deposit; therefore we assume that each consumer has a probability of 1/2 of being "the first in line" to withdraw the \$1; therefore the expected t = 1 consumption of each depositor is $Ec_1 = 1/2$.

The second set of rows in Table 1 deals with two depositors in cases such that only one

of them experiences a liquidity need. In this case, as long as the bank maintains at least 50% reserves, no bank run occurs, as the bank possesses at least \$1 in stored cash.

The third set of rows in Table 1 deals with a single depositor. In this case, a bank run may occur as long as the bank maintains less than 100% reserves.

Table 1 highlights the role played by our assumption of consumption indivisibility, Assumption 1. We interpret a liquidity shock as a possible significant expense faced by the consumer in t = 1. Acquisitions of an apartment, a car, and education are examples of significant expenses in the sense of our model. In this case, the consumer must have the entire amount before purchase. Thus, if the bank lends out some of the consumer's money, the consumer will not be able to purchase the good and will therefore not withdraw any partial amount (less than \$1 in the present case).

3.3 Equilibrium Reserves

We now proceed to characterize the subgame-perfect Nash equilibrium capturing the interaction between the consumers, the bank and the central bank in this economy. Even without explicitly solving the entire game by backwards induction, we can easily conjecture how the commercial bank sets its profit-maximizing amount of reserves kept liquid. The reader is reminded that ρ^{\min} , set in stage -1 (see subsection 2.3), denotes the regulatory minimum reserve requirement decided by the central bank.

Proposition 1

The bank's profit-maximizing reserve ratio is given by

$$\rho = \begin{cases}
\frac{1}{2} & \text{if } \rho^{\min} \leq \frac{1}{2} \\
\rho^{\min} & \text{otherwise.}
\end{cases}$$
(3)

Proof. The proof follows three steps. First, we show that the bank will never find it profitable to choose a reserve ratio $\rho < 1/2$. By way of contradiction, suppose that $\rho < 1/2$. If one consumer makes a deposit and faces a liquidity shock, Table 1 shows that this consumer will not be able to satisfy his liquidity need as the bank maintains less than \$1 of reserves. If two consumers deposit and one or two consumers face a liquidity need, the bank will not be

able to satisfy any liquidity need as it keeps $2\rho < 1$ reserves. Hence, given that $\alpha R > 1$, consumers will not deposit in the bank; instead consumers will invest their entire amount in the investment project (1). In other words, when $\rho < 1/2$ the bank can never supply the liquidity needs of any consumer restricted to the feasible consumption bundles exhibiting intertemporal indivisibility.

Second, we show that the profit-maximizing bank will never maintain 100% reserves ratio. Suppose the bank does maintain $\rho = 1$. Then, the bank does not allocate any funds deposited by consumers to the investment project and stores the entire amount, thereby receiving no return. Hence, maintaining $\rho = 1$ yields zero profits to the bank.

Third, it remains to show that when $\rho^{\min} > 1/2$, a profit-maximizing bank will not set $\rho^{\min} < \rho < 1$. Suppose that it does select a particular $\bar{\rho} \in (\rho^{\min}, 1)$. We now show that the bank can increase its profit by reducing the reserve ratio to $\rho = \rho^{\min}$. The second row of Table 1 shows that when both consumers make a deposit, they are indifferent between any $\rho = 1/2$ and $\bar{\rho}$. The sixth row of Table 1 shows that a single depositor is indifferent between all reserve ratios satisfying $\rho < 1$. These two results follow directly from the assumed intertemporal indivisibility, i.e., the risk of a bank run is the same for all levels of the reserve ratio in the range. Altogether, consumers' decisions whether to make a deposit is unaffected by reserve rations $\rho^{\min} < \rho < 1$.

Comparing the bank's profit under these reserve ratios when two consumers make a deposit (Table 1 showed that a single consumer will not make a deposit as long as $\rho < 1$) yield

$$E\pi|_{\rho^{\min}} = \beta(1-\rho^{\min})2\alpha R > \beta(1-\bar{\rho})2\alpha R \quad \text{for all} \quad \bar{\rho} \in (\rho^{\min}, 1).$$

Proposition 1 implies that even without any mandatory reserve requirement, the bank will voluntarily never set a reserve ratio $\rho < 1/2$, because setting a low reserve ratio would deter consumers from making deposits. In addition, setting $\rho = 1$ would not generate any profits. Proposition 1 also implies that the expected profit monotonically increases as the reserve ratio ρ falls from 1 to 1/2. Since there are only two types of agents, consumers and the bank, we can directly apply the Pareto criterion to draw the following welfare conclusion.

Corollary 1

Any mandatory reserve requirement which does not satisfy $\rho^{\min} = 1/2$ or $\rho^{\min} = 1$ is socially inefficient in the Pareto sense.

Proof. As already indicated in the proof of Proposition 1, from the point of view of depositors, the liquidity risks are the same for all reserve ratios in the range $1/2 < \rho < 1$, while the bank's profit monotonically decreases over this range. Proposition 1 also implies that the bank sets $\rho = 1/2$ whenever $0 \le \rho^{\min} \le 1/2$, hence yielding a higher profit to the bank without sacrificing consumers' liquidity needs.

Corollary 1 implies that there is no loss of generality by assuming that the minimum reserve requirement imposed by a welfare-maximizing central bank (policymaker) is confined to the two values: $\rho^{\min} = 1/2$ or $\rho^{\min} = 1$. We therefore make the following definition. DEFINITION 1

We say that the central bank adopts a **narrow-banking policy** if it requires 100% reserve requirement ($\rho^{\min} = 1$). Otherwise, we say that the central bank adopts a **wide-banking policy**.

4. Wide-banking equilibrium

We now focus on an economy where the central bank exercises a wide-banking policy (i.e., $\rho^{\min} = 1/2$). Hence, by Proposition 1 the commercial bank chooses the reserve ratio $\rho = 1/2$. In period t = 0 each consumer must decide whether to deposit the entire endowment of \$1 in the bank, or whether to exercise the investment opportunity.

Lemma 1

Under wide banking, there does not exist an equilibrium where only one consumer makes a deposit with the bank.

Proof. The sixth row of Table 1 implies that if only one consumer makes a deposit he will not be able to satisfy his consumption need if he experiences a t = 1 shock. Hence, a

single depositor can consume only in t = 2 provided that the bank is not bankrupt (i.e., the investment project yields a return). Therefore, expected utility of a single depositor is $\beta \alpha$ which is lower than $\beta \alpha R$ obtained if the consumer invests in the project rather than making a deposit with the bank.

Lemma 1 implies that in equilibrium it must hold that either both consumers make a deposit or none.

In our model we rule out a coordination failure between the consumers. This can be easily justified by extending the model by dividing period t = 0 into two subperiods with sequentially-entering consumers. In this case, the unique subgame-perfect equilibrium is such that both consumers make a deposit if and only if their utility when both make a deposit exceeds their utility when both pursue the outside investment project. In order to find the condition under which, in equilibrium, each consumer makes a deposit we need to compare the utility of a representative consumer when both make a deposit to his utility when no one makes a deposit.

4.1 Expected consumer utility

We first calculate the expected utility associated with the situation when both consumers make a deposit with the bank. Table 2 demonstrates how expected period 1 and period 2 consumption levels as well as expected utilities conditional on realizations are calculated. Table 2 is constructed in the following way.

# Shocks	Prob.	t = 1 Run?	Ec_1	Ec_2	$EU _{\#\text{shocks}}$
2	θ^2	Yes	1/2	$\alpha/2$	$(1+\beta\alpha)/2$
1	$\theta(1-\theta)$	No	1	0	1
1	$(1-\theta)\theta$	No	0	α	$\beta \alpha$
0	$(1-\theta)^2$	No	0	$1/2 + \alpha/2$	$\beta(1+\alpha)/2$

Table 2: Wide banking: Expected consumption and utility given different realizations

Both consumers experience a liquidity need: with probability θ^2 . Since the bank has only \$1 of reserves, only one consumer can withdraw from demand deposit and satisfy

his consumption need. The other consumer encounters forced saving where his t = 2 consumption is contingent on the bank not being bankrupt (probability α). Since each consumer has 50% chance of being first in line to withdraw in period t = 1, t = 1 expected consumption is 1/2 and t = 2 expected consumption is $\beta \alpha/2$.

- Only one consumer experiences a liquidity need: with probability $\theta(1-\theta)$. The bank maintains the exact amount to satisfy the liquidity need of one consumer, hence t = 1consumption equals 1.
- Only one consumer does not experience a liquidity need: This consumer attempts to withdraw money only in t = 2, where the money will be available only with probability α .
- No consumer faces a liquidity shock: with probability $(1 \theta)^2$. Both consumers attempt to withdraw in t = 2 only. The first consumer to stand in line will always get his \$1 deposit as the bank maintained \$1 of reserves. The second consumer will manage to withdraw his \$1 only with probability α (that the bank is not bankrupt in case the investment project fails). With each consumer as an equal probability of being the first in line, expected t = 2 consumption is $1/2 + \alpha/2$.

Summing up, in an event that both consumers make a deposit with the bank in t = 0, the expected lifetime utility of each consumer is

$$V_2|_{\rho=\frac{1}{2}} = \frac{\beta\alpha + \beta(\theta - 1)^2 + \theta(2 - \theta)}{2}.$$
 (4)

Now, suppose that none of the consumers makes any deposit with the bank. Then, both consumers invest in the project described by equation (1) and each gains an expected utility given by

$$V_0 = \beta \alpha R. \tag{5}$$

Comparing (4) and (5), we can conclude that the configuration with both consumers making a deposit constitutes the unique equilibrium outcome if

$$V_2|_{\rho=\frac{1}{2}} > V_0$$
 hence if $R < \frac{\beta\alpha + \beta(\theta - 1)^2 + \theta(2 - \theta)}{2\beta\alpha}$. (6)

Clearly, (6) indicates that the parameter range where both consumers make deposits increases with a decline in the investment yield, R, and with a decline in the investment probability parameter α as well as with the discount parameter, β . Appendix A provides a detailed interpretation for condition (6).

4.2 Social welfare in a wide-banking equilibrium

Proposition 1 implies that when the reserve requirement is $\rho^{\min} = 1/2$, it is profitable for the bank to lend out half of its available funds to the investment project characterized by (1). Clearly, the bank can make above normal expected profit only if consumers make a deposit. If no consumer makes any deposit, the bank does not earn any profit. Hence, the expected profit levels of the bank when both consumers make a deposit and when none makes a deposit are

$$E\pi_2|_{\rho=\frac{1}{2}} = \beta \alpha (R-1), \text{ and } \pi_0|_{\rho=\frac{1}{2}} = 0,$$
 (7)

respectively. Thus, in the case where both consumers make a deposit (total \$2 of deposits), the bank lends out \$1, and therefore makes an expected net profit of $\beta \alpha (R-1)$.

We define the social welfare function to be the sum of consumers' expected utilities and the banks' expected profit level. Formally, let $W = 2EU + E\pi$. Hence, social welfare levels when both make a deposit and none makes a deposit are given by

$$W_2|_{\rho=\frac{1}{2}} = \theta(2-\theta) + \beta(\theta-1)^2 + \beta\alpha R \text{ and } W_0|_{\rho=\frac{1}{2}} = 2\beta\alpha R.$$
 (8)

The second equation in (8) is the social welfare when both consumers invest their money in the outside investment project, which equals their discounted t = 2 expected income. Appendix B provides a detailed interpretation of (8) and a formal proof of the following proposition.

Proposition 2

Under wide banking, an equilibrium in which both consumers make deposits is socially optimal.

Proposition 2 follows from the fact that the social welfare function (8) includes the expected profit of the bank which is strictly positive when both consumers make a deposit, whereas consumers compare their utilities only. That is, consumers choose to make a deposit whenever $2EU > 2\beta\alpha R$, which implies that $2EU + \pi > 2\beta\alpha R$, hence socially optimal.

5. Narrow-Banking Equilibrium

In this section we turn to an environment with narrow banking where the central bank mandates that the commercial bank maintains one-hundred percent reserves ($\rho^{\min} = 1$). Under a narrow banking policy, the bank is not allowed to lend out any amount of deposited money and therefore does not make above normal profit. Hence,

$$\pi_2|_{\rho=1} = \pi_0|_{\rho=1} = 0. \tag{9}$$

In the event that both consumers make a deposit with the narrow bank at t = 0, the expected lifetime utility of each consumer is now given by

$$V_2|_{\rho=1} = \theta + \beta(1-\theta), \tag{10}$$

which equals exactly to the expected consumption.

Alternatively, if both consumers do not make any deposit with the bank, they invest in the project described by (1) and each gains an expected utility $V_0|_{\rho=1} = \beta \alpha R$ (which is the same utility as under the wide banking policy (5) since consumers do not utilize the banks at all). Comparing (10) and (5), we find that the unique narrow-banking equilibrium will be characterized by both consumers making deposits if

$$V_2|_{\rho=1} > V_0$$
 hence if $R < \frac{\theta + \beta(1-\theta)}{\alpha\beta}$. (11)

It is easy to verify that the return requirement incorporated in (11) is a necessary condition for that in (6). This means that under one-hundred percent reserve requirement the parameter range where both consumers make a deposit constitutes the unique equilibrium includes the parameter range under which the same unique equilibrium is realized under fifty-percent reserve requirement. This result is not surprising since under one-hundred percent reserves consumers are fully insured against all liquidity shocks whereas no interest is paid by the bank in either case. Finally, since the narrow bank does not make a profit, social welfare under narrow banking is merely the sum of consumers' expected utilities, that is,

$$W_2|_{\rho=1} = 2\left[\theta + \beta(1-\theta)\right], \text{ and } W_0|_{\rho=1} = 2\beta\alpha R.$$
 (12)

Hence, consumers choose to make deposits if and only if it is socially optimal to do so. Recall that under wide banking, social welfare was not entirely captured by consumers' incentives since under wide banking the bank makes a strictly positive profit (see Proposition 2).

6. Narrow Banking or Wide Banking?

We now approach the main argument of this paper. We ask the following question: Will a narrow-banking policy generate a welfare gain relative to the wide-banking policy? Clearly, there is no difference between these two policies under the parameter range where consumers choose not to make deposits. Therefore, it remains to compare the welfare levels under these two policies in those circumstances when both consumers make a deposit with the bank in t = 0. For this comparison, it is sufficient to restrict the comparison to parameter configurations satisfying condition (6). Comparing the social welfare under narrow banking (12) with the social welfare under wide banking (8) shows that

$$W_2|_{\rho=1} > W_2|_{\rho=\frac{1}{2}}$$
 if $R < \frac{\beta + (1-\beta)\theta^2}{\beta\alpha}$. (13)

Appendix C provides a detailed interpretation of the condition in (13). We have established the following Proposition.

Proposition 3

Under condition (13), a narrow-banking policy is socially preferred to a wide-banking policy.

Proposition 3 places an upper bound on the investment return, or alternatively, a lower bound on the probability θ of a liquidity need in period t = 1, for narrow banking to socially dominate wide banking. However, note that for sufficiently high R, consumers will prefer the investment project and in that case there is no difference between narrow and wide banking (as banks are not used at all). Thus, it is possible that under condition (13) banks do not attract deposits at all and hence under such circumstances there is no difference between the narrow- and the wide-banking policies. Therefore, we conclude this section by analyzing the entire parameter range in order to characterize precisely the conditions under which the narrow-banking policy generates a strictly higher social welfare compared with the wide-banking policy.

There are three conditions on the investment returns, R, to be compared: Condition (6) under which two consumers will make a deposit under wide banking, condition (11) under which two consumers will make a deposit under narrow banking, and condition (13) under which narrow banking is socially preferred over wide banking. As was pointed out earlier, if consumers choose to make a deposit under wide banking they will definitely choose to make a deposit under narrow banking; that is, condition (6) implies condition (11). Figure 1 illustrates the parameter range under the assumption that

$$\beta > \frac{\theta(2-3\theta)}{1-\alpha+\theta(2-3\theta)}.$$
(14)

Condition (14) is derived by comparing the parameter range where (13) is greater than (6).



Figure 1: High probability for a t = 1 liquidity need [(14) holds]

Notice that condition (14) always holds when $\theta > 2/3$ and hence we interpret this condition as a high probability for a liquidity need in t = 1. In this case,

Proposition 4

Under condition (14), wide banking yields a strictly lower social welfare than narrow banking whenever consumers choose to make a deposit with the bank.

Figure 2 illustrates the parameter range for the case that condition (14) is reversed. In



Figure 2: Low probability for a t = 1 liquidity need [(14) reversed]

this case, there is a parameter range where consumers make a deposit and wide banking yields a higher welfare than narrow banking. This case occurs since the probability of realizing a liquidity need is low.

7. Deposit Insurance

So far, our analysis concentrated on comparing wide banking with narrow banking under the assumption that the government does not institute a national deposit insurance system. In reality, most countries have instituted such systems. For that reason it is extremely important to investigate whether our analysis extends to an economy with a deposit insurance system.

Assume that the central bank maintains a wide-banking policy thereby setting a minimum reserve requirement of $\rho^{\min} = 1/2$. But, suppose now that, in addition, the central bank establishes a national deposit insurance program which guarantees a supply of liquidity to the bank in the case of a bank run. Formally, in the context of the present model, a deposit insurance program will be activated in period t = 1 if and only if the bank has insufficient funds to liquidate its demand deposit obligations. Under $\rho = 1/2$, a bank run occurs in t = 1 with probability θ^2 , when the two consumers realize liquidity shocks.

Now the following question arises: How does a national deposit insurance program obtain funds to 'rescue' a bank during a bank run? We will assume that in the event of a bank run, the rescue money is generated by levying a 'rescue' tax on consumers. Let T denote the amount of tax levied on each consumer in period t = 1. Clearly, if both consumers demand their deposits in t = 1, the bank is short of \$1 relative to its liabilities to consumers (as it keeps only \$1 as reserve for a total \$2 liabilities for the two demand deposits). Thus, in an event of a bank run, the tax burden on each consumer must be T = 1/2. As commonly assumed in the public finance literature, we also assume that this contingent tax does not affect consumers' decisions whether to make a deposit with the bank, or whether to pursue an investment.

Suppose that both consumers make a deposit with the bank. With probability θ^2 both will experience a liquidity need in t = 1. With deposit insurance in force, both consumers will be able to withdraw their \$1 from the bank in t = 1. However, since each consumer is taxed by T = 1/2, under the consumption indivisibility assumption, not even one consumer will be able to make a purchase at t = 1. Hence, with probability θ^2 both consumers, experiencing a liquidity need in t = 1, will be forced to delay consumption to period t = 2. In this case, since the bank maintains \$1 in cash and \$1 in the investment project, expected period 2 consumption of each consumer is $c_2 = (1 + \alpha)/2$.

With probability $\theta(1-\theta)$ a consumer will realize a liquidity shock alone and will consume $c_1 = 1$. With probability $(1-\theta)\theta$ a consumer will not realize a liquidity shock whereas the other will, and will have an expected consumption level of $c_2 = \alpha$ at t = 2. Finally, with probability $(1-\theta)^2$ both consumers will not experience a t = 1 liquidity shock, so one consumer will consume $c_2 = 1$ where as the other $c_2 = \alpha$, each with probability 1/2. Hence, when both consumers make a deposit, the expected utility of each consumer is

$$V_2|_{\mathrm{DI}} = \beta \theta^2 \frac{1+\alpha}{2} + \theta(1-\theta) + \beta(1-\theta)\theta\alpha + \beta(1-\theta)^2 \frac{1+\alpha}{2}.$$
 (15)

Since the bank is not subjected to any penalties, its expected profit is not affected by the deposit insurance and, since $\rho = 1/2$, its expected profit is $E\pi = \beta \alpha (R - 1)$. Hence, under a deposit insurance and a wide-banking regime, social welfare is given by

$$W_2|_{\rm DI} = \theta^2 \beta (1+\alpha) + 2\theta (1-\theta) + 2\beta (1-\theta)\theta \alpha + \beta (1-\theta)^2 (1+\alpha) + \beta \alpha (R-1).$$
(16)

Comparing (16) with the social welfare under wide banking without deposit insurance, given in (8), yields the following proposition.

Proposition 5

Under the wide-banking policy, the introduction of deposit insurance will always be welfare reducing.

The intuition for explaining Proposition 5 goes as follows. In our model a deposit insurance is welfare reducing since in an event of a bank run, taxation will cause both consumers to unwillfully postpone their consumption until t = 2, whereas without a deposit insurance, only one consumer is forced to delay consumption.⁴

In policy discussions a narrow-banking policy is often considered to represent a substitute for a deposit insurance system. In light of Proposition 5 our model has very strong implications concerning the relationship between narrow-banking and deposit insurance policies. Namely,

Corollary 2

Whenever narrow-banking policy is socially optimal, the introduction of deposit insurance under the wide-banking regime cannot substitute for a narrow-banking policy.

A common criticism of national deposit insurance policies is based on the distortions created through the inevitable moral hazard problems facing such insurance systems like all forms of insurance arrangements. Against this background it should be emphasized that the present model has its focus on mechanisms completely different from moral hazard. Thus, our model offers criticism of deposit insurance policies along a completely different dimension. The present model centers around the mechanism whereby deposit insurance cannot substitute for narrow banking as a policy instrument concerned with social welfare, since deposit insurance merely provides a transfer of funds from consumers to banks when the banks fail to maintain sufficient liquidity. Of course, this effect would be weakened if we

 $^{{}^{4}}A$ more 'mathematical' intuition for this result goes as follows. The restriction of the consumption set to require a minimum expenditure of \$1 (Assumption 1) generates a convex utility function, which yields the result that 'lotteries' yield a higher expected utility than insurance.

were to model risk-averse consumers, but, nevertheless, the basic mechanism would remain unchanged in the presence of risk averse consumers.

8. Concluding Remarks

This paper points out a distortion due to a missing financial product in the form of riskless banking. From time to time, consumers wish that they had a method of depositing money and obtaining basic services without having to bear any risk. In almost all countries such a service does not exist since the wide-banking policy adopted by central banks bundles these basic services with risk-taking generated by banks lending out a large fraction of depositors' money. The major difference between most of the literature supporting the wide-banking policy and the present paper is that in our model consumers have access to investment opportunities outside the banking industry, whereas in the literature supporting the widebanking policy consumers can save only via the banks.

This paper also points out a potential drawback with the commonly used depositinsurance programs (in addition to the well-known distortion created by moral hazard). We demonstrate that the introduction of deposit insurance cannot substitute for a narrowbanking policy. Moreover, we also demonstrate that deposit insurance can be welfare reducing due to the extra burden exerted by taxing all consumers to pay for the bank run. This extra burden imposes additional liquidity constraints even on consumers who manage to withdraw their money during a bank run.

Our paper can be easily extended to incorporate correlation in consumers' liquidity shocks. In fact, our argument will become stronger if we assume that consumers' liquidity shocks are positively correlated as such a correlation will increase the likelihood of bank runs. In contrast, our argument will not hold if the liquidity shocks are perfectly negatively correlated, because in such a case bank runs will not be realized as the consumers would never simultaneously demand their money back from the bank.

Finally, the fact that bundling of deposits with risk is widely observed indeed implies that there might be substantial real economic synergies between deposit-taking and lending activities from the point of view of the banking industry. However, in order to *justify* the integration of the two activities as a liquidity-enhancing policy instrument one needs to establish convincing evidence of such synergies. Only then, banks can be chartered to integrate loans with deposit-taking. Therefore, there seems to be a clear analogy between bank regulation and antitrust legislation. The antitrust policy literature tells us that since an extensive horizontal merger creates an incentive to abuse market power by a merged firm, such a merger should be approved only if the merging firms are able to establish strong evidence of efficiency-creating synergies.

Appendix A. Interpretation of Condition (6)

The RHS(6) increases with θ and is drawn in Figure 3.



Figure 3: Interpretation of condition (6)

Recall that (1) implies that $R > 1/\alpha$. Figure 3 reveals that for $R = R_1 < (1+\beta\alpha)/(2\beta\alpha)$ the region in which condition 6 is satisfied is nonempty. The figure also reveals that in this region there is a tradeoff between a larger R and a larger value of θ , which means that a when the probability of the t = 1 liquidity need (θ) increases, deposits become more attractive and hence consumers will continue to make deposits even if the investment return parameter R increases slightly. Also, notice that for very small values of θ condition (6) fails to hold implying that making deposits is not an equilibrium. This happens since if the liquidity need is unlikely to be realized, the investment project dominates making deposits.

For $R = R_2 > (1 + \beta \alpha)/(2\beta \alpha)$ the investment project becomes sufficiently attractive as it dominates making deposits even if the probability t = 1 liquidity need in $\theta = 1$.

The border line case,

$$R = \frac{1 + \beta \alpha}{2\beta \alpha}, \quad \text{or equivalently} \quad \beta \alpha R = \frac{1 + \beta \alpha}{2}$$

is interesting since it highlights the precise difference between making deposits and investing in the project. This shows that, given that both consumers realize liquidity shocks with probability 1, the discounted expected return on the project, $\beta \alpha R$ is equal to the value of $EU|_2$ given in Table 2 (expected utility given a t = 1 bank run).

Finally, an increase in the discount parameter β or the probability of success of the investment project α shift downward the curve in Figure 3 and therefore, for a given θ , decreases the parameter range where making deposits constitutes an equilibrium. Thus, an increase in β or α must be accompanied with an increase in θ in order to maintain the equilibrium where consumers make deposits.

Appendix B. Interpretation of (8) and a proof of Proposition 2

We investigate under what conditions making deposits with the bank socially dominates investing in the project, i.e., $W_2|_{\frac{1}{2}} > W_0|_{\frac{1}{2}}$. Equation (8) implies that this occurs when

$$R < \frac{\theta(2-\theta) + \beta(\theta-1)^2}{\beta\alpha},\tag{17}$$

which is drawn in Figure 4.

Therefore, the interpretation of (17) is qualitatively the same as for the equilibrium condition (6), given in Appendix A. For $R = R_1 < 1/(\beta \alpha)$ there exists a θ in which making deposits is a social optimum. In contrast, for $R = R_2 > 1/(\beta \alpha)$ making deposits can never be a social optimum.

We now prove Proposition 2. It can be easily verified that if R satisfies (6) then it also satisfies (17). Hence, the equilibrium in which both consumers make deposits is socially



Figure 4: Making deposits is a social optimum

optimal. However, note that for intermediate values of R, more precisely

$$\frac{\beta\alpha + \beta(\theta - 1)^2 + \theta(2 - \theta)}{2\beta\alpha} < R < \frac{\theta(2 - \theta) + \beta(\theta - 1)^2}{\beta\alpha}$$

there is a market failure since in equilibrium consumers choose not to deposit where making deposits constitute a social optimum.

Appendix C. Interpretation of (13)

Condition (13) is drawn in Figure 5.

The interpretation of condition (13) is qualitatively the same as the interpretations given in Appendix A and Appendix B. For $R = R_1 < 1/(\beta\alpha)$ there always exists a sufficiently high θ in which (13) is satisfied. $R = R_2 > 1/(\beta\alpha)$ condition (13) is never satisfied. The borderline case $R = 1/(\beta\alpha)$ is interesting as it implies that a necessary condition for (13) to hold is $\beta\alpha R \leq 1$ meaning that given that a t = 1 liquidity shock occurs with probability 1, consumers prefer to be able to withdraw \$1 in t = 1 over receiving an expected discounted return of $\beta\alpha R$ on their investment. Clearly, this is a weak restriction which drives the entire intuition of our results, namely, that consumers derive a high utility level from being able to withdraw money in case of a liquidity need.



Figure 5: Interpretation of (13)

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