# Behaviour of Multiple Generalized Langton's Ants

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## Abstract

Since their introduction by C. Langton, virtual ants have intrigued people with their behaviour. The system is governed by very simple local rules but gives rise to intricate patterns and, to date, no general theory of its dynamics has been found. Virtual ants have been mainly studied experimentally by computer simulation. In this work we have pursued the experimental simulation of an extension of Langton's ant: the generalized ant. We give a first rough classification of the patterns that arise as a consequence of their dynamical behaviour in the case of single ants, drawing on the work of others as well as on our own simulations. We then go on to the study of multiple generalized ants and we show under what conditions some new interesting behaviours arise.

## 1 Langton's Ant

C. Langton[1,2] is the inventor of this amazingly simple automaton. The virtual ant moves in a planar grid. There are two kinds of cells in the grid: white cells and black cells. Initially the ant is on the central square, for example, and is given a direction (N, S, E, W) towards which it is heading. The ant moves one cell forward in that direction according to the following rule: if it finds a black square then the the cell changes its color to white and the ant turns 90 degrees to the left. Conversely, if the cell is white the color changes to black and the ant turns 90 degrees to the right. Starting from an all-white grid the ant returns periodically to the origin during the first 500 steps leaving a more or less symmetrical trail behind it. Afterwards, the patterns that are traced become somewhat chaotic[1] but suddenly a straight diagonal pattern is traced that has been called a "highway" by J. Propp, who discovered it. On the highway the ant repeatedly follows the same sequence of 104 steps (see fig. 1). The orientation of the highway depends on the initial ant direction.

If the initial configuration of black and white cells is different, for example if there are scattered black cells in the grid at the beginning, the ant always ends up building the highway, at least on the many simulations that have been done, although nobody exactly knows if this is necessarily always the case. The only rigorous result on the system to date is due to X.P. Kong and E.G.D. Cohen and says that *an ant's trajectory is necessarily unbounded and escapes from any finite region.* A demonstration of the theorem can be found in ref.[3].

Several variations on the basic ant's rules have been tried. For example, if the ant goes straight ahead instead of turning right, then an horizontal or vertical highway two cells wide is built. Another variation proposed in [3] introduces a third type of cell, a grey cell, that never changes its color. The added rule is that if the ant lands on a grey cell it continues undisturbed in its current direction. In this model one also gets an highway construction but, due to the fact that the Kong-Cohen theorem no longer applies, periodic patterns can be obtained from some initial configurations.

Figure 1 : Highway construction

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1.Virtual ants as described here are perfectly deterministic systems. In this work we employ the term 'chaotic' in a intuitive sense to describe patterns that appear to be random. There is no implication that the system is chaotic in the technical sense of non-linear dynamical systems theory.

## 2 Generalized Ants

Generalized ants are an extension of Langton's ant. A generalized ant is defined to have n states numbered from 0 to n-1 instead of just two. In our pictorial representation the n states are represented by n different colors. Generalized ants have been proposed by G. Turk at Stanford and independently by L. Bunimovich and S. Troubetzkoy [4].

A generalized ant will be described by a rule string of n bits $S_k = \{0,1\}$ with k = 0..n-1. At each iteration the ant advances by one step in a given direction, looks at the state k of the cell at this position and turns right if bit k-th in the rule string is 1 and left otherwise. At the same time, the state of the cell changes to (k+1) mod n, where mod stands for the modulus operation. In this notation Langton's ant is represented by the string (1 0). Figure 2 graphically depicts the behavior of such a generalized virtual ant.

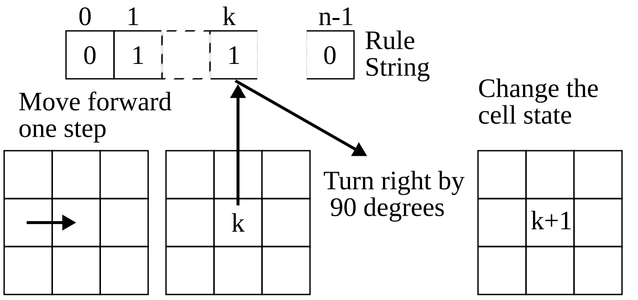


Figure 2 : Generalized ant's step

Rule strings consisting of all 1's or all 0's are trivial since the ant will simply turn in the same direction (right or left respectively) all the time. Similarly, complementing all the bits in a rule gives a mirror image of the original pattern. This can be seen for example in the case of ant 5 (101) in fig. 3.

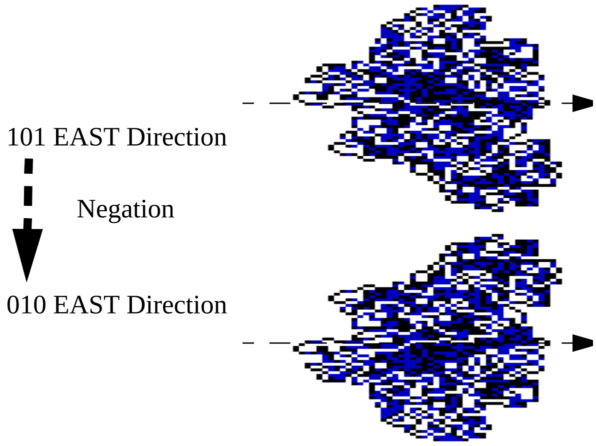


Figure 3 : Complementing a rule string.

It is interesting to examine the behaviour of a few non-trivial generalized ants to see if some general common pattern emerges. In order to simplify the notation and for classification purposes, we will use the decimal number corresponding to the rule string taken as an unsigned binary number. For example, Langton's ant (1 0) will be ant 2 in this notation. Unless otherwise specified the initial state of the grid will be 0 everywhere. Furthermore, the grid will be of finite size in practice. That is, we will assume that when an ant reaches the grid border it stops there. The observations described here are basically a systematization of those reported in [5] plus some remarks from our own experiments. Each simulation has been performed a few times only and a high but still limited number of steps has been done in each simulation. These results are therefore to be considered as indicative rather than definitive.

**Ant 1 (0 1)**

This ant behaves in the same way as Langton's but the trace is reversed with respect to the latter (see fig.3 above). For example, if it starts heading East then the highway is built in the south-west direction instead of north-west. Otherwise the behaviour is the same.

**Ant 2 (1 0)**

This is the well-known Langton's ant and does not need further comment.

**Ant 4 (1 0 0)**

This ant begins by tracing the same symmetrical patterns as ant 2 but with a bilateral symmetry instead of polar. This is followed by a chaotic phase and after having tracked 150 millions steps no recognizable structure emerges.

**Ant 5 (1 0 1)**

Same behavior as ant 2. At the beginning the patterns traced present an axis of symmetry of order two but after a while any symmetry disappears and after 150 million steps the behaviour is still apparently chaotic.

**Ant 6 (1 1 0)**

This ant builds a highway after 150 steps only but the pattern is different from that of ant 2. Each successive piece of the highway takes 18 steps to be built instead of 104.

**Ant 8 (1 0 0 0)**

This ant did not build any highway and the patterns traced did not show any signs of regularity.

**Ant 9 (1 0 0 1)**

Ant 9 does not build a highway. On the other hand, it generates growing symmetrical patterns.

**Ant 10 (1 0 1 0)**

This ant behaves in the same way as ant 2 with the same highway building period but it leaves a four colors trail instead of a bicolored one. This is a manifestation of the general phenomenon by which a rule string having two or more repetitions of a shorter rule will behave in the same way as the shorter one. Figure 4 depicts ant 10.

Figure 4 : Ant 10: chaos and highway building.

**Ant 11 (1 0 1 1)**

This ant, not described in [5], always behaves in a chaotic manner. Fig. 5 shows the pattern generated after 100,000 iterations.

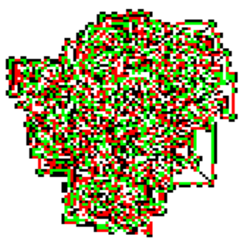

Figure 5 : Constant chaos.

**Ant 12 (1 1 0 0)**

This ant builds bilaterally symmetrical patterns that persist in time. Actually, total bilateral symmetry is observed when the ant visits its initial position.

**Ant 13 (1 1 0 1)**

This ant starts chaotically but after about 250,000 steps a highway appears. The period of the highway is 388. Fig. 6 shows the state of the automaton shortly after the building of the highway.

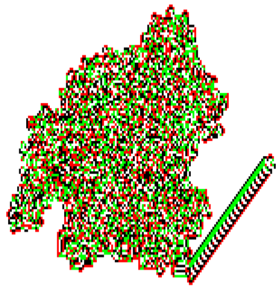

Figure 6 : Highway built by ant 13.

**Ant 14 (1 1 1 0)**

Ant 14 seems to be an hybrid of ants 2 and 6 [5]. It builds a highway like ant 2 but the period is 52 i.e., half the period of ant 2 and the shape of the highway is similar to that of ant 6.

Jimm Propp [5] did a simulation of a few other generalized ants with rule strings of length 5 and 6 finding some new behaviors. In general, ants having five bits rule-strings do not trace bilaterally symmetrical patterns but a new spiral pattern behavior appears with ant 27. With rule strings of length 6 it was found in [5] that all the ants that generate bilaterally symmetrical patterns have numbers that are divisible by three. Clearly, more systematic simulations and analyses are needed to make sense of these regularities.

## 2.1 Summary of the behavior of some generalized ants

The following table gives a summary of the behavior of the non-trivial ants previously described. Ants are classified according to their highway-building capabilities and according to the symmetry of the patterns they generate. The symmetry is indicated provided that it stays stable for some time even in those cases in which it eventually disappears. When a given ant builds a highway, its period i.e., the number of steps needed for buiding a repetitive segment, is also indicated.

Table 1. Rule strings of length four: summary of observations

| Ant Number | Rule String | Highway | Symmetry |
|---|---|---|---|
| 2 | ***1 0*** | yes , 104 | polar |
| 4 | ***1 0 0*** | no | bilateral |
| 5 | ***1 0 1*** | no | polar |
| 6 | ***1 1 0*** | yes, 18 | no |
| 8 | ***1 0 0 0*** | no, | no |
| 9 | ***1 0 0 1*** | no | bilateral |
| 10 | ***1 0 1 0*** | yes, 104 | polar |
| 11 | ***1 0 1 1*** | no | no |
| 12 | ***1 1 0 0*** | no | bilateral |
| 13 | ***1 1 0 1*** | yes, 388 | no |
| 14 | ***1 1 1 0*** | yes, 52 | no |

From the preceding table it can be seen that ants 6 and 14 have the same overall behaviour. They only differ by the fact that the common pattern 1 1 0 is preceded by a 1 in the case of ant 14. Starting from this observation, we have found experimentally that all ants whose rule string contains this same pattern preceded by all 1's (1 1 0, 1 1 1 0, 1 1 1 1 0, ...) always build a highway and behave in a manner similar to that of ant (1 1 0).

## 3 Collective behaviour

We are now going to describe the collective behaviour of more than one ant of the same species (i.e., having the same rule string). The ants will evolve independently of each other; that is, they will ignore each other in case of collisions. However, they influence their mutual behaviour through the trace they leave behind them since this corresponds to state changes of some cells, which amounts to a dynamical change of the environment the ants will be confronted with. Thus the dynamical evolution of a single ant will implicitly depend on the history of all the other ants, giving rise in general to unpredictable collective dynamics. Furthermore, in the following the ants are treated sequentially one after the other unless otherwise stated. In terms of automata transition rules, this means that the state of a given ant is updated before the following ant in the sequence does its transition.

We begin our study by the simulation of a two Langton's ant system and then proceed to the description of a four ants collection, followed by a pair of generalized (1 0 1 0) ants. To our knowledge, only C. Langton has performed some multiant simulation [1]. However, we have studied different ant collections and we have found some interesting, previously unknown behaviors that may emerge under appropriate conditions.

### 3.1 Two Langton's ants

C. Langton briefly described some behaviours of a two ant system in [1]. We have tested several configurations and will report here about the two typical behaviors that were observed: periodic interactions and chaotic evolution. When the system enters a periodic attractor, patterns repeat themselves with a constant period. On the other hand, chaotic evolutions give rise to intricate structures that fill the whole available space if sufficient time is available and in which no regular space-time pattern can be recognized. It is to be noted that for the following simulations the grid space has cyclic boundary conditions i.e., it is toroidal.

After several attempts, we found that the following configuration gives rise to an interesting behavior of the two ants. On a 500 by 500 grid the first ant was placed at X=200 , Y= 250 heading south and the second one is on the same line at X=300, Y=250 heading east. We then observed the following stages in the system behavior:

1. Individual development: in this phase both ants leave their trails independently of each other and after 10,000 steps they build their highways separately.
2. Encounter: one of the ants meets the highway of the other and "walks" along it until it meets the other ant, which is still busy building its own highway. After that the behavior of the system is again complex.
3. Withdrawal: after meeting several times the ants undo their highways and come back to their starting points. During this phase, the ant that walked on the highway to meet its companion erases its trail in such a way that the second ant finds its own highway in the same state as it was when it built it.

Figure 7 shows the construction phase and the withdrawal. However, these static pictures do not do justice of the nice dynamics of the system: our computer simulations are much more explicit and easy to follow.

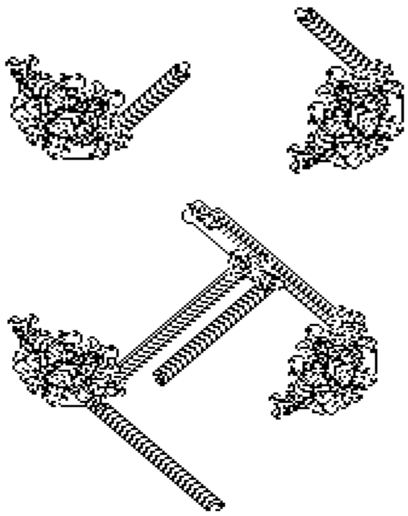

Figure 7 : Individual development and withdrawal.

Once the two ants are again at their starting points they begin to produce their traces followed by the highways but the pattern is rotated by 180 degrees with respect to the original one since they came back to the original points with their directions changed. Normally the traces should not meet since they now diverge. However, due to the cyclic structure of the grid space they go through the same phases 1-3 as described in section 3.1. At the end of this second cycle the ants find themselves again in the same position and with the same orientations as in step 0. From there on the system evolves periodically going through the same states. The pattern period is thus one complete cycle whereas the true period is two cycles. Therefore, this behaviour seems to contradict Kong-Cohen theorem since the trajectory will stay bounded in the case of a periodic multi-ant system.

We have tried a number of cases with the two ants initially on the same line. As a general rule, we have observed that if the ants directions are initially shifted by 90 degrees and the difference of their X coordinates is even (a distance of two suffices) then the interaction will be periodic, otherwise if the difference is odd it will be chaotic. If the ants are close enough to each other on the line they might meet before they start building their respective highways. However, if the original separation is even the system still shows a periodic behavior but without the highways. When the behaviour of the ant pair is chaotic there is no “highway following” behavior on the part of one of the ants. Instead, the ant goes through the highway undisturbed, after having left a complicated trail mark on it.

At least another non-collinear initial configuration of the two ants among those that have been tried has given rise to a periodic interaction in our simulations. The starting points are at X=99 , Y= 192 (heading north) and X=401, Y=217 (heading north).

## 3.2 Four Langton’s ants

We did several simulations starting from different configurations of the ants. Here we will describe only one particular four ants configuration which, under suitable conditions, gives rise to an amazing periodic pattern development. The ants are placed initially at the following points on the grid: X=200 , Y= 200 (ant1, heading south), X=300, Y=200 (ant2, heading east), X=300, Y=300 (ant3, heading west) and X=300, Y=300 (ant 4, heading north). Fig. 8 shows the periodic pattern that emerges after 28,000 time steps. Again, the visual effect is much more interesting when watching the simulation running than on a static picture.

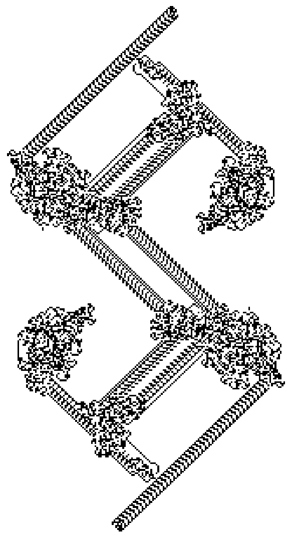

Figure 8 : Periodic interactions of four ants

For this periodic behavior to manifest itself it is necessary that some conditions be satisfied. First, the ants are updated sequentially one after the other in a round-robin manner as described in section 3.1. Second, the order in which the update takes place matters. Assuming that we always start with ant 1, there are six possibilities. We found experimentally that successively updating the ants in the order ant 1-ant 2-ant 3-ant 4 does not produce a periodic behavior. There are some transient quasi-periodic patterns but they disappear and afterwards the behavior is unstructured. In fact, all the combinations give rise to non-periodic behavior except those that imply a crossing between the ants as they are placed in the grid (see fig. 9).

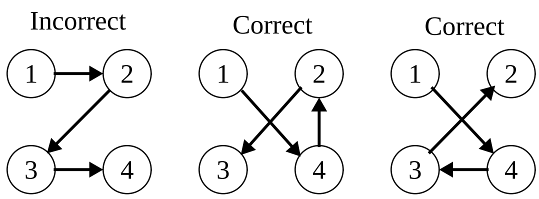


Figure 9 : Updating orders for the four ants system.

### 3.3 Two (1 0 1 0) ants

Single ants 10 (1 0 1 0) have already been seen as behaving identically to Langton's (1 0) ants in section 2.1. It is thus interesting to see if this similarity is maintained in the case of the joint evolution of two ants. It turns out that the behavior is indeed the same under the same initial conditions, with the emergence of an identical periodical pattern. The only difference is that the period of the system is four phases (see section 3.1) instead of two. Since four is the length of the rule string in the present case, it seems that there is a direct relationship between the period and the rule string length. This indication has been indeed confirmed by simulating two (1 0 1 0 1 0) ants. Fig. 10 shows a stage of the highway undoing.

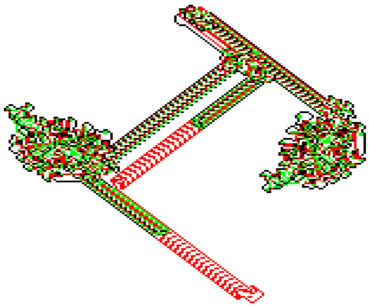

Figure 10 : Two (1 0 1 0) ants: pattern construction and withdrawal.

The collective behaviors described in this section have all been observed by imposing a sequential update of the virtual ants. It would be natural to ask whether the same or similar patterns emerge when updating all the ants in parallel. We did not have time to explore this research avenue a great deal. However, we have recently come across the work of B.Chopard [6] who carried out simulations of a two-ant system using a parallel updating rule. Under some well-defined initial conditions of the vant's position and orientation, he also reported periodic behaviour. He attributed the phenomenon to the crossing of ant trajectories and to the reversible character of the rule.

## 4 Conclusions

This experimental study of virtual ants behavior has shown that sometimes a slight rule modification may give rise to completely different dynamical patterns. This is a general phenomenon that has been observed in many systems where the global dynamics arise as a collective effect of local interactions. Indeed, in spite of the apparent simplicity of the ant automata, very little is known about their detailed dynamics and long-term behavior, computer simulations being at present the only practical way to study these ants. We have tried to classify the behavior of single generalized ants up to rule string length four according to highway building capabilities and the nature of the generated patterns, i.e. whether they possess some long-lasting symmetry or are chaotic. This has allowed to pinpoint some regularity in the general behavior of the ants.

Collections of virtual ants are even more interesting since they interact indirectly through their traces and give rise to puzzling emergent phenomena. We have studied in particular two and four-ants collections. We have empirically been able to show that under some well-specified conditions these ant collections present complicated cyclic behaviors. It has also been shown that these regular patterns are very fragile as they disappear, giving way to unstructured motifs, if some parameters of the simulations are slightly modified. Pursuing the artificial insect analogy, this can be seen as a manifestation of Wilson's *multiplication effect* whereby even a small change in the individual's behavior may cause large social effects at the level of the collectivity.

In spite of the large number of simulations performed, we have only scratched the surface of the complex world of multiple generalized ants and more simulation work would be needed to uncover new phenomena and to discover possible hidden regularities. However, besides more simulations, firmer mathematical basis are required to guide the search and to further advance our knowledge of these automata.

The present work has been made possible through the construction of a software tool with simple but effective graphical capabilities. All the pictures shown here have been obtained with this program. The program will be made available to all those interested.

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