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Implementing the maximum likelihood methodology to measure a driver's critical gap

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Abstract

Most of the capacity calculation procedures for two-way stop-controlled (TWSC) intersections are based on gap acceptance models. Critical gap is one of the major parameters for gap acceptance models. The accuracy of capacity estimation is mainly determined by the accuracy of the critical gap. This paper focuses on the implementation of the maximum likelihood technique to measure a driver's critical gap using field data. A methodology to define gap events is proposed, so that the accepted gaps and maximum rejected gaps required by the maximum likelihood technique could be obtained. Specific issues regarding multi-lane situations and major street right turn movement are discussed. Special conditions observed during the research are addressed when the proposed method cannot be applied directly, such as the existence of a mid-block refuge area where minor street drivers can seek gaps in a two-stage process, pedestrian blockage, and downstream queue spill back. The proposed method was adopted in measuring critical gap under US conditions during a research project, described by Kyte et al. (1996). © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Most of the capacity calculation procedures for two-way stop-controlled (TWSC) intersections are based on gap-acceptance models. *Critical gap* is one of the major parameters for gap acceptance models. The accuracy of capacity estimation is mainly determined by the accuracy of the critical gap. Unfortunately, the critical gap cannot be measured directly in the field. There have

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been numerous studies and techniques developed for estimating the critical gap. Among these methods, the maximum likelihood technique proves to be the most accurate and reliable (see Troutbeck, 1992 and Brilon, 1995).

All of the available methods require similar information, such as accepted gaps and rejected gaps. The maximum likelihood method requires information about the accepted gap and the largest rejected gap for each driver. However, all the studies so far were based on one major stream and one minor stream. No study to date has addressed how to define gap events when several traffic streams are involved—that is, how to extract the accepted gap and rejected gap information from the field data. Without a clear definition of the gap events, it is impossible to implement the various techniques for measuring the critical gap using field data.

This paper focuses on the implementation of the maximum likelihood technique to measure critical gap using data collected in the field. A methodology to define gap events is proposed, so that the accepted gaps and the largest rejected gaps could be obtained. The measurement at multilane sites and the treatment of major-street right-turn movement are specifically discussed. The paper points out some unusual situations where the proposed method cannot be applied directly. The proposed methodology has been used by Kyte et al. (1996) to measure critical gaps under conditions in the US.

2. Theory of the maximum likelihood technique

The maximum likelihood method of estimating critical gap is based on the fact that a driver's critical gap is between the range of his largest rejected gap and his accepted gap. A probabilistic distribution for the critical gaps must be assumed. Troutbeck (1992) used a log-normal distribution for the critical gaps. This distribution is skewed to the right and has non-negative values, as would be expected in these circumstances. Brilon (1995) used a hyper-Erlang distribution. Similar results were reported between the two approaches. The background theory is discussed below; all derivations are based on Troutbeck (1992).

The following notations are used for subsequent equations:

y_i = the logarithm of the gap accepted by the ith driver
x_i = the logarithm of the largest gap rejected by the ith driver. $x_i = 0$ if no gap was rejected
μ = mean of the distribution of the logarithms of the individual driver's critical gaps
σ^2 = variance of the distribution of the logarithms of the individual driver's critical gaps
f( ) = probability density function for the normal distribution
F( ) = cumulative distribution function for the normal distribution

The maximum likelihood of a sample of n drivers having an accepted gap and a largest rejected gap of (y_i, x_i) is

$$\prod_{i=1}^{n}[\mathrm{F}(y_i) - \mathrm{F}(x_i)]$$

The logarithm, L, of the likelihood is then

$$L = \sum_{i=1}^{n} \ln\left[\mathrm{F}(y_i) - \mathrm{F}(x_i)\right] \tag{1}$$

The likelihood estimators μ and σ^2 that maximize L are the solutions to the two equations:

$$\frac{\partial L}{\partial \mu} = 0 \tag{2}$$

$$\frac{\partial L}{\partial \sigma^2} = 0 \tag{3}$$

That is, they are solutions of

$$\frac{\partial L}{\partial \mu} = \sum_{i=1}^{n} \frac{\frac{\partial \mathrm{F}(y_i)}{\partial \mu} - \frac{\partial \mathrm{F}(x_i)}{\partial \mu}}{\mathrm{F}(y_i) - \mathrm{F}(x_i)} = 0 \tag{4}$$

$$\frac{\partial L}{\partial \sigma^2} = \sum_{i=1}^{n} \frac{\frac{\partial \mathrm{F}(y_i)}{\partial \sigma^2} - \frac{\partial \mathrm{F}(x_i)}{\partial \sigma^2}}{\mathrm{F}(y_i) - \mathrm{F}(x_i)} = 0 \tag{5}$$

It can then be shown that

$$\frac{\partial \mathrm{F}(x)}{\partial \mu} = -\mathrm{f}(x) \tag{6}$$

$$\frac{\partial \mathrm{F}(x)}{\partial \sigma^2} = -\frac{x - \mu}{2\sigma^2}\mathrm{f}(x) \tag{7}$$

This then leads to the two equations that must be solved iteratively using numerical methods:

$$\sum_{i=1}^{n} \frac{\mathrm{f}(x_i) - \mathrm{f}(y_i)}{\mathrm{F}(y_i) - \mathrm{F}(x)} = 0 \tag{8}$$

$$\sum_{i=1}^{n} \frac{(x_i - \mu)\mathrm{f}(x_i) - (y_i - \mu)\mathrm{f}(y_i)}{\mathrm{F}(y_i) - \mathrm{F}(x_i)} = 0 \tag{9}$$

where $\mathrm{f}(x_i)$, $\mathrm{f}(y_i)$, $\mathrm{F}(x_i)$, and $\mathrm{F}(y_i)$ are also functions of μ and σ^2

A computer program was developed by Troutbeck to solve these equations (1992). The mean critical gap t_c and the variance s^2 can then be computed by:

$$t_c = e^{\mu + 0.5\sigma^2} \tag{10}$$

$$s^2 = t_c^2(e^{\sigma^2} - 1) \tag{11}$$

It is this mean critical gap that has been used in various gap acceptance capacity and delay models.

3. Definition of gap events

Gap events are the time events used to define the beginning and end of each major-stream gap. Another term used to describe the gap-acceptance process is *lag*. A lag is actually the first gap that faces the minor-stream driver. The size of a lag is measured from the time when the minor-stream driver arrives at the stop line until the next major-stream vehicle passes. For further discussions in this paper, gap events are also used to define lags.

When there are only two traffic streams at a TWSC intersection, the gaps perceived by the minor-stream drivers are clear. The passage times of the major-stream vehicles at the conflicting area can be used to define gap events. The size of a gap is determined by the duration between two gap events. However, under most conditions, there are usually several traffic streams of different priorities. The order of priority among different traffic streams is generally described as follows: major-street through and right-turn movement; major-street left-turn movement and minor-street right-turn movement; minor-street through movement; and minor-street left-turn movement. Drivers with lower priority have to seek gaps formed by other traffic streams with higher priorities. When the major-street approach has multiple lanes, minor-street drivers will only seek gaps among those traffic streams that have physical conflict potential with the minor-street drivers. For example, a minor-street right-turning vehicle may only seek gaps among vehicles travelling on the outermost lane. Gap events need to be defined differently for different traffic streams, and they should truly reflect drivers' gap-acceptance behavior. The following sections discuss the proposed method for defining gap events.

Gap events for a minor-stream are defined in the following manner: the passage time of any vehicle that conflicts directly with a subject vehicle can be defined as a *begin gap event*. A subject vehicle is a vehicle in the traffic stream of interest. If it is a "lag", the begin gap event is the time at which the subject vehicle arrives at the stop line. The end gap event must be the passage of a major-street vehicle of higher priority that conflicts with the subject vehicle. As a result, only the major-street through and right-turn vehicles can be defined as an end gap event for the major-street left-turn movement; whereas all of the major-street vehicles can be defined as the end gap event for the minor-street movements. The end gap event is defined as such because only the major-street vehicles of higher priority would affect the minor-street driver in any significant way. More specifically, what actually determines a driver's decision about whether to enter the intersection is when the next major-street vehicle will arrive at the intersection. Vehicles on the opposing minor street generally do not exhibit clear priority over the subject approach, no matter what kind of movement they are making. For example, a minor-street left-turning vehicle does not have to wait for a minor-street through vehicle on the other approach if there is an acceptable gap on the major street. Rather, drivers were observed to enter the intersection on a first-come, first-served basis. Three examples to illustrate the importance of correctly defining the key events are shown in Table 1.

Table 1
Examples to illustrate begin and end gap events for maximum likelihood method for critical gap

Example 1:
*The subject approach is southbound.

	PassTime	Movement	EnterQ	FirstQ	ExitQ
	00:10:50	EBTH			
	00:10:51	WBLT	00:10:51	00:10:51	00:10:51
	00:10:52	EBTH			
Begin Gap	00:10:54	NBTH	00:09:31	00:10:48	00:10:54
	00:10:55	SBLT	00:08:42	00:10:48.	00:10:54
End Gap	00:10:58	EBTH			
	00:10:59	WBTH			
	00:11:00	EBRT			
	00:11:02	EBRT			
	00:11:03	EBTH			

Example 2:
*The subject approach is northbound.

	PassTime	Movement	EnterQ	FirstQ	ExitQ
	00:10:50	EBTH			
	00:10:51	WBT11			
	00:10:51	WBLT	00:10:25	00:10:26	00:10:51
	00:10:52	WBTH			
Begin Gap	00:10:55	SBT11	00:09:31	00:10:30	00:10:54
	00:10:55	NBLT	00:08:42	00:10:24	00:10:54
	00:10:58	SBT11			
End Gap	00:10:58	EBTH			
	00:11:00	EBRT			
	00:11:02	EBRT			
	00:11:03	EBTH			

Example 3:
*The subject approach is northbound.

	PassTime	Movement	EnterQ	FirstQ	ExitQ
	00:09:45	WBT11			
	00:09:50	WBT11			
Begin Lag	00:10:55	NBLT	00:10:24	00:10:24	00:10:54
	00:10:58	SBTH			
End Lag	00:10:58	EBTH			
	00:11:00	EBRT			
	00:11:02	EBRT			
	00:11:03	EBTH			

In Example 1, the southbound left-turning (SBLT) vehicle reached the stop line at 00:10:48. Fig. 1 shows the northbound through (NBTH) vehicle and the SBLT vehicle at the stop line. The SBLT vehicle rejected several gaps before accepting a gap. The rejected gaps were defined by the eastbound through (EBTH) vehicle at 00:10:50 (Fig. 2), which was a lag; the westbound left-turn (WBLT) vehicle at 00:10:51 (Fig. 3); and the EBTH at 00:10:52 (Fig. 4). The accepted gap was defined by the NBTH vehicle (Fig. 5; passage time at 00:10:54) and the EBTH vehicle (Fig. 7,

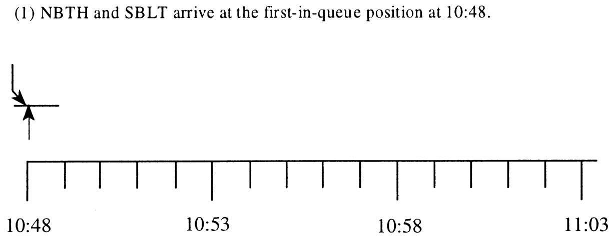

Fig. 1. Illustration of gap events (a).

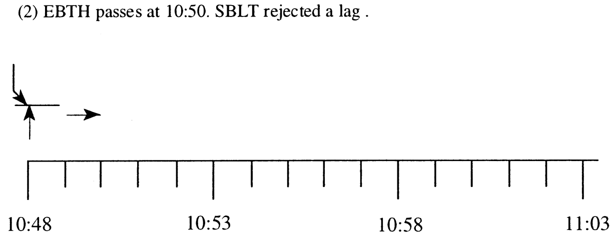

Fig. 2. Illustration of gap events (b).

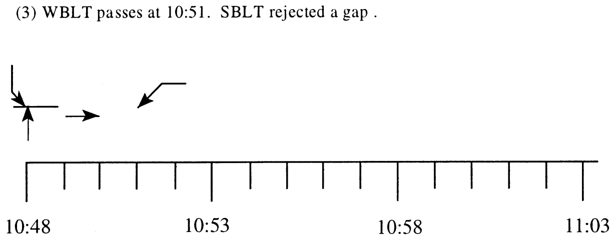

Fig. 3. Illustration of gap events (c).

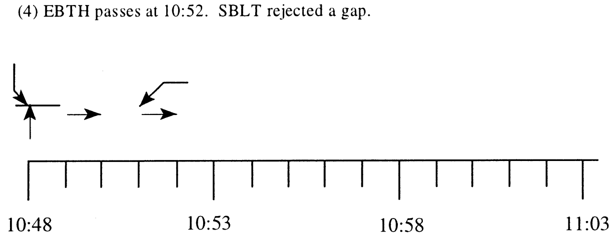

Fig. 4. Illustration of gap events (d).

passage time at 00:10:58). The accepted gap equalled 4 s. Note that the NBTH vehicle was included in defining the begin gap event (the accepted gap), but not included in defining the end gap event. Fig. 6 shows the successful passage of the SBLT vehicle into the intersection.

In Example 2 (Table 1), an EBTH vehicle (00:10:59) was defined as an end gap vehicle rather than the southbound through (SBTH) (00:10:58), because only vehicles on the major street can be defined as an end-gap vehicle. Example 3 shows a situation where a "lag" exists.

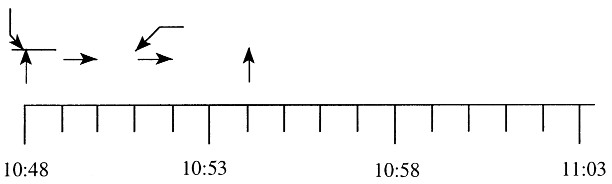
(5) NBTH passes at 10:54. This is not defined as a rejected gap of SBLT .

Fig. 5. Illustration of gap events (e).

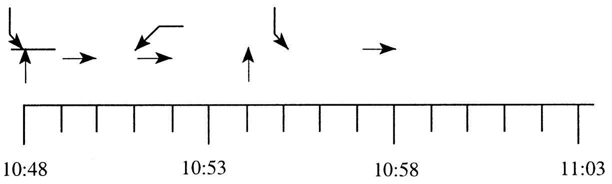
(6) SBLT passes at 10:55, accepted a gap of 4 seconds. NBTH defines the begin gap and EBTH defines the end gap .

Fig. 6. Illustration of gap events (f).

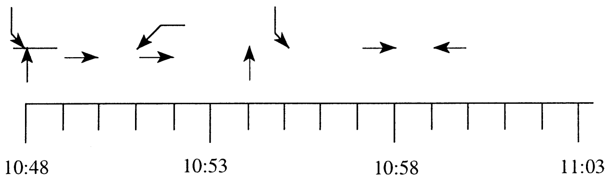
(7) EBTH passes at 10:58, defining the end gap event .

Fig. 7. Illustration of gap events (g).

Computer software was developed to extract these gap events using data collected in the field (Kyte et al., 1996). The software generates a data file in which all the gaps rejected and accepted by each minor-street vehicle are listed. The software further processes the data file and extracts all of the accepted and the maximum rejected gaps for a particular movement. The information is then used as input for the maximum likelihood estimation program developed by Troutbeck (1992). The software provides a report of the mean critical gap, the standard deviation, and the number of observations.

4. Gap events at multi-lane intersections

At intersections with multiple lanes on the major street, vehicles of a conflicting movement are distributed among all of the available travel lanes. However, only vehicles in a specific lane may have physical conflict potential with the minor-street vehicle. Fig. 8 illustrates such a situation. In Fig. 8, the major street has two through lanes in each direction. For the minor-street right-turn movement, only vehicles in lane 2 from the left side may have physical conflict potential with the

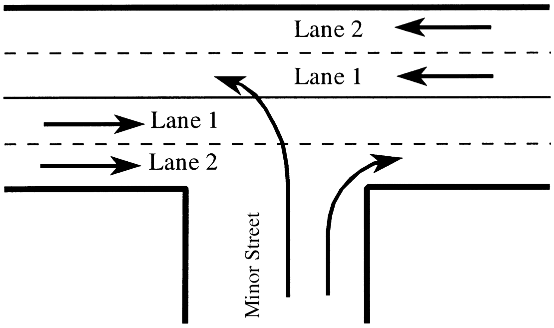

Fig. 8. Illustration of gap events at multi-lane intersections.

minor-street right-turn vehicle because a right-turn vehicle tends to merge into the outermost lane. All the vehicles from the left side of the minor street left-turn movement have physical-conflict potential with the minor-street vehicle. However, only those vehicles in lane 1 from the right side may have physical-conflict potential with the minor-street left-turn vehicle because a left-turn vehicle tends to merge into the inner lane.

Issues arise regarding how to define gap events at multi-lane sites. Two alternatives were investigated while defining gap events in this case. The first was to include all the vehicles in a conflicting movement, no matter which lane the vehicle travels in. The second was to include only those vehicles in the lane which have physical conflict potential for the minor-street movement.

The first approach would result in a large number of smaller rejected and accepted gaps due to the inclusion of vehicles on all the travel lanes. Therefore, a smaller critical gap would result when compared with an intersection with a single lane on the major street. The second alternative seems to be appropriate to better describe gap-acceptance characteristics at multi-lane intersections. This approach would eliminate vehicles on the non-conflicting lane when defining gap events. As a result, larger gaps would result. Vehicles in the non-conflicting lane also have some effect on the minor-street driver; however, this effect has been taken into account implicitly by the set of accepted and rejected gaps used to obtain the critical gap estimate.

5. Treatment of major-street right-turn movement

The major-street right-turn movement is similar in nature to a through movement on a multi-lane road. The major-street right-turning vehicles do not physically conflict with the minor-street movement. Similarly, vehicles in the non-conflicting lane at a multi-lane intersection do not physically conflict with a minor-street movement. However, the major-street right-turn movement was treated differently while defining gap events. The major-street right-turn movement was included when defining gap events, while vehicles in the non-conflicting lane at a multi-lane site were not. The main reason for taking this approach is to look for a better explanation of the gap-acceptance characteristics.

At multi-lane intersections, vehicles on the non-conflicting lane are excluded when defining gap events. As a result, a larger critical gap is obtained than at single-lane intersections. The result is more intuitive and can be easily explained. For example, it makes sense that larger critical gaps are expected at multi-lane intersections because the minor-street driver hesitates more when faced with vehicles in the non-conflicting lane. Larger critical gap results in smaller capacity, and the multi-lane effect has been taken into account in the critical gap calculation itself. Therefore, in calculating conflicting flow rates for the capacity calculation procedure, as described in the HCM (Transportation Research Board,1994), only those vehicles in the conflicting lane are considered.

If the major-street right-turn movement is dealt with in the same way—that is, major-street right-turn vehicles are excluded when defining gap events—a larger critical gap would result. When there are more right-turning vehicles on the major street, the critical gap is larger: conversely, if major-street right-turning vehicles are included when defining gap events, the critical gap is smaller. Both scenarios lead to confusion in interpreting the gap-acceptance process. To resolve this issue, one alternative is to consider the right-turn movement effect in calculating conflicting flow rates, as is done in the HCM. Specifically, the right-turn effect is not included in calculating the critical gap, as described below.

First, major-street right-turn vehicles are included when defining gap events. A smaller critical gap will result if major-street right-turn vehicles are included. Second, a regression analysis is conducted to identify the effect of the right-turn movement, i.e. to consider the proportion of right-turn movement as an independent parameter in the regression analysis. An equation similar to Eq. (12) can be obtained from the regression analysis.

$$t_c = \mathrm{f}(X) - a \times P_{\mathrm{rt}} \tag{12}$$

where

$\mathrm{f}(X)$ = function of other related parameters, such as time in queue, vehicle type
P_{rt} = proportion of major-street right-turn movement
a = coefficient of the regression analysis for the major-street right-turn movement

To yield a critical gap value without the effect of major-street right-turn movement, simply set P_{rt} to zero in the equation. If the critical gap resulting from such an approach is used in the capacity calculations, the right-turn movement must be considered somewhere in the procedure. In this analysis, the right-turn movement is taken into account in the calculation of conflicting flow rates. In the HCM procedure, half of the right-turn volume is included in the calculation of conflicting flow rates for the minor-stream movements.

As discussed above, the treatment of major-street right-turn movement and major-street vehicles at a multi-lane intersection differ. The only reason is to allow easier interpretation of the gap-acceptance process, and to be consistent with traditional findings, which indicate that (1) the critical gap is larger at multi-lane intersections, and (2) the critical gap is not related to the proportion of major-street right-turn movements.

6. Specific issues

While the method proposed above may be used to measure the critical gap under normal conditions, some unusual situations must be addressed so that potential bias can be avoided. Specifically, the proposed procedure may not be appropriate under the following conditions.

6.1. Two-stage gap acceptance

Two-stage gap acceptance refers to the situation where a mid-block refuge area exists on the major street that allows the minor-street drivers to seek gaps in a two-stage process. For example, when a two-way left-turn lane (TWLTL) or a raised median exists, some minor-street drivers cross one major-street approach, then seek gaps in the second approach while waiting in the middle refuge area. The gap event definition becomes more complicated when a two-stage gap-acceptance process is involved. First of all, not all drivers are willing to use a two-stage gap-acceptance process. Those who are willing to make a two-stage gap acceptance are more aggressive drivers; those who are not are characterized as more conservative. In addition to the aggressiveness factor, a two-stage gap-acceptance also allows the driver to seek smaller gaps, because he or she faces traffic streams from only one direction at a time. The mid-block area also presents a wide variety of conditions that may significantly affect driver behavior. For example, the mid-block area may be wide enough to allow two or three vehicles to stay parallel, which makes it possible for all of the waiting vehicles to use the same gap. Another difficulty in characterizing this process is determining the direction from which a vehicle will first occupy the mid-block area.

6.2. Pedestrian blockage

Pedestrian blockage refers to the situation in which a pedestrian is crossing at the intersection. A pedestrian can cause major-street vehicles to stop if the pedestrian is crossing the major street, or eliminate the opportunity for a minor-street vehicle to seek gaps while the path is being blocked by the pedestrian. When a pedestrian blocks the major street, the minor-street vehicles may move through the intersection at the same time, causing drivers to accept large gaps. When a pedestrian crosses the minor street, the minor-street driver has no chance to judge major-street gaps, which may cause him or her to reject large gaps. If the proposed method is used, data obtained during pedestrian blockages should be eliminated.

6.3. Downstream queue spill back

Downstream queue spill back refers to the situation when an over-saturated downstream intersection causes queue backup to the TWSC intersection. When downstream queue spill back exists, no gap events can be defined to reflect gap-acceptance characteristics; consequently, it is impossible to measure critical gap under such conditions.

6.4. Travel lane preference

The existence of a downstream intersection may cause drivers to select a preferred lane to turn into. This is often observed when a driver tries to avoid changing lanes to make a turn at the downstream intersection. Such a situation only exists at multi-lane intersections. For example, if the driver is going to make a left turn at the downstream intersection, he or she may prefer to merge into the inner lane (the one close to the median) to avoid additional lane changes while approaching the downstream intersection. If critical gap is to be estimated under such conditions, the gap events related to the two types of drivers have to be differentiated. One would be for those who make normal turns, and the other would be for those who would select their preferred lane.

7. Summary and conclusions

This paper documents the background of the maximum likelihood technique for measuring critical gaps at TWSC intersections. The application of this technique requires a clear definition of gap events that would reasonably reflect a driver's gap-acceptance characteristics. A methodology for defining and extracting gap events is proposed. The paper addresses how the multi-lane situations and the major-street right-turn movement are handled while applying the critical gap procedure. Special conditions for which the proposed procedure cannot be directly applied include two-stage gap-acceptance process, pedestrian blockage, downstream queue spill back, and driver lane preference.

The method discussed in this paper was used in measuring critical gaps under conditions in the US. The results of capacity and delay testing show that model accuracy was significantly improved when using the newly measured critical gaps (Kyte et al., 1996).

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