# Demand Deposit Contracts and the Probability of Bank Runs✧

Itay Goldstein* and Ady Pauzner**

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## ABSTRACT

We study a model of bank runs based on Diamond and Dybvig [1983]. We assume that agents do not have common knowledge regarding the fundamentals of the economy, but rather receive slightly noisy signals. The new model has a unique equilibrium in which the fundamentals determine whether a bank run will occur. This lets us compute the ex-ante probability of a bank run and relate it to the parameters of the demand deposit contract. We find that offering a higher return to agents who demand early withdrawal makes the bank more vulnerable to runs. We construct an optimal demand deposit contract that trades off the benefits from risk sharing against the costs of bank runs. Under this contract, there is a positive probability of panic-based bank runs. Nevertheless, it improves welfare relative to the autarkic regime. Finally, being able to make welfare computations, we assess the desirability of regimes that are intended to prevent bank runs: suspension of convertibility and deposit insurance.

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* The Fuqua School of Business, Duke University. E-mail: itayg@mail.duke.edu

** The Eitan Berglas School of Economics, Tel Aviv University. E-mail: pauzner@post.tau.ac.il.

## 1. Introduction:

Banks often finance long-term investments with short-term liabilities. While this can have obvious welfare benefits, it makes banks vulnerable to panic based runs. A panic based run occurs when investors rush to withdraw their deposits, believing that other depositors are going to do the same and that the bank will fail. As a result, the bank is forced to liquidate investments at a loss and indeed fails.

In a seminal paper, Diamond and Dybvig (1983, henceforth D&D) provided a coherent explanation as to why a bank might hold such a risky portfolio. By offering 'demand deposit' contracts, the bank enables investors who may have early liquidity needs to participate in profitable long-term investments. Since the bank deals with many investors, it can respond to their idiosyncratic liquidity needs and thereby provide a form of risk sharing that increases welfare. D&D show that their model has two equilibria. In the first, only investors who face liquidity needs demand early withdrawal and risk sharing is achieved. The second equilibrium, however, involves a 'bank run' in which all investors, including those with no liquidity need, demand early withdrawal. This equilibrium provides no risk sharing and is even inferior to the allocation agents can obtain without the bank.

A difficulty in the D&D model is that it cannot predict which equilibrium will occur or how likely each equilibrium is. This leaves a few open issues. First, testable empirical predictions cannot be derived since the model does not identify factors that affect the probability of bank runs. Second, the relation between the parameters of the demand deposit contract and the likelihood of runs cannot be studied. As a result, an optimal demand deposit contract that trades off the benefits from risk sharing against the costs of bank runs cannot be designed. Third, as bank runs generate a worse outcome relative to the autarchic one, whether demand deposit contracts improve welfare at all depends on the probability of runs. Since the probability of a bank run cannot be determined, this issue cannot be addressed. Finally, the applicability of the model to policy issues is limited: the probability of bank runs is a key element in assessing policies that are intended to prevent runs, or in comparing demand deposits to alternative types of contracts.

In this paper, we try to address all these issues. We modify D&D's model by assuming that the fundamentals of the economy (specifically, the return on long-term investments) are stochastic. Moreover, we assume that investors do not have common knowledge about them, but rather obtain private signals that are very close to the real value. In many cases, the assumption that investors observe noisy signals is more realistic than the assumption that they all share precisely the same information and opinions. We show that the modified model has a unique Bayesian equilibrium, in which bank runs occur if and only if the realization of the fundamentals in the economy is lower than some critical value.

It is important to stress, however, that although in our model the fundamentals uniquely determine whether a bank run will occur, runs are still panic-based, that is, driven by bad expectations. In most scenarios, each investor would like to take the action she believes that others take – she demands early withdrawal just because she fears others would. The key point, however, is that the beliefs of investors are uniquely determined by the realization of the fundamentals. In other words, the fundamentals do not determine agents' actions directly, but rather serve as a device that coordinates agents' beliefs on a particular outcome. Thus, our model provides testable predictions that reconcile two seemingly contradictory views: that bank runs occur following negative *real* shocks,[1] and that bank runs sometimes result from coordination failures,[2] in the sense that they occur even when the economic environment is sufficiently strong that depositors would not have run had they thought other depositors would not run.

Knowing whether a run occurs at each realization of the fundamentals, we can compute the probability of a bank run. We find that this probability depends positively on the degree of risk sharing embodied in the banking contract. The intuition is simple: an agent's incentive to demand early withdrawal is greater when she is offered a higher short-term return. This incentive is even further increased since the agent knows that other agents are also attracted by the higher return. Thus, a bank that offers more risk sharing becomes more vulnerable to runs.

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[1] For empirical evidence, see Gorton (1988), Demirguc and Detragiache (1998), and Kaminsky and Reinhart (1999).

[2] See Radelet and Sachs (1998) and Krugman (2000) for descriptions of recent international financial crises.

When the bank designs the demand deposit contract, it must take into account the positive impact that an increase in the short-term return (i.e., an increase in the degree of liquidity that the bank provides) has on the likelihood of runs. One may now wonder whether demand deposit contracts are still desirable when their destabilizing effect is considered. We answer this affirmatively: banks would still offer risk sharing, although this would cause runs to occur for some realizations of the fundamentals. As bank runs occur in equilibrium, however, demand deposit contracts do not achieve the first best allocation. Given this constraint, we analyze the optimal demand deposit contract in which there is an optimal tradeoff between the benefit from risk sharing and the cost of bank runs. This contract does not exploit all the potential gains from risk sharing, as doing so would cause too many bank runs.

Being able to make welfare computations, we go on and analyze the desirability of measures intended to prevent bank runs: suspension of convertibility and deposit insurance. Each one prevents panic-based runs, but has drawbacks of its own. We first look at suspension of convertibility. Here, the bank commits to stop paying at the short term after the number of agents who demanded early withdrawal has reached the (known) number of agents who have liquidity needs. Since the return in period 2 is now guaranteed, panic-based bank runs do not occur. However, when (non panic-based) runs do occur, they are more costly. This is because the agents with liquidity needs will not necessarily be among those who are granted early withdrawal. Thus, when agents are sufficiently averse to not consuming at all when they have liquidity needs, the ex-ante expected welfare under the suspension of convertibility regime might be lower than the welfare achieved by demand deposit contracts. Another disadvantage of suspension of convertibility is that agents are not allowed to withdraw their deposits even when fundamentals are so bad that investment projects should better be terminated prematurely.

Next, we look at deposit insurance. Here, the government promises each small bank to cover its short-term liabilities in case of a bank run (the payments will be financed by taxing all agents, regardless of whether their bank is subject to a run). This policy measure solves the coordination problem within each bank, and thus it reduces the probability of bank runs, given the short-term return that is promised by the banks. However, this policy measure also generates a moral-hazard problem: when it is implemented, small banks do not take into account the cost of the insurance (which is financed by taxing *all* agents), and thus tend to promise short term returns that are excessively high. As a result, the probability of bank runs under this regime might be even higher than without government intervention, and the overall welfare might be lower.

Our paper is closely related to many papers in the bank-run literature. Postlewaite and Vives (1987), Goldfajn and Valdes (1997), and Allen and Gale (1998) study models where bank runs happen in equilibrium. However, bank runs in these models are never panic-based since, whenever agents run, they would do so even if other agents didn't. Cooper and Ross (1998) present a model with panic-based bank runs, but assume that the probability of these bank runs is exogenous and unaffected by the form of the banking contract.[3] Temzelides (1997) employs an evolutionary model for equilibrium selection. His model, however, does not deal with the relation between the probability of runs and the banking contract.

A number of authors have studied models in which only some agents receive information concerning the prospects of the bank. Jacklin and Bhattacharya (1988) (see also Alonso (1996), Loewy (1998), and Bougheas (1999)) explain bank runs as an equilibrium phenomenon, but again ignore the possibility of panic-based runs. Chari and Jagannathan (1988) and Chen (1999) study panic-based bank runs, but of a different kind. Here, runs occur when uninformed agents interpret the fact that others run as an indication that fundamentals are bad.[4]

The method used in our paper to obtain a unique equilibrium is closely related to Carlsson and van Damme (1993) and Morris and Shin (1998). They show how the introduction of noisy signals to multiple-equilibria games leads to a contagion effect that generates a unique equilibrium.[5] Importantly, the proof of uniqueness in these papers builds crucially on the assumption of full strategic complementarities between agents' actions – a property that is not satisfied in standard bank run models. (The reason is that an agent's incentive

[3] Chang and Velasco (2001) use this approach in the context of international financial crises.

[4] For more developments of the D&D model see: Waldo (1985), Jacklin (1987), Bental and Eckstein and Peled (1988), Williamson (1988), Wallace (1988,1990), Engineer (1989), Chari (1989), and Garber and Grilli (1989). For surveys see: Calomiris and Gorton (1991), Dowd (1992), and Freixas and Rochet (1997).

[5] The idea that predictions can be very different when the assumption that agent have common knowledge of the payoffs is weakened to 'almost common knowledge' was first explored in Rubinstein (1989).

for early withdrawal is highest not when all agents do so, but rather when the number of agents demanding withdrawal reaches the level at which the bank goes bankrupt.) Thus, in order to extend the uniqueness result to situations with only partial strategic complementarities, we develop a different proof technique. (For further generalizations of the uniqueness results, see Morris and Shin (2001).) It is important to note, however, that the derivation of a unique equilibrium is not the focus of our paper. The main contributions of our model are the demonstration of the tradeoff between the provision of liquidity and the probability of a bank run, the computation of the optimal contract given this tradeoff, the result that the provision of liquidity via demand deposits is welfare improving even though bank runs sometimes occur, and the policy analysis.

In a parallel work, Rochet and Vives (2002) apply the Carlsson and van Damme technique to a model of bank runs. They simplify the payoff structure by assuming that agents can deposit money in banks only through intermediaries (fund managers). They assume that the intermediaries have objectives which are different than those of their investors; these objectives do satisfy the strong form of strategic complementarities, and thus the technical problem that we face in this paper is avoided. Importantly, the focus of their paper is completely different from ours, as they do not analyze the optimal tradeoff between risk sharing and bank runs, but rather look at the issue of a 'Lender of Last Resort'.

The remainder of this paper is organized as follows. Section 2 presents a benchmark model which is a slight modification of the D&D model. This model has multiple equilibria. In section 3 we introduce private signals into the model and obtain a unique equilibrium. Section 4 studies the relationship between the demand deposit contract and the likelihood of runs, computes the optimal contract and shows that demand deposit contracts are welfare improving. In section 5 we analyze policy measures that are intended to avoid bank runs. Concluding remarks appear in section 6. Proofs are relegated to the appendix.

## 2. The Benchmark Model.

**The economy**

Our benchmark model is a slight variation of D&D's model. There are three periods (0,1,2), one good, and a continuum [0,1] of agents. Each agent is born in period 0 with an endowment of 1. Consumption occurs only in periods 1 or 2 ($c_1$ and $c_2$ denote an agent's consumption levels). Each agent can be of two types: with probability $\lambda$ the agent is impatient and with probability 1-$\lambda$ she is patient. Agents' types are i.i.d.; we assume no aggregate uncertainty. Agents learn their types (which are their private information) at the beginning of period 1. Impatient agents can consume only in period 1. They obtain utility of $u(c_1)$. Patient agents can consume at either period; their utility is $u(c_1 + c_2)$.[6] $u$ is twice continuously differentiable, increasing, and has a relative risk-aversion coefficient, $-cu''(c)/u'(c)$, greater than 1.

Agents have access to a productive technology that yields higher returns in the long run: for each unit of input in period 0, the technology generates 1 unit of output in period 1 *or* $R$ units of output in period 2. In contrast to D&D, we assume that $R$ is not a fixed parameter, but rather depends on the random state of world $\theta$. (An alternative interpretation is that the long-run return is fixed, but the probability it would materialize depends on $\theta$.) The state $\theta$ is drawn according to a uniform distribution on [0,1] and is unknown to agents before period 2. The long-run return $R(\theta)$ is an increasing function of $\theta$, and satisfies $E_\theta u(R(\theta)) > u(1)$, so that for patient agents it is superior to the short-run return.

Note that although our technology has stochastic returns, the essence of our benchmark model is the same as that of D&D's model. When contemplating their actions, agents simply consider $E_\theta u(R(\theta))$ instead of D&D's fixed $u(R)$. The modification will be important in the next section, where agents receive partial information about $R$ in period 1.

[6]Equally, one can assume that patient agents consume only in period 2, and that goods can be stored with no depreciation.

**Autarky**

We first consider the behavior of agents when they act in isolation from each other. Since the long-term investment can be liquidated in period 1 without cost, the optimal behavior of each agent in period 0 is to invest her endowment in the long-term technology. In period 1 an agent who discovers that she is impatient liquidates her investment and consumes her endowment. Since $E_\theta\, u(R(\theta)) > u(1)$, a patient agent would keep the investment until period 2 and consume the return then. Thus, the expected utility of an agent as perceived in period 0 is:

$$U_{AUTARKY} = \lambda u(1) + (1-\lambda) E_\theta\, u(R(\theta))$$

When agents act in isolation from each other they are not insured against the risk of being impatient. Because of the high coefficient of risk aversion, a transfer of consumption from patient agents to impatient ones could be beneficial, ex-ante, to all agents, although it would necessitate the early liquidation of long-term investments.

**The first-best allocation**

Suppose there is a social planner who can verify agents' types when they are realized. The planner determines the period 1 consumption level $c_1$ of impatient agents and the (random) period-2 consumption level $\tilde{c}_2$ of patient agents. Since $\lambda c_1$ units of investment need to be liquidated in period 1 to satisfy the consumption of the impatient agents, patient agents get to consume

$$c_2 = \frac{(1-\lambda c_1)}{1-\lambda} R\,.$$

Agents' ex-ante expected payoff is, therefore:

$$\lambda u(c_1) + (1-\lambda) E_\theta\, u\left(\frac{1-\lambda c_1}{1-\lambda} R(\theta)\right)$$

The planner chooses $c_1$ to maximize this expression. The first order condition is ($c_1^{FB}$ denotes the first-best $c_1$):

$$u'(c_1^{FB}) = E_\theta R(\theta) u'\left(\frac{1-\lambda c_1^{FB}}{1-\lambda} R(\theta)\right)$$

This condition equates the benefit and cost from the early liquidation of the marginal unit of investment. The left-hand side is the marginal benefit to impatient agents, while the right-hand side is the marginal cost borne by the patient agents. The early liquidation involves a loss since the short-term return of 1 is lower, on average, than the long-term return of *R*. However, impatient agents have a lower level of consumption and thus enjoy a higher marginal utility. At the optimum, the gain from risk sharing exactly offsets the loss of return.

Since the marginal benefit is decreasing in $c_1$ and the marginal cost is increasing, the first best $c_1$ is more than 1 as long as, at $c_1$=1, the marginal benefit is greater than the marginal cost: $1 \cdot u'(1) > E_\theta R(\theta) u'\big(R(\theta)\big)$. Since $cu'(c)$ is a decreasing function of *c* (recall that the coefficient of relative risk aversion is more than 1), increasing $c_1$ slightly above 1 generates a gain from risk sharing if $R(\theta) > 1$, and a loss if $R(\theta) < 1$. The expected gain outweighs the expected loss if the probability that $R(\theta) < 1$ is not too large.

To give an exact bound on this probability, we need some notation. Let *F* be the cumulative distribution function of *R*: $F(R) = prob[R(\theta) \le R]$. Setting $c_1$ above 1 is desirable as long as:

$$\frac{F(1)}{1-F(1)} < \frac{A}{B}$$

Where $A = E[u'(1) - Ru'(R) \mid R > 1]$ is the marginal gain from increasing $c_1$ above 1, conditional on *R* being more than 1, and $B = E[Ru'(R) - u'(1) \mid R < 1]$ is the marginal loss, conditional on *R* being less than 1. (As explained above, both *A* and *B* are positive.) Note that the above condition, that *F*(1) is sufficiently small, is in accord with our assumption that $E_\theta u(R(\theta)) > u(1)$.

**Banks**

The above analysis presumed that agents' types were observable. When types are private information, the payments to agents cannot be made contingent on their types. To achieve risk sharing, we need a mechanism that induces agents to reveal their types. This creates a possible role for banks. A bank could offer agents 'demand deposit' contracts, which give the depositor the right to withdraw her investment in period 1, and receive a fixed payment which is higher than the scrap value of 1. As long as the (expected) period-2 payment is higher, all patient agents would prefer to wait till period 2, and agents' types would be revealed.

We assume that the economy has a banking sector with free entry, and that all banks have access to the same investment technology. Since banks earn no profits, they offer the same contract as the one that would be offered by a single bank that maximizes the welfare of agents.[7] We restrict banks to offer demand deposit contracts only.[8] Such a contract takes the following form: each agent deposits her endowment in the bank in period 0. If she demands withdrawal in period 1, she receives a fixed payoff of $r_1>1$; if she waits until period 2 she receives a stochastic payoff of $\tilde{r}_2$ which is the proceeds of the non liquidated investments divided by the number of remaining depositors.[9] These payments are maintained as long as the bank has enough resources to pay every agent who demands early withdrawal. If not, the bank liquidates all the investments and divides the proceeds among the agents who demanded withdrawal in period 1.[10] In that case, an agent who comes in

[7] This equivalence follows from the fact that there are no externalities between different banks. The contract that one bank offers to its investors does not affect the payoffs to agents who invest in another bank. (We assume an agent cannot invest in more than one bank.)

[8] This means that, as long as it is solvent, the bank cannot make the period-1 payment contingent on the number of agents demanding withdrawal. This restriction is common in the literature; for possible justifications see, e.g., Jacklin and Bhattacharya (1988) and Calomiris and Kahn (1991).

[9] Note that if we interpret the stochastic long-term return $R(\theta)$ as a fixed return with a probability of default that decreases in $\theta$, then as long as there is no run ($n=\lambda$), the second period payoff $\tilde{r}_2$ can also be though as a fixed payment with a probability of default. Thus, our assumption that $\tilde{r}_2$ is variable is not in conflict with the observation that real-world demand deposit contracts promise a fixed long-term interest rate.

[10] An alternative specification that appears in many models is the sequential service constraint. We chose the alternative of equal division for analytical convenience; however, all our results hold also under sequential service.

period 2 receives 0. The payments are depicted in table 1 ($n$ denotes the proportion of agents demanding early withdrawal):

| Withdrawal in period | $n<1/r_1$ | $n \geq 1/r_1$ |
|---|---|---|
| 1 | $r_1$ | $1/n$ |
| 2 | $\frac{(1-nr_1)}{1-n}R(\theta)$ | 0 |

Table 1: Ex post payoffs of a patient agent

Suppose the bank sets $r_1$ to be $c_1^{FB}$. If only impatient agents demand early withdrawal, the expected utility of patient agents will be $E_\theta\, u\left(\frac{1-\lambda r_1}{1-\lambda} R(\theta)\right)$. As long as this is more than $u(r_1)$, there will be an equilibrium in which, indeed, only impatient agents demand early withdrawal. However, as was shown by Diamond and Dybvig, this banking contract makes the bank vulnerable to runs. There is a second equilibrium in which *all* agents demand early withdrawal. When they do so, the first period payment is 1 and the second period payment is 0, so that it is indeed optimal for agents to demand early withdrawal. This equilibrium is inferior to the autarky case, since all long-run investments are liquidated and each agent consumes 1.

Our benchmark model (like D&D's) provides no tools to predict which equilibrium is more likely to occur. D&D devised the optimal demand-deposit contract under the assumption that the 'good' equilibrium is always selected; under this assumption, the optimal $r_1$ in our model is $c_1^{FB}$. This approach has two drawbacks. First, the contract is not optimal if the probability of bank runs is not negligible. It is not even obvious that risk sharing is desirable in that case. Second, the computation of the banking contract presumes away any possible relation between the degree of insurance provided by the banking contract and the likelihood of a bank run. If such a relation exists, the optimal $r_1$ will not be $c_1^{FB}$. These drawbacks will be tackled in the next section, where we extend the model so as to obtain firmer predictions.

## 3. Agents with Private Signals: Unique Equilibrium.

We now extend the model by assuming that at the beginning of period 1 each agent receives a very precise private signal regarding the fundamentals of the economy. (A second important modification that concerns the technology will be introduced later). We will see that these signals force agents to coordinate their actions. They will run on the bank when the fundamentals are in one range, and select the 'good' equilibrium in another range. Consequently, we will be able to determine the probability of bank runs for any *given* banking contract. Knowing how this probability is affected by the amount of risk sharing provided by the contract, we will revert to period 0 and characterize the optimal contract.

Specifically, we assume that state $\theta$ is realized at the beginning of period 1. At this point, $\theta$ is not publicly revealed. Rather, each agent $i$ obtains a signal $\theta_i = \theta + \varepsilon_i$, where $\varepsilon_i$ are small error terms that are independently and uniformly distributed over the interval $[-\varepsilon, \varepsilon]$. An agent's signal can be thought of as private information available to her, or as her private opinion regarding the prospects of the long-term return on the investment project. Note that while each agent has different information, none has an advantage in terms of the quality of the signal. We will analyze the equilibrium behavior of agents as signals become arbitrarily precise, i.e., as $\varepsilon$ approaches 0.

The introduction of private signals changes the results considerably. A patient agent's decision whether to demand withdrawal in period 1 or 2 would depend on her signal. (Abusing both English and decision theory, we will sometimes refer to demanding early withdrawal as 'running on the bank'.) The effect of the signal is twofold. The signal provides information concerning the expected second period payment: the higher the signal, the higher is the posterior distribution attributed by the agent to the true value of $R(\theta)$, and the lower the incentive to run on the bank. Perhaps more importantly, an agent's signal provides information about other agents' signals, which allows an inference regarding their actions. Observing a high signal makes the agent believe that other agents obtained high signals as well. Consequently, she attributes a low likelihood to the possibility that they will run on the bank. Since strategies are complementary, this makes her incentive to run even smaller.

We start by analyzing the events in period 1, assuming that the banking contract that was chosen in period 0 offers $r_1$ to agents demanding withdrawal in period 1, and that all agents have chosen to deposit their endowments in the bank.[11] While all impatient agents demand early withdrawal, patient agents need to compare the expected payoffs from going to the bank in period 1 or 2. The ex-post payoff of a patient agent from these two options depends on both $\theta$ and the proportion $n$ of agents demanding early withdrawal (see Table 1 on page 8). Since the agent's signal gives her (partial) information regarding both $\theta$ and $n$, it affects her calculation of her expected payoffs. Thus, her action will depend on her signal.

We assume that there are feasible signals that are extremely good or extremely bad, for which a patient agent's best action is independent of her belief concerning other patient agents' behavior. As will be seen in the sequel, the possibility of observing such extreme signals suffices to ignite a contagion effect that leads to a unique outcome for any realization of the fundamentals. Importantly, we need not assume that such events are likely; any small positive probability suffices. Moreover, the outcome will not depend on the exact ranges of fundamentals at which such signals are observed.

We start with the lower range. When the state of the world is very bad ($\theta$ very low), the long term return will be lower than the short term return, even if all patient agents were to wait ($n=\lambda$). If, given her signal, a patient agent is sure that this is the case, her best action is to run regardless of her belief about the behavior of the other patient agents. More precisely, we denote by $\underline{\theta}(r_1)$ the value of $\theta$ for which $r_1 = \frac{1-\lambda r_1}{1-\lambda} R(\theta)$, and refer to the interval $[0, \underline{\theta}(r_1))$ as the *lower dominance region*. Since the difference between an agent's signal and the true $\theta$ is no more than $\varepsilon$, we know that she will demand early withdrawal if she observes a signal $\theta_i < \underline{\theta}(r_1) - \varepsilon$. We assume that such extremely bad states are feasible: that for any $r_1 \geq 1$ there are feasible values of $\theta$ for which all agents receive signals that assure them that $\theta$ is in the lower dominance region. Since $\underline{\theta}$ is increasing in $r_1$, the condition that guarantees this for any $r_1 \geq 1$ (and for sufficiently small $\varepsilon$) is: $\underline{\theta}(1) > 0$ (or, equivalently, $R(0)<1$).

[11] Clearly, $r_1$ must be at least 1 but less than $1/\lambda$.

Similarly, we assume an *upper dominance region* of parameters: a range $(\bar{\theta},1]$ in which no patient agent would demand early withdrawal. To this end, we need to modify the investment technology available to the bank. Instead of assuming that the short-term return is fixed at 1, we assume that it equals 1 in the range $[0,\bar{\theta}]$, and equals $1/\lambda$ in the range $(\bar{\theta},1]$. ($\bar{\theta}$ will be taken to be arbitrarily close to 1.) The interpretation of this assumption is that when the fundamentals are extremely high, not only the long-term return improves, but also the short-term return does. We also assume that for $\theta \in (\bar{\theta},1]$, the long-term return $R(\theta)$ is larger than the short-term return.[12,13] When a patient agent knows that the fundamentals are in the region $(\bar{\theta},1]$, she will not run, whatever her belief regarding the behavior of other agents is. Why? Since the return from a single investment unit exceeds the maximal possible value of $r_1$ (which is $1/\lambda$), there is no need to liquidate more than one unit of investment in order to pay one agent in period 1. As a result the return to agents who come at the period 2 is guaranteed.[14]

An alternative assumption that generates an upper dominance region is that there exists an external large agent, who would be willing to buy the bank and pay its liabilities if she knew for sure that the long-run return was very high. This agent need not be a governmental institute; it can be a private agent, since she can be sure of making a large profit. Note however, that while such an assumption is very plausible if we think of a single bank (or country) being subject to a panic run, it is less plausible if we think of our bank as representing the whole world economy, whereas all sources of liquidity are already exhausted. (Our assumption on technology is not subject to this critique.)

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[12] This specification of the short-term return is the simplest that guarantees the existence of an upper dominance region. A maybe more natural assumption, that the short-term return increases gradually over [0,1] will lead to the same results but would complicate the exposition.

[13] Note that the definition of the lower dominance region is not affected by the jump in technology as long as $r_1$ is not too large. Otherwise, $\bar{\theta} = \underline{\theta}$.

[14] More formally, when $\theta > \bar{\theta}$, an agent who demands early withdrawal receives $r_1$, whereas an agent who waits receives $\frac{1-n\lambda r_1}{1-n} R(\theta)$ (which must be higher than $r_1$ because $r_1$ is smaller than $1/\lambda$, and $R(\theta)$ is larger than $1/\lambda$).

Note that we did not introduce this modification in the technology in the benchmark model in order to keep the presentation as simple as possible. It is clear, however, that since we take $\bar{\theta}$ to be arbitrarily close to 1, the analysis would have been unchanged.

Importantly, our model can be analyzed even if we do not assume the existence of an upper dominance region. In spite of the fact that in this case there are multiple equilibria, several equilibrium selection criteria (refinements) show that the more reasonable equilibrium is the same as the unique equilibrium that we obtain when we assume the upper dominance regions. We discuss these refinements in Section 6.

The two dominance regions are just extreme ranges of the fundamentals at which agents' behavior is known. This is illustrated in Figure I. The dotted line represents a lower bound on $n$, implied by the lower dominance region. This line is constructed as follows. The agents who would definitely demand early withdrawal are all the impatient agents, plus the patient agents who get signals below the threshold level $\underline{\theta}(r_1)-\varepsilon$. Thus, when $\theta < \underline{\theta}(r_1)-2\varepsilon$, all patient agents get signals below $\underline{\theta}(r_1)-\varepsilon$ and $n$ must be 1. When $\theta > \underline{\theta}(r_1)$, no patient agent gets a signal below $\underline{\theta}(r_1)-\varepsilon$ and no patient agent must run. As a result, the lower bound on $n$ in this range is only $\lambda$. As $\theta$ grows from $\underline{\theta}(r_1)-2\varepsilon$ to $\underline{\theta}(r_1)$, the proportion of patient agents observing signals below $\underline{\theta}(r_1)-\varepsilon$ decreases linearly, since the distribution of signal errors is uniform. The solid line is the upper bound, implied by the upper dominance region. It is constructed in a similar way, using the fact that patient agents would not run if they observed a signal above $\overline{\theta}+\varepsilon$.

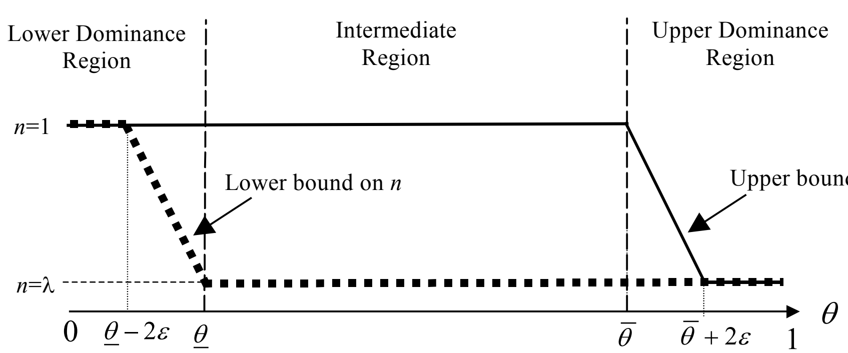


Figure I: Direct implications of the dominance regions on agents' behavior

Because the two dominance regions represent very unlikely scenarios, where fundamentals are so extreme that they determine uniquely what agents would do, their existence gives little *direct* information regarding agents' behavior. The two bounds can be far apart, generating a large intermediate region in which an agent's optimal strategy depends on her beliefs regarding other agents' actions.

However, the beliefs of agents in the intermediate region are not arbitrary. Since agents observe only noisy signals of the fundamentals, they do not exactly know the signals that other agents observed. Thus, in the choice of the equilibrium action at a given signal, an agent must take into account the equilibrium actions at nearby signals. Again, these actions depend on the equilibrium actions taken at further signals, and so on. Eventually, the equilibrium must be consistent with the (known) behavior at the dominance regions. Thus, our information structure places stringent restrictions on the structure of the equilibrium strategies and beliefs.

Theorem 1 says that the model with noisy signals has a unique equilibrium. A patient agent's action is uniquely determined by her signal: she demands early withdrawal if and only if her signal is below a certain threshold. The corollary that follows computes the resulting aggregate behavior for any realization of the fundamentals.

THEOREM 1: The model has a unique equilibrium in which patient agents run if they observe a signal below threshold $\theta^{*}(r_1)$ and do not run above.

Before we continue to describe the implications of our equilibrium, we want to discuss the theoretical contribution of our proof. The usual argument that shows that with noisy signals there is a unique equilibrium (see Carlsson and van-Damme (1993) and Morris and Shin (1998)) builds on the property of full strategic complementarities – that an agent's incentive to take an action is higher when more other agents take that action. In our model, this property does not hold, since a patient agent's incentive to run is highest when $n=1/r_1$, rather than when $n=1$. This is a general feature of standard bank-run models: Once the bank is already bankrupt, the larger the number of agents that demand early withdrawal, the smaller is the amount that will be paid to each one of them, and the lower the incentive to withdraw early. Because of this feature, the approach that is used in the literature to prove uniqueness of equilibrium cannot be used in our case. In particular, if full strategic complementarities exist, uniqueness can be shown to be a result of iterative elimination of dominated strategies, whereas in our case we cannot show that. Nevertheless, using a different technical approach, we are able to show the uniqueness of equilibrium. Morris and Shin (2001) provide more discussion of our result.

COROLLARY: Given $r_1$, the proportion of agents demanding early withdrawal depends only on the fundamentals. It is given by:

$$n(\theta,\theta^*(r_1)) = \begin{cases} 1 & \text{if } \theta \le \theta^*(r_1)-\varepsilon \\ \lambda + (1-\lambda)(\frac{1}{2} + \frac{\theta^*(r_1)-\theta}{2\varepsilon}) & \text{if } \theta^*(r_1)+\varepsilon \le \theta \le \theta^*(r_1)-\varepsilon \\ \lambda & \text{if } \theta \ge \theta^*(r_1)+\varepsilon \end{cases}$$

The calculation of $n$ is as follows. All impatient agents (proportion $\lambda$) go to the bank in period 1. Among the patient agents only those who receive a signal below $\theta^*(r_1)$ go. When $\theta$ is below $\theta^*(r_1)-\varepsilon$ all patient agents receive signals below $\theta^*(r_1)$ and demand early withdrawal. We call this scenario 'total run'. When $\theta$ is above $\theta^*(r_1)+\varepsilon$, they all receive signals above $\theta^*(r_1)$ and wait till period 2. We label this case 'no run'. When $\theta$ is between $\theta^*(r_1)-\varepsilon$ and $\theta^*(r_1)+\varepsilon$ there is a 'partial run'. Proportion $n(\theta)$ decreases linearly as fewer agents observe signals below $\theta^*(r_1)$ when $\theta$ is higher. (Recall that agents' signals are independent and uniformly distributed between $\theta-\varepsilon$ and $\theta+\varepsilon$.) This is shown in Figure II:

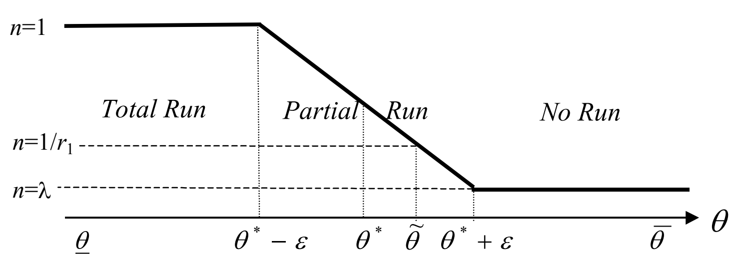


Figure II: The unique equilibrium as a function of the fundamentals

The range of partial run can be divided into two: when $\theta$ is below $\tilde{\theta}(r_1) \equiv \theta^*(r_1) + \varepsilon(1 - 2\frac{1-\lambda r_1}{(1-\lambda)r_1})$, the proportion of agents demanding early withdrawal $n(\theta,\theta^*(r_1))$ is greater than $1/r_1$. In this case the bank does not have enough resources to pay its period 1 liabilities. Agents demanding withdrawal in period 1 receive $1/n < r_1$, and those coming in period 2 receive 0. When $\theta > \tilde{\theta}(r_1)$, agents who come in period 1 receive the promised payment $r_1$, while those coming in period two receive $\frac{1-r_1 n}{1-n} R(\theta)$.

It is important to stress that although the realization of $\theta$ uniquely determines how many patient agents will run on the bank, most run episodes – those that occur in the intermediate region – are still driven by bad expectations. Since running on the bank is not a dominant action in this region, the reason patient agents do run is that they believe others will do so. Because they are driven by bad expectations, we refer to bank runs in the intermediate region as 'panic-based' runs. Thus, the fundamentals serve as a coordination device for the expectations of agents, and thereby indirectly determine how many agents will run on the bank. The crucial point is that this coordination device is not just a sunspot, but rather a payoff-relevant variable. This fact, and the existence of dominance regions, forces a unique outcome; in contrast to sunspots, there can be no equilibrium in which agents ignore their signals.

To conclude this section, we compute the threshold signal $\theta^*(r_1)$. A patient agent who receives signal $\theta^*(r_1)$ must be indifferent between going to the bank in period 1 or period 2. That agent's posterior distribution of $\theta$ is uniform over the interval $[\theta^*(r_1)-\varepsilon, \theta^*(r_1)+\varepsilon]$. Given $\theta$, she believes that the proportion of agents withdrawing in period 1 is $n = n(\theta, \theta^*(r_1))$. Thus, her posterior distribution of $n$ is uniform over $[\lambda,1]$. Equating her expected payoff from withdrawing in period 1 to that of waiting for period 2 we obtain an implicit definition of $\theta^*(r_1)$:

$$\int_{n=\lambda}^{1/r_1} u(r_1) + \int_{n=1/r_1}^{1} u(\frac{1}{n}) = \int_{n=\lambda}^{1/r_1} u(\frac{1-r_1 n}{1-n} R(\theta(n))) + \int_{n=1/r_1}^{1} u(0),$$

where $\theta(n) = \theta^*(r_1) + \varepsilon(\frac{1+\lambda}{2} - n)$ is the inverse function of $n(\theta, \theta^*(r_1))$.[15] Note that the expression is further simplified as the signal noise $\varepsilon$ converges towards 0, since then $\theta(n)$ is simply $\theta^*$.

[15] More precisely, this implicit definition is correct as long as the resulting $\theta^*$ is below $\overline{\theta} - \varepsilon$, which holds as long as $r_1$ is not too large; otherwise, $\theta^*$ would be close to $\overline{\theta}$. (In section 4, we show that when $\overline{\theta}$ is close to

## 4. The Banking Contract and the Likelihood of Runs

Having characterized the unique equilibrium, we are now equipped with the tools to study the effect of the banking contract on the likelihood of bank runs. Theorem 2 says that when $r_1$ is larger, patient agents will demand early withdrawal for a larger set of signals. This means that the banking system becomes more vulnerable to bank runs when it offers more risk sharing. The intuition is simple: if the payment in period 1 is increased and the payment in period 2 is decreased, the incentive of patient agents to withdraw in period 1 is higher. This incentive is further increased since, knowing that other agents are more likely to withdraw in period 1, the agent assigns a higher probability to a bank run.

THEOREM 2: $\theta^*(r_1)$ is increasing in $r_1$.

Knowing the effect of $r_1$ on the behavior of agents in period 1, we can revert to period 0 and compute the optimal banking contract. The bank chooses $r_1$ to maximize the ex-ante expected utility of a representative agent, which is given by the following expression:

$$\begin{aligned}
EU(r_1) = & \int_0^{\theta^*(r_1)-\varepsilon} u(1)\,d\theta \\
& + \int_{\theta^*(r_1)-\varepsilon}^{\tilde{\theta}(r_1,\theta^*(r_1))} n(\theta,\theta^*(r_1)) \cdot u\Big(\frac{1}{n(\theta,\theta^*(r_1))}\Big) + (1-n(\theta,\theta^*(r_1))) \cdot u(0)\,d\theta \\
& + \int_{\tilde{\theta}(r_1,\theta^*(r_1))}^{\theta^*(r_1)+\varepsilon} n(\theta,\theta^*(r_1)) \cdot u(r_1) + (1-n(\theta,\theta^*(r_1))) \cdot u\Big(\frac{1-n(\theta,\theta^*(r_1))r_1}{1-n(\theta,\theta^*(r_1))} R(\theta)\Big)\,d\theta \\
& + \int_{\theta^*(r_1)+\varepsilon}^{\overline{\theta}} \lambda \cdot u(r_1) + (1-\lambda) \cdot u\Big(\frac{1-\lambda r_1}{1-\lambda} R(\theta)\Big)\,d\theta \\
& + \int_{\overline{\theta}}^{1} \lambda \cdot u(r_1) + (1-\lambda) \cdot u\Big(\frac{1-\lambda \cdot \lambda r_1}{1-\lambda} R(\theta)\Big)\,d\theta
\end{aligned}$$

The ex-ante expected utility depends on the payments under all possible values of $\theta$. In the range below $\theta^* - \varepsilon$ there is a total run. All investments are liquidated in period 1, and agents of both types receive 1. In the range between $\theta^* - \varepsilon$ and $\theta^* + \varepsilon$ a measure $n$-$\lambda$ of

1, the bank will choose $r_1$ sufficiently small so that there is a nontrivial region where agents don't run, i.e., such that $\theta^*$ is below $\overline{\theta}$ .)

patient agents join the $\lambda$ impatient ones and demand early withdrawal. The exact payments in that range depend on whether the bank is able to meet all period 1 demands. In the range above $\theta^* + \varepsilon$ there is no run. Impatient agents receive $r_1$ (in period 1), and all patient agents wait till period 2 and receive $\frac{1-\lambda r_1}{1-\lambda} R(\theta)$. If the fundamentals are in the upper dominance region, patient agents receive an even higher payoff, because of the jump in the return on investments that are terminated prematurely (which permits termination of fewer investments to pay the impatient agents' demands).

It is worth noting that when $\varepsilon$ and 1-$\overline{\theta}$ approach 0, the above expression converges to:

$$\lim_{\substack{\overline{\theta} \to 1 \\ \varepsilon < 1-\overline{\theta}}} EU(r_1) = \int_0^{\theta^*(r_1)} u(1)\, d\theta + \int_{\theta^*(r_1)}^{1} \lambda \cdot u(r_1) + (1-\lambda) \cdot u(\frac{1-\lambda r_1}{1-\lambda} R(\theta))\, d\theta \,.$$

The computation of the optimal $r_1$ is different than in the benchmark case. The main difference is that now the bank needs to consider an extra cost associated with increasing $r_1$. This cost results from the increase in the likelihood of a bank run that follows the increase in $r_1$. This raises the question whether demand deposit contracts, which pay impatient agents more than the liquidation value of their investments, are still desirable when the destabilizing effect of increasing $r_1$ is taken into account. That is, whether provision of liquidity via demand deposit contracts is desirable even when the cost of bank runs that result from this type of contract is considered. Theorem 3 gives a positive answer: $r_1$ should be set above the short term return of 1, although this will cause runs to occur with positive probability even when the realization of $R(\theta)$ is above 1.[16]

THEOREM 3: For small enough $\varepsilon$, the optimal $r_1$ must be larger than 1. (A sufficient condition under which the conclusion holds also for large $\varepsilon$ is: $\frac{F(1)}{1-F(1)} < \frac{\lambda A}{B}$.)

The intuition is as follows: Increasing $r_1$ slightly above 1 will cause patient agents to demand early withdrawal when $R(\theta)$ is slightly above 1. However, in this scenario the run

[16] We interpret the theorem as saying that the optimal first-period payment is higher than the short-term return, since the theorem holds even at the limit as 1-$\overline{\theta}$ (and $\varepsilon$) converge to 0, i.e., in the case where the short-term return is always 1.

causes almost no harm, as the liquidation value of the investment is close to the long-term return. It follows that, at $r_1$=1, a small increase in the probability of a bank run that results from increasing $r_1$ entails almost no cost. Thus, at that point, the bank need only consider the effect that the increase of $r_1$ has on the gain from risk sharing, which is positive.[17] In the case where $\varepsilon$ is larger, there is a small opposite effect: when $r_1$=1 some patient agents will wait till period 2 even when $R$ is somewhat below 1. This is because their signals mistakenly teach them that $R$ is above 1. These agents suffer from the fact that more than $\lambda$ agents run, when their marginal utilities are high, and are further harmed if $r_1$ is increased. This effect does not change the results when the sufficient condition for large $\varepsilon$ in the theorem holds.

Interestingly, because the optimal level of $r_1$ is higher than 1, panic-based bank runs occur at the optimum. This can be seen by comparing $\theta^*(r_1)$ with $\underline{\theta}(r_1)$, and noting that the first is larger than the second when $r_1$ is above 1. Thus, the optimal demand deposit contract achieves a better welfare than is achieved under autarky, but it still does not achieve the first best allocation, as it generates panic-based bank runs.

Having shown that 'demand deposit' contracts improve welfare, we now analyze the forces that determine the optimal $r_1$, i.e., the optimal degree of liquidity that is provided by the banking contract. For simplicity we consider the case where $\varepsilon$ and 1-$\overline{\theta}$ approach 0 (again, with $\varepsilon$ < 1-$\overline{\theta}$). First, we note that the optimal $r_1$ must be lower than 1/λ: if $r_1$ were larger, a bank run always occurs, and the ex-ante welfare is lower than in the case where $r_1 = 1$. In fact, the optimal $r_1$ must be such that $\theta^*(r_1) < \overline{\theta}$, for the exact same reason. Thus, we must have an interior solution for $r_1$. The first order condition that determines the optimal $r_1$ will be (recall that $\varepsilon$ and 1-$\overline{\theta}$ approach 0):

$$\lambda\int_{\theta^*(r_1)}^{1}\left[u'(r_1)-R(\theta)u'\left(\frac{1-\lambda r_1}{1-\lambda}R(\theta)\right)\right]d\theta-\frac{\partial\theta^*(r_1)}{\partial r_1}\left[\left(\lambda u(r_1)+(1-\lambda)u\left(\frac{1-\lambda r_1}{1-\lambda}R(\theta^*)\right)\right)-u(1)\right]=0$$

[17] Note that this gain will be higher than in the benchmark model since when $R(\theta)$<1, there will be a run, and the losses from increasing $r_1$ at these levels of fundamentals will not be faced. Thus, when $\varepsilon$ approaches 0, we don't even need the condition that was needed in the benchmark model.

The first term is the marginal gain from risk sharing that follows the transfer of consumption from patient to impatient agents, as a result of an increase in $r_1$. The second term is the marginal cost that results from the destabilizing effect of increasing $r_1$. If the second term were 0, as in the benchmark model where the bank ignored the possibility of runs, the optimal $r_1$ would have been higher than in the benchmark case. This is because here the integral is taken over the range $[\theta^*(r_1),1]$, rather than over all possible values of $\theta$, and because the benefit from risk sharing is increasing in $R(\theta)$ (recall that agents have increasing relative risk aversion). However, since the second term is positive, there is an extra force that makes the bank prefer a lower $r_1$. Thus, our optimal $r_1$ might be lower or higher than in the benchmark model.

Importantly, the first order condition demonstrates that the optimal demand deposit contract does not exploit all the potential gains from risk sharing. Since the second term in the first order condition is positive, at the optimum, the first term will also be positive. As a result, the bank can increase the gains from risk sharing by increasing $r_1$ above its optimal level. However, because of the cost of more bank runs the bank chooses not to do this. Thus, in the model with noisy signals, the optimal contract demonstrates the tradeoff between risk sharing and costs of bank runs, a point that could not be addressed by our benchmark model or by the D&D framework.

## 5. Measures to prevent bank runs

In Section 4, we characterized the optimal demand deposit contract, and showed that the promised short-term payoff reflects the optimal tradeoff between the benefit from improved risk sharing and the cost resulting from a higher probability of bank runs. As we showed, this contract does not achieve the first-best allocation, as it yields non-efficient bank runs in equilibrium. In this section, we analyze the desirability of measures to prevent bank runs. First, we analyze suspension of convertibility. Then, we analyze deposit insurance. For simplicity, in this section we focus on the limit case in which $\varepsilon$ and 1-$\overline{\theta}$ approach 0.

**Suspension of Convertibility:**

Under this regime, the bank commits to pay $r_1$ in period 1 to no more than $\lambda$ agents. As a result, agents who wait are guaranteed a payoff of $\frac{1-\lambda r_1}{1-\lambda}R(\theta)$ in period 2, independently of what other agents do. The advantage of such a contract relative to the simple demand-deposit contract is that (as $\varepsilon$ converges to 0) it prevents bank runs when $\theta$ is between $\underline{\theta}(r_1)$ and $\theta^*(r_1)$. That is, panic-based runs do not occur. However, this contract also has a drawback: in the case of a run which is not panic-based (when $\theta < \underline{\theta}(r_1)$), since the bank cannot distinguish between patient and impatient agents, some impatient agents will not be among the $\lambda$ who get $r_1$ in period 1 and thus will consume 0. This might be a worse outcome than the outcome of bank run in the simple demand-deposit case, since there everybody consumes 1. (Note that this drawback exists also relative to a benchmark with a sequential service constraint – see Footnote 18.)

Thus, the choice between a simple demand deposit contract and a contract with suspension of convertibility introduces a non-trivial tradeoff. In the first case, bank runs occur under more circumstances, but in the second case the runs that do occur are more costly. To clarify our argument, we write down the ex-ante expected utility of a representative agent in both cases.

In the case of a simple demand deposit contract, as $\varepsilon$ and 1-$\overline{\theta}$ converge to 0, the ex-ante expected utility becomes:

$$\int_0^{\theta^*(r_1^{DD})} u(1)d\theta + \int_{\theta^*(r_1^{DD})}^1 \lambda \cdot u(r_1^{DD}) + (1-\lambda)\cdot u(\frac{1-\lambda r_1^{DD}}{1-\lambda}R(\theta))\, d\theta$$

($r_1^{DD}$ denotes the optimal $r_1$ in the case of the simple demand-deposit contract). Here, below $\theta^*(r_1^{DD})$ a bank run occurs and all agents consume 1, while above $\theta^*(r_1^{DD})$ there is no run; impatient agents consume $r_1^{DD}$ and patient ones consume $\frac{1-\lambda r_1^{DD}}{1-\lambda}R(\theta)$. In the case of suspension of convertibility, the ex-ante expected utility is ($r_1^S$ denotes the optimal $r_1$ in this case):

$$\int_0^{\underline{\theta}(r_1^S)} \lambda \cdot u(r_1^S) + (1-\lambda)\cdot\left(\lambda\cdot u(0) + (1-\lambda)\cdot u\left(\frac{1-\lambda r_1^S}{1-\lambda}R(\theta)\right)\right)d\theta$$
$$+\int_{\underline{\theta}(r_1^S)}^1 \lambda\cdot u(r_1^S) + (1-\lambda)\cdot u\left(\frac{1-\lambda r_1^S}{1-\lambda}R(\theta)\right)d\theta \quad .$$

Below $\underline{\theta}(r_1^S)$ there is a run. $\lambda$ agents receive and consume $r_1^S$ in period 1, while the remaining 1-$\lambda$ receive $\frac{1-\lambda r_1^S}{1-\lambda}R(\theta)$ in period 2. Assuming that the probability of being denied payoff in period 1 is independent of the agent's type, proportion $\lambda$ of them are impatient and consume 0, while proportion 1-$\lambda$ are patient and consume $\frac{1-\lambda r_1^S}{1-\lambda}R(\theta)$.

By comparing the above two expressions, we can easily see that suspension of convertibility might yield lower welfare. Indeed, for a given $r_1$, $\underline{\theta}(r_1)$ must be below $\theta^*(r_1)$. This means that for any fixed degree of risk sharing, bank runs are less likely under this regime. If this were the only consideration, suspension of convertibility would be the superior contract even when $r_1$ is chosen optimally in each case. However, the drawback is that when a run does occur, fraction *(1-*$\lambda$*)*$\lambda$ of the agents consumes 0 (the others consume $r_1^s$ or $\frac{1-\lambda r_1^S}{1-\lambda}R(\theta)$). If agents are very risk averse, this might be far worse than the outcome under the demand-deposit contract, where in case of a run all agents consume 1.[18]

Another disadvantage of suspension of convertibility is that it does not allow full liquidation of the investment asset even in cases in which it is efficient to do so. That is, when the long term return *R* is below the short-term return of 1, the bank will not liquidate the whole long-term asset, but rather only a portion of it.

Another point of comparison between the two regimes is the amount of liquidity that is provided, that is, which mechanism yields a higher level of $r_1$. Since for a fixed $r_1$ runs occur less frequently with suspension of convertibility, one might expect that the optimal level of liquidity be higher in that case. However, since bank runs can be more costly (as

[18] For simplicity and tractability, the demand-deposit contract that we studied earlier specifies that in the case of a full bank run ($n$=1), the bank's resources are split equally among all agents. The alternative assumption is the 'sequential service constraint'. Here, $1/r_1$ agents are paid the promised return of $r_1$, while the remaining are paid 0. Even under this specification, our conclusion that suspension of convertibility might yield lower expected welfare still holds. The reason is that under this regime, there are circumstances in which when there is a run, *more* agents are forced to consume 0.

we saw above), the bank might find it optimal to offer less liquidity. To see this more formally, consider the first order conditions that determine $r_1^{DD}$ and $r_1^{S}$ (again, in the limit as $\varepsilon$ and 1-$\overline{\theta}$ and converge to 0). The condition that determines $r_1^{DD}$ is:

$$\lambda\int_{\theta^*(r_1^{DD})}^{1} u'(r_1^{DD}) - R(\theta)u'(\frac{1-\lambda r_1^{DD}}{1-\lambda}R(\theta))\,d\theta$$

$$-\frac{\partial\theta^*(r_1^{DD})}{\partial r_1^{DD}}\left[\left(\lambda u(r_1^{DD}) + (1-\lambda)u(\frac{1-\lambda r_1^{DD}}{1-\lambda}R(\theta^*(r_1^{DD})))\right) - u(1)\right] = 0$$

The benefit of increasing $r_1^{DD}$ is the difference in marginal utilities, and is obtained for realizations of $\theta$ above $\theta^*(r_1^{DD})$. The cost (second line) is that $\theta^*(r_1^{DD})$ rises: runs occur for more realizations of $\theta$, and for these realizations all agents receive 1 (rather than $r_1^{DD}$ or $u(\frac{1-\lambda r_1^{DD}}{1-\lambda}R(\theta^*(r_1^{DD}))$, depending on their type).

while $r_1^{S}$ is determined by:

$$u'\left(r_1^S\right) - \int_0^{\underline{\theta}(r_1^S)}(1-\lambda)R(\theta)\cdot u'\left(\frac{1-\lambda r_1^S}{1-\lambda}R(\theta)\right)d\theta - \int_{\underline{\theta}(r_1^S)}^{1} R(\theta)\cdot u'\left(\frac{1-\lambda r_1^S}{1-\lambda}R(\theta)\right)d\theta$$

$$-\frac{\partial\underline{\theta}(r_1^S)}{\partial r_1^S}(1-\lambda)\left(u\left(\frac{1-\lambda r_1^S}{1-\lambda}R(\underline{\theta}(r_1^S))\right) - u(0)\right) = 0$$

Under high degrees of risk aversion, the cost associated with the increased probability of a bank run (second line) will be higher than the one in the former case, since now some agents receive 0 rather than $\frac{1-\lambda r_1^S}{1-\lambda}R(\underline{\theta}(r_1^S))$. Thus, the optimal level of liquidity provided by the bank under suspension of convertibility might be lower. (Note that while both $\frac{\partial\underline{\theta}(r_1^S)}{\partial r_1^S}$ and $\frac{\partial\theta^*(r_1^{NS})}{\partial r_1^{NS}}$ are positive, they cannot be ranked as their magnitudes depend on the derivatives of $R(\theta)$ in the appropriate ranges).

**Deposit insurance:**

The idea of deposit insurance is that the government promises to collect taxes and provide liquidity to the bank in case the bank faces financial distress (i.e., when the number of agents demanding early withdrawal $n$ exceeds the number of impatient agents $\lambda$. With deposit insurance, patient agents know that if they wait they will receive the promised return independently of the number of agents who run. Hence, panic-based runs are prevented: patient agents withdraw their deposits only when this is their dominant action, i.e., when $\theta$ is below $\underline{\theta}(r_1)$ (rather then below the higher threshold $\theta^*(r_1)$). However, deposit insurance also has a drawback, as it creates moral hazard: when the bank designs the optimal contract, it does not internalize the cost of the taxes that might be required to pay the insurance. Thus, the bank has an incentive to over-exploit the deposit insurance by setting $r_1$ higher than the socially optimal level.

To see this tradeoff explicitly, consider a continuum of identical banks, each choosing its own liquidity level $r_1$. The government promises to pay each bank, in period 1, a subsidy of $(n-\lambda)(r_1 - \frac{1-\lambda r_1}{1-\lambda})$. In this way, the bank can still pay patient agents who demand early withdrawal the promised payoff of $r_1$, but it only needs to liquidate that agent's share $\frac{1-\lambda r_1}{1-\lambda}$ of the investment left for period 2. Consequently, bank's resources suffice exactly to pay the promised return to agents who wait (that is, $\frac{1-\lambda r_1}{1-\lambda} R(\theta)$). The government's expenditure is financed by equally taxing all agents (i.e., the depositors of all the banks). Since banks are small and competitive, they offer contracts that maximize the expected welfare of the representative agent without taking into account the effect of the contract on the taxes that agents have to pay.

Let $r_1^*$ denote the equilibrium level of $r_1$ chosen by all banks. When $\theta < \underline{\theta}(r_1^*)$, an economy-wide bank run occurs; the tax that needs to be collected from each agent amounts to $(1-\lambda)(r_1^* - \frac{1-\lambda r_1^*}{1-\lambda}) = r_1^* - 1$. (Note that while subsidy is given to the 1-$\lambda$ patient agents who run, all agents share the tax.) Now consider a single bank choosing its own $r_1$, taking the tax burden as given. A run on this specific bank will occur when $\theta < \underline{\theta}(r_1)$. The bank will then set $r_1$ to maximize the following expression:

$$EU(r_1)=\begin{cases}\int_0^{\underline{\theta}(r_1)} u\left(r_1+1-r_1^*\right)d\theta \\ +\int_{\underline{\theta}(r_1)}^{\underline{\theta}(r_1^*)} \lambda\cdot u(r_1+1-r_1^*)+(1-\lambda)\cdot u\left(\frac{1-\lambda r_1}{1-\lambda}R(\theta)+1-r_1^*\right)d\theta & if\ r_1\geq r_1^* \\ +\int_{\underline{\theta}(r_1^*)}^{1} \lambda\cdot u(r_1)+(1-\lambda)\cdot u\left(\frac{1-\lambda r_1}{1-\lambda}R(\theta)\right)d\theta \\ \\ \int_0^{\underline{\theta}(r_1^*)} u\left(r_1+1-r_1^*\right)d\theta \\ +\int_{\underline{\theta}(r_1^*)}^{\underline{\theta}(r_1)} u(r_1)\ d\theta & if\ r_1\leq r_1^* \\ +\int_{\underline{\theta}(r_1)}^{1} \lambda\cdot u(r_1)+(1-\lambda)\cdot u\left(\frac{1-\lambda r_1}{1-\lambda}R(\theta)\right)d\theta \end{cases}$$

It is easy to see that the function is differentiable with respect to $r_1$ (the lower and upper derivatives at $r_1^*$ are equal). Thus, the equilibrium condition is that derivative be 0 when $r_1=r_1^*$:

$$0=\frac{\partial EU(r_1)}{\partial r_1}\Big|_{r_1=r_1^*}=\int_0^{\underline{\theta}(r_1^*)} u'(1)d\theta+\int_{\underline{\theta}(r_1^*)}^{1} \lambda\cdot u'(r_1^*)-\lambda R(\theta)\cdot u'\left(\frac{1-\lambda r_1^*}{1-\lambda}R(\theta)\right)d\theta$$

The above first order condition defines the equilibrium level of $r_1$. We can compare the equilibrium level of $r_1$ with the optimal level of $r_1$ (under the constraint that bank runs occur when $\theta<\underline{\theta}(r_1)$). Thus, a central planer who takes into account the cost of taxes will set $r_1$ to maximize the following expression:

$$\int_0^{\underline{\theta}(r_1)} u(1)d\theta+\int_{\underline{\theta}(r_1)}^{1} \lambda\cdot u(r_1)+(1-\lambda)\cdot u\left(\frac{1-\lambda r_1}{1-\lambda}R(\theta)\right)d\theta.$$

This yields the following first order condition that determines the optimal value of $r_1$:

$$-\frac{d\underline{\theta}(r_1)}{dr_1}\left[\lambda\cdot u(r_1)+(1-\lambda)\cdot u\left(\frac{1-\lambda r_1}{1-\lambda}R(\underline{\theta}(r_1))\right)-u(1)\right]. \\ +\int_{\underline{\theta}(r_1)}^{1} \lambda\cdot u'(r_1)-\lambda R(\theta)\cdot u'\left(\frac{1-\lambda r_1}{1-\lambda}R(\theta)\right)d\theta=0$$

Comparing the two first order conditions, we can see that in equilibrium banks offer short-term returns that are too high. This is the drawback of a deposit-insurance regime: because of the moral hazard, banks promise too high returns in the short term. These high returns are covered in turn by the agents themselves, in the form of taxes. The reason for this moral-hazard problem is that banks do not take into account the cost of the insurance when they set the promised short-term return.

We can now study the welfare implications of deposit insurance. Denoting by $r_1^{DD}$ and $r_1^{I}$ the short-term payoff offered under the standard demand-deposit contract and under deposit insurance, respectively, the agent's expected utility in the deposit-insurance case is:

$$\int_0^{\underline{\theta}(r_1^I)} u(1)d\theta + \int_{\underline{\theta}(r_1^I)}^1 \lambda \cdot u(r_1^I) + (1-\lambda)\cdot u\left(\frac{1-\lambda r_1^I}{1-\lambda} R(\theta)\right) d\theta ,$$

while in the standard demand-deposit case it is:

$$\int_0^{\theta^*(r_1^{DD})} u(1)d\theta + \int_{\theta^*(r_1^{DD})}^1 \lambda \cdot u(r_1^{DD}) + (1-\lambda)\cdot u\left(\frac{1-\lambda r_1^{DD}}{1-\lambda} R(\theta)\right) d\theta .$$

We can see that if the moral hazard is so severe that $r_1^{I}$ is very high, the welfare under deposit insurance might be lower than under a standard demand deposit contract. This is because of two reasons: First, if $r_1^{I}$ is very high, $\underline{\theta}(r_1^I)$ can be higher than $\theta^*(r_1^{DD})$. Second, the risk sharing that is provided under such a regime might be non-efficient: i.e. $\lambda \cdot u(r_1^{DI}) + (1-\lambda)\cdot u\left(\frac{1-\lambda r_1^{DI}}{1-\lambda} R(\theta)\right)$ will be less than $\lambda \cdot u(r_1^{DD}) + (1-\lambda)\cdot u\left(\frac{1-\lambda r_1^{DD}}{1-\lambda} R(\theta)\right)$ for a large range of the fundamentals.

One way to prevent the moral hazard problem under a regime of deposit insurance is to supervise the banks, and to prevent them from setting $r_1$ above the optimal level. This policy measure is indeed used, as governments usually restrict insured banks from having a large mismatch between short-term liabilities and long term assets. (Another moral hazard consideration that requires government monitoring is the riskiness of the bank's investment portfolio.) However, in reality such a policy measure might have its own drawbacks, since the government does not always have all the information that banks have, and thus does not necessarily know what is the optimal level of $r_1$.

# 6. Concluding remarks

We studied a model of bank runs, which is based on D&D's framework. While their model has multiple equilibria, ours has a unique equilibrium in which a run occurs if and only if the fundamentals of the economy are below some threshold level. Nonetheless, there are panic-based runs: runs that occur when the fundamentals are good enough that agents would not run if they believed that others would not.

Knowing when runs occur, we could compute their probability. We found that this depends on the contract offered by the bank: banks become more vulnerable to runs when they offer more risk sharing. However, even when this destabilizing effect is taken into account, banks still improve the welfare of agents by offering demand deposit contracts. We characterized the optimal demand deposit contract, and showed that this contract trades off the benefits from risk sharing against the cost of bank runs. We then used the tools that we developed to assess regimes that are intended to prevent bank runs.

In the rest of this section, we discuss the necessity of three main assumptions that we made in the paper: the existence of an upper dominance region, the uniform distribution of the fundamentals, and the uniform distribution of noise.

**Upper dominance region**

A crucial condition that leads to a unique equilibrium in our model is the existence of an upper dominance region (implied by the assumption on the technology or by the alternative of an external lender). Indeed, the range of fundamentals in which not running is the agents' dominant action can be taken to be arbitrarily small, thereby representing extreme situations where fundamentals are extraordinarily good. Thus, this assumption is rather weak. Nevertheless, we now explore the model's predictions when this assumption is not made.

First, we note that without an upper dominance region, our model has multiple equilibria. Two are easy to point out: one is the trivial, bad equilibrium, in which agents run for any signal. The other is the threshold equilibrium that was characterized as the unique equilibrium of our model. In addition, we cannot preclude the existence of other equilibria in which agents run at all signals below $\theta^*$, while above they sometimes run and sometimes don't.

Importantly, the unique equilibrium that was characterized in Theorem 1 is the only equilibrium that survives three different equilibrium selection criteria (refinements). Thus, if we adopt any of these selection criteria, we can still analyze the model without the assumption of an upper dominance region, and obtain the same results. We now list these refinements:

<u>Equilibrium selection criterion a:</u> *The patient agents coordinate on the best equilibrium*.

By "best" equilibrium we mean the equilibrium that Pareto-dominates all others. In our model, this is also the equilibrium at which the set of signals at which agents run is the smallest.[19]

<u>Equilibrium selection criterion b:</u> *The equilibrium on which patient agents coordinate has the property that when they observe signals that are extremely high* ($\theta_i \in [1-2\varepsilon, 1]$)*, they do not run.*

The idea is that a panic-based run does not happen when agents know that the fundamentals are excessively good. As with the assumption of the upper dominance region, this is sufficient in order to rule out runs in a much larger range of parameters.

<u>Equilibrium selection criterion c:</u> *The equilibrium on which patient agents coordinate has monotonic strategies and is non-trivial (i.e., agents' actions depend on their signals).*

Importantly, while some other papers in the literature use equilibrium selection criteria (the "best equilibrium" criterion) they always select equilibria with no panic-based runs. That is, in their best equilibrium runs occur only when early withdrawal is the agents' dominant action: when $\theta < \underline{\theta}(r_1)$ (See for example Goldfajn and Valdes (1997), and Allen and Gale

[19] An equilibrium with this property exists (and "smallest" is well defined) if $\varepsilon$ is small enough, and is the same as the equilibrium characterized in Theorem 1. The reason is that we can show by iterative dominance that patient agents must run below $\theta^*$, and thus that equilibrium, which has the property that patient agents run below $\theta^*$ and never run above, is the one with the smallest set of signals at which agents run. (Note also that since patient agents are small and identical, they all behave in the same manner.)

(1998)). In our model, by contrast, the equilibrium does have panic-based bank runs (even if it is not a unique equilibrium): agents run whenever $\theta<\theta^{*}(r_1)$. Therefore, we can analyze the interdependence between the banking contract and the probability that agents will lose their confidence in the solvency of the bank. This is the main novelty of our paper and it is thus maintained even if the model had multiple equilibria (which would be the case had the assumption of an upper dominance region been dropped).

**Uniform distribution of the fundamentals**

This assumption does not limit the generality of the model in any important way. This is because there are almost no restrictions on the function $R(\theta)$ which relates the state $\theta$ to the long-term return. Thus, any distribution of the long-term return on investments is allowed. Moreover, in analyzing the case of small noise ($\varepsilon\to 0$), this assumption can be dropped.

**Uniform distribution of noise**

This assumption is important in deriving a unique equilibrium when the payoff structure does not satisfy full strategic complementarities (as is the case in our model). However, Morris and Shin (2001), who discuss our result, show that if one limits the analysis to monotone strategies, then a unique equilibrium exists for a broader class of distributions.

# Appendix

## Proof of Theorem 1

We say that a patient agent *acts according to threshold* $\theta$ if she goes to the bank in period 1 (2) when her signal is below (above) $\theta$. We denote by $n(\theta,\theta')$ the number of agents demanding early withdrawal when the state is $\theta$ and each agent acts according to threshold $\theta'$. By $n(\theta)$ we denote an agent's arbitrary *feasible* belief regarding the proportion of agents demanding early withdrawal as a function of the true state $\theta$. By 'feasible' we mean that belief $n(\theta)$ must be derived from a belief as to how individual agents act as a function of their signals. This places a restriction that the derivative of $n(\theta)$ with respect to $\theta$ is always in the interval $[-(1-\lambda)/2\varepsilon,(1-\lambda)/2\varepsilon]$. (This is because $n(\theta)=\lambda+\frac{(1-\lambda)}{2\varepsilon}\cdot\int_{\theta_i=\theta-\varepsilon}^{\theta+\varepsilon}(\text{proportion of agents who run at signal }\theta_i)d\theta_i$ ).

Let $v(\theta,n)$ denote the difference in a patient agent's utility from going to that bank in period 2 vs. period 1, if she knew that the state is $\theta$ and that the number of agents demanding early withdrawal is $n$. For $\theta<\overline{\theta}$, it is given by:[20]

$$v(\theta,n)=\begin{cases} u(0)-u(\frac{1}{n}) & \text{if } 1\geq n\geq \frac{1}{r_1} \\ u(\frac{1-nr_1}{1-n}R(\theta))-u(r_1) & \text{if } \frac{1}{r_1}\geq n\geq \lambda \end{cases}$$

Finally, let $\Delta^{r_1}(\theta_i,n(\theta))$ denote a patient agent's expected difference in utility from going to the bank in period 2 rather than 1, when she observes signal $\theta_i$ and holds belief $n(\theta)$. When an agent observes signal $\theta_i$, her posterior distribution of $\theta$ is uniform over the interval $[\theta'-\varepsilon,\theta'+\varepsilon]$. Thus, $\Delta$ is given by:

$$\Delta^{r_1}(\theta_i,n(\theta))=\int_{\theta=\theta_i-\varepsilon}^{\theta_i+\varepsilon}v(\theta,n(\theta))\,d\theta\,.$$

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[20] For brevity, in the rest of the proof we use this definition of $v$ even if $\theta$ is above $\overline{\theta}$. It is trivial to check that all our arguments go through even if the correct definition of $v$ for that range is used.

Lemma 1 states two properties of $\Delta^{r_1}(\theta_i, n(\theta))$. First, it is continuous in $\theta_i$, because small changes in an agent's signal $\theta_i$ slightly shifts the interval $[\theta_i\text{-}\varepsilon, \theta_i+\varepsilon]$ over which the agent calculates expectations. Second, a positive shift in the observed signal, accompanied by the same shift of *n*, increases the gain from waiting. The reason is that the agent sees the same distribution of *n*, but expects $\theta$ to be higher.

LEMMA 1: $\Delta^{r_1}(\theta_i, n(\theta))$ is continuous in $\theta_i$ and in *n*(▶ )[21]. Moreover, If *a*>0 and there exists $\theta \in [\theta_i - \varepsilon, \theta_i + \varepsilon]$ for which $n(\theta) < 1/r_1$, then $\Delta^{r_1}(\theta_i, n(\theta)) < \Delta^{r_1}(\theta_i + a, n(\theta + a))$.

**Proof of Lemma 1:**

Continuity holds because a change in $\theta_i$ only changes the limits of integration in the computation of Δ, and because the integrand is bounded in *n*. For the second claim, note that the only difference between the integrals that define $\Delta^{r_1}(\theta_i, n(\theta))$ and $\Delta^{r_1}(\theta_i + a, n(\theta + a))$ is that in the first we use $R(\theta)$ while in the second we use $R(\theta+a)$, which is larger. This will make the second integral strictly larger if, over the limits of integration, there is a segment where $n(\theta) < 1/r_1$. QED.

We first show that a threshold equilibrium, i.e., an equilibrium in which all agents act according to some common threshold, exists. By Lemma 1, $\Delta^{r_1}(\theta', n(\theta, \theta'))$ is continuous in $\theta'$. By the existence of dominance regions, it is negative below $\underline{\theta}(r_1) - \varepsilon$ and positive above $\overline{\theta}(r_1) + \varepsilon$. Thus, there exists some $\theta^*$ at which it equals 0. Now assume that all agents to act according to the threshold $\theta^*$. This is an equilibrium if:

(1) $\Delta^{r_1}(\theta_i, n(\theta, \theta^*)) > \Delta^{r_1}(\theta^*, n(\theta, \theta^*)) = 0$ for $\theta_i > \theta^*$, and

(2) $\Delta^{r_1}(\theta_i, n(\theta, \theta^*)) < \Delta^{r_1}(\theta^*, n(\theta, \theta^*)) = 0$ for $\theta_i < \theta^*$.

To see why (1) holds, note that the intervals over which the two integrals are computed can be decomposed into a (maybe empty) common part $c = [\theta_i - \varepsilon, \theta_i + \varepsilon] \cap [\theta^* - \varepsilon, \theta^* + \varepsilon]$, and disjoint intervals $d^i = [\theta_i - \varepsilon, \theta_i + \varepsilon] \setminus c$ and $d^* = [\theta^* - \varepsilon, \theta^* + \varepsilon] \setminus c$. Thus,

[21] To define continuity in *n* let the distance between functions $n_1(\theta)$ and $n_2(\theta)$ be $\sup\{n_1(\theta) - n_2(\theta)\}$.

$\Delta^{r_1}(\theta_i, n(\theta,\theta^*)) - \Delta^{r_1}(\theta^*, n(\theta,\theta^*)) = \int_{\theta\in d^i} v(\theta, n(\theta,\theta^*)) - \int_{\theta\in d^*} v(\theta, n(\theta,\theta^*))$. The intervals $d^i, d^*$ have the same length. However, $v(\theta, n(\theta,\theta^*)$ is higher when $\theta$ is in the interval $d^i$ (where both $n(\theta,\theta^*) = \lambda$ and $\theta$ is large) then when $\theta$ is in the interval $d^*$ (where both $n(\theta,\theta^*) \geq \lambda$ and $\theta$ is small).

To see why (2) holds, we use another lemma. Lemma 2 shows that the game between the agents satisfies a weak form of *strategic complementarities*: under some restrictions, an agent's incentive to run on the bank is higher if she knows that other agents tend to run more often (i.e., if the function $n(\theta)$ is higher).[22]

LEMMA 2: Assume that $0 \leq \theta_2 - \theta_1 \leq 2\varepsilon$ and that for all $\theta \in [\theta_1, \theta_2]$, $f(\theta) \leq \theta_1$ and $n'(\theta) \geq n(\theta) = \lambda + (1-\lambda)(\theta_2 - \theta)/2\varepsilon$. If $\int_{\theta_1}^{\theta_2} v(\theta, n(\theta))\, d\theta \geq 0$ then $\int_{\theta_1}^{\theta_2} v(\theta, n(\theta))\, d\theta > \int_{\theta_1}^{\theta_2} v(f(\theta), n'(\theta))\, d\theta$.

**Proof of Lemma 2:**

The lemma clearly holds if $v(\theta, n(\theta))$ is positive for all $\theta \in [\theta_1, \theta_2]$, since whenever $v$ is positive it must be decreasing in $n$ and increasing in $\theta$. Otherwise, let $\theta_0$ be (the unique) $\theta \in [\theta_1, \theta_2]$ at which $v(\theta, n(\theta)) = 0$, and let $\theta_0'$ be the smallest $\theta \in [\theta_1, \theta_2]$ at which $v(\theta, n'(\theta)) = 0$ (if $v(\theta, n'(\theta))$ is negative for all $\theta \in [\theta_1, \theta_2]$ the lemma trivially holds). Now, $\int_{\theta_1}^{\theta_0} v(\theta, n(\theta))\, d\theta \geq \int_{\theta_1}^{\theta_0'} v(f(\theta), n'(\theta))\, d\theta$ since $n'(\theta_1) \geq n(\theta_1)$ and $n(\theta)$ decreases in the fastest feasible rate (see page 31) from $\theta_1$ to $\theta_0$, implying that $n'(\theta)$ ranges over all the negative values that $n(\theta)$ reaches, but 'stays longer' at each[23]. (Recall also that $f(\theta') \leq \theta''$

[22] In fact, for (2) we need less than Lemma 2. The lemma is needed, however, later in the proof.

[23] More formally, by changing variables of integration we have $\int_{\theta_1}^{\theta_0} v(\theta, n(\theta))\, d\theta = \int_{n(\theta_0)}^{n(\theta_1)} v(\theta, n(\theta)) \left|\frac{\partial\theta}{\partial n}\right| dn$ and $\int_{\theta_1}^{\theta_0'} v(f(\theta), n'(\theta))\, d\theta = \int_{\min\{n'(\theta):\theta\in[\theta_1,\theta_0']\}}^{\max\{n'(\theta):\theta\in[\theta_1,\theta_0']\}} v(f(\theta), n'(\theta)) \left|\frac{\partial\theta}{\partial n'}\right| dn' + \int_{\{\theta:\frac{\partial n'}{\partial\theta}=0\}} v(f(\theta), n'(\theta)) d\theta$. The second integral is at least as negative because it is computed over a range which is at least as large, because $\left|\frac{\partial\theta}{\partial n'}\right| \geq \left|\frac{\partial\theta}{\partial n}\right|$, and because $v$ is negative.

for all $\theta',\theta''\in[\theta_1,\theta_2]$ and that $v$ is non decreasing in $\theta$.) Now, since $n(\theta)$ decreases faster, $\theta_0'$ is no smaller than $\theta_0$. Thus, and since $v$ is decreasing in $n$ and increasing in $\theta$ when $v$ is positive, we also have $\int_{\theta_0}^{\theta_2} v(\theta,n(\theta))\,d\theta \geq \int_{\theta_0'}^{\theta_2} v(\theta,n(\theta))\,d\theta > \int_{\theta_0'}^{\theta_2} v(f(\theta),n'(\theta))\,d\theta$.

Thus, $\int_{\theta_1}^{\theta_2} v(\theta,n(\theta))\,d\theta > \int_{\theta_1}^{\theta_2} v(f(\theta),n'(\theta))\,d\theta$. QED.

Decomposing the integrals as in part 1, and denoting the interval $d^*$ as $[\theta_1,\theta_2]$, we have $\Delta^{r_1}(\theta_i,n(\theta,\theta^*))-\Delta^{r_1}(\theta^*,n(\theta,\theta^*))=\int_{\theta=\theta_1}^{\theta_2} v(\theta+\theta_i-\theta^*,n(\theta+\theta_i-\theta^*,\theta^*))-\int_{\theta=\theta_1}^{\theta_2} v(\theta,n(\theta,\theta^*))$ Now, $\theta+\theta_i-\theta^*<\theta_1$ and $n(\theta+\theta_i-\theta^*,\theta^*)\geq n(\theta,\theta^*)=\lambda+(1-\lambda)(\theta_2-\theta)/2\varepsilon$ for all $\theta\in[\theta_1,\theta_2]$, and $\int_{\theta=\theta_1}^{\theta_2} v(\theta,n(\theta,\theta^*))=0$. By Lemma 2, $\int_{\theta=\theta_1}^{\theta_2} v(\theta,n(\theta+\theta_i-\theta^*,\theta^*))<0$.

It is easy to see that there must be only one threshold equilibrium. This simply follows from the fact that $\Delta^{r_1}(\theta',n(\theta,\theta'))$ is increasing in $\theta'$ (Lemma 1). Thus, to conclude the proof that the equilibrium is unique, we need to show that it must be a threshold equilibrium.

Let $\theta_B=\sup\{\theta_i:\Delta^{r_1}(\theta_i,n(\theta))\leq 0\}$. By the existence of the upper dominance region, $\theta_B<1-2\varepsilon$. By continuity of Δ, $\Delta^{r_1}(\theta_B,n(\theta))=0$. First note that $\frac{\partial}{\partial\theta_B}\Delta^{r_1}(\theta_B,n(\theta))=v(\theta_B+\varepsilon,n(\theta_B+\varepsilon))-v(\theta_B-\varepsilon,n(\theta_B-\varepsilon))>0$, since $n(\theta_B-\varepsilon)\leq n(\theta_B+\varepsilon)=\lambda$ and $\theta_B-\varepsilon<\theta_B+\varepsilon$ (note that given $\theta$, $v$ is maximized when $n=\lambda$). This means that $\Delta<0$ along some nontrivial segment to the left of $\theta_B$. If the equilibrium is a threshold equilibrium, this segment is simply $[0,\theta_B]$. We now show that this must be the case.

Assume to the contrary that there are signals below $\theta_B$ at which $\Delta\geq 0$. Denote $\theta_A=\sup\{\theta_i<\theta_B:\Delta^{r_1}(\theta_i,n(\theta))\geq 0\}$. By continuity, $\Delta^{r_1}(\theta_A,n(\theta))=\Delta^{r_1}(\theta_B,n(\theta))=0$. Let $c=(\theta_A-\varepsilon,\theta_A+\varepsilon)\cap(\theta_B-\varepsilon,\theta_B+\varepsilon)$, $d^A=[\theta_A-\varepsilon,\theta_A+\varepsilon]\setminus c$ and $d^B=[\theta_B-\varepsilon,\theta_B+\varepsilon]\setminus c$. Denote the interval $d^B$ as $[\theta_1,\theta_2]$, and consider the transformation $\overleftarrow{\theta}=\theta_1+\theta_2-2\varepsilon-\theta$. (Note that as $\theta$ moves from the lower end of $d^B$ to its upper end, $\overleftarrow{\theta}$ moves from the upper end of $d^A$ to its lower end.) We have $\Delta^{r_1}(\theta_B,n(\theta))-\Delta^{r_1}(\theta_A,n(\theta))=$

$\int_{\theta=\theta_1}^{\theta_2} v(\theta, n(\theta)) - \int_{\theta=\theta_1}^{\theta_2} v(\overleftarrow{\theta}, n(\overleftarrow{\theta}))$. Now, for $\theta \in [\theta_1, \theta_2]$ we have $n(\overleftarrow{\theta}) \geq n(\theta) = \lambda + (1-\lambda)(\theta_2 - \theta)/2\varepsilon$. This is because $n(\overleftarrow{\theta}_1) \geq n(\theta_1)$ and $n(\theta)$ decreases at the fastest feasible rate between $\theta_1$ and $\theta_2$. We also know that $\int_{\theta=\theta_1}^{\theta_2} v(\theta, n(\theta)) \geq 0$. (If $\int_{\theta=\theta_1}^{\theta_2} v(\theta, n(\theta))$ were negative, we would also have $\int_{\theta \in c} v(\theta, n(\theta)) < 0$, contradicting the fact that their sum, $\Delta^{r_1}(\theta_B, n(\theta))$ equals 0.) Moreover, $\overleftarrow{\theta} < \theta_1$ for all $\theta \in (\theta_1, \theta_2]$. Thus, by Lemma 2, $\int_{\theta=\theta_1}^{\theta_2} v(\overleftarrow{\theta}, n(\overleftarrow{\theta})) < \int_{\theta=\theta_1}^{\theta_2} v(\theta, n(\theta))$. This implies that $\Delta^{r_1}(\theta_B, n(\theta)) > \Delta^{r_1}(\theta_A, n(\theta))$, a contradiction. QED.

**Proof of Theorem 2**

The equation that defines $\theta^*(r_1)$ is $f(\theta^*, r_1) = 0$, where

$$f(\theta^*, r_1) = \frac{1}{2\varepsilon}\left( \int_{\theta=\theta^*(r_1)-\varepsilon}^{\tilde{\theta}(r_1)} u(\frac{1}{n(\theta, \theta^*(r_1))}) - u(0)\, d\theta + \int_{\theta=\tilde{\theta}(r_1)}^{\theta^*(r_1)+\varepsilon} u(r_1) - u(R(\theta)\frac{1 - r_1 n(\theta, \theta^*(r_1))}{1 - n(\theta, \theta^*(r_1))})\, d\theta \right)$$

Performing the change of variables $n = \lambda + (1-\lambda)(\frac{1}{2} + \frac{\theta^*(r_1) - \theta}{2\varepsilon})$ , we obtain:

$$f(\theta^*, r_1) = \frac{1}{1-\lambda}\left( \int_{n=\lambda}^{1/r_1} u(R(\theta(\theta^*, n))\frac{1 - r_1 n}{1-n}) - u(r_1)\, dn + \int_{n=1/r_1}^{1} u(0) - u(\frac{1}{n})\, dn \right) = 0$$

where $\theta(\theta^*, n) = \theta^* + \varepsilon(1 - 2\frac{n-\lambda}{1-\lambda})$. Differentiating with respect to $\theta^*$ and $r_1$ we get:

$$\frac{\partial f(\theta^*, r_1)}{\partial \theta^*} = \frac{1}{1-\lambda}\int_{n=\lambda}^{1/r_1} u'(R(\theta(\theta^*, n))\frac{1 - r_1 n}{1-n}) \cdot R'(\theta(\theta^*, n)) \cdot \frac{\partial \theta(\theta^*, n)}{\partial \theta^*}\, dn > 0$$

$$\frac{\partial f(\theta^*, r_1)}{\partial r_1} = \frac{1}{1-\lambda}\int_{n=\lambda}^{1/r_1} u'(R(\theta(\theta^*, n))\frac{1 - r_1 n}{1-n}) \cdot R(\theta(\theta^*, n))\frac{-n}{1-n} - u'(r_1)\, dn < 0$$

(Note that the derivatives with respect to the integral limits cancel out.)

Hence, by the implicit function theorem, $\frac{\partial\theta^*}{\partial r_1} = -\frac{\partial f / \partial r_1}{\partial f / \partial\theta^*} > 0$. QED.

**Proof of Theorem 3**

When $r_1$=1, the derivative of $EU(r_1)$ with respect to $r_1$ is:

$$\begin{aligned}\frac{\partial EU(r_1)}{\partial r_1} = {} & \tfrac{1-\lambda}{2\varepsilon}\frac{\partial\theta^*}{\partial r_1}\int_{\theta^*(1)-\varepsilon}^{\theta^*(1)+\varepsilon} u(1) - u(R(\theta))\,d\theta \\ & + \int_{\theta^*(1)-\varepsilon}^{\theta^*(1)+\varepsilon} n(\theta,\theta^*(r_1))\cdot\left(u'(1) - u'(R(\theta))R(\theta)\right)d\theta \\ & + \int_{\theta^*(1)+\varepsilon}^{\overline{\theta}} \lambda\left(u'(1) - u'(R(\theta))R(\theta)\right)d\theta \\ & + \int_{\overline{\theta}}^{1} \lambda(u'(1) - \lambda u'(\tfrac{1-\lambda^2}{1-\lambda}R(\theta))R(\theta))d\theta\end{aligned}$$

We first check the case where $\varepsilon$ and 1-$\overline{\theta}$ approach 0. The first term is 0 by the definition of $\theta^*$. Thus, the expression converges to $\int_{\theta^*(1)}^{1} \lambda\left(u'(1) - u'(R(\theta))R(\theta)\right)d\theta$ , which is positive because $-cu''(c)/u'(c)$ is greater than 1, and because $\theta^*(1)$ converges to $\underline{\theta}(1)$, and $R(\underline{\theta}(1)) = 1$. Thus, in this case, it is always optimal to increase $r_1$ above 1.

Now consider the case of an arbitrary $\varepsilon$. The first term is again 0. Let $\hat{\theta}$ be the value of $\theta$ at which $R(\theta) = 1$, and note that $u'(R(\theta))R(\theta) < u'(1)$ exactly when $\theta > \hat{\theta}$. The second term is at least

$$\int_{\theta^*(1)-\varepsilon}^{\hat{\theta}} n(\theta,\theta^*(r_1))\cdot\left(u'(1) - u'(R(\theta))R(\theta)\right)d\theta + \int_{\hat{\theta}}^{\theta^*(1)+\varepsilon} \lambda\cdot\left(u'(1) - u'(R(\theta))R(\theta)\right)d\theta.$$

The sum of the third and fourth terms is greater than:

$$\int_{\theta^*(1)+\varepsilon}^{1} \lambda\left(u'(1) - u'(R(\theta))R(\theta)\right)d\theta$$

Thus, the whole expression is greater than:

$$\int_{\theta^*(1)-\varepsilon}^{\hat{\theta}} n(\theta,\theta^*(r_1))\cdot\left(u'(1)-u'(R(\theta))R(\theta)\right)d\theta+\int_{\hat{\theta}}^{1}\lambda\cdot\left(u'(1)-u'(R(\theta))R(\theta)\right)d\theta\,.$$

Which is at least:

$$\int_{0}^{\hat{\theta}} 1\cdot\left(u'(1)-u'(R(\theta))R(\theta)\right)d\theta+\int_{\hat{\theta}}^{1}\lambda\cdot\left(u'(1)-u'(R(\theta))R(\theta)\right)d\theta$$
$$=-B\cdot F(1)+A\cdot(1-F(1))$$

Which is positive by the condition of the theorem. Thus, at $r_1$=1, the derivative of $EU(r_1)$ with respect to $r_1$ is positive. QED.

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