ON SNR GAIN OF POLARIZATION MATCHING

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ABSTRACT

Recently, results demonstrating the feasibility of polarization matching within mobile environments have been presented. In this paper, the SNR gain from polarization matching is analyzed assuming a simple Rayleigh fading channel model, and it is observed that the SNR gain from polarization matching is moderate even when the mobile polarization is perfectly known in the base station. However, if the cross–polarization coupling ratio is high, large losses caused by polarization mismatch can be avoided in individual radio links when using polarization matching.

1 INTRODUCTION

Several multi–antenna techniques, such as beamforming and transmit diversity, increasing the uplink and downlink capacity and coverage have been presented for both 2G and 3G systems. Given the strict complexity requirements of handsets it is of great importance to find multi–antenna solutions that are based on algorithms which are implemented in base station rather than in mobile station. Beamforming is a classical example of this kind of solution providing average link performance improvement. In beamforming, the goal is to increase the average strength of the electric field nearby the mobile station by transmitting the same linearly preprocessed signal from strongly correlated antennas in such a way that signals add up constructively in the direction of the mobile. While beamforming is based on the estimated direction of mobile station, it is also possible to increase link performance if the polarization of the transmitter and receiver are matched properly. In this paper, we study the so–called polarization matching between transmit and receive antennas.

Recently, it has been demonstrated [1, 2] that polarization matching can be feasible in mobile environments. Roughly, the polarization matching is a method where base station employs the information concerning the mobile polarization when selecting downlink transmit weights. The mobile polarization can be known either from uplink measurements or it can be estimated based on the feedback signaling from mobile station.

The paper is structured as follows: Section 2 presents the system model, and Section 3 analyzes the gain achievable from polarization matching. The paper is concluded in Section 4.

2 SYSTEM MODEL

Consider a system utilizing two dual–polarized transmit antennas in the base station and a single receive antenna in the mobile station. The polarization of the mobile station antenna is denoted by $\mathbf{u}$, where $\mathbf{u}$ is a two-dimensional complex vector such that $\|\mathbf{u}\| = 1$. For linear vertical polarization we have $\mathbf{u}_V = (1, 0)^T$ while the linear horizontal polarization is of the form $\mathbf{u}_H = (0, 1)^T$. We assume that the base station is equipped with vertically and horizontally polarized transmit antenna branches. In the following we adopt the model used in [3]. In a single path fading environment the mobile station receives the signal

$$r = \mathbf{u}^* \cdot \Big(\mathbf{u}_V H_V + \mathbf{u}_H H_H\Big) s + n, \tag{1}$$

where s is the transmitted symbol, n is zero–mean Gaussian noise, $H_V = w_V h_{VV} + w_H h_{HV}$, $H_H = w_H h_{HH} + w_V h_{VH}$ and $\mathbf{w} = (w_V, w_H)^T$ is the transmit weight vector. We remark that h_{XY} is the impulse response corresponding to the signal transmitted using X–polarization and received using Y–polarization. In the analysis we assume that coefficients h_{XY} are uncorrelated. This assumption is well valid in the broadside of the antenna system. It is emphasized that a single antenna receiver can receive only part of the two–dimensional signal while using dual–polarized antennas the mobile would be able to receive all the available signal energy. All coefficients h_{XY} are assumed to be complex zero-mean Gaussian variables and we denote $\mathrm{E}\{|h_{XY}|^2\} = 2\sigma_{XY}^2$. Using the notation $\mathbf{a} = (\mathbf{u}^* \cdot \mathbf{u}_V, \mathbf{u}^* \cdot \mathbf{u}_H)$ the received signal can be expressed in the form

$$r = \Big(\mathbf{aCw}\Big) s + n, \qquad \mathbf{C} = \begin{pmatrix} h_{VV} & h_{HV} \\ h_{VH} & h_{HH} \end{pmatrix}.$$

3 POLARIZATION MATCHING

Let us study the selection of the optimal transmit weight vector $\mathbf{w}$. We denote by u_X the projection of mobile polarization to the X–polarization, *i.e.*, $u_X = \mathbf{u}^* \cdot \mathbf{u}_X$. The channel autocorrelation matrix is given by

$$\mathbf{R} = \mathrm{E}\{\mathbf{C}^*\mathbf{R}_p\mathbf{C}\}, \quad \mathbf{R}_p = \mathbf{a}^*\mathbf{a} = \begin{pmatrix} |u_V|^2 & u_V^* u_H \\ u_H^* u_V & |u_H|^2 \end{pmatrix},$$

where the expectation has been taken elementwise. The problem of polarization matching is the following: Find weight vector $\mathbf{w}$ such that $\mathbf{w}^*\mathbf{R}\mathbf{w}$ is maximized. The solution to this problem is obvious. If mobile polarization is $\mathbf{u}$, then we select $\mathbf{w}$ such that $\mathbf{u} = w_V\mathbf{u}_V + w_H\mathbf{u}_H$. Thus, we match the polarization of the transmitted signal with the receiver polarization. Since $\mathbf{u}_V$ and $\mathbf{u}_H$ are orthogonal unit vectors, the above matching operation is always possible.

In order to illustrate the benefits given by the polarization matching we study first the single antenna transmission utilizing a vertically polarized antenna, hence we take $w_V = 1$, $w_H = 0$. Assume that mobile polarization is linear. Then we can write $\mathbf{u} = (\sin\theta, \cos\theta)^T$ and the received signal becomes

$$r = H(\theta)s + n = (h_{VV}\sin\theta + h_{VH}\cos\theta)s + n.$$

For a given θ the received SNR is

$$\gamma(\theta) = \mathrm{E}\{|H(\theta)|^2|\theta\} = 2\sigma_{VV}^2\sin^2\theta + 2\sigma_{VH}^2\cos^2\theta.$$

If receiver polarization is vertical we have $\theta = \pi/2$ while $\theta = 0$ for horizontal polarization. The ratio

$$XPR_{VH} = \frac{\sigma_{VV}^2}{\sigma_{VH}^2}$$

between received vertical power and horizontal power is referred to as cross–polarization coupling ratio. The usage of the subscript emphasizes that XPR_{VH} presents a leakeage of power from vertical to horizontal polarization. This notation is adopted since the leakeage from horizontal to vertical polarization is not necessarily the same as XPR_{VH}. Moreover, XPR_{VH} depends on the environment and values $0 - 15$ dB have been given in literature. It is seen that if XPR_{VH} is high, then the mismatch between transmitter and receiver polarizations can cause large losses in received SNR.

Let us compute the expected SNR when θ is uniformly distributed on $(-\pi/2, \pi/2)$, *i.e.*, the orientation of the mobile polarization is random. Since $\mathrm{E}\{\sin^2\theta\} = \mathrm{E}\{\cos^2\theta\} = 1/2$ we find that $\gamma = \mathrm{E}\{\gamma(\theta)\} = \sigma_{VV}^2 + \sigma_{VH}^2$. In the sequel we scale the transmit powers such that $\sigma_{VV}^2 + \sigma_{VH}^2 = 1$. Then $\gamma = 1$ for the single antenna case. Similarly, we set $\sigma_{HH}^2 + \sigma_{HV}^2 = 1$.

Consider next the random matching. This corresponds to the system where w_V and w_H are random but $\|\mathbf{w}\| = 1$. Now

$$H(\theta) = w_V(h_{VV}\sin\theta + h_{VH}\cos\theta) + w_H(h_{HH}\cos\theta + h_{HV}\sin\theta)$$

and after some elementary operations we find that $\gamma = 1$ as in the case of single antenna transmission. However, there is a major difference between random matching and a single antenna system since the transmit weights can be changed continuously in random matching. This implies that polarization mismatch — as well as perfect match — occurs only occasionally. This provides interleaving gain when channel coding is employed.

Consider the polarization matching assuming that mobile polarization is perfectly known in the transmitter. Now we take $w_V = \sin\theta$, $w_H = \cos\theta$ and the received signal in the mobile is given by

$$H(\theta) = h_{VV}\sin^2\theta + h_{HH}\cos^2\theta + (h_{VH} + h_{HV})\sin\theta\cos\theta.$$

Since $\mathrm{E}\{\sin^4\theta\} = \mathrm{E}\{\cos^4\theta\} = 3/8$ and $\mathrm{E}\{\sin^2 2\theta\} = 1/2$ we obtain

$$\gamma = \frac{3}{4}(\sigma_{VV}^2 + \sigma_{HH}^2) + \frac{1}{4}(\sigma_{VH}^2 + \sigma_{HV}^2).$$

We remark that if $XPR_{VH}, XPR_{HV} \rightarrow \infty$, then the achieved SNR gain against single antenna case and random matching is 1.76 dB which is the same as in the case of fast (within channel coherence time) mobile directed antenna selection with two transmit antennas. Contrary to transmit diversity designed to mitigate fast fading, polarization matching does not provide diversity gain against fast fading but it gives polarization matching gain against polarization mismatch. The difference is essential: while signal attenuation caused by fast fading may last only a short time depending on the mobile speed, the polarization mismatch can last very long time depending on the mobile antenna orientation. Finally, we note that if $XPR_{VH} = XPR_{HV} = 0$dB, $\gamma = 1$ and polarization matching does not provide any gain.

4 CONCLUSIONS

The polarization matching proposed in [1, 2] was studied, and it was found out that the SNR gain from polarization matching is moderate even when the mobile polarization is perfectly known in the base station. However, if the cross–polarization coupling ratio is high, large losses caused by polarization mismatch can be avoided in individual radio links when using polarization matching. This results as an improved network performance since the co-channel interference is attenuated. It is also expected that the practical gain from polarization matching is higher than indicated by our analysis since there is usually a correlation between transmit antenna branches when using cross-polarized antennas.

References

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