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Abstract

We investigate in a spatial Cournot competition setting how much product R&D investment firms engage in in order to differentiate their goods. Comparing to the setting with no spatial connotation, the firms tend to under-invest in product R&D. This result holds even if marginal cost reducing process innovation is possible after product innovation. We also find the complementarity in product differentiation, process innovation, and lack of transport cost.

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1 Introduction

Since Hotelling (1929), what type of product attributes firms choose in a strategic setting has been analyzed in various models. Hotelling presented

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the principle of minimum differentiation, in which firms tend to produce similar products. On the other hand, d'Aspremont, Gabszewicz, and Thisse (1979) showed that in a game where firms select locations in a linear city in the first stage and then compete in price in the second stage, the equilibrium outcome entails firms locating at the far edges of the city. This illustrated that in order to curtail the fierce price competition with close product characteristics, firms choose to follow the principle of maximum differentiation. In this manner, whether maximum, minimum, or intermediate differentiation takes place became a topic of interest thereafter.

Product differentiation has been analyzed in the literature mainly in two ways, using the differentiated demand function as in Singh and Vives (1984) and in the spatial context. Lin and Saggi (2002) endogenized the level of differentiation in the former setting. In the latter literature, Neven and Thisse (1990) showed that when firms can differentiate their goods vertically and horizontally, there are equilibria where maximum differentiation takes place in one dimension and minimum differentiation in the other dimension. Then, Tabuchi (1994) established that in a location-then-price competition model in a rectangular (two-dimensional) market, two firms maximize differentiation in one dimension and minimize differentiation in the other dimension. To follow this up, Irmen and Thisse (1998) showed that in an n dimensional setting, firms maximize differentiation in just one dimension, whereas minimum differentiation results in all other dimensions. Thus, when firms can choose the level of differentiation, they tend to choose similar characteristics in all but one dimension, as differentiation in one dimension is enough to alleviate the price competition.

In determining the level of differentiation, the firms should be able to choose non-spatial differentiation characteristic as well. There have not been much analyses in combining the two differentiation literature. This is due to the predominance of price competition model in the spatial literature. However, as quantity competition models are gaining prominence in spatial literature, there have been works combining the two differentiation aspects.

The analysis of location-quantity competition started with the works of Anderson and Neven (1991) and Hamilton, Thisse, and Weskamp (1989).

They considered a linear city market where markets exist throughout the line segment $[0, 1]$ and the firms must bear the cost of shipping as they supply to each market. The authors concluded that central agglomeration occurs, supporting the principle of minimum differentiation. Thereafter, Pal (1998) extended the model to the duopoly case of circular city and showed that the firms locate apart from each other in equilibrium, supporting the principle of maximum differentiation. Gupta, et al. (2004) and Shimizu and Matsumura (2003) solved for equilibrium in an n firm case, showing that maximum differentiation is an equilibrium, but there exist many other, including ones where some, but not all, firms agglomerate at some point(s).[1]

As for combining the location-then-quantity competition setting with the differentiated demand function, Shimizu (2002) and Yu and Lai (2003) introduced exogenous non-spatial differentiation in a two-firm setting and showed that if the products are sufficiently differentiated such that they are considered complements, the equilibrium outcomes are different from when the products are substitutes. Later, Gupta, Pal, and Sarkar (2006) extended this setting and showed multiple equilibria when there are two sets of firms and the goods produced by each group are differentiated. All these papers assumed that the level of non-spatial differentiation is exogenous.

In this paper, we adopt the location-quantity competition setting with two firms in a circular city market, as used in Shimizu (2002). Then we endogenize the level of non-spatial differentiation using the setting of Lin and Saggi (2002). We abstract from endogenizing location choice, as Matsumura and Shimizu (2006) showed that for any increasing cost functions for two firms (may be distinct for each firm), the equilibrium location outcome is maximum differentiation. Thus the motivation for this paper is to consider whether this setting strengthens or counters the arguments in the earlier literature. Since the location choice is maximum differentiation, we should expect minimum differentiation in the non-spatial dimension. Our result is that, while the differentiation is not minimal, the firms tend to lower the

[1] For other results on the location-quantity competition models in a circular city, see Chamorro-Rivas (2000), Matsushima (2001), and Pal and Sarkar (2006).

level of differentiation than when the firms do not face spatial differentiation created by transport cost.

There are two likely effects in this model. The first is the same as in Tabuchi (1994) and Irmen and Thisse (1998), where firms' incentive to differentiate becomes very low once there is one dimension with already maximized differentiation. The second deals with local monopolization. As the unit transport cost increases, the markets near the firm's home location becomes more advantageous for the firm, since the other firm must pay a larger amount of transport cost. Thus the firms can exercise more market power close to the home location. This potential local monopolization would encourage the firms to increase product differentiation to increase demand. These two effects counter each other, and in our paper the former effect is stronger, resulting in lower differentiation for the case with transport than without.

We also consider the game where in the stage between endogenous product innovation selection and Cournot competition, firms can also engage in cost-reducing process innovation.[2] Even when this option is added, our result does not change: firm's incentive to differentiate non-spatially is lower when transport cost exists. We also examine the effect the existence of process innovation has on product innovation R&D and find that they are complementary. Combining the results, among the regimes in our model, the level of product innovation R&D is highest with process R&D and without transport cost, and is lowest without process R&D and with transport cost. Thus we find a complementary relationship between product and process innovation investments and lack of transport cost. This implies that product differentiation and differentiation by the existence of transport cost act as substitutes, not complements, for the firms' product selection. This result is consistent with the aforementioned literature.

The rest of the paper proceeds as follows. Section 2 provides the model. Section 3 analyzes the setting where the firms do not engage in process

[2] For an example of cost-reducing investment in a spatial setting, see Matsumura and Matsushima (2004).

innovation. Then in section 4 the firms are allowed to engage in process innovation R&D. Section 5 concludes the paper.

2 Model

Let there be two firms in an industry. There is a circular city of length 1, where markets exist throughout the perimeter. Each market in $[0, 1]$ is denoted x. The firms' locations are denoted x_1 and x_2. Each firm produces differentiated goods and supplies them to all markets on the perimeter. Transport cost is linear with respect to distance and amount shipped, and shipping q goods from x_i to x requires transport cost $t|x - x_i|q$, where $t \geq 0$ is a constant.

Consumers gain utility from two differentiated goods and numeraire m. Consumer located at market x has utility as given in the following equation.

$$u(q_1, q_2, m) = a(q_1+q_2) - \frac{q_1^2 + q_2^2}{2} - sq_1q_2 - t(|x_1 - x|q_1 + |x_2 - x|q_2) + m, \ 0 \leq s \leq 1.$$

Solving the utility maximization problem for the representative consumer, we have the following inverse demand function.

$$p_1 = a - q_1 - sq_2 - t|x_1 - x| \quad \text{and} \quad p_2 = a - sq_1 - q_2 - t|x_2 - x|. \tag{1}$$

Here, q_i and p_i are output and price of firm i, respectively, and s represents the degree of product differentiation between the goods provided by the two firms.[3] When $s = 1$ the two goods are perfect substitutes, and when $s = 0$ they are independent. For the intermediate values, as s decreases, the degree of differentiation between the goods increases, the demand for both goods shifting outward. Finally, a is the demand intercept, signifying the size of demand. In the literature, a is assumed to be sufficiently large as to ensure positive output at every market for both firms. We follow this and assume that a is sufficiently large so that $a > c + 3t$ must hold.

[3] This setting, except for the transport cost part, is from Singh and Vives (1984). The restriction $s \leq 1$ exists so that the own effect of change in q is larger than the cross effect, making the analysis tractable.

The degree of differentiation can be increased by engaging in an R&D. In other words, s is decreased in such a case. Let the level of product R&D for firm i be denoted d_i. Then, following Lin and Saggi (2002), s is given by $s = \bar{s} - (d_1 + d_2)$, where $\bar{s} \leq 1$ is the initial degree of differentiation, and $0 \leq d_i \leq \bar{s}/2$ holds. Cost of product R&D is given by $F(d_i)$, and we assume $F(0) = 0$ and that it is increasing and strictly convex, that is, $F' > 0$, $F'' > 0$ and $F''' > 0$. We also assume $\lim_{d_i \to +0} F'(d_i) = 0$, $F(\bar{s}/2)$ is sufficiently large in order to obtain interior solution. Let the marginal cost for product be equal for both firms at c.

The game proceeds as follows. In stage 0, each firm decides its location x_i in order to maximize the total profit Π_i. However, as Matsumura and Shimizu (2006) show, in many cases of circular city location-quantity competition game including this one, equilibrium location outcome is always where firms locate as far apart as possible. Since the principle of maximum differentiation is very strong regarding this side of product differentiation, we do not consider endogenous location choice and set exogenously that firms are located opposite to each other, as in $(x_1^*, x_2^*) = (0, 1/2)$ without loss of generality.

Thus the game we analyze starts at stage 1. Here, firms independently and simultaneously decide the level of R&D investment d_i. Note that cost $F(d_i)$ is needed for investment. In stage 2, firms engage in process R&D, where marginal cost c can be lowered. If firm i lowers its marginal cost by ε_i to $c - \varepsilon_i$, it must pay investment cost $\gamma\varepsilon_i^2/2$. The two firms simultaneously and independently decide the levels of ε_i. In stage 3, firms engage in Cournot competition, with equation (1) the inverse demand function.

Throughout the analysis, we compare two cases. One is the model as described up to here. The other is the case without transport cost. That is, we would like to determine the effect the existence of transport cost has on various outcomes of the model. In order to do so, we must normalize the two cases to make them comparable. Here, we use the "equal average cost" criterion. In the model up to here, which we call the "transport case" and denote the outcomes by using superscript "T" (note the use of capital letter), the cost is low when serving to location close to the home location and high in the locations opposite to the home. The average transport cost is ($\int_0^1 |0-x|dx$) $= 1/4$. In the alternate model, which we call the "no transport case" and denote the results by using superscript "N", we ignore the spatial effect and set $|x_i - x| = 1/4$. Note that there is no spatial differentiation in this setting. Since the average cost is the same, we should be able to compare the two cases without much bias.

In the next section, we analyze the game without process innovation. Thus we only have stages 1 and 3. In section 4, we include process innovation into analysis.

3 Product differentiation only

Let us now consider the game without the second stage of process innovation. We compare the "Transport" and "No-transport" cases and determine in which setting investment for differentiation is greater. The results in this section are denoted by superscript "n" (note the use of lower-case letter), which stands for no-process innovation.

The equilibrium concept used is subgame perfect Nash equilibrium. Therefore, we solve the game using backward induction. In the third stage subgame, profit π_i for firm i at market x is given by

$$\pi_i(s, q_1, q_2; x) = \{a - q_i - sq_j - t|x - x_i| - c\}q_i.$$

Then we have Cournot equilibrium outcome as follows:

$$\begin{aligned}
q_i^n(s;x) &= \frac{(a-c)(2-s) - 2t|x_i - x| + st|x_j - x|}{4 - s^2}, \\
p_i^n(s;x) &= \frac{a(2-s) + c(2+s-s^2) - 2t|x_i - x| + st|x_j - x|}{4 - s^2}, \\
\pi_i(s, q_1^n(s;x), q_2^n(s;x); x) &= \left[\frac{(a-c)(2-s) - 2t|x_i - x| + st|x_j - x|}{4 - s^2}\right]^2 \\
&= [q_i^n(s,x)]^2,
\end{aligned}$$

where $q_i^n(s;x)$, $p_i^n(s;x)$, and $\pi_i(s, q_1^n(s;x), q_2^n(s;x); x)$ are Cournot output, price, and local profit, respectively.

Let the gross profit $\Pi_i^n(s)$ be the summation of all local profits in the subgame. Now we incorporate the two cases into the setting. $\Pi_i^{Tn}(s)$ represents the gross profit for the case with transport cost. On the other hand, $\Pi_i^{Nn}(s)$ denotes the gross profit level where $|x_i - x| = 1/4$ is substituted before local profits are added. To sum up, we have the following.

$$\Pi_i^{Tn}(s) = 2\int_0^{1/2} \left[\frac{(a-c)(2-s) - 2tx + st(1/2-x)}{4-s^2}\right]^2 dx$$
$$= \frac{12a^2(2-s)^2 + 12c^2(2-s)^2 + 6c(2-s)^2 t - 6a(2-s)^2(4c+t) + (4-2s+s^2)t^2}{12(4-s^2)^2},$$
$$\Pi_i^{Nn}(s) = 2\int_0^{1/2} \left[\frac{(a-c)(2-s) - 2t(1/4) + st(1/4)}{4-s^2}\right]^2 dx$$
$$= \frac{(4a-4c-t)^2}{16(2+s)^2}.$$

Here, for all $s \in [0,1]$, the following holds.

Lemma 1 *The profit level in the transport case is higher than that in the no transport case. Also, the difference in profits decreases as the degree of differentiation increases and as unit transport cost decreases.*

Proof. Given $t > 0$, the difference in profits between the transport model and the no transport model is

$$\Pi_i^{Tn}(s) - \Pi_i^{Nn}(s) = \frac{t^2}{48(2-s)^2} > 0.$$

Differentiating this with respect to s, we have

$$\frac{\partial(\Pi_i^{Tn}(s) - \Pi_i^{Nn}(s))}{\partial s} = \frac{t^2}{24(2-s)^3} > 0,$$
$$\frac{\partial^2(\Pi_i^{Tn}(s) - \Pi_i^{Nn}(s))}{\partial s^2} = \frac{t^2}{8(2-s)^4} > 0.$$

Thus, as the differentiation degree becomes higher (smaller s), the difference in profits between the models decreases. Also, differentiating the profit difference with respect to t we have

$$\frac{\partial(\Pi_i^{Tn}(s) - \Pi_i^{Nn}(s))}{\partial t} = \frac{t}{24(2-s)^2} > 0. \qquad \textbf{(Q.E.D.)}$$

Note that the second order derivative of the profit differential with respect to s is also positive. Therefore, the marginal product R&D investment leads to a larger profit difference when the degree of differentiation is higher.

Now, let us consider the first stage. Firm i decides d_i in order to maximize its own profit less investment cost for product R&D. Thus we need to solve the following.

$$\max_{d_i} \Pi_i^{kn}(s) - F(d_i).$$

From now on, let us use k as an index such that $k \in \{T, N\}$. Then the FOCs for the two models are as follows:

$$F'(d_i) = \frac{(4a - 4c - t)^2}{8(2+s)^3} - \frac{t^2}{24(2-s)^3}, \tag{2}$$

$$F'(d_i) = \frac{(4a - 4c - t)^2}{8(2+s)^3}, \tag{3}$$

where $s = \bar{s} - d_i - d_j$. The RHS of (2) is the marginal gross profit due to additional R&D investment. Using relation $s = \bar{s} - d_i - d_j$, we can denote this as $MP^{Tn}(d_i)$. Similarly, the RHS of (3) is $MP^{Nn}(d_i)$. Finally, denote by D_i^{Tn} the set of investment levels where (2) is satisfied, and D_i^{Nn} be that where (3) is satisfied. In other words, $D_i^{Tn} \equiv \{d_i | F'(d_i) = MP^{Tn}(d_i)\}$ and $D_i^{Nn} \equiv \{d_i | F'(d_i) = MP^{Nn}(d_i)\}$. Let $d_i^{Tn} \in D_i^{Tn}$ and $d_i^{Nn} \in D_i^{Nn}$ be elements of the sets, if these elements exist.

Let us now examine in which case the product R&D is greater. The following holds for all s.

Proposition 1 *Assume that $t > 0$ and there is no process R&D investment stage. Then the product R&D investment level is higher in the no transport case than in the transport case.*

Proof. First, we show that d^{Tn} and d^{Nn} are unique. Then we prove the statement in the proposition. From equations (2) and (3),

$$MP^{Nn}(d_i) - MP^{Tn}(d_i) = \frac{t^2}{24(2-s)^3}.$$

Thus we have $MP^{Nn}(d_i) > MP^{Tn}(d_i) > 0$ when $d_i \in [0, \bar{s}/2]$, and these values are bounded above. Now, we show that MP^{Tn} and MP^{Nn} are strictly increasing convex functions in the relevant region.

Using equation (3) and noting that $s = \bar{s} - d_i - d_j$, we have

$$\frac{\partial MP^{Nn}(d_i)}{\partial d_i} = \frac{3(4a-4c-t)^2}{8(2+s)^4} > 0,$$
$$\frac{\partial^2 MP^{Nn}(d_i)}{\partial d_i^2} = \frac{3(4a-4c-t)^2}{2(2+s)^5} > 0.$$

From equations (2), we also have

$$\begin{aligned}
\frac{\partial MP^{Tn}(d_i)}{\partial d_i} &= \frac{3(4a-4c-t)^2}{8(2+s)^4} + \frac{t^2}{8(2-s)^4} > 0, \\
\frac{\partial^2 MP^{Tn}(d_i)}{\partial d_i^2} &= \frac{3(4a-4c-t)^2}{2(2+s)^5} - \frac{t^2}{2(2-s)^5} \geq \frac{(4a-4c-t)^2}{162} - \frac{t^2}{2} \\
&= \frac{1}{162}\{(4a-4c-t)^2 - 81t^2\} \\
&= \frac{1}{162}(4a-4c-10t)(4a-4c+8t) > 0.
\end{aligned}$$

In the latter derivation, the first inequality is due to the LHS being decreasing in s and thus takes the least value at $s = 1$, which is the highest value of s relevant in our model. The last inequality is due to the assumption $a > c + 3t$. Thus we have shown that $MP^{Tn}(d_i)$ and $MP^{Nn}(d_i)$ are both strictly increasing and strictly convex.

The assumption on F states $F(0) = 0$, $\lim_{d_i \to +0} F'(d_i) = 0$, $F(\bar{s}/2)$ is sufficiently high, and $F', F'', F''' > 0$. Thus, both MP^{Tn} and MP^{Nn} cross $F'(d_i)$ exactly once each in the domain $d_i \in [0, \bar{s}/2]$. Then, from $MP^{Nn}(d_i) > MP^{Tn}(d_i) > 0$, $d^{Tn} < d^{Nn}$ must hold.

(Q.E.D.)

The RHSs of (2) and (3) represent marginal gross profit of an additional one unit of process R&D investment. The LHSs are the marginal investment cost in such cases. For all $s \in [0, 1]$, the no transport case has a larger marginal gross profit. On the other hand, function F' is the same for both equations. Thus, for FOCs to be satisfied, the investment level must be higher in the no transport case. (See Figure 1.)

[Figure 1 around here.]

We provide an intuition for this result. Both t and s can be considered parameters affecting the degree of differentiation. An increase in t and a decrease in s both make the goods supplied by the two firms more differentiated, albeit in a different sense. Thus, as t increases, the firms' incentive to further differentiate the goods lessens. This effect is similar to Tabuchi (1994) and Irmen and Thisse (1999), where given that one dimension is maximally differentiated, the other dimensions are minimally differentiated. Our result does not make the differentiation be fully minimum, but the incentive for these cases should work in the same direction.

4 Interaction between process and product R&D

In this section, we introduce process R&D into the model in the second stage. The focus is on how the firms' incentives for product R&D changes due to this introduction. As noted in section 2, process innovation investment by ε_i allows firm i to lower its marginal cost to $c-\varepsilon_i$, at the expense of investment cost $\gamma\varepsilon_i^2/2$. We assume that $\gamma > 8/9$ so that the relevant second order conditions are satisfied.[4] Then, the local profit for firm i at market x is given as follows.

$$\begin{aligned}\pi_i(s,\varepsilon,&q_1(s,\varepsilon;x),q_2(s,\varepsilon;x);x)\\ &= \{a-q_i(s,\varepsilon,x)-sq_j(s,\varepsilon,x)-t|x-x_i|-(c-\varepsilon_i)\}q_i(s,\varepsilon,x),\end{aligned}$$

where $\varepsilon \equiv (\varepsilon_1,\varepsilon_2)$. We denote by superscript "p" (note the lower case) the results with process R&D. Thus the third stage Cournot equilibrium outcomes are given by $q_i^p(s,\varepsilon;x)$, $p_i^p(s,\varepsilon;x)$, and $\pi_i(s,\varepsilon,q_1^p(s,\varepsilon;x),q_2^p(s,\varepsilon;x);x)$

[4] This condition is the same as in the process innovation section in Lin and Saggi (2002).

, and they are derived as follows:

$$
\begin{aligned}
q_i^p(s,\varepsilon;x) &= \frac{a(2-s)-2(c-\varepsilon_i)+s(c-\varepsilon_j)-2t|x_i-x|+st|x_j-x|}{4-s^2},\\
p_i^p(s,\varepsilon;x) &= \frac{a(2-s)+(2-s^2)(c-\varepsilon_i)+s(c-\varepsilon_j)-2t|x_i-x|+st|x_j-x|}{4-s^2},\\
\pi_i(s,\varepsilon,\ &q_1^p(s,\varepsilon;x),q_2^p(s,\varepsilon;x);x)\\
&= \left[\frac{a(2-s)-2(c-\varepsilon_i)+s(c-\varepsilon_j)-2t|x_i-x|+st|x_j-x|}{4-s^2}\right]^2\\
&= (q_i^p(s,\varepsilon;x))^2.
\end{aligned}
$$

Similar to in the last section, we sum the local profit $\pi_i^p(s,\varepsilon,q_1^p(s,\varepsilon;x),q_2^p(s,\varepsilon;x);x)$ to derive the gross profit,

$$
\begin{aligned}
\Pi_i^{Tp}(s,\varepsilon) &= 2\int_0^{1/2}\left[\frac{a(2-s)-2(c-\varepsilon_i)+s(c-\varepsilon_j)-2tx+st(1/2-x)}{4-s^2}\right]^2 dx\\
&= \frac{2[\{-a(2-s)+2(c-\varepsilon_i)-s(c-\varepsilon_j)+t\}^3]}{3(2+s)(4-s^2)^2t}\\
&\qquad + \frac{2[a(2-s)+2(c-\varepsilon_i)-s(c-\varepsilon_j)+st/2\}^3]}{3(2+s)(4-s^2)^2t},\\
\Pi_i^{Np}(s,\varepsilon) &= 2\int_0^{1/2}\left[\frac{a(2-s)-2(c-\varepsilon_i)+s(c-\varepsilon_j)-2t(1/4)+st(1/4)}{4-s^2}\right]^2 dx\\
&= \left[\frac{4a(2-s)-8(c-\varepsilon_i)+4s(c-\varepsilon_j)-(2-s)t}{4(4-s^2)}\right]^2.
\end{aligned}
$$

Let us now consider the second stage. We derive the optimal process R&D investment level by solving the following profit maximization problem for firm i.

$$
\max_{\varepsilon_i}\ \Pi_i^{kp}(s,\varepsilon)-\frac{\gamma\varepsilon_i^2}{2}. \tag{4}
$$

Letting $\varepsilon_i^{kp}(s)$ be the equilibrium process R&D investment level for firm i in stage 2 subgame, we have

$$
\varepsilon_i^{kp}(s) = \frac{4a-4c-t}{\gamma(2-s)(2+s)^2-4}. \tag{5}
$$

Thus in both models the firms respectively select the same level of process innovation, given the level of s. Note that the second order condition is satisfied by the assumption $\gamma > 8/9$.[5]

Let the vector of innovations be denoted $\varepsilon^{kp}(s) = (\varepsilon_1^{kp}(s), \varepsilon_2^{kp}(s))$. Then the equilibrium profit as derived in the second stage is given by $\Pi_i^{kp}(s, \varepsilon^{kp}) - \gamma(\varepsilon_i^{kp})^2/2$.[6] To sum, we have the following lemma.

Lemma 2 *For any $s \in [0, 1]$, process R&D investment levels for transport and no transport models are equal. The investment levels are decreasing in s when $s < 2/3$ and increasing in s when $s > 2/3$.*

Proof. The first part is from equation (5). The latter is derived by differentiating (5) with respect to s.

$$\frac{\partial \varepsilon_i^{kp}(s)}{\partial s} = \frac{\gamma(4a - 4c - t)(-4 + 4s + 3s^2)}{\{\gamma(2 - s)(2 + s)^2 - 4\}^2}.$$

Except for $(-4+4s+3s^2)$, all subexpressions above are positive. $(-4+4s+3s^2) = (3s - 2)(s + 2)$ is positive when $s > 2/3$ and negative when $s < 2/3$, yielding the result.

(Q.E.D.)

This lemma exhibits the same outcome as in that in Lin and Saggi (2002), including the threshold 2/3. Thus, as mentioned there, the effect of product differentiation on process innovation is non-monotone. In particular, when the degree of differentiation is high (low s), the incentive to engage in process R&D investment increases as product differentiation increases.

We now examine how the process R&D investment is affected by a change in the unit transport cost t.

Lemma 3 *The process R&D investment decreases as the unit transport cost t increases.*

[5] For the SOC to be satisfied, $\gamma > 8/(4 - s^2)^2$ must hold. Let the RHS of this equation be denoted $f(s)$. Then it is shown to be increasing in $s \in (0, 1]$ because $f'(s) = 32s(4 - s^2)^{-3} > 0$. Thus, assuming $\gamma > f(1) = 8/9$ is enough for the SOC.

[6] This profit is net of process investment cost, but gross of product investment cost.

Proof. By taking the derivative of process R&D level $\varepsilon_i^{kp}(s)$ with respect to t, we have

$$\frac{\partial \varepsilon_i^{kp}(s)}{\partial t} = -\frac{1}{\gamma(2-s)(2+s)^2 - 4}.$$

The denominator of this equation takes its minimum value $8\gamma - 4 > 0$ when $s = 0$. Therefore, since $\gamma > 8/9$, $\partial \varepsilon_i^{kp}(s)/\partial t < 0$ holds.

(Q.E.D.)

From Lemma 2 and Lemma 3, process R&D in the second stage is large when t is small and when product R&D in stage 1 is high, leading to a small s.

Finally, let us examine stage 1. Firm i solves the following profit maximization problem.

$$\max_{d_i} \; \Pi_i^{kp}(s, \varepsilon^{kp}(s)) - \frac{\gamma(\varepsilon_i^{kp}(s))^2}{2} - F(d_i).$$

The first order condition is where $F'(d_i)$ is equal to marginal gross profit, including the effect on the second stage investment. We denote the marginal gross profit for the transport and the no transport cases by $MP^{Tp}(d_i)$ and $MP^{Np}(d_i)$, respectively. Let the set of investment level d_i such that the marginal gross profit and marginal investment cost are equal in each case be denoted D_i^{Tp} and D_i^{Np}. Formally, $D_i^{Tp} \equiv \{d_i | MP^{Tp}(d_i) = F'(d_i)\}$ and $D_i^{Np} \equiv \{d_i | MP^{Np}(d_i) = F'(d_i)\}$. An element in each set is denoted d_i^{Tp} and d_i^{Np}, respectively.[7] After tedious calculation, function MP^{Np} can be shown to be convex in the region $d_i \in [0, \bar{s}/2]$, but not necessarily increasing.[8] (See Appendix for proof on their convexity.) Therefore, there are possibly multiple elements in D_i^{Tp} and D_i^{Np} depending on the shape of function F'. Given this, we select the maximum and minimum elements that satisfy $MP^{kp}(d_i) =$

[7] Since $MP^{Tp}(0)$, $MP^{Tp}(\bar{s}/2)$, $MP^{Np}(0)$, and $MP^{Np}(\bar{s}/2)$ are greater than 0 and bounded above, and from the assumptions on F, D_i^{Tp} and D_i^{Np} are not empty.

[8] The function MP^{Tp} is also likely convex, as seen by numerical calculations. We do not prove its convexity, since it is more tedious and it is not necessary in the analysis.

$F'(d_i)$ in each model and compare their relative size.[9] Let us define as follows:

$$\tilde{d}_i^{Tp} \equiv \min D_i^{Tp} \quad \text{and} \quad \bar{d}_i^{Tp} \equiv \max D_i^{Tp},$$
$$\tilde{d}_i^{Np} \equiv \min D_i^{Np} \quad \text{and} \quad \bar{d}_i^{Np} \equiv \max D_i^{Np}.$$

Now we are ready to compare the product innovation R&D levels. First, we examine how the investment levels are affected by the existence of transport cost, given the existence of process R&D stage.

Proposition 2 *Let $t > 0$ and process R&D possibly takes place in the second stage. Then the product R&D investment level is higher in the no transport model than in the transport model.*

Proof. Let us consider the shape of function MP^{Np}. From $\partial^2 MP^{Np}(d_i)/\partial d_i^2 > 0$, it is convex (see Appendix). From definition, $\tilde{d}_i^{Tp} \leq \bar{d}_i^{Tp}$ and $\tilde{d}_i^{Np} \leq \bar{d}_i^{Np}$ hold.

In addition, for any $s \in [0, 1]$, the difference in marginal gross profit is positive:

$$MP^{Np}(d_i) - MP^{Tp}(d_i) = \frac{t^2}{24(2-s)^3} > 0.$$

Thus, from the assumptions on F', we obtain $\bar{d}_i^{Tp} < \tilde{d}_i^{Np}$. Combining, $\tilde{d}_i^{Tp} \leq \bar{d}_i^{Tp} < \tilde{d}_i^{Np} \leq \bar{d}_i^{Np}$ holds. Therefore, marginal gross profit for product R&D investment is higher in the no transport model.

(Q.E.D.)

From Propositions 1 and 2, we have our main result.

Proposition 3 *Let $t > 0$. Then, whether firms have the opportunity to engage in the process innovation investments in the second stage, product R&D investment levels are higher in the no transport model than in the transport model.*

[9] Strictly speaking, even if $MP^{kp}(d_i) = F'(d_i)$ holds, the outcome may not be an equilibrium depending on how the marginal gross profit and marginal investment curves cross. Namely, the slope of the marginal gross profit function must be less than that of F'. We also may need to decide which equilibrium outcome is most likely. However, here we would just like to compare the sizes of investment in each regime, and defining maximum and minimum levels within each regime is sufficient for this cause.

The following proposition concerns with how the incentive for product R&D is affected by the existence of the process R&D opportunity.

Proposition 4 *In both the transport model and the no transport model, the product innovation R&D level is higher when the opportunity for process R&D investment is present.*

Proof. See Appendix.

This complementarity result between product and process innovations is the same as in Lin and Saggi (2002).

From Propositions 3 and 4, the following easily follows.

Proposition 5 *The level of product R&D investment is the highest in the no transport model with process R&D,* d^{Np}*. It is the lowest in the transport model without process innovation,* d^{Tn}*.*

Unfortunately, we can construct numerical examples that has either of d^{Nn} and d^{Tp} in equilibrium to be larger than the other.[10]

Now we will provide intuition for Propositions 3, 4, and 5. We have Figures 2 and 3 to illustrate the situation. When γ is small and as it shrinks to near 8/9, MP^{Np} and MP^{Tp} shift to upper right, while the curves are convex but not monotone and takes a minimum value somewhere in $s \in (0, \bar{s})$ as an interior solution. On the other hand, as γ is made larger, MP^{Np} and MP^{Tp} shift to lower left, and after γ passes some threshold value, the minimum occurs at $s = 0$ and the curves become increasing and convex.

When γ becomes sufficiently large, MP^{Np} and MP^{Tp} coincide with MP^{Nn} and MP^{Tn}. This is because as γ rises, firms lose incentive to engage in the process R&D investment due to high investment cost. Thus ε_i becomes lower and the corresponding marginal gross profits become closer.

Similarly, as the unit transport cost t decreases, MP^{Tn} and MP^{Nn} move closer, as do MP^{Tp} and MP^{Np}. This is obvious, as lowering t implies lowering the difference between the transport setting and the no transport setting. When $t = 0$, the difference vanishes.

[10] Some kind of sufficient conditions that determine the relative sizes may exist. We are currently examining this issue.

[Figure 2 around here.]

[Figure 3 around here.]

5 Concluding Remarks

Our investigation focused on how the existence of transport cost may affect the product innovation and process innovation incentives for a duopoly. We find that firms tend to lessen the degree of differentiation in the non-spatial dimension in the case with transport cost than in the case without. This is in line with the literature in spatial price competition, in that maximization in one dimension is enough to curtail fierce competition between the firms. Our result did not exhibit minimum differentiation because our setting is different from the previous results in that the competition is in quantity rather than price. However, the general message is the same.

We also find the complementarity between high degree of product differentiation, high level of process innovation, and low unit transport cost. The complementarity between the first two elements in the case without transport cost is consistent with the result of Lin and Saggi (2002). A possible extension of our paper is to endogenize unit transport cost. Instead of or in addition to the marginal cost reducing process innovation, t can be reduced in a similar fashion in the second stage. Following Eaton and Schmitt (1994) and Norman and Thisse (1999), we can interpret transport cost as difficulty in creating differentiated goods, and lowering transport cost implies implementing a more flexible manufacturing system. We conjecture that our general result of complementarity of high product differentiation and low transport cost would still hold.

Appendix

Proof that MP^{Np} is convex: Throughout the proof, note that $\gamma > 8/9$, $s = \bar{s} - d_i - d_j$, and $0 \leq s \leq \bar{s} \leq 1$. We first derive MP^{Np}. Plugging in equation (5) into the expression being maximized in (4) and then taking the derivative with respect to d_i, we obtain

$$MP^{Np}(d_i) = \frac{\gamma^2(2+s)^2\{8-8s-\gamma(2-s)^3(2+s)\}(4a-4c-t)^2\}}{8\{4-\gamma(2-s)(2+s)^2\}^3}.$$

Taking two derivatives with respect to d_i yields

$$\begin{aligned}
\frac{\partial^2 MP^{Np}(d_i)}{\partial d_i^2} &= \frac{3\gamma^2 K(4a-4c-t)^2}{2\{\gamma(2-s)(2+s)^2-4\}^5}, \\
\text{where } K &= \gamma^3(4-s^2)^5 - 2\gamma^2(2+s)^3(16+16s-96s^2+60s^3-25s^4) \\
&\quad -8\gamma(32-24s+20s^2+70s^3+25s^4)+64(1+s).
\end{aligned}$$

Let us examine the denominator. Noting that $(2-s)(2+s)^2$ takes its minimum at $s=0$ when $s \in [0,1]$, we have

$$\gamma(2-s)(2+s)^2 - 4 > \frac{8}{9}\cdot 2^3 - 4 = \frac{28}{9} > 0.$$

Therefore, if K is positive, the whole is positive, and we have the desired result.

Now we examine K. Taking the first, second, and third derivatives of K with respect to γ yields

$$\begin{aligned}
\frac{\partial K}{\partial \gamma} &= 3\gamma^2(4-s^2)^5 - 4\gamma(2+s)^3(16+16s-96s^2+60s^3-25s^4) \\
&\quad -8(32-24s+20s^2+70s^3+25s^4), \\
\frac{\partial^2 K}{\partial \gamma^2} &= 6\gamma(4-s^2)^5 - 4(2+s)^3(16+16s-96s^2+60s^3-25s^4), \\
\frac{\partial^3 K}{\partial \gamma^3} &= 6(4-s^2)^5 > 0.
\end{aligned}$$

Since the third derivative is positive, the second derivative is minimized when γ takes its minimum, which we set at $\gamma = 8/9$. Then

$$\begin{aligned}
\left.\frac{\partial^2 K}{\partial \gamma^2}\right|_{\gamma=8/9} &> \frac{16}{3}(4-s^2)^5 - 4(2+s)^3(16+16s-96s^2+60s^3-25s^4), \\
&= \frac{4}{3}(2+s)^3(464-816s+416s^2+140s^3-85s^4-16s^5+24s^6-4s^7) > 0,
\end{aligned}$$

since $464-816s+416s^2 = 416(1-s)^2+16s+48 > 0$. Since the second derivative is positive, the first derivative is minimized when γ takes its minimum. Then

$$\begin{aligned}\left.\frac{\partial K}{\partial \gamma}\right|_{\gamma=8/9} &> \frac{64}{27}(4-s^2)^5 - \frac{32}{9}(2+s)^3(16+16s-96s^2+60s^3-25s^4) \\ &\quad - 8(32-24s+20s^2+70s^3+25s^4), \\ &= \frac{8}{27}(5792-3192s-5020s^2+4830s^3 \\ &\quad + 4925s^4+432s^5-200s^6+300s^7+160s^8-8s^{10}) > 0,\end{aligned}$$

which holds because $5792-3192s-5020s^2+4830s^3 = 5792-4092s-1420s^2+1230s^3+900(1-2s)^2 s > 0$. Since the first derivative is positive, K is minimized when γ takes its minimum. Then

$$\left. K \right|_{\gamma=8/9} > \frac{64}{729} J > 0,$$

where $J \equiv (4025-3087s-3220s^2+4410s^3+3815s^4+648s^5+340s^6+450s^7+160s^8-8s^{10})$. This holds because $4025-3087s-3220s^2+4410s^3 = 4025-3571s-316s^2+54s^3+484(1-3s)^2 s > 0$. Thus finally, we have $K > 0$, which implies MP^{Np} is convex.

(Q.E.D.)

Proof of Proposition 4: In the transport model, the difference in marginal gross profit between when process R&D is present or not is, for any $s \in [0,1]$,

$$\begin{aligned}&MP^{Tp}(d_i) - MP^{Tn}(d_i) \\ &= \frac{(4a-4c-t)^2\{16-12\gamma(2-s)(2+s)^2+\gamma^2(2+s)^4(8-10s+5s^2)\}}{2(2+s)^3\{\gamma(2-s)(2+s)^2-4\}^3}.\end{aligned}$$

In the no transport model, the difference is as follows.

$$\begin{aligned}&MP^{Np}(d_i) - MP^{Nn}(d_i) \\ &= \frac{(4a-4c-t)^2\{16-12\gamma(2-s)(2+s)^2+\gamma^2(2+s)^4(8-10s+5s^2)\}}{2(2+s)^3\{\gamma(2-s)(2+s)^2-4\}^3}.\end{aligned}$$

Since these equations are the same, we need to show that this is positive for all s.

First, the denominator is positive from the previous proof. Thus, if the numerator is positive, the whole is also positive.

Let L be equal to $16 - 12\gamma(2-s)(2+s)^2 + \gamma^2(2+s)^4(8-10s+5s^2)$. Then we have

$$\begin{aligned}
\frac{\partial L}{\partial \gamma} &= 2\gamma(2+s)^4(8-10s+5s^2) - 12(2-s)(2+s)^2 \\
&> \frac{16}{9}(2+s)^4(8-10s+5s^2) - 12(2-s)(2+s)^2 \\
&= \frac{4(2+s)^2}{9}\{4(2+s)^2(8-10s+5s^2) - 27(2-s)\} \\
&= \frac{4(2+s)^2}{9}(74 - 5s - 48s^2 + 40s^3 + 20s^4) > 0.
\end{aligned}$$

The first inequality holds because $(8-10s+5s^2) = 5(1-s)^2 + 3 > 0$. Thus L is minimized at $\gamma = 8/9$.

$$\begin{aligned}
L\Big|_{\gamma=8/9} &= 16 - \frac{32}{3}(2-s)(2+s)^2 + \frac{64}{81}(2+s)^4(8-10s+5s^2) \\
&= \frac{16}{81}\{81 - 54(2-s)(2+s)^2 + 4(2+s)^4(8-10s+5s^2)\} \\
&= \frac{16}{81}(161 + 168s - 84s^2 - 10s^3 + 192s^4 + 120s^5 + 20s^6) > 0.
\end{aligned}$$

Thus L is positive for all $\gamma > 8/9$, and we have the desired result.

(Q.E.D.)

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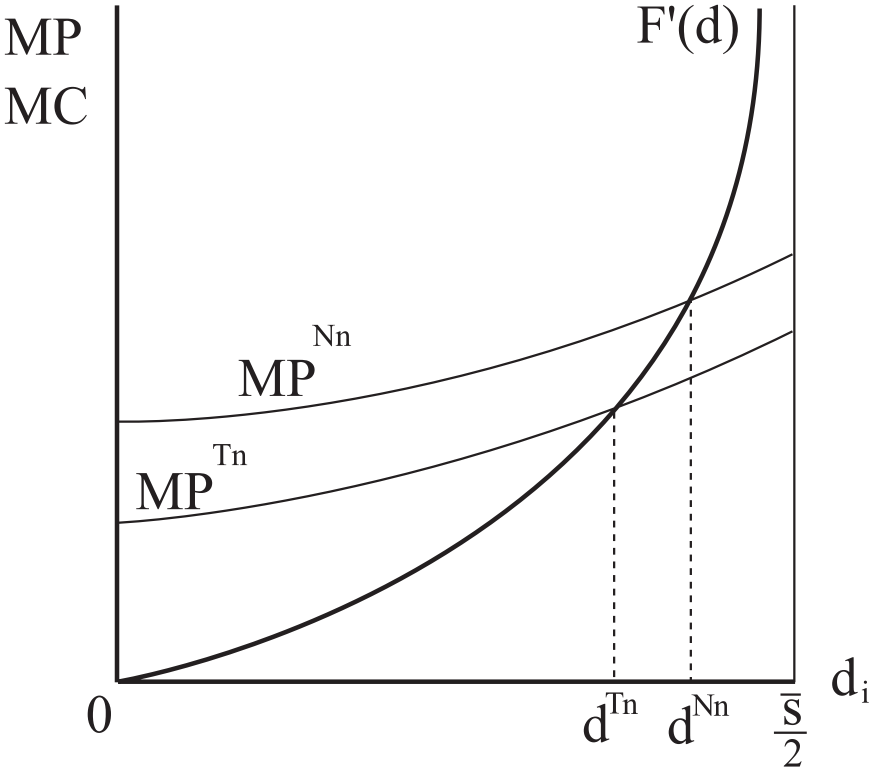

Figure 1: In the game without process innovation stage, investment in the transport case is less than in the no transport case. That is, $d^{Tn} < d^{Nn}$.

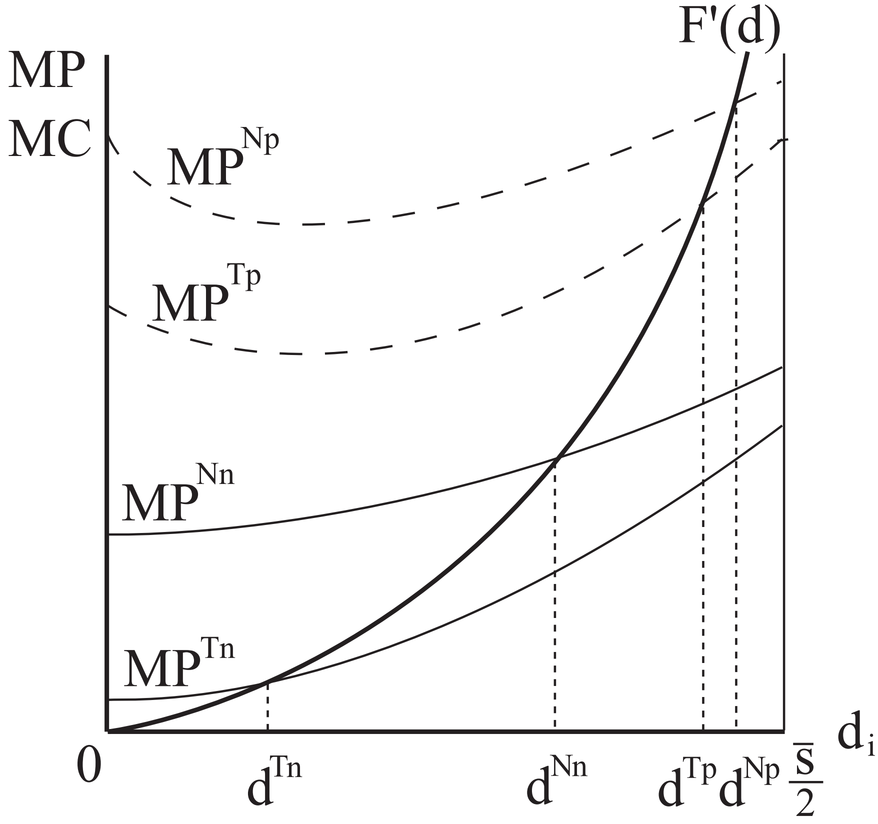

Figure 2: In the game with process innovation stage, when γ is low, the Np and Tp curves tend to be high, leading to high investment levels in those cases.

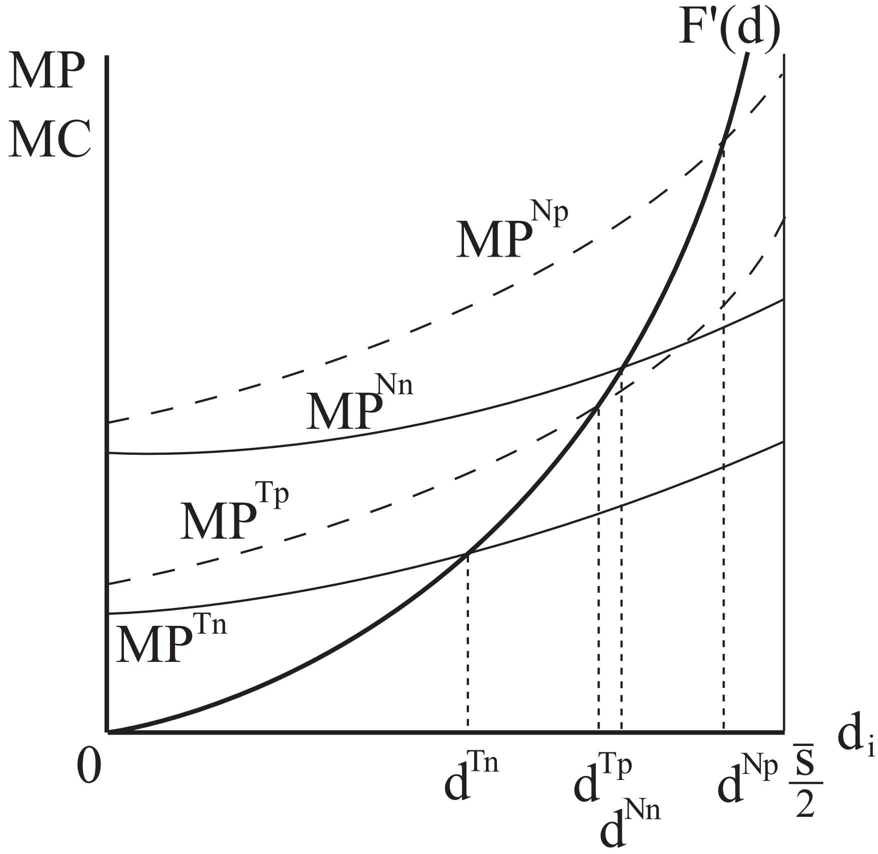

Figure 3: In the game with process innovation stage, when γ becomes higher, the Np and Tp curves drop, and the relative size of investment for the Tp and Nn cases may reverse.