Portfolio Adjustment on Jump Risk: Evidences from Asian Emerging Markets

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Abstract

This study investigates the measurement of investment weight adjustment on jump risk of five Asian emerging markets. Considering the risk of rare event, we develop an adjusted jump-diffusion measurement model for dynamic asset allocation in an international diversified portfolio. This paper endeavors to find a solution to the optimal weight on jump risks that prior research has stuck on an unknown expectation item. By simulating the price process and constructing multiple dimension jump diffusion on weight adjustment, a covariance of international assets investment is found. Then, we show how to diversify the jumps risk by using international assets portfolios.

Keywords: Rare events, dynamic asset allocation, international diversified portfolio, jump risk.
1. Introduction

The emergence of developing countries in East Asia is a main characteristic of worldwide economic change in the last 30 years. The reason is that Asian emerging markets have shown strong economic growth patterns in the world's economic powers. Even though economic developments in developed countries have decreased since the 1990s, many countries in East Asia have maintained high economic growth and development. According to a report by World Bank (2006), in 1996, the economic growth rate of east Asian countries was 8.1%, while on average the rest of the world was 3.2%, and the average of OECD countries was 2.9%. Moreover, the global economic growth rate between 1995~2004 was 3.3%, the OECD countries was 2.8%, and the East Asian countries was 7.7%.

Since recently, many companies in developed countries have expanded operations into emerging markets; investors are now willing to look for investment opportunities in Asia. Jumps are clearly of importance for portfolio weight adjustment and risk management. Rare events in the market also cause the price of an asset to jump. With respect to the above prospects, attention has been given to the growth prospects of these five emerging Asian markets for different reasons. Even though these markets have quite different economic histories, all of them have shown high growth rates and have the potential to continue these trends.

The centre of economic prosperity in the world has turned to the East. The U.K. and Canada fell off the top seven as calculated by PPP method and were replaced by two Asian countries—China and India. Specifically in 2004, the total GDP of China, Japan and South Korea exceeded U.S. $7 trillion, which accounted for 1/5 of the world economy. During the same period, the trade amount of the Asian-Pacific region climbed up to $15 billion, whereby China and India accounted for $3 billion among them. Exports, as a share gross domestic product, have doubled to 30 percent in 2000. After joining the World Trade Organization (WTO) in 2001, China has been required to liberalize its trade and financial services sector. Comparatively, India’s growth is largely driven by domestic consumption which is similar to that of many advanced economies of the West. India’s vast educated workforce helps its industries such as retail, finance, construction and hospitality to benefit from the buoyant domestic demand.

South Korea and Taiwan belonged to the group of “East Asian Tigers”. These economies were frequently noted for high economic growth rates and rapid
industrialization between the 1960s and 1990s. After experiencing the impact of the Asian financial crisis in 1997, South Korea rose up to become the 9th largest economy, while Taiwan remained as the 23rd largest economy in the world. However, the criticism of these economies focuses on the structural weaknesses in the regulatory framework.

Thailand has maintained an export-dependent economy position, where exports accounted for 60% of GDP over the past several years. This keeps Thailand as the second largest economy in Southeast Asia. Thailand’s recovery from the Asian financial crisis in 1997 relied on exports, largely from external demands, namely from the U.S. Since then, Thailand’s economic policy combines domestic stimulus with the country’s traditional promotion of open markets and foreign investment. Even though weak export demand held their GDP growth to 1.9% in 2001, domestic stimulus and export revival fuelled a better performance, with real GDP growth at 5.3% and 6.3% in 2002 and 2003, respectively.

There is a tendency to invest in Asian emerging countries. The investor may be interested in optimal portfolio weights among Asian countries. This study selects those five emerging Asian markets’ stock indexes (as previous mentioned, China, India, South Korea, Taiwan, and Thailand) into the portfolio. Specifically, the monthly returns on stock indexes of these five emerging Asian countries are conducted.

This paper aims to establish an efficient and acceptable method for the weight changes of investment caused by jump risk in the emerging markets. Therefore, we investigate the fact that rare event risk jump have strong effects and excess returns in the emerging stock market. The method of dynamic asset allocation is widely applied in deciding the optimal portfolio weights (see, Merton, 1990). However, there are still many problems still need to be solved using this method, particularly for event-driven uncertainty, evade intensities, and jump sizes. Therefore, the observations are difficult to measure, assess and adjust in a portfolio. Moreover, one of the obstacles in the prior studies of jump diffusion portfolio model is that there exists an expectation in the solution of the optimal portfolio weight (e.g., Das and Uppal, 2003; Liu et al., 2003; Runggaldier, 2002; and Wu, 2003). Additionally, previous research was hindered by the covariance between each two assets and covariance between each two assets’ jumps (e.g., Aït-Sahalia, 2004; Bentzen and Sellin, 2003). Therefore, this study develops a method to bridge research gaps.

We arranged this paper in other sections as follows: in section 2, the basic model
for the solution has been found for the optimal portfolio weight. Section 3 calibrates
an empirical evidence for the emerging markets. We analyze the result of investment
weight adjustment in section 4, and in Section 5, we arrive at a conclusion.

2. Optimal Portfolio Model

This study sets a model to find the optimal investment weight in five emerging Asian
markets. There are two kinds of asset including a riskless asset bond, \( B \), and risky
asset \( S \). The return of bond follows the process

\[
\frac{dB}{B} = r dt , \quad \text{.....................................................(1)}
\]

where \( B \) represents a riskless asset bond with return \( r \).

There are five risky assets whose prices \( S = [S_1, S_2, ..., S_5]' \) are subject to
event-related jumps. In continuous time finance, a popular way to generate
discontinuity is to apply the compound Poisson Jump Model. Therefore, the returns of
five risky assets, \( \frac{dS_i}{S_i}, i = 1, ..., 5 \), follow a jump-diffusion process using the following
formula:

\[
\frac{dS}{S} = (\mu - r - \lambda \theta) dt + \sqrt{V} dZ + X dq \quad \text{.................................(2)}
\]

where \( \frac{dS}{S} = [\frac{dS_1}{S_1}, \frac{dS_2}{S_2}, ..., \frac{dS_5}{S_5}]' \) is the returns matrix of the five risky assets,
\( \mu = [\mu_1, \mu_2, ..., \mu_5]' \) is the vector of the mean return of five risky assets and
\( r = [r_1, ..., r_5]' \) is the vector of riskless interest rate. \( \theta = [\theta_1, \theta_2, ..., \theta_5]' \) is the mean of
jumps size of the five risky assets. The term \( \lambda = \begin{bmatrix} \lambda_1 & 0 \\ \vdots & \ddots \\ 0 & \lambda_5 \end{bmatrix} \) captures the mean
percentage jump in the asset price conditional on jump occurrence and different jump
arrivals. The probability from a Poisson process with stochastic arrival intensity \( \lambda_i \)
is \( \text{Pr}(dq_i = 1) = \lambda_i dt \) and \( E[dq_i] = \lambda_i, \quad i = 1, ..., 5 \) and the matrix of the jumps arrival
index is \( Q = \begin{bmatrix} dq_1 & 0 \\ \vdots & \ddots \\ 0 & dq_5 \end{bmatrix} \). The event-related jumps size of five risky assets is
denoted by vector \( X = [X_1, X_2, ..., X_5]' \).

Therefore,
\[ V = \sigma' \rho \sigma \]
\[
\begin{bmatrix}
\sigma_1 & 0 & \cdots & 0 \\
0 & \sigma_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_5
\end{bmatrix}
\begin{bmatrix}
1 & \rho_{1,2} & \cdots & \rho_{1,5} \\
\rho_{2,1} & 1 & \cdots & \rho_{2,5} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{5,1} & \rho_{5,2} & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
\sigma_1 & 0 & \cdots & 0 \\
0 & \sigma_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_5
\end{bmatrix}
\]
stands for the 5×5 variance-covariance matrix of 5 risky assets which standard deviation is denoted by \( \sigma \). The correlation between asset i and asset j is indicated by \( \rho_{ij} \) and the vector of standard Brownian motion of five risky assets is defined as
\[ dZ = [dZ_1, dZ_2, \ldots, dZ_5]' \sim N(0, I dt) . \]

Subsequently, this study turns to a portfolio decision by applying the Taylor expansion to the Euler equation and approximating the optimal portfolio choice. Maximizing the expected utility of the terminal wealth \( W_T \), i.e. \( \max_{\Phi} E[U(W_T)] \), and the return of wealth process satisfy the self-financing condition. This study arrives at the dynamic wealth process equation:
\[ \frac{dW}{W} = \phi_0 \frac{dB}{B} + \Phi \frac{dS}{S} \] .................................

where \( \phi_0 \) represents the weight on risk-free asset, \( \Phi = [\phi_1, \phi_2, \ldots, \phi_5]' \) is a vector of a portfolio which indicates the investor’s portfolio choice among the available investment opportunities.

In searching for the optimal portfolio strategy, this study adopts the standard stochastic control approach and assumes that the jump arrival probability is constant. The principle of optimal stochastic control leads to the following Hamilton-Jacobi-Bellman (HJB) equation by Merton (1990, Chapter 5) for the indirect utility function \( J \):
\[
\max_{\phi} \left\{ \frac{\partial J}{\partial t} + \frac{\partial J}{\partial W} E(dW) + \frac{1}{2} \frac{\partial^2 J}{\partial W^2} Var(dW) + E[J(W(1+\Phi'QX),t)] - J(W,t) \right\} = 0 \] .

This study searches for the optimal portfolio strategy \( \Phi^* \) by first conjecturing that the indirect utility function is of the form
\[ J(W,t) = \frac{1}{1-\gamma} W^{1-\gamma} \exp(A(t)) , \]
where \( \gamma > 0, \, \gamma \neq 1 \) and \( A(t) \) is a function of time but not of the state variables \( W \). Given this function form, this study takes derivatives of \( J(W,t) \) with respect to its arguments, \( J_w = W^{-\gamma} e^{A} , \, J_{ww} = -\gamma W^{-\gamma-1} e^{A} \), substituting it into the HJB equation in
Equation (4). Subsequently we arrive at

\[
\max_{\Phi} \{ J_t + W^{-\gamma} e^{\gamma t} [r + \Phi'(\mu - r - \lambda \theta)] + \frac{1}{2} (-\gamma W^{-\gamma} e^{\gamma t}) e^{\gamma t} \Phi V \Phi \} dt + E[\frac{1}{1-\gamma} (W(1 + \Phi'QX)^{-\gamma} e^{\gamma t} - \frac{1}{1-\gamma} W^{-\gamma} e^{\gamma t})] = 0 \ldots \ldots (5)
\]

We differentiate Equation (5) with respect to the portfolio weight of the risky asset, \( \Phi \), and divide by \( W^{-\gamma} e^{\gamma t} dt \) to obtain the following first-order condition:

\[
(\mu - r - \lambda \theta) - \gamma V \Phi + E[(1 + \Phi''QX)^{-\gamma} QX] = 0 \ldots \ldots \ldots \ldots \ldots (6)
\]

From Equation (6) the optimal portfolio weight can be expressed by

\[
\Phi^* = V^{-\gamma} \left[ \frac{\mu - r - \lambda \theta}{\gamma} + \frac{E[(1 + \Phi''QX)^{-\gamma} QX]}{\gamma} \right] \ldots \ldots \ldots \ldots \ldots (7)
\]

for each \( \phi^*_i \in \Phi^* \).

The traditional solution for the optimal portfolio weight is inadequate because there is an unknown expectation value in Equation (7) as an implicit function (Das and Uppal, 2003; Liu et al., 2003; and Wu, 2003). In general solving for the unclosed form after setting the jump size distribution, traditional research usually applies a numerical method (i.e. simulation) to find the value of optimal portfolio weight, \( \Phi^* \).

Here, in the solution for optimal portfolio weight in Equation (7), we place little attention to the joint distribution to the jump size \( X \). For the expectation \( E[(1 + \Phi''QX)^{-\gamma} QX] \), if each element in \( \Phi''QX \) is small (i.e. \( \phi^* < 1, X < 1 \)), then it follows that:

\[
E[(1 + \Phi''QX)^{-\gamma} QX] \approx E[e^{-\gamma \Phi''QX}. QX] = (\lambda \theta - \gamma \Phi'' \lambda \Omega \lambda \Phi^*) \exp[-\gamma \Phi'' \lambda \Omega + \frac{\gamma^2}{2} \Phi'' \lambda \Omega \lambda \Phi^*] \ldots (8)
\]

The 5×5 variance-covariance matrix of jumps size of 5 risky assets can be expressed by

\[
\Omega = \begin{bmatrix}
\delta_1 & 1 & \rho_{1,2} & \ldots & \rho_{1,5} \\
0 & \delta_2 & 1 & \ldots & \rho_{2,5} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \ldots & \delta_5 & 1 \\
0 & \rho_{3,1} & \rho_{3,2} & \ldots & 1 \\
0 & 0 & \ldots & \delta_5 & 1 \\
\end{bmatrix}
\]

Substitution in Equation (8), gives the following expression for the optimal portfolio weight
\[ \Phi^* = \mathbf{V}^{-1}\left[ \frac{\mu - \mathbf{r} - \lambda^0}{\gamma} + \frac{\lambda^0}{\gamma} \mathbf{\Omega} \Phi^* \right] \exp\left[ -\gamma \Phi^0 \mathbf{0} + \frac{\gamma^2}{2} \Phi^0 \mathbf{\Omega} \Phi^* \right] \] ....(9)

Thus, this study finds a closed form solution \( \Phi^* = f(\mu, \mathbf{V}, \lambda^0, \mathbf{\Omega}, \gamma, \mathbf{r}) \) for the optimal portfolio weight of 5 risky assets as shown in Equation (9) which improves the earlier work in Equation (7).

3. Empirical Results

For the purpose of simplicity, this study investigates investment in risky and riskless assets without any transaction costs and restrictions. The monthly data was retrieved from Morgan Stanley Capital International Inc. (MSCI) database using the sample period from January 1993 to November 2006. Table 1 summarizes the statistical properties of the stock returns for the five emerging Asian markets and the world market without considering event risk jump.

### Table 1. Statistics of stock return for five emerging Asian markets and the world

<table>
<thead>
<tr>
<th>Market Indices</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Max</th>
<th>Min</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Bera-Jarque</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>0.0231</td>
<td>1.3147</td>
<td>0.4650</td>
<td>-0.2767</td>
<td>0.7928</td>
<td>5.7531</td>
<td>246.32</td>
</tr>
<tr>
<td>India</td>
<td>0.1403</td>
<td>0.9915</td>
<td>0.2200</td>
<td>-0.1774</td>
<td>-0.0136</td>
<td>2.5797</td>
<td>46.03</td>
</tr>
<tr>
<td>South Korea</td>
<td>0.1597</td>
<td>1.4530</td>
<td>0.7059</td>
<td>-0.3126</td>
<td>1.2629</td>
<td>9.3175</td>
<td>644.60</td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.0986</td>
<td>1.1490</td>
<td>0.4644</td>
<td>-0.2187</td>
<td>1.0155</td>
<td>6.0433</td>
<td>281.14</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.0474</td>
<td>1.4977</td>
<td>0.4318</td>
<td>-0.3405</td>
<td>0.4279</td>
<td>4.7172</td>
<td>158.98</td>
</tr>
</tbody>
</table>

Note: Sample period for MSCI index return monthly data is retrieved from January 1993 to December 2006. The stock indices are retrieved from the world index in MSCI.

From Table 1 we read that the five emerging Asian markets all have positive mean returns during that period. Compared with a normal distribution, we have an empirical distribution with a negative skewness in world index return and Indian stock index return. The kurtosis measures indicate that the return distribution peaked more (or fat tails) than the normal, China of 5.7889, Taiwan of 6.0788 and South Korea of 9.3746. These abnormal protruding curves have a significant event jump of stock returns.

Simple bivariate correlations on stock indexes between the five emerging Asian markets are presented in Table 2. In general, there are positive and low correlations between each of the two countries in the five emerging Asian markets. The mean returns correlation especially between Thailand and South Korea of 0.5694 and
China and Taiwan of 0.5086 are positively significant.

Table 2. Bivariate correlations between five emerging Asian markets

<table>
<thead>
<tr>
<th>Indices</th>
<th>China</th>
<th>India</th>
<th>South Korea</th>
<th>Taiwan</th>
<th>Thailand</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>1.0000</td>
<td>0.2794</td>
<td>0.2412</td>
<td>0.5086</td>
<td>0.4897</td>
</tr>
<tr>
<td>India</td>
<td>0.2794</td>
<td>1.0000</td>
<td>0.2413</td>
<td>0.3233</td>
<td>0.22597</td>
</tr>
<tr>
<td>South Korea</td>
<td>0.2412</td>
<td>0.2413</td>
<td>1.0000</td>
<td>0.3620</td>
<td>0.56941</td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.5086</td>
<td>0.3233</td>
<td>0.3620</td>
<td>1.0000</td>
<td>0.47454</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.4897</td>
<td>0.2260</td>
<td>0.5694</td>
<td>0.4745</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Note: Simple bivariate correlations between five emerging Asian markets are presented.

With respect to verifying the jumps that exists in the emerging markets, this study observes the estimation of weight $\Phi^i$ by the endogenous variables $\mu, V, \lambda, \theta, \Omega$. A popular method for jump risk parameters estimation is developed by Press (1967) and Beckers (1981), who employ a version of the method of moments known as the cumulant matching method. In their specification, the mean jump amplitudes and the remaining parameters, $\mu_i, \theta_i, \sigma_i, \delta_i$ ($i=1,2,\ldots,5$) and $\lambda$ are endogenous. Therefore,

$$\ln \frac{S(T)}{S(t)} \sim \sum_{i=1}^\infty \frac{e^{-\lambda \tau}}{i!} N\left(\mu \tau - i \theta, \sigma^2 \tau + i \delta^2\right),$$

where $\tau = T-t$, the log return distribution is described as a Poisson mixture of normal distribution. Using the relationship between cumulants and moments (Kendall and Stuart, 1963), it can be proved that the log return distribution is leptokurtic and therefore might be better to describe the actual stock price return than the pure lognormal model. The formulae for the solution of parameters, $\mu_i, \theta_i, \sigma_i, \delta_i$ ($i=1,2,\ldots,5$) and $\lambda$ in Press (1967) and Beckers (1981).

However, the formulae for estimating the parameters demand unreasonable assumptions of mean return zero or jump mean return zero, either in Press (1967) or Beckers (1981). The alternative method for estimation in jump risk portfolio model is the Maximum Likelihood Estimates (MLE). This study follows the process in Bentzen and Sellin (2003) that investigates jump risk in the market portfolio with different types of log-likelihood function.

If there are no jumps in each country, the log-likelihood function can be written as
\[ \ln L_d = -\frac{T}{2} \ln(2\pi\sigma^2 h) - \sum_{t=1}^{T} \frac{(\ln s_t - \mu h)^2}{2\sigma^2 h}, \]

where \( T \) is the number of observations, \( h \) is the increment of time between observations and \( s_t = \frac{S_t}{S_{t-1}} \).

Otherwise, if there are jumps, the log-likelihood function in each country can be written as follows:

\[ \ln L_n = -\frac{T}{2} \ln(2\pi\lambda h) + \sum_{t=1}^{T} \ln \left[ \sum_{j=0}^{\infty} \frac{(\lambda h)^j}{j!} \exp \left( -\frac{(\ln s_t - \mu h - \theta j)^2}{2(\sigma^2 + \delta^2 j)} \right) \right]. \]

The likelihood ratio statistic, \( LR = -2(\ln L_d - \ln L_n) \), is distributed asymptotically \( \chi^2 \) with the degree of freedom 3. This paper follows Bentzen and Sellin (2003) which selects the value of \( j \) from zero to ten. The estimates are given for the combined diffusion and jump process. A simple likely ratio test of the jump parameter indicates that for some of the daily sample periods a statistically significant jump component exists. This is confirmed by the likelihood ratio test. All of the daily sample periods reject the null hypothesis of a continuous sample path process at a 99% significance level. The significance levels for this distribution are given the probability 0.5, 0.9, 0.95 and 0.99 of the null rejecting hypothesis for the \( \chi^2 \) value 2.37, 6.25, 7.81, and 11.35, respectively.

Thus, this study uses the log-likelihood function for the estimation of the mixed Poisson jump diffusion model and the simple Wiener process model of monthly returns to the stock market indices of the five emerging Asian markets. A likelihood ratio test is available to be used to test the hypothesis that jump risk exists. By investigating the monthly data under the condition of event instability, this study presents the parameter estimates for jump-diffusion model of emerging markets as estimated by MLE. The jump frequencies estimation of the Indian return shows a low value, 0.0230. This result implies that there is near no jump arrival in India in Table 3. The estimated standard deviation of stock return jump size with zero, induces the rejection to the null hypothesis of existing jumps on India return significantly.

It is found that the expected values of monthly stock returns of five emerging markets are all positive when not considering the effect of jump diffusion. However, the expected value of monthly stock returns of China, Taiwan, and Thailand become negative if the effect of jump diffusion is considered.
Table 3. Estimates of monthly returns of the five emerging Asian stock markets by MLE method

<table>
<thead>
<tr>
<th>Indices</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\sigma}$</th>
<th>$\hat{\lambda}$</th>
<th>$\hat{\theta}$</th>
<th>$\hat{\delta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>-0.0741</td>
<td>0.1047</td>
<td>0.4028</td>
<td>0.0000</td>
<td>0.2415</td>
</tr>
<tr>
<td></td>
<td>(0.0957)</td>
<td>(0.0132)</td>
<td>(0.1009)</td>
<td>(0.0315)</td>
<td>(0.3281)</td>
</tr>
<tr>
<td>India</td>
<td>0.1403</td>
<td>0.0814</td>
<td>0.0230</td>
<td>0.0042</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>(0.1639)</td>
<td>(0.0289)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
<td>(7.1583**)</td>
</tr>
<tr>
<td>South Korea</td>
<td>0.0027</td>
<td>0.0676</td>
<td>0.0521</td>
<td>0.0312</td>
<td>3.0096</td>
</tr>
<tr>
<td></td>
<td>(0.0986)</td>
<td>(0.0148)</td>
<td>(0.0382)</td>
<td>(0.0155)</td>
<td>(1.5104)</td>
</tr>
<tr>
<td>Taiwan</td>
<td>-0.0525</td>
<td>0.0712</td>
<td>0.1906</td>
<td>0.0125</td>
<td>0.7926</td>
</tr>
<tr>
<td></td>
<td>(0.1208)</td>
<td>(0.0167)</td>
<td>(0.5493)</td>
<td>(0.0604)</td>
<td>(2.7398**)</td>
</tr>
<tr>
<td>Thailand</td>
<td>-0.1541</td>
<td>0.1223</td>
<td>0.3314</td>
<td>0.0000</td>
<td>0.6079</td>
</tr>
<tr>
<td></td>
<td>(0.1474)</td>
<td>(0.0160)</td>
<td>(0.1232)</td>
<td>(0.0273)</td>
<td>(0.8049)</td>
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</table>

<table>
<thead>
<tr>
<th>Indices</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\sigma}$</th>
<th>L. R. test</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>0.0231</td>
<td>0.1431</td>
<td>21.8840</td>
</tr>
<tr>
<td></td>
<td>(0.1089)</td>
<td>(0.0109)</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>0.14035</td>
<td>0.0814</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>(0.0765)</td>
<td>(0.0101)</td>
<td></td>
</tr>
<tr>
<td>South Korea</td>
<td>0.1597</td>
<td>0.1749</td>
<td>36.7240</td>
</tr>
<tr>
<td></td>
<td>(0.1247)</td>
<td>(0.0104)</td>
<td></td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.0986</td>
<td>0.1094</td>
<td>21.7550</td>
</tr>
<tr>
<td></td>
<td>(0.0994)</td>
<td>(0.0085)</td>
<td></td>
</tr>
<tr>
<td>Thailand</td>
<td>0.0474</td>
<td>0.1858</td>
<td>15.1340</td>
</tr>
<tr>
<td></td>
<td>(0.1185)</td>
<td>(0.0153)</td>
<td></td>
</tr>
</tbody>
</table>

Note: $\hat{\mu}$ is the estimated mean of stock return; $\hat{\sigma}$ is the estimated standard deviation of stock return; and, a Poisson process with stochastic estimated arrival intensity $\hat{\lambda}$. A random percentage jump size with estimated mean $\hat{\theta}$, and $\hat{\delta}$ is the estimated standard deviation of random percentage jump size of stock return. Standard errors are reported in the parenthesis below the point estimates. The last column indicates the likelihood ratio test for the hypothesis that there are jumps versus the alternative hypothesis of no jumps. The significance levels for this distribution are given the probability 0.5, 0.9, 0.95 and 0.99 of the null rejecting hypothesis for the $\chi^2$ value 2.37, 6.25, 7.81, and 11.35, respectively. The * stands for 0.5 and ** stands for 0.9 of the probability significance levels, respectively.
In previous work, the optimal portfolio weight cannot be found due to an unknown expected value in the solution of portfolio weight and the covariance of pure diffusion and covariances of jump diffusion unidentified. In this paper, we attempt to solve the expectation term by joint probability density function and the investigation of the covariance between each two of the five emerging Asian markets through the approach of 1000 times Monte Carlo Simulation with jump diffusion and pure diffusion.

Table 4 shows that considering no jump versus jump produces the change of covariance correlation from negative to positive in the stock return of China--India and South Korea--India. Moreover, the stock index return with considering jump of Taiwan has a negative covariance with those of India, South Korea and Thailand markets. The monthly return with considering jump of the Thailand stock market has a negative covariance with the rest of the emerging markets. However, if jump-diffusion is not considered, China stock return has merely a positive covariance with South Korea.

**Table 4. Estimates of covariance correlations between two emerging Asian markets by simulation**

<table>
<thead>
<tr>
<th>Indices</th>
<th>China</th>
<th>India</th>
<th>South Korea</th>
<th>Taiwan</th>
<th>Thailand</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: with jump-diffusion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>1</td>
<td>0.00405</td>
<td>0.00049</td>
<td>0.00134</td>
<td>-0.00001</td>
</tr>
<tr>
<td>India</td>
<td>0.00405</td>
<td>1</td>
<td>0.00078</td>
<td>-0.00004</td>
<td>-0.00470</td>
</tr>
<tr>
<td>South Korea</td>
<td>0.00049</td>
<td>0.00078</td>
<td>1</td>
<td>-0.00537</td>
<td>-0.00232</td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.00134</td>
<td>-0.00004</td>
<td>-0.00537</td>
<td>1</td>
<td>-0.00031</td>
</tr>
<tr>
<td>Thailand</td>
<td>-0.00001</td>
<td>-0.00470</td>
<td>-0.00232</td>
<td>-0.00031</td>
<td>1</td>
</tr>
<tr>
<td><strong>Panel B: without jump-diffusion (pure diffusion)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>1</td>
<td>-0.00234</td>
<td>0.00254</td>
<td>-0.00089</td>
<td>-0.00050</td>
</tr>
<tr>
<td>India</td>
<td>-0.00234</td>
<td>1</td>
<td>-0.00131</td>
<td>-0.00003</td>
<td>-0.00428</td>
</tr>
<tr>
<td>South Korea</td>
<td>0.00254</td>
<td>-0.00131</td>
<td>1</td>
<td>0.00416</td>
<td>0.00075</td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.00089</td>
<td>-0.00003</td>
<td>0.00416</td>
<td>1</td>
<td>0.00141</td>
</tr>
<tr>
<td>Thailand</td>
<td>-0.00050</td>
<td>-0.00428</td>
<td>0.00075</td>
<td>0.00141</td>
<td>1</td>
</tr>
</tbody>
</table>
This study finds that investment weight adjustment to portfolio fraction in India has been reduced when the jump risk is underestimated. The portfolio weight on the South Korea index return has a 0.0101 increase which is the only positive effect in considering jumps among the five markets in Table 5.

Table 5. Investment portfolio weight adjustment of the five emerging Asian markets

<table>
<thead>
<tr>
<th>Asia Region indices</th>
<th>Weights with no jump $\phi$</th>
<th>Weights with jumps $\phi$</th>
<th>Weights adjustment $\phi - \overline{\phi}$</th>
<th>Effect on Jump size $\phi - \overline{\phi}\theta$</th>
<th>Effect on Jump risk $\phi - \overline{\phi}\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>-0.8406</td>
<td>-0.9028</td>
<td>-0.0622</td>
<td>-0.0065</td>
<td>-0.0000</td>
</tr>
<tr>
<td>India</td>
<td>2.6233</td>
<td>2.5260</td>
<td>-0.0974</td>
<td>-0.0079</td>
<td>-0.0064</td>
</tr>
<tr>
<td>South Korea</td>
<td>1.6531</td>
<td>1.6632</td>
<td>0.0101</td>
<td>0.0007</td>
<td>0.0018</td>
</tr>
<tr>
<td>Taiwan</td>
<td>1.0919</td>
<td>0.8426</td>
<td>-0.2493</td>
<td>-0.0177</td>
<td>-0.0279</td>
</tr>
<tr>
<td>Thailand</td>
<td>-1.0596</td>
<td>-1.2105</td>
<td>-0.1509</td>
<td>-0.0185</td>
<td>-0.0000</td>
</tr>
</tbody>
</table>

Note: the positive value of $\phi - \overline{\phi}$ represents the portfolio weights increased and negative value represents the portfolio weights decreased.

Table 5 shows that South Korea stock markets have a positive effect on mean return by jump. Inversely, the stock markets in China, India, Taiwan and Thailand show a negative jump effect on both jump size and jump risk.

4. Conclusion

In previous work, the optimal portfolio weight cannot be found due to two reasons: (1) there is still an unknown expected value in the final solution of portfolio weight; and, (2) the covariance of pure diffusion and covariances of jump diffusion are not identified. Here, this paper attempts to overcome the existence of an expectation value in solving the optimal portfolio issue. It also develops the weight adjustment on rare event jump for investment decisions. Moreover, this study investigates the covariance between each two from five emerging markets through the approach of 2000 times Monte Carlo Simulation and obtains a reasonable value of covariance for jump size.

The empirical results conclude that whenever a jump appears, the investors reduce their equities weight in emerging markets. The results reaffirm that hedging by adjusting the portfolio weight should be encouraged to reduce the impacts caused
by rare event jumps.

References


Merton, R. C., 1990, Continuous-Time Finance, Blackwell Cambridge MA &
Oxford UK. Ch. 5 and 6. (p.147 and p.204).


