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Majority rule when voters like to win

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Abstract

I analyze symmetric majority rule voting equilibria when voters wish to elect the better candidate *and* to vote for the winner. When voters care only about the winning candidate (the standard formulation) a unique responsive equilibrium exists. The addition of a desire to win creates multiple equilibria, some with unusual properties. In most of these equilibria information is not aggregated effectively, and I uncover the novel possibility of *negative* information aggregation in which information aggregated in equilibrium is used to select the worse rather than the better candidate.

I then characterize the efficiency of optimal equilibria as the population becomes large and show that a discontinuity arises in the information aggregation capabilities of the majority rule voting mechanism: in elections without a dominant front-running candidate the better candidate is almost surely elected, whereas in races with a front-runner information cannot be effectively aggregated in equilibrium.

JEL classification: D72

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1. Introduction

People are sheep. This truism, in addition to being a common observation on everyday life, reflects a basic finding of psychology. Beginning with the famous conformity experiments of Asch, a large literature has arisen documenting the fact that, in a variety of circumstances and environments, people exhibit a tendency to conform with group behavior (Asch, 1951, 1956; see Aronson et al., 1997 for a general discussion).

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A prominent explanation of conformity is information, and in particular the social learning of asymmetric information. According to this explanation, people, when faced with a decision problem, learn from the actions of others and adjust their behavior accordingly (such as in information cascades). Although information surely drives a significant fraction of conformist behavior, it does not explain all such behavior. Rather, psychologists have established that a large part of conformist behavior is non-informational, what may best be described as the result of social pressures to conform.[1] Moreover, psychologists have shown that conformity induced by social pressure is robust to experimental conditions and can arise in a wide variety of practical situations.

An important environment where an incentive to conform manifests itself is political decision making. In work that motivated Asch's experiments, Newcomb (1943) showed that political ideologies are malleable to group social pressures. More recently, empirical and experimental work in political science has shown that voters are motivated in part by a desire to vote for the winning candidate (see Bartels, 1985 and Niemi and Bartels, 1984, respectively). Indeed, one may argue that social pressures to conform apply a fortiori to majority voting decisions as, by definition, conforming with a majority choice corresponds with joining the winning team. Thus, any incentive an individual has to conform with majority choice is reinforced by their enjoyment of winning.[2]

The objective of this paper is to understand the impact that an incentive to win (or conform) has on political outcomes. I take as a primitive of the model that voters wish to conform with the majority and then explore the implications. In particular, I seek to understand how an incentive to conform affects individual voting behavior and, more importantly, what it implies for election outcomes and the ability of the voting mechanism to aggregate dispersed information.

The model I consider is a variant of the now-standard model of voting with incomplete information made prominent by Austen-Smith and Banks (1996). Two candidates compete for election, one of whom is competent and the other incompetent, and voters are imperfectly informed about which candidate is preferred (there is a common interest over competence). In the model of Austen-Smith and Banks (1996) it is assumed that voters are concerned only with selecting the better candidate (or, in their jury context, reaching the correct verdict). I amend their formulation and suppose that, in addition to preferring the better candidate, voters receive a utility bonus should they support the winning candidate.

I show here that the addition of a desire to conform has a significant impact on both behavior and outcomes. For the standard formulation (with no desire to conform), Feddersen and Pesendorfer (1998) prove that there exists a unique symmetric *responsive* equilibrium, and that as the population becomes large this equilibrium almost surely selects the better of the two candidates.[3] In contrast, I show that if voters additionally possess a desire to conform then neither of these results holds, regardless of the relative strength of the conformity preference. Moreover,

[1] Although more careful experimental work has been done, the non-informational property of conformity can be inferred from Asch's original experiments. Subjects were asked to select from several choices the line that was longest. Considering the significant differences in the alternatives offered, it is hard to argue that informational asymmetries played much of a role in subject choice.

[2] Just as a football fan likes to support a winning team, one may conclude that voters derive enjoyment from supporting a winning candidate. As the correspondence between winning and conforming is exact under simple majority rule, I use the terms interchangeably throughout the paper.

[3] A strategy is responsive if voter behavior is determined, at least in part, by their private information.

I show that the impact of a desire to conform is uneven, yielding widely divergent outcomes that depend in a systematic way on the nature of the electoral contest.

The inclusion of a desire of voters to *win* creates competing incentives for each voter: wanting to select the better candidate *and* ensuring they are part of the winning majority. I show that these dueling incentives can be balanced in equilibrium in multiple ways, and that uniqueness of responsive equilibria no longer generally holds. Indeed, I find that some surprising and unusual behavior can be supported in equilibrium. For example, I uncover a generic class of equilibria in which the ex ante underdog candidate (according to prior beliefs) is more likely to win election than the candidate that is ex ante favored. Even more strikingly, I prove the existence of *unordered* equilibria. In these unintuitive equilibria, voters who receive private information favoring a particular candidate are actually less likely to support this candidate and more likely to vote for the *opposing* candidate.

The multiplicity of equilibria that arise are interesting not only for the peculiar behavior they entail, but also for what is implied for efficiency. The new equilibria vary continuously as the desire to conform varies in strength, and are *close* to equilibria in the standard model when the desire to conform is small. The equilibria that they are close to, however, are often the unresponsive equilibria in which no information is aggregated (when all voters support the same candidate), including the inefficient equilibrium in which all voters coordinate on the ex ante underdog candidate. Consequently, information aggregation is not only imperfect, but it can be dramatically inefficient. In many equilibria voters are more likely to select the better candidate if they abandon the election altogether and instead appoint the front-running candidate directly. In fact, I show for unordered equilibria that information can be aggregated negatively. In these extremely inefficient equilibria information is aggregated but is used counterproductively to select the *worse* rather than the better candidate. Consequently, voters are better served appointing even the underdog candidate rather than holding the election and playing an unordered equilibrium.

Despite the undesirable possibilities, I show that a desire of voters to win does not always inhibit the aggregation of information, and in some circumstances can reinforce it. These differences depend in a systematic way on the nature of the electoral contest according to whether the election is *tight* or *lopsided*. In a lopsided election there exists a strong front-running candidate, and this necessarily inhibits information aggregation. The presence of a front-runner skews expectations: if information is to be aggregated efficiently then the front-runner must win more frequently than the underdog, but if this is the case then voters are induced to abandon their private information and vote for the likely winner. In contrast, in tight elections a strong front-runner does not exist and the previous logic does not necessarily apply. In tight elections neither candidate dominates and this allows voters to vote informatively, even when they possess a desire to join the winning majority.

The differences between tight and lopsided elections are magnified as the population grows. The final results of the paper consider equilibrium behavior in large populations and show that a discontinuity arises in the efficiency of majority rule voting. Large populations offer the advantage of more aggregate information available to voters, but also simultaneously the disadvantage that each voter has a smaller probability that they will prove pivotal to the outcome. In lopsided elections the incentive to use private information is increasingly dominated by the desire to conform as the population grows, and in equilibrium the effective aggregation of information is not possible. In contrast, for tight elections the incentive to conform reinforces informative voting, and at the limit perfect information aggregation is possible, creating the discontinuity.

In addition to describing the level of information aggregation (*informational efficiency*), I also consider the ability of majority rule voting to maximize the sum of voter utility, what I refer to as *utilitarian efficiency*. This measure differs from informational efficiency in that it includes the utility boost voters receive from conforming. I show that these dual measures are often in conflict. Informative voting enables the effective use of information, but at the cost of a sizable losing minority. For lopsided elections I show that utilitarian and informational efficiency coincide on the unresponsive equilibria, whereas in tight elections utilitarian efficiency is optimized at different equilibria, depending on the relative strength of voters' desire to conform.

The results I present are largely, but not entirely, negative with respect to the effectiveness of majority rule voting and are perhaps best interpreted as cautionary about its general applicability. Although majority rule voting offers in some instances the prospect of a miraculous aggregation of dispersed private information, such success is far from guaranteed, and it is possible that outcomes are significantly worse than if the election had not been held at all. The results presented here are also instructive. The multitude of equilibrium possibilities suggest an explanation for why voting produces good outcomes in some circumstances, but puzzling results in others. Moreover, in addition to documenting this variety, I take a first small step toward explaining it, and by so doing provide recommendations for when majority rule voting should and should not be used.

1.1. Related literature

The extension of individual utility functions to include conformity has made little headway into economics, despite an exhortation by Akerlof (1997) that not only can such an extension be made, but that it should be made (see also Jones, 1984). Remarkably, however, and considerably predating Akerlof, the formulation I adopt was proposed by Hinich (1981, p. 135) who surmises that an assumption that voters desire voting for the winner "is no less plausible than the assumption that voters believe they can be pivotal." Hinich's contribution predates not only Akerlof, but both the game theoretic advance to voting of Palfrey and Rosenthal (1983) and the strategic voting advance of Austen-Smith and Banks (1996). My model embeds Hinich's insight into a considerably richer environment and, not surprisingly, derives considerably different conclusions.

The approach taken here is to ask, given an incentive of voters to conform, how are behavior and aggregate outcomes affected. A related and equally important question is where the preference to conform comes from. In the voting context many explanations are possible. First is the pure psychological motivation. There is no obvious reason why an innate preference for conformity that exists elsewhere does not also manifest in the vote decision. Indeed, given the small incentives provided by informational considerations, only a small preference to conform is needed to alter behavior.

Alternatively, an indirect preference for conformity may be induced by expanding the model and studying the rational response of voters to external pressures. In this vein, Bernheim (1994) constructs a model of social status and conformity that applies equally to politics.[4] More particularly, winners of elections typically control resources that can be used to reward supporters (jobs, contracts, etc.), providing an incentive for voters to conform.[5] Expanding the model to

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[4] Similarly, Prendergast (1993) develops a model of "yes men" which induces conforming behavior endogenously.

[5] Another example arises in the US Supreme Court where the writing of decisions is assigned to a member of the majority voting group. The power of authorship provides the holder with considerable influence over the final ruling, providing an incentive for justices to join a majority they may otherwise not.

allow for these or similar effects is relatively straightforward, but is not the focus here and is left for another time.

Several other papers follow Hinich (1981) and expand utility functions along the lines described here. Meirowitz and Wiseman (2005) consider a campaign contribution game and suppose that donors have a desire to support the winning candidate in addition to supporting their ideologically more preferred candidate. Glazer and Rubinstein (1998) study a related setting in which experts possess a desire to see their recommendation implemented, although they take a general mechanism design approach rather than studying a particular mechanism as done here.[6]

Companion to the current paper is Callander (2007) that explores the same model when voting is sequential and the population countably infinite. In that paper I show how a desire to conform induces momentum and bandwagons, two phenomena commonly observed in real sequential elections. In sequential voting—as here for simultaneous voting—the incentive to conform does not swamp the informational incentive, even in large populations. Voting bandwagons do not begin immediately with the amount of informative voting varying in the desire to conform. Surprisingly, informative voting is supported in equilibrium precisely because a bandwagon begins. Precisely because later voters follow the lead of earlier voters, earlier voters have the incentive to vote informatively. Thus, in sequential voting information aggregation is possible but it is necessarily incomplete.

Sequential voting differs from simultaneous voting also in the importance of tight versus lopsided elections. Although information is aggregated imperfectly in sequential voting, the quality of aggregation is independent of whether the election is tight or lopsided. Combining this with the discontinuity found here for simultaneous voting reveals an important conclusion about vote timing: that in tight elections simultaneous voting is better at aggregating information, whereas for lopsided elections sequential voting is preferred.

The discontinuity in information aggregation between tight and lopsided elections brings to the forefront the distinction between public and private information in elections. The addition of another private signal (by including another voter) has only a minimal impact on outcomes and behavior, yet the same information, if revealed publicly, can radically transform voting patterns if it switches the election from lopsided to tight or vice versa. This distinction resonates well with intuitions about real political campaigns (in that candidates must be credible for voters to reward them with their support) and is consistent with the emerging importance of public information in other settings (Morris and Shin, 2002; Chwe, 2001). Although I cannot address these issues directly here, I briefly consider the possibilities in the concluding discussion and speculate on how they would affect political behavior.

The remainder of the paper is organized as follows. The model is described in the following section and in Section 3 results for arbitrary populations are presented. Section 4 considers equilibrium behavior in large populations. I conclude in Section 5 with a discussion of the results and related issues. The proofs of all formal results are relegated to the appendix.

2. The model

The model is one of voting with incomplete information. $2n + 1$ voters simultaneously cast ballots for one of two candidates, A or B, where n is any positive integer (and so the number

[6] Also related are several papers that study incomplete information voting games when voters attempt to signal information to candidates to affect policy outcomes. Piketty (2000) and Shotts (2006) study repeated election settings, and Razin (2003) a single election where policy outcomes depend on the size of the winning majority (the "mandate").

of voters is odd). The winning candidate is determined by majority rule and abstention is not allowed (the election cannot end in a tie).

There are two possible states of the world, also labeled A and B. The state of the world can be interpreted as specifying which candidate is unambiguously *better* in the minds of the voters. The voters share a common prior, π, that the true state is A, where $\pi \in (0, 1)$.

Voters have identical preferences dependent upon whether the better candidate is chosen (A in state A and B in state B) and whether they vote for the winning candidate (reward of $k \geqslant 0$). The utility of voting for candidate J is given by:

$$\begin{array}{lll} 1+k & & J \text{ wins in state } J, \\ 1 & \text{if} & J \text{ loses in state } H \neq J, \\ k & & J \text{ wins in state } H \neq J, \\ 0 & & J \text{ loses in state } J. \end{array}$$

Voters each receive an independent, private signal, s_i, about the true state of the world. The signal is either α or β and is sent accurately with probability $p > \frac{1}{2}$. That is, $P(\alpha|A) = P(\beta|B) = p$. Bayes' Rule is used to update beliefs after all s_i. Denote by $\varphi(s_i|\pi)$ a voter's posterior belief that the true state is A and observe that $\varphi(\alpha|\pi) > \varphi(\beta|\pi)$; the argument π is hereafter omitted where obvious.

A *strategy* σ_i for voter i maps her information into the probability of voting for either candidate; denote the strategy profile by σ. I restrict attention to strategy profiles symmetric across voters and of the form $\sigma = \{q_\alpha, q_\beta\}$, where q_α and q_β denote the probability that an α observer votes for A, and that a β observer votes for B, respectively. I refer to the strategy $\{1, 1\}$ as *informative voting* for obvious reasons; similarly, the strategies $\{0, 1\}$ and $\{1, 0\}$ represent *uninformative voting*. A strategy is *unresponsive* if $q_\alpha = 1 - q_\beta$, and *responsive* otherwise. I say loosely that two strategies σ and σ' are *close* if $|q_\alpha - q'_\alpha|$ and $|q_\beta - q'_\beta|$ are small. The equilibrium concept used is Bayes–Nash.

The relationship between π and p determines the nature of the election. An election is said to be *tight* if $\pi \in (1 - p, p)$ and *lopsided* if $\pi \notin [1 - p, p]$.[7] In lopsided elections an ex ante favored candidate exists, equating to a strong-front runner (before voting begins). In tight races such a front-runner does not exist.

I consider two measures of efficiency: informational and utilitarian. The *informational efficiency* of a voting strategy measures the quality of information aggregation. Denote by $P_n(\sigma)$ the probability that the better candidate is elected given the strategy profile σ and population size $2n + 1$, and by $P_n^{\mathrm{A}}(\sigma)$ and $P_n^{\mathrm{B}}(\sigma)$ the probabilities that A and B win, respectively, given they are the better candidate; therefore, $P_n(\sigma) = \pi P_n^{\mathrm{A}}(\sigma) + (1 - \pi) P_n^{\mathrm{B}}(\sigma)$.

In contrast, *utilitarian efficiency* measures total voter utility, capturing both the quality of information aggregation and the benefit voters receive from supporting the winner. Denote by $E_n(\sigma)$ the expected sum of voter utility for strategy profile σ.

In measuring social welfare, the appropriate efficiency metric depends on the circumstances. For example, in settings where k reflects a material benefit—such as rewards from the winning

[7] The critical values $\pi \in \{p, 1 - p\}$ lead to substantively similar behavior, but with several additional technicalities that are not worth reporting in detail.

candidate—receiving k is a distributional rather than productive benefit, and social welfare is optimized by maximizing informational rather than utilitarian efficiency.[8]

3. Equilibria in arbitrary populations

The limiting case of $k = 0$ corresponds to the standard model studied by Austen-Smith and Banks (1996), McLennan (1998), and Feddersen and Pesendorfer (1998). In addition to allowing $k > 0$, the analysis here proceeds along slightly different lines. The previous papers fix prior beliefs to $\frac{1}{2}$ and, maintaining the assumption of common preference, allow the payoffs from the different outcomes to vary, what they refer to as varying the *threshold of doubt* (for example, allowing the utility from acquitting a guilty defendant to differ from the utility of convicting an innocent defendant). In contrast, I fix the threshold of doubt and allow prior beliefs to vary. This adjustment allows me to focus explicitly on front-runner and underdog candidates, and does not substantively change the analysis.

Conditional on A being the true state, the probability that a voter using strategy $\{q_\alpha, q_\beta\}$ votes for A, denoted by γ_{A}, is:

$$\gamma_{\mathrm{A}} = pq_\alpha + (1-p)(1-q_\beta).$$

Likewise, if B is the true state the probability the voter votes for B, denoted by γ_{B}, is:

$$\gamma_{\mathrm{B}} = pq_\beta + (1-p)(1-q_\alpha).$$

For the symmetric strategy profile, $\{q_\alpha, q_\beta\}$, votes are independent and the distribution of outcomes is binomial. For candidate $J \in \{A, B\}$, define the probability he receives m out of $2n$ votes cast by:

$$f_{2n}^{m}(\gamma_J) = \binom{2n}{m}(\gamma_J)^m(1-\gamma_J)^{2n-m},$$

and let

$$F_m^{m'}(\gamma_J|2n) = \sum_{l=m}^{m'} f_{2n}^{l}(\gamma_J)$$

be the probability that the number of votes J receives is between m and m'.

Voter i's utility from voting for either candidate depends on whether she supports the winner and whether the better candidate wins. Her utility from voting for candidate A is given by:

$$\begin{aligned}&\varphi(s_i)F_n^{2n}(\gamma_{\mathrm{A}}|2n) + \big(1-\varphi(s_i)\big)F_{n+1}^{2n}(\gamma_{\mathrm{B}}|2n)\\&\quad + k.\big[\varphi(s_i)F_n^{2n}(\gamma_{\mathrm{A}}|2n) + \big(1-\varphi(s_i)\big)F_0^{n}(\gamma_{\mathrm{B}}|2n)\big],\end{aligned}$$

and the utility of voting for candidate B is:

$$\begin{aligned}&\varphi(s_i)F_{n+1}^{2n}(\gamma_{\mathrm{A}}|2n) + \big(1-\varphi(s_i)\big)F_n^{2n}(\gamma_{\mathrm{B}}|2n)\\&\quad + k.\big[\varphi(s_i)F_0^{n}(\gamma_{\mathrm{A}}|2n) + \big(1-\varphi(s_i)\big)F_n^{2n}(\gamma_{\mathrm{B}}|2n)\big].\end{aligned}$$

[8] In some settings, consequently, the benefit k may more appropriately be modeled as dependent on the size of the winning majority. For ease of exposition I consider here only the constant value case, although the results should not substantively change.

The relative size of these utilities drives the behavior of voters. Subtracting the utility of voting for B from that of voting for A, and defining this difference by $\Phi_n(q_\alpha, q_\beta, s_i)$, we have:

$$\Phi_n(q_\alpha, q_\beta, s_i) = \Delta_n(q_\alpha, q_\beta, s_i) + k.\Psi_n(q_\alpha, q_\beta, s_i),$$

where Δ and Ψ represent the components of Φ from electing the better candidate and from conforming with the majority, respectively. These terms are given by (noting that $f_{2n}^m(\gamma) = f_{2n}^{2n-m}(1-\gamma)$):

$$\begin{aligned}
\Delta_n(q_\alpha, q_\beta, s_i) &= \varphi(s_i) f_{2n}^n(\gamma_{\mathrm{A}}) - \bigl(1-\varphi(s_i)\bigr) f_{2n}^n(\gamma_{\mathrm{B}}),\\
\Psi_n(q_\alpha, q_\beta, s_i) &= \varphi(s_i)\bigl[F_0^{n-1}(1-\gamma_{\mathrm{A}}|2n) - F_0^{n-1}(\gamma_{\mathrm{A}}|2n)\bigr]\\
&\quad - \bigl(1-\varphi(s_i)\bigr)\bigl[F_0^{n-1}(1-\gamma_{\mathrm{B}}|2n) - F_0^{n-1}(\gamma_{\mathrm{B}}|2n)\bigr].
\end{aligned}$$

The variables Φ, Δ, and Ψ form the basis of the analysis to follow. In particular, Φ determines the voter preference: if $\Phi > 0$ the voter strictly prefers to vote for candidate A, and if $\Phi < 0$ she strictly prefers candidate B. The indifference case, $\Phi = 0$, will play a central role in the mixed strategy equilibria. Note that Φ collapses to the standard formulation if $k = 0$, depending only on the event that a voter is pivotal.

It would be intuitive, and attractive, if in equilibrium $q_\alpha \geqslant 1 - q_\beta$; that is, a voter receiving the signal α is more likely to vote for A than one receiving the signal β. Despite the appeal of this property, and that it holds when $k = 0$, it is not necessarily true when $k > 0$. The two cases $q_\alpha \geqslant 1 - q_\beta$ and $q_\alpha < 1 - q_\beta$ are, however, useful in characterizing equilibria. An equilibrium strategy is said to be *ordered* if $q_\alpha \geqslant 1 - q_\beta$ and *unordered* otherwise ($q_\alpha < 1 - q_\beta$). The remainder of this section is demarcated according to this distinction.

3.1. Ordered equilibria

3.1.1. Unresponsive equilibria and preliminaries

A well-known result in standard voting games is that a pair of equilibria exist in which all voters support the same candidate and no information is aggregated. These unresponsive equilibria also exist when $k > 0$.

Lemma 1. *For each n and $k \geqslant 0$, the strategies $\{0, 1\}$ and $\{1, 0\}$ are equilibria*; $P_n(0, 1) = 1 - \pi$ *and* $P_n(1, 0) = \pi$.

These unresponsive equilibria are strict when $k > 0$, but are only weak if $k = 0$. As such, when $k > 0$ they cannot be eliminated by a restriction to weakly dominated strategies, as is often done when $k = 0$. In the current setting a second reason exists for not dismissing these equilibria: they are no longer the most inefficient outcomes possible (as they are when $k = 0$) as they produce better outcomes than in some responsive equilibria. Thus, in this setting uninformative voting provides a non-vacuous lower bound on efficiency under majority rule.

The next lemma rules out the possibility in an ordered equilibrium that observers of both signals play mixed strategies. An implication of this result is that there are no unresponsive equilibria in addition to those in Lemma 1.

Lemma 2. *For every $k \geqslant 0$ the strategy $\{q_\alpha, q_\beta\}$ is not an equilibrium if $q_\alpha, q_\beta \in (0, 1)$ and $q_\alpha \geqslant 1 - q_\beta$.*

Hereafter I focus attention on responsive equilibria. The characterization of ordered responsive equilibria varies on whether an election is tight or lopsided. The analysis is broken down along this line, beginning with equilibria in lopsided elections.

3.1.2. Responsive equilibria in lopsided elections

In lopsided elections the private beliefs of voters favor a common candidate—the front-runner—regardless of the private signal that is received. This imbalance leads to the famous result of Austen-Smith and Banks (1996) that voting informatively does not constitute equilibrium behavior. If all other voters are voting informatively then, conditional on being pivotal, a voter optimally abandons her own information and votes for the front-runner (as her private signal is outweighed by prior beliefs). Feddersen and Pesendorfer (1998) show in this case that a unique symmetric responsive equilibrium exists if the voting population is sufficiently large. This result is stated as Lemma 3, the proof of which is standard and subsumed here into the proof of Theorem 1. For simplicity, and without loss of generality, I assume hereafter in lopsided elections that A is the ex ante preferred candidate.

Lemma 3. *Set $\pi \in (p, 1)$ and $k = 0$. If $n > \lambda$ a unique symmetric responsive equilibrium exists: $q_\alpha = 1, q_\beta \in (0, 1)$ (α observers vote A, β observers mix), where λ satisfies*

$$\frac{(1-\pi)p^{\lambda+1}}{(1-\pi)p^{\lambda+1} + \pi(1-p)^{\lambda+1}} = \frac{1}{2}.$$

A responsive equilibrium does not exist if $n \leqslant \lambda$.

With observers of β occasionally voting for A in equilibrium, voters interpret being pivotal as a favorable signal about candidate B, and sufficiently favorable that β observers are indifferent over the candidates and prepared to mix. The definition of λ requires the population to be large enough such that a voter's posterior beliefs can reach $\frac{1}{2}$ if she is pivotal. For population sizes $n \leqslant \lambda$ the number of private signals is insufficient to outweigh prior beliefs, and voters strictly prefer to vote for A regardless of their private signal if others are voting responsively (note that λ is not required to be an integer). Fig. 1 depicts these incentives, showing the value of $\Phi_n(1, q_\beta, \beta)$ as q_β varies from 0 to 1. The responsive equilibrium corresponds to the point where $\Phi_n(1, q_\beta, \beta) = 0$: for smaller values of q_β observers of signal β strictly prefer voting for B, and for larger values they strictly prefer voting for A. Note that $q_\beta = 0$ represents an unresponsive strategy, and corresponds to one of the uninformative voting equilibria (Lemma 1).

The responsive equilibrium aggregates information imperfectly but beneficially: it is efficient in the class of symmetric strategy profiles and it is shown in Feddersen and Pesendorfer (1998) that the quality of information aggregation approaches perfection as the population grows arbitrarily large.[9]

The addition of a desire to conform changes the set of equilibria in lopsided elections, with the impact taking two forms. Firstly, the balance of beliefs in the responsive equilibrium of Lemma 3 is affected by the desire to conform, and an informationally efficient equilibrium no longer exists. Indeed, if the preference to conform grows sufficiently (or the population is large enough), a responsive equilibrium in which α observers support A and β observers mix no longer exists. Secondly, and more strikingly, several new responsive equilibria emerge and these equilibria are far from informationally efficient, further weakening the quality of information aggregation.

[9] Optimality within the class of symmetric strategies follows from an adjustment of the arguments of McLennan (1998).

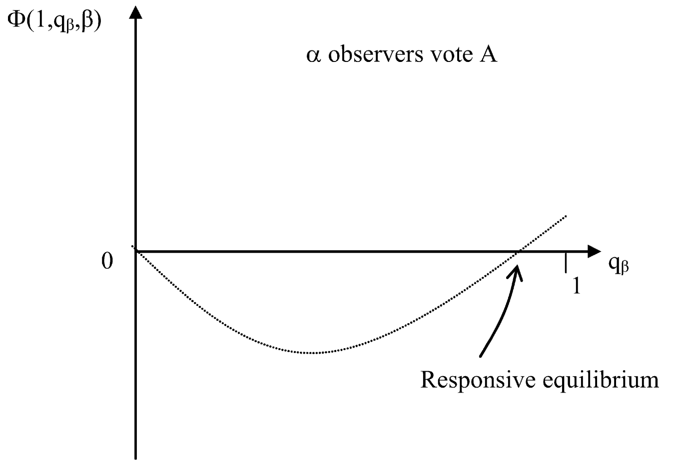

Fig. 1. Unique responsive equilibria in lopsided elections for $k = 0$; $\pi \in (p, 1)$.

Theorem 1. *Set $\pi \in (p, 1)$ and $k > 0$. The following properties hold*:

(i) *Informative voting is not an equilibrium for any n.*
(ii) *At least one equilibrium of the form $q_\beta = 1, q_\alpha \in (0, 1)$ exists for each n.*
(iii) *An equilibrium of the form $q_\alpha = 1, q_\beta \in (0, 1)$ exists if and only if $n > \lambda$ and $k \in (0, \tilde{k}_n]$, for some $\tilde{k}_n$. If $k \in (0, \tilde{k}_n)$ then at least two such equilibria exist. Moreover, $\tilde{k}_n \to 0$ as $n \to \infty$.*

The equilibria are depicted graphically in Fig. 2. The different lines represent the different components of voter utility. For simplicity I have drawn the Ψ component as linear. This is the case when there are three voters, but is not so generally (although it can be shown that Ψ is always strictly monotonic when $k > 0$). Also, the figure depicts three mixed equilibria (two of type iii and one of type ii), although more mixed equilibria may generally exist.[10]

The type (ii) equilibrium is representative of the type of behavior that is supportable as equilibrium when voters have a preference for winning. As in the standard model equilibrium (Lemma 3), this new equilibrium is responsive and in mixed strategies. However, the mixing is the reverse of normal and requires that voters observing signals in favor of the front-runner mix, whereas observers of signals favoring the underdog play a pure strategy.

The unusual behavior in this equilibrium is maintained by a balance of voters' dueling incentives. Although candidate A is the nominal front-runner, the behavior of other voters induces each voter to believe that candidate B is more likely to win. This creates what may be called a belief-bandwagon in favor of candidate B. This bandwagon leads to more support for candidate B, but not all the way to the unresponsive equilibrium (all voting for B) as voters must weigh the payoff from joining the majority against the desire to elect the better candidate. As more voters support candidate B, being pivotal provides strong evidence that candidate A is in fact the better candidate. This provides a countering incentive to vote for candidate A, and one sufficiently strong that an equilibrium exists in responsive strategies.

[10] I have been unable in arbitrary populations to rule out additional equilibria, although in numerical exercises additional equilibria have not appeared. The analytical difficulty arises because Φ may not be quasiconcave or quasiconvex.

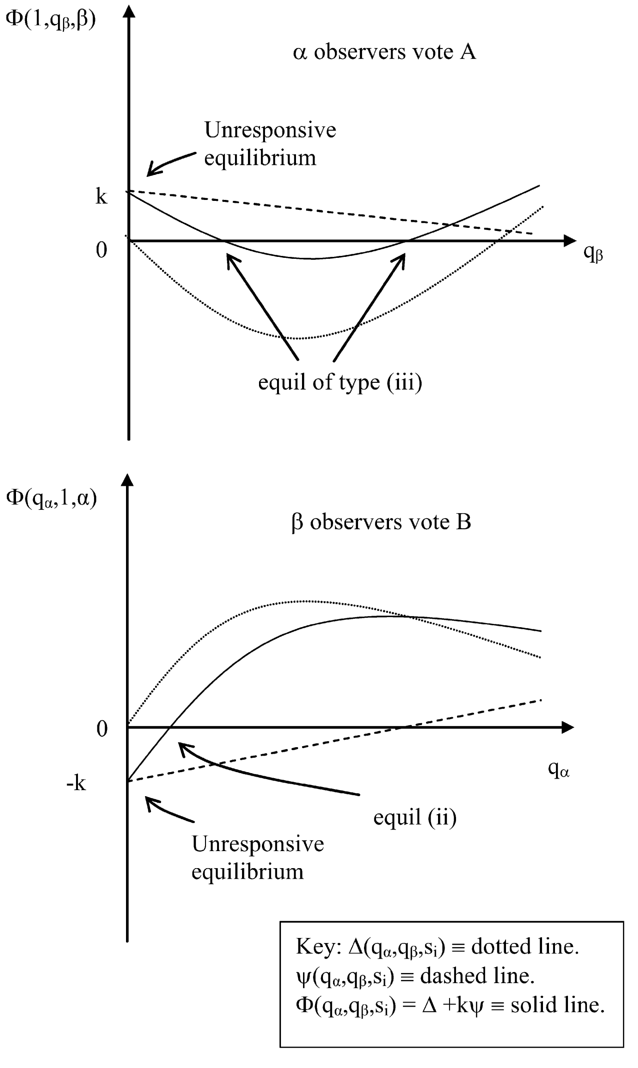

Fig. 2. Ordered equilibria in lopsided elections; $\pi \in (p, 1)$.

Equilibrium (ii) is far from the responsive equilibrium in Lemma 3, but it is not entirely new. Rather, when k is small type (ii) equilibrium behavior is close to the unresponsive equilibrium in which all voters vote for candidate B (Lemma 1). Consequently, information aggregation is extremely poor in equilibrium (ii), despite voters using responsive strategies. A consolation of this misuse of information is that the winning majority is likely to be large, and the outcome may be preferred from the utilitarian perspective.

The desire to conform also impacts the more standard equilibrium of Lemma 3 in which β observers mix. The effect on this equilibrium is small when the desire to conform is small, and the equilibrium remains effective at aggregating information. The standard equilibrium is now joined by an additional responsive equilibria of a similar form (in which β observers mix). The additional equilibrium is not close to the informationally efficient equilibrium, but rather is close the unresponsive equilibrium in which all voters support candidate A.

This pair of equilibria involve the same trade-off in equilibrium, although of different degree, and this trade-off is the reverse to that underlying equilibrium (ii). In these equilibria the voters prefer to vote for B if pivotal as the equilibrium value of q_β is less than in the equilibrium of Lemma 3, and the voters trade this off against an expectation that candidate A is more likely to win. The equilibrium with lower q_β provides stronger voter preferences than the other equilibrium, both in the belief that A is more likely to win and that B is the better candidate. This trade-off also shows why an equilibrium is not possible for values of q_β greater than in Lemma 3: observers of β would prefer to vote A if pivotal *and* believe A the likely winner, thereby ruling out the required mixing behavior.

As either the population or the desire to conform (n or k) increase, the equilibrium set continues to change. Ultimately the pair of responsive equilibria in which β observers mix disappear and an equilibrium of the standard form cannot be supported in equilibrium. Rather, the only responsive equilibria that persist are of the opposite form in which observers of signals in favor of the front-runner mix. Therefore, for large populations the set of responsive equilibrium behavior when $k > 0$ is very different relative to the standard formulation of $k = 0$.

A consequence of Theorem 1 is that the quality of information aggregation does not improve with an increase in population size, unlike the standard formulation of $k = 0$. In fact, not only does the addition of voters—and their additional information—not improve the quality of decisions, but it inhibits the aggregation of information held by voters already in the system. This finding is important for determining the optimal size of a voting population. In contrast to when $k = 0$, larger is not always better when $k > 0$ and the optimal population size is finite. To achieve informational efficiency, therefore, a social planner must restrict the franchise. This requirement leads to the unusual conclusion that positive voting costs may be socially beneficial.

3.1.3. Responsive equilibria in tight elections

The distinction between lopsided and tight elections is whether a voter's estimate as to which candidate is better is determined more by her prior belief or the private signal she receives. In tight elections neither candidate is preeminent ex ante and a voter's private signal dominates her beliefs. In this environment, informative voting is not subject to the same tension as in lopsided elections, and for $k = 0$ informative voting is the unique responsive equilibrium (Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1998). This result is stated in Lemma 4.

Lemma 4. *Set $\pi \in (1 - p, p)$ and $k = 0$. Informative voting, $\sigma = \{1, 1\}$, is the unique responsive equilibrium.*

As is the case for lopsided elections, the addition of a desire to conform expands the set of equilibria. Significantly, the informative voting equilibrium is not affected, and in fact the desire to conform reinforces the incentive of voters to vote informatively. Theorem 2 describes equilibrium properties for tight elections.

Theorem 2. *Set $\pi \in (1 - p, p)$ and $k > 0$. For each n an ordered responsive equilibrium exists of each of the following forms*:

(i) $q_\alpha = 1$, $q_\beta = 1$.
(ii) $q_\alpha = 1$, $q_\beta \in (0, 1)$.
(iii) $q_\alpha \in (0, 1)$, $q_\beta = 1$.

The equilibria are depicted in Fig. 3 for when three ordered responsive equilibria exist. To see why the incentive to conform reinforces informative voting it is best to consider the competing incentives separately. If all other voters are voting informatively then the standard logic applies when a voter is pivotal: vote according to her private information. Simultaneously, informative voting implies that the better candidate is more likely to emerge and win (as there is likely to be a majority of signals in his favor) and the incentive to conform leads voters to vote for who they believe to be better. In tight elections this again leads a voter to act according to her private information. Therefore, both the informational incentive and the desire to conform push voters to vote informatively, and informative voting remains an equilibrium for any $k > 0$.

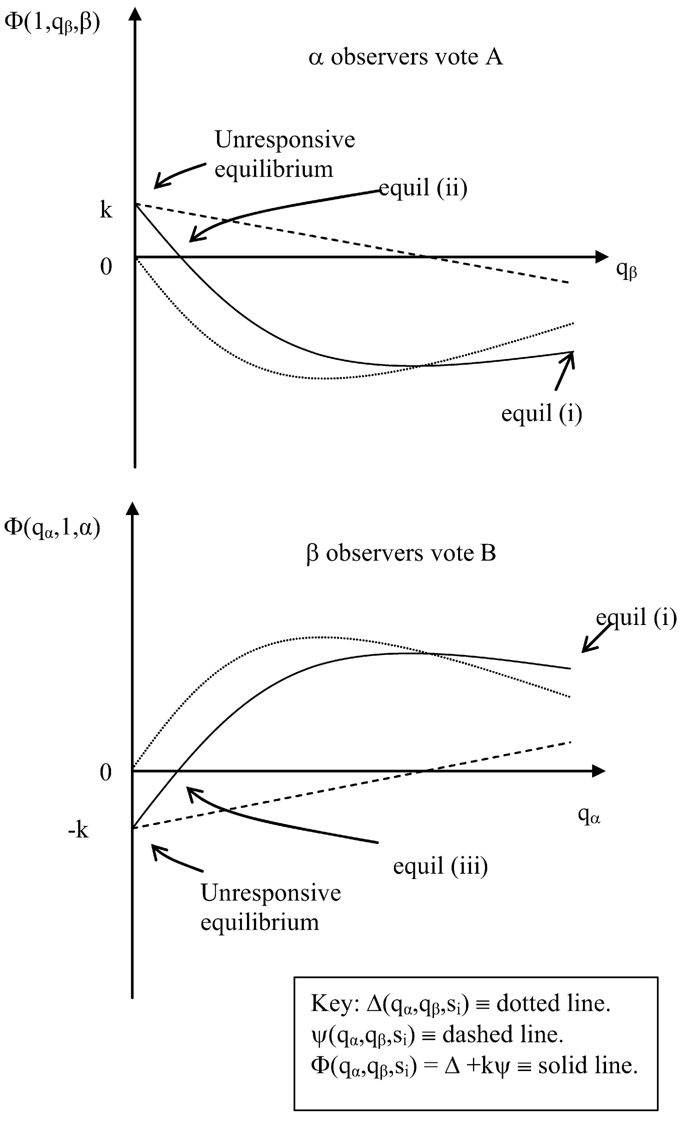

Fig. 3. Ordered equilibria in tight elections; $\pi \in (1-p, p)$.

The logic of informative voting relies critically on the belief that all other voters are voting informatively. For other strategies, the dueling motivations of voters do not lead to informative voting and, as with lopsided elections, additional responsive equilibria are possible. As candidates in tight elections are (nearly) symmetric, responsive mixed equilibria are possible with observers of either signal mixing. The informational inefficiency of these equilibria is notable here as it arises in an environment in which perfect information aggregation is possible. Moreover, the existence of these equilibria imply that in tight elections the refinement to responsive equilibria no longer guarantees a unique prediction.

The results for tight elections are similar in several respects to those for lopsided elections, but in one important respect they contrast sharply. In all elections a desire by voters to conform expands the equilibrium set, and the addition of new responsive equilibria suggest that the efficient use of information is far from guaranteed. In tight elections, however, the desire to conform does not corrupt behavior in all equilibria, and in fact in the most efficient equilibrium the revelation of information is reinforced by the desire to conform. These differences lead to differences in the ability of majority rule to aggregate information in tight versus lopsided elections, and in Section 4 I show that these differences are particularly stark for large populations. Before doing so, I prove the existence of unordered equilibria and study their properties.

3.2. Unordered equilibria

The previous results establish that a preference to conform may lead to equilibria in which information is not aggregated effectively. Here I show how things can be much worse, and that equilibrium behavior may produce outcomes more inefficient than if the election were not held at all. These equilibria use unordered strategies and their existence is demonstrated in Theorem 3. First, I confirm in Lemma 5 that equilibria in unordered strategies produce remarkably inefficient outcomes.

Lemma 5. $P_n(q_\alpha, q_\beta) < \min[\pi, 1-\pi]$ *if* $q_\alpha < 1 - q_\beta$.

The degree of inefficiency in unordered equilibria—both informational and utilitarian—is sufficiently extreme that better outcomes are produced by abandoning the election and appointing a candidate directly. Note that superior outcomes are produced not only by appointing the front-runner, but even if the underdog is directly appointed. Such extreme outcomes require, paradoxically, a negative aggregation of information. Information is aggregated in unordered equilibria, but this accumulated information is used to select the worse rather than the better candidate. Theorem 3 proves that such equilibria exist when the election is tight.

Theorem 3. *Set* $\pi \in (1-p, p)$ *and* $k > 0$. *For some* $\hat{n}$, *at least two unordered equilibria exist if* $n > \hat{n}$. *They are of the form*:

(i) $q_\alpha = 0$, $q_\beta \in (0, 1)$.
(ii) $q_\beta = 0$, $q_\alpha \in (0, 1)$.

An unordered equilibrium of type (i) is depicted in Fig. 4; the value $\Psi_n(0, q_\beta, \alpha)$ for α observers is also plotted for reference. In unordered equilibria a rather perverse balance of beliefs is required to maintain equilibrium. Consider the type (i) equilibrium in which β observers mix and α observers always vote for B. An observer of β believes that B is the better candidate

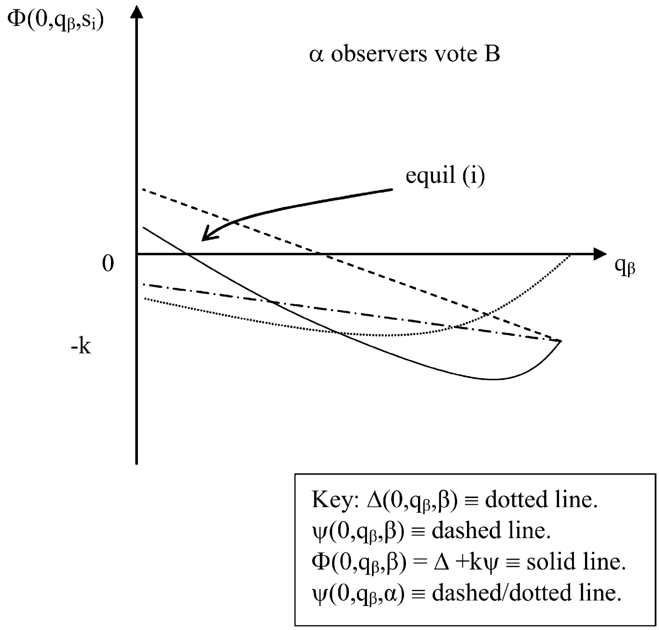

Fig. 4. Unordered equilibria in tight elections; $\pi \in (1-p, p)$.

(as the election is tight) and conditional on being pivotal prefers to vote for B. However, because she believes B to be the better candidate, she also believes that a majority of private signals favor candidate B. And as β observers often vote for A, she wishes to vote for A as she believes that candidate A is more likely to win. In contrast, α observers engage in the reverse logic. Upon observing α, a voter believes candidate A to be better and that a majority of signals favor A. Again, however, as α observers vote for B, the voter believes B is the more likely winner and votes for B herself.

Establishing the existence of unordered equilibria in lopsided elections is more difficult, although I cannot rule them out generally. For unordered strategies it can be verified that mixing ratios exist to ensure the mixing type of voter is indifferent. However, I have been unable to confirm that the other type of voter is simultaneously acting optimally. The difficulty is in ordering the relative payoffs of the voters. This can be seen in Fig. 5 which depicts the general relationship between components of voter utility; as is shown, the relative size of Ψ and φ for observers of different signals leaves the net effect on Φ indeterminate.

Theorem 4 considers large populations and establishes a non-existence result. For lopsided elections and large populations, the theorem rules out maximally inefficient equilibria in which voters observing a signal in favor of the front-running candidate always vote for the underdog.

Theorem 4. *Set $\pi \in (p, 1)$ and $k > 0$. For some $\tilde{n}$ an equilibrium of the form $q_\alpha = 0, q_\beta \in (0, 1)$ does not exist if $n > \tilde{n}$.*

4. Optimal equilibria in large populations

In this section I consider behavior as the population grows arbitrarily large. The focus is on determining and comparing the optimal equilibria for tight and lopsided elections, both in terms

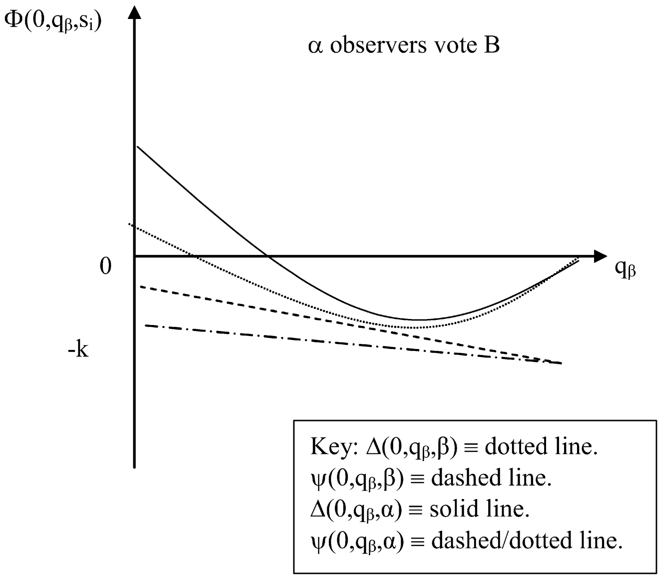

Fig. 5. Unordered equilibria in lopsided elections; $\pi \in (p, 1)$.

of informational and utilitarian efficiency. These equilibria provide a best case scenario of what is possible under majority rule voting in large elections.[11] Surprisingly, in large populations responsive voting is still possible in lopsided as well as tight elections. This is despite the payoff from conforming remaining fixed and each voter's pivot probability becoming vanishingly small as the population grows. I consider first the limiting behavior in mixed responsive equilibria, and then use the results to characterize the optimal equilibria for tight and lopsided elections.[12]

4.1. Limiting behavior in mixed responsive equilibria

Lemma 2 shows that in equilibrium observers of at least one type of signal must play a pure strategy. In large populations this implies that the candidate whom these voters vote for wins election almost surely when he is better. The following lemma proves that the opposing candidate cannot also win almost surely when he is better, and describes the limiting probabilities of each candidate winning when better.

Lemma 6. *For a sequence of strategy profiles $\sigma_n = \{q_\alpha^n, q_\beta^n\}$ as $n \to \infty$:*

(i) *Suppose $q_\alpha^n = 1$ for all n.*
Then $P_n^{\mathrm{A}}(\sigma_n) \to 1$, and $\Phi_n(1, q_\beta^n, \beta) = 0$ implies $P_n^{\mathrm{B}}(\sigma_n) \to \frac{1}{2(1-\varphi(\beta))}$.

(ii) *Suppose $q_\beta^n = 1$ for all n.*
Then $P_n^{\mathrm{B}}(\sigma) \to 1$, and $\Phi_n(q_\alpha^n, 1, \alpha) = 0$ implies $P_n^{\mathrm{A}}(\sigma_n) \to \frac{1}{2\varphi(\alpha)}$.

---

[11] I focus on optimal equilibria as a general ranking of the mechanisms is precluded by the generic existence of unresponsive equilibria (and here the refinement to responsive equilibria does not yield uniqueness as in the $k = 0$ case).

[12] As unordered equilibria are dominated by unresponsive equilibria, I consider here only ordered mixed strategy equilibria.

For mixed strategy equilibria Lemma 6 implies that significant uncertainty about the outcome persists even as the population grows large (with the exception of knife-edge cases). This uncertainty is critical to enabling the aggregation of information in equilibrium. If instead the better candidate always won, voters would abandon their desire to affect the outcome and act in order to conform, thereby undermining their incentive to vote informatively and aggregate information. The presence of uncertainty in the outcome precludes full information aggregation, but by so doing it allows some information to be aggregated. The desire to conform provides only a weak incentive when the outcome is uncertain, and this allows even a small informational incentive to influence voter behavior.

Several additional properties of the mixed equilibria are also of interest. First, the limiting behavior is independent of k (although behavior as the limit is approached depends on k). Second, as the private signals become more informative (p increases), it is required that the quality of information aggregation declines. Thus, the more information held by each individual voter, the less information that is aggregated into electoral outcomes, highlighting again the unintuitive trade-offs required to support equilibrium when voters possess dual incentives.

Finally, the size of the voting coalitions in a mixed equilibria can take on two extreme forms: either a close cliff-hanger or a landslide. If β observers mix in equilibrium and candidate A is better, then A wins in a landslide with a vote share strictly greater than p. In contrast, when candidate B is better the outcome is uncertain and vote shares approach $\frac{1}{2}$ for each candidate.[13]

4.2. *Tight elections*

Three types of equilibria exist in tight elections: informative voting, uninformative voting, and the mixed equilibria. Informational efficiency here is straightforward and maximized by the informative voting equilibrium. Utilitarian efficiency is less straightforward. The cost of informative voting is that a large losing minority is created (of size $1-p$). The unresponsive equilibrium avoids altogether a losing minority but at the expense of information aggregation. The mixed equilibria provide a middle ground between these two extremes. Theorem 5 shows that the mixed strategy trade-off is never optimal, and that the utilitarian optimal equilibrium must be in pure strategies when the election is tight.

Theorem 5. *For $\pi \in (1-p, p)$ and $n \to \infty$,*

(i) *Informational efficiency*: *Informative voting is the optimal equilibrium for $k \geqslant 0$, where $P_n(1,1) \to 1$.*
(ii) *Utilitarian efficiency*: *Informative voting is the optimal equilibrium if $k < \bar{k}$, and an uninformative voting equilibrium* (*voting for the slight front-runner*) *is optimal if $k > \bar{k}$, where $\bar{k} = \min[\frac{1-\pi}{1-p}, \frac{\pi}{1-p}]$.*

4.3. *Lopsided elections*

In lopsided elections one of two equilibria must be optimal: uninformative voting for the front-running candidate, or a mixed equilibrium in which α observers mix. In the first equilibrium no information is aggregated but a losing minority is avoided, whereas in the second equilibrium

[13] The vote shares cannot equal $\frac{1}{2}$ in expectation as this would be sufficient for β observers to vote A.

some information is aggregated but at the cost of sizable losing minorities. The aggregation of information in the mixed equilibrium is profitable for some parameters (it does better than pick the front-runner) and so this equilibrium is also utilitarian optimal for sufficiently small k.

Theorem 6. *Set $\pi \in (p, 1)$, $k > 0$, let $n \to \infty$, and define $\hat{k} = \frac{(1-\pi)-p(2\pi-1)}{(1-\pi)+p(2\pi-1)}$. The following equilibria are optimal*:

	Optimal equilibria	
	Informational	Utilitarian
(i) $p \geqslant \frac{1}{\sqrt{2}}$, or $p \in (\frac{1}{2}, \frac{1}{\sqrt{2}})$ and $\pi > \frac{1+p}{1+2p}$,	$\{1, 0\}$	$\{1, 0\}$
(ii) $p \in (\frac{1}{2}, \frac{1}{\sqrt{2}})$ and $\pi < \frac{1+p}{1+2p}$	$\{q_\alpha, 1\}$	$\begin{Bmatrix} \{1,0\} & \text{if } k \geqslant \hat{k} \\ \{q_\alpha, 1\} & \text{if } k \leqslant \hat{k} \end{Bmatrix}$
(iii) $p \in (\frac{1}{2}, \frac{1}{\sqrt{2}})$ and $\pi = \frac{1+p}{1+2p}$	$\{1, 0\}, \{q_\alpha, 1\}$	$\{1, 0\}$

Effective information aggregation is possible in lopsided elections if and only if the front-runner is not excessively strong. This ranking is intuitive. In the mixed equilibrium the underdog always wins when better whereas the front-runner often loses. In comparison with the unresponsive equilibrium, therefore, the outcome is better when the underdog is the better candidate and worse when the front-runner is better. Consequently, when prior beliefs are not overly lopsided in favor of one candidate, the benefit of the mixed equilibrium exceeds the cost and it is optimal. For more lopsided priors, the aggregation of information leads to worse outcomes in expectation, and voters are better off abandoning their private information and voting unanimously for the front-running candidate.

Fig. 6 captures graphically the relationships described in Theorems 5 and 6 for informational efficiency, plotting the optimal value of P_n as prior beliefs vary. The left side panel shows the

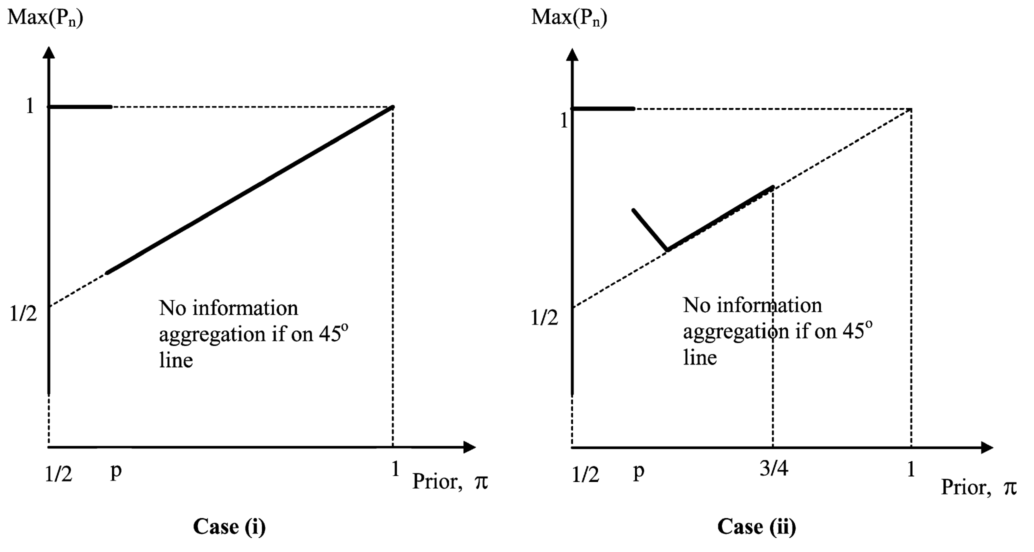

Fig. 6. Informational efficiency of optimal equilibria as $n \to \infty$ when (i) $p \in (\frac{1}{2}, \frac{1}{\sqrt{2}})$ and $\pi \geqslant \frac{1+p}{1+2p}$, or $p \geqslant \frac{1}{\sqrt{2}}$; (ii) $p \in (\frac{1}{2}, \frac{1}{\sqrt{2}})$ and $\pi < \frac{1+p}{1+2p}$.

situation described by case (i) of Theorem 6, and the right side panel the situation described by case (ii); note that the conditions of case (ii) imply $\pi \leqslant \frac{3}{4}$ (case (iii) of the theorem is the point at which the curve kinks).

The main finding is that a discontinuity exists in the information aggregation capability of majority rule voting, with the discontinuity at the point of demarcation between tight and lopsided elections. Moreover, the difference between the cases is stark: in tight elections information aggregation is perfect, whereas in lopsided elections little information is aggregated, and in many cases all private information is optimally ignored.

These results can also be compared to the informational efficiency of voting in the standard model ($k = 0$) where information aggregation is perfect for all elections, both tight and lopsided. This creates a discontinuity in informational efficiency at $k = 0$ also when the population is large and the election lopsided. Consequently, the information aggregation results of the standard model are robust to a desire to conform *only* if the election is tight. In particular, the effective aggregation of information disintegrates entirely in lopsided elections with even an arbitrarily small desire to conform.

5. Discussion

I conclude with a brief discussion of the results and several issues that emerge.

5.1. Lopsided versus tight elections: why the difference?

The boundary between tight and lopsided elections causes a change in behavior at $k = 0$ as well as when $k > 0$, although in the former case the change is neither as sharp nor for the same reason. The behavioral change when $k = 0$ flows from what might best be called an integer problem: with a finite number of voters behavior must be adjusted to allow for lopsided prior beliefs. Significantly, and not surprisingly, as the population size grows this integer problem is obviated and the adjustment in behavior required to support equilibrium declines as a fraction of overall behavior. For larger populations, as a result, efficiency is achieved in both tight and lopsided elections.

The results presented here show that the difference between tight and lopsided elections is much more fundamental than a simple integer problem. Rather, the more important difference between tight and lopsided elections is one of expectations. In lopsided elections expectations are skewed in favor of one candidate, and this skewness proves critical when voters are motivated by concerns other than simply electing the better candidate. The incentive to conform pushes a wedge between expectations and the revelation of private information. Critically, this wedge grows as the population grows and for large populations a discontinuity arises in the ability of majority rule voting to aggregate information.

5.2. Public versus private information

In addition to describing voter behavior in a particular setting, the model and results are of interest to political behavior more broadly. In particular, the results stress the importance of public information in politics, and the differences between it and private information. The discontinuity in the characteristics of majority rule voting at the boundary between tight and lopsided elections imply that small amounts of public information can make a big difference to outcomes (for example, if it changes an election from lopsided to tight). Significantly, the same impact does not follow from the addition of comparable private information. An implication of this property is that, ironically, starting with less public information about the candidates can actually produce better outcomes if it were to change an election from lopsided to tight. Significantly, this distinction between public and private information does not arise when $k = 0$ as in that case small amounts of information have only a small impact on outcomes, regardless of whether the information is privately or publicly conveyed.

The discontinuity can also be interpreted as creating critical thresholds for candidates to be considered "credible." This feature resonates well with intuitions about real political campaigns, and also suggests a foundation to examine more closely the behavior of strategic candidates. In particular, the difference between public and private information suggests when candidates may undertake high profile common-knowledge generating campaign activities and when they may instead campaign more at an individual level. A candidate must not only convince voters that he is better, but he must also convince each voter that other voters believe he is better, and therefore that his candidacy is credible. Moreover, this incentive is asymmetric: underdogs wish to campaign in high publicity situations to move the election from lopsided to tight (even if such tactics are risky), whereas front-runner candidates prefer to campaign "door-to-door" to avoid high publicity events that may take away their advantage in expectations and push the election back to being tight.[14]

To the best of my knowledge, a model distinguishing between public and private campaign signals would be novel. Yet such possibilities are consistent with the emerging importance of public information in other settings. Morris and Shin (2002) make this point in a simple theoretical market model setting, and Chwe (2001) provides numerous real world examples.

5.3. *Voter motivations*

In addition to wanting to win, voters may also have other desires that affect their voting behavior. I consider two possibilities briefly here. The first possibility is that voters like to be "right." That is, voters like to personally vote for the better candidate.[15] Such an incentive differs from standard motivations in that it is an *individual* incentive to be right rather than a desire that the group choice be right.

The impact of a desire to be right depends on voter patience. Presumably a winner's type is not revealed until sometime during the term of office, and behavior will depend on whether voters are prepared to wait for vindication. If voters are impatient then they may instead look for a more immediate cue. In the ordered equilibria the outcome of the election is just such a cue as the better candidate wins more than half of the time (and in most cases more frequently than that). Thus, voting for the winning candidate is a positive signal that a voter is more likely right than wrong, and for impatient voters the desire to be right transforms into an incentive to vote for the winner. This suggests that the ordered equilibria described here also hold, at least in approximate form, when voters like to be right. Unordered equilibria do not carry over in this way as in these equilibria winning the election is a signal that a candidate is *worse*, and voters who like to be right would rather vote for the loser rather than the winner.

[14] In Callander (2007) I show how early voters serve this purpose when voting is sequential, as the public revelation of their vote decision effectively converts their private signals into public information.

[15] As the co-editor of this journal suggests, voters may like to sport a bumper sticker proclaiming "Don't blame me: I voted for B."

If voters are patient behavior differs from here, although important properties persist. When voters like to be right there are fewer mixed equilibria.[16] In fact, for both tight and lopsided elections there exists a unique symmetric equilibrium when the population is large, even without the restriction to responsive strategies. In tight elections both unresponsive equilibria disappear and informative voting is the unique symmetric equilibrium. The unresponsive equilibria disappear as they aggregate no information, leaving a voter's private signal as his best guess as to which candidate is better (whereas the same logic reinforces informative voting). In lopsided elections, on the other hand, voters believe the front-runner more likely to be better, regardless of their private signal, and the unique symmetric equilibrium is for all to vote for the front-runner.

Although the set of equilibria differs when voters like to be right rather than win, the two preferences share the critical property of a discontinuity in information aggregation between tight and lopsided elections. Under both preferences, information is aggregated perfectly in large tight elections, whereas it is not aggregated at all when an election is lopsided.

Another possible preference is that voters are *contrarian* rather than conformist (corresponding to $k < 0$). Two properties emerge quickly for this case, one positive and one negative (that can be deduced by adjusting Figs. 2–3). On the positive side, contrarianism of any degree breaks down unresponsive equilibria as a certain winner encourages voters to switch to the underdog. The negative side of contrarianism is that if it is sufficiently strong it also breaks down the informationally-efficient informative voting equilibrium in tight elections.

Equilibrium behavior more generally is difficult to ascertain when voters are contrarian. The difficulty (similar to that for unordered equilibria) is that while indifference for observers of one signal is straightforward, confirming that this is equilibrium behavior for observers of the other signal does not follow immediately from a dominance relationship (as in ordered equilibria when $k > 0$).[17]

A more interesting variation would examine heterogeneous voters, with some voters conformist and others contrarian. This heterogeneity resonates empirically, for while psychological studies confirm conformism is the norm, anecdotal evidence suggests that contrarianism exists. The open question then is whether such heterogeneity delivers the best-of-both-worlds: contrarianism sufficiently strong so as to eliminate unresponsive equilibria but at the same time not strong enough to destroy the informative voting equilibrium (and the attendant effective aggregation of information).

Many other voter motivations exist beyond those described here, although examining them is beyond the scope of this paper. Examples include a preference to end the contest quickly, such as in presidential primaries as this provides the winner with a better chance in the general election. Another alternative is that voters seek to communicate with the candidates through the size of the vote share (Piketty, 2000; Razin, 2003, and Shotts, 2006 present models of this sort). Exploring these and other possibilities further is a worthwhile direction of research.

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16 Equilibrium behavior can be deduced by altering Figs. 2 and 3, with the Ψ curve now a constant value. The value of Ψ depends on a voter's prior belief and private signal and is independent of the strategies of others (the sign is determined by $\varphi(s_i|\pi)$).

17 A connection between contrarianism and wanting to be right is provided in Callander and Horner (2007) that shows contrarianism is often the "right" strategy in a social learning environment.

5.4. Stability

The precise balance of beliefs and expectations necessary for equilibrium gives rise to a question of equilibrium stability. An equilibrium of the form $\{1, q_\beta\}$ is *stable* if in some neighborhood of q_β, the strategy $\{1, q'_\beta\}$ implies $\Phi_n(1, q'_\beta, s_i) \lesseqqgtr 0$ when $q'_\beta \lesseqqgtr q_\beta$ (with analogous definitions holding for other strategies). Intuitively, stability requires that if, say, q_β is perturbed to a higher value (representing more votes for B) then voters strictly prefer voting for candidate A, leading them to lower q_β and return to the equilibrium level.[18] Stability can be determined in Figs. 2–5 by noting the slope of $\Phi_n(1, q_\beta, s_i)$ at each mixed strategy equilibrium.

Given this definition, it is easy to establish that all equilibria in pure strategies are stable whereas some in mixed strategies are not. In particular, the new equilibria found here are generally those that are unstable (assuming multiple equilibria of the types reported in Theorems 1 and 2 do not exist). This change to the equilibrium set leaves some conclusions from the previous sections unchanged, and puts a new spin on others.

Left unchanged by a restriction to stable equilibria is the discontinuity in information aggregation between tight and lopsided elections. In making the comparison between Theorems 5 and 6 I compared, with one exception, two pure strategy equilibria, both of which are stable. Moreover, removing the one mixed-strategy unstable equilibrium from the comparison makes the discontinuity only starker. In this case the optimal equilibria are now described entirely by the left side panel of Fig. 6.

Equilibrium stability begs the question of how likely it is (if at all) that an iterative process will converge to each stable equilibrium? Surprisingly, the new mixed equilibria revealed here, although they themselves may no longer provide predictions of behavior, are an important element in answering this question. Each mixed equilibrium defines the *basin of attraction* for a stable equilibrium. The basin of attraction is the largest region around a stable equilibrium such that, starting from any point in this region, best reply dynamics lead back to the equilibrium.

For example, the mixed equilibrium depicted in the second panel of Fig. 2 is not stable, yet it delineates the boundary of the basin of attraction for the stable unresponsive equilibrium in which everyone votes B. This particular example is notable as it implies the basin of attraction for the unresponsive equilibria may be substantial. In contrast, the unresponsive equilibria are unstable when $k = 0$ and typically ignored. When voters like to conform these equilibria cannot be so easily dismissed and they play a key role in predicting behavior.

The discussion here is preliminary and the issue of stability warrants closer investigation in the voting literature. In particular, a characterization of stability and the corresponding basins of attraction may prove useful to experimental studies of voting in which players compete repeatedly, potentially learning and adjusting their behavior.

5.5. Concluding remarks

In this paper I have studied voting behavior and aggregate outcomes for an incomplete information environment in which voters possess an incentive to vote for the winner in addition to electing the better candidate. Although the model is simple, many conclusions emerge. The most basic of these is that majority rule voting, despite its seductive possibilities, does not provide a

[18] Formally, for a continuous time adjustment process in which the time derivative of each agent's mixed strategy is proportional to $\Phi_n(1, q'_\beta, s_i)$, stability requires that at nearby mixed strategies the adjustment process generates motion in the direction of the equilibrium.

perfect panacea to the difficulties of aggregating diverse information and preferences, even in an environment with only two alternatives.

I have focused the analysis exclusively on simple majority rule as it is the most widely used and studied. Of course, other voting rules are employed in practice and in some situations the voting rule itself is subject to choice. Analyzing the impact of conformity in these environments, as well as the broader question of mechanism design, are worthy topics of study but beyond the scope of the current paper. In exploring these broader questions the results presented here, in addition to solving an important special case, should provide insight into how informational and conformity incentives interact more generally.

Acknowledgments

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Appendix A

I omit the subscript n for arbitrary populations (on Δ, Ψ, and Φ), including it explicitly only in results for which the population size is allowed to vary.

Proof of Lemma 1. For strategy $\{0, 1\}$, $\Delta(0, 1, \alpha) = \Delta(0, 1, \beta) = 0$, and $\Psi(0, 1, \alpha) = \Psi(0, 1, \beta) = -1$. Therefore $\Phi(0, 1, \alpha) = \Phi(0, 1, \beta) = -k$, and always voting for B is an equilibrium. The proof for $\{1, 0\}$ is identical. □

Proof of Lemma 2. Simple algebra establishes:

$$\begin{aligned}\Delta(q_\alpha, q_\beta, \alpha) - \Delta(q_\alpha, q_\beta, \beta) &= \big[\varphi(\alpha) - \varphi(\beta)\big]\big[f_{2n}^n(\gamma_{\mathrm{A}}) + f_{2n}^n(\gamma_{\mathrm{B}})\big] \\ &> 0,\end{aligned}$$

as $\varphi(\alpha) > \varphi(\beta)$. Similarly:

$$\begin{aligned}\Psi(q_\alpha, q_\beta, \alpha) - \Psi(q_\alpha, q_\beta, \beta) &= -\big[\varphi(\alpha) - \varphi(\beta)\big]\left\{\begin{array}{l}\big[F_0^{n-1}(\gamma_{\mathrm{A}}|2n) - F_0^{n-1}(1-\gamma_{\mathrm{B}}|2n)\big] \\ +\big[F_0^{n-1}(\gamma_{\mathrm{B}}|2n) - F_0^{n-1}(1-\gamma_{\mathrm{A}}|2n)\big]\end{array}\right\} \\ &\geqslant 0\end{aligned}$$

as $q_\alpha \geqslant 1 - q_\beta$ by assumption, which implies $\gamma_{\mathrm{A}} \geqslant 1 - \gamma_{\mathrm{B}}$ and $\gamma_{\mathrm{B}} \geqslant 1 - \gamma_{\mathrm{A}}$. Therefore, $\Phi_n(q_\alpha, q_\beta, \alpha) > \Phi_n(q_\alpha, q_\beta, \beta)$ and a mixed equilibrium is not possible. □

Several properties recur in the proofs to follow. I state and prove these as Lemma 7, and refer to them as facts (a), (b), and (c), hereafter.

Lemma 7.

(a) $\Phi(q_\alpha, q_\beta, \alpha) > \Phi(q_\alpha, q_\beta, \beta)$,
(b) $\Phi(q_\alpha, q_\beta, s_i)$ *is continuous in* q_α *and* q_β,
(c) $\varphi(s_i) \gtreqless \frac{1}{2} \Rightarrow \psi(1, 1, s_i) \gtreqless 0$, $\Delta(1, 1, s_i) \gtreqless 0$, *and* $\Phi(1, 1, s_i) \gtreqless 0$, *for both* s_i.

Proof of Lemma 7. Part (a) is proven in the proof of Lemma 2 and (b) is obvious. For (c), note that $q_\alpha = q_\beta = 1$ implies $\gamma_A = \gamma_B = p$, giving:

$$\begin{aligned}\Psi(1,1,s_i) &= \varphi(s_i)\big[F_0^{n-1}(1-p|2n) - F_0^{n-1}(p|2n)\big] \\ &\quad - \big(1-\varphi(s_i)\big)\big[F_0^{n-1}(1-p|2n) - F_0^{n-1}(p|2n)\big] \\ &= \big(2\varphi(s_i)-1\big)\big[F_0^{n-1}(1-p|2n) - F_0^{n-1}(p|2n)\big],\end{aligned}$$

and,

$$\Delta(1,1,s_i) = f_{2n}^n(p)\big(2\varphi(s_i)-1\big),$$

where the sign of both expressions is determined by the sign of $2\varphi(s_i)-1$ (as $p > \frac{1}{2}$). □

Proof of Theorem 1. Part (i) follows from fact (c) of Lemma 7.

To prove part (ii), fact (a) of Lemma 7 implies it is sufficient to show $\Phi(q_\alpha,1,\alpha)=0$ for some $q_\alpha \in (0,1)$. By facts (b) and (c) of the lemma, this reduces to showing $\Phi(q'_\alpha,1,\alpha)<0$ for some $q'_\alpha \in (0,1)$, which is true as Lemma 1 proved $\Phi(0,1,s_i)=-k$ for both s_i.

Turn now to part (iii) and consider the components of Φ. From above we have $\Psi(1,1,s_i)>0$ for both s_i, and it is obvious that $\frac{\partial\Psi}{\partial q_\beta}(1,q_\beta,s_i)<0$; thus, $\Psi(1,q_\beta,s_i)>0$ for all q_β, and $\Phi(1,q_\beta,s_i)$ is strictly increasing in k. For Δ we have $\Delta(1,0,\beta)=0$ and $\Delta(1,1,\beta)>0$. The result requires the following two properties, which I prove in turn:

(I) $\Delta(1,q'_\beta,\beta)\geqslant 0$ implies $\Delta(1,q''_\beta,\beta)>0$ if $q''_\beta > q'_\beta > 0$,
(II) $\Delta(1,q_\beta,\beta)<0$ for some q_β if and only if $n>\lambda$.

Proof of Property (I). $\Delta(1,q'_\beta,\beta)\geqslant 0$ implies $\frac{\varphi(\beta)}{1-\varphi(\beta)}R^n \geqslant 1$, where $R=\frac{(1-\gamma_A)\gamma_A}{(1-\gamma_B)\gamma_B}$. If $q_\alpha=1$, then $\gamma_A = 1-q_\beta(1-p)$ and $\gamma_B = pq_\beta$, and several steps of algebra leads to $\frac{\partial R}{\partial q_\beta}>0$, establishing the result.

Proof of Property (II). Property (I) implies that $\Delta(1,q_\beta,\beta)<0$ for some q_β only if $\Delta(1,q_\beta,\beta)<0$ for q_β small. This requires $\lim_{q_\beta\to 0}\frac{\partial\Delta}{\partial q_\beta}(1,q_\beta,\beta)<0$. Differentiating and rearranging gives:

$$\begin{aligned}\frac{\partial\Delta}{\partial q_\beta}(1,q_\beta,\beta) &= -(1-p)\varphi(\beta)\binom{2n}{n}\gamma_A^{n-1}(1-\gamma_A)^{n-1}n(1-2\gamma_A) \\ &\quad - p\big(1-\varphi(\beta)\big)\binom{2n}{n}\gamma_B^{n-1}(1-\gamma_B)^{n-1}n(1-2\gamma_B) \\ &= n\binom{2n}{n}\gamma_A^{n-1}(1-\gamma_A)^{n-1}\begin{bmatrix} -(1-p)\varphi(\beta)(1-2\gamma_A) \\ -p(1-\varphi(\beta))\frac{\gamma_B^{n-1}(1-\gamma_B)^{n-1}}{\gamma_A^{n-1}(1-\gamma_A)^{n-1}}(1-2\gamma_B)\end{bmatrix}.\end{aligned}$$

Substituting in for $q_\alpha=1$ and q_β:

$$\begin{aligned}\lim_{q_\beta\to 0}\frac{(1-\gamma_B)\gamma_B}{(1-\gamma_A)\gamma_A} &= \lim_{q_\beta\to 0}\frac{p}{1-p}\frac{(1-pq_\beta)}{(1-q_\beta(1-p))} \\ &= \frac{p}{1-p}.\end{aligned}$$

As $\gamma_A \to 1$ and $\gamma_B \to 0$, the limit of the bracketed term in $\frac{\partial \Delta}{\partial q_B}$ is then

$$\left[(1-p)\varphi(\beta) - p\big(1-\varphi(\beta)\big)\left(\frac{p}{1-p}\right)^{n-1}\right],$$

which is negative if and only if $n > \lambda$ (where λ is as defined in Lemma 3).

Putting the pieces together, properties (I) and (II) show for all $q_\alpha > 0$ that $n < \lambda$ implies $\Delta > 0$; hence, $\Phi > 0$, and no equilibrium exists. Similarly, (I) and (II) show for k sufficiently large that $k.\Psi_n > |\Delta|$, for all $q_\alpha > 0$, again giving $\Phi > 0$ and the nonexistence of equilibrium.

For the final claim note that $\Delta_n(1, q_\beta, s_i)$ is composed of the probability difference between two distinct binomial events. As $n \to \infty$ these probabilities become arbitrarily small and $\Delta_n(1, q_\beta, s_i) \to 0$ for all q_β. $\Psi(1, q_\beta, s_i)$ is minimized at $q_\beta = 1$ (as this maximizes the probability of B winning) and $\lim_{n\to\infty} \Psi_n(1, 1, s_i) = [2\varphi(s_i) - 1]$ (as by the law of large numbers the better candidate always wins). Therefore, for any k and $q_\beta > 0$, $\Phi_n(1, q_\beta, \beta) > 0$ for sufficiently large n, and the result follows. □

Proof of Theorem 2. (i) This is immediate from fact (c) in Lemma 7 as in tight elections $\varphi(\alpha) > \frac{1}{2} > \varphi(\beta)$.

(ii) From Lemma 1, $\Phi(1, 0, \beta) > 0$. By the continuity of utility, and part (i), there exists at least one $q_\beta \in (0, 1)$ such that $\Phi(1, q_\beta, \beta) = 0$, and as $\Phi(1, q_\beta, \alpha) > \Phi(1, q_\beta, \beta)$, $\{1, q_\beta\}$ is an equilibrium. Case (iii) is analogous to (ii). □

Proof of Lemma 5. Omitting the strategy argument for simplicity, $q_\alpha < 1 - q_\beta$ implies a voter is more likely to vote for candidate A following an observation of β rather than α. This implies candidate A is more likely to win in state B than state A: $P_n^A < 1 - P_n^B$. Applying the same logic for candidate B: $P_n^B < 1 - P_n^A$. Therefore:

$$\begin{aligned} P_n &= \pi P_n^A + (1-\pi) P_n^B \\ &< \pi\big[1 - P_n^B\big] + (1-\pi) P_n^B \\ &< \pi + P_n^B(1-2\pi) \\ &< 1-\pi. \end{aligned}$$

Substituting instead for P_n^A produces $P_n < \pi$, and the result follows. □

Proof of Theorem 3. Part (i). I prove for n sufficiently large that $\Phi_n(0, q_\beta, \alpha) < 0$ for all q_β, whereas $\Phi_n(0, q_\beta, \beta) = 0$ for some $q_\beta \in (0, 1)$, from which the result follows (recall that fact (a) in Lemma 7 requires ordered strategies). For $q_\alpha = q_\beta = 0$, $\gamma_A = 1 - p$ and $\gamma_B = 1 - p$, giving:

$$\Psi_n(0, 0, s_i) = \big[F_0^{n-1}(1-p|2n) - F_0^{n-1}(p|2n)\big]\big[\varphi(s_i) - \big(1 - \varphi(s_i)\big)\big],$$

which, as $p > \frac{1}{2}$, implies:

$$\Psi_n(0, 0, s_i) \gtreqless 0 \quad \text{if} \begin{cases} \varphi(s_i) < \frac{1}{2} \\ \varphi(s_i) = \frac{1}{2} \\ \varphi(s_i) > \frac{1}{2} \end{cases}, \quad \text{and,}$$

$$\Psi_n(0, 0, s_i) \to 1 - 2\varphi(s_i) \quad \text{as } n \to \infty.$$

As $\Psi_n(0,1,\alpha) = \Psi_n(0,1,\beta) = -1$ for all n (from Lemma 1) and $\Psi_n(0, q_\beta, s_i)$ is strictly decreasing in q_β for both s_i, we have for all q_β that $\Psi_n(0, q_\beta, \alpha) < \varepsilon < 0$, whereas $\Psi_n(0, q_\beta, \beta)$ crosses from positive to negative for some $q_\beta \in (0,1)$.

The components of $\Delta_n(q_\alpha, q_\beta, s_i)$ are individual events in a binomial distribution and for all q_α, q_β, $\Delta_n(q_\alpha, q_\beta, s_i) \to 0$ as $n \to \infty$. Therefore, for large enough n, $\Phi_n(0, q_\beta, \alpha) < 0$ for all q_β, and by continuity of utility there is at least one $q_\beta \in (0,1)$ such that $\Phi_n(0, q_\beta, \beta) = 0$, proving part (i). The proof of (ii) is analogous as the election is tight. □

Proof of Theorem 4. The method of proof is similar to Theorem 3. As the election is lopsided, there is an ε such that $\Psi_n(0, q_\beta, s_i) < \varepsilon < 0$ for all s_i, n, and q_β. As Δ consists of the probability of two binomial events, $\Delta_n(0, q_\beta, s_i) \to 0$ as $n \to \infty$, and for sufficiently large n, $\Phi_n(0, q_\beta, \beta) < 0$ for all q_β. Thus, β observers mixing cannot support an equilibrium if $q_\alpha = 0$ and the result follows. □

Proof of Lemma 6. Part (i). $q_\alpha = 1$ implies $\gamma_A \geqslant p > \frac{1}{2}$, which for $n \to \infty$ gives $P_n^A(\sigma_n) \to 1$ by the law of large numbers. As $n \to \infty$, $\Delta_n \to 0$, and so $\Phi_n \to \Psi_n$, which is given by (omitting the argument σ_n where obvious):

$$\begin{aligned}\Psi_n\big(1, q_\beta^n, \beta\big) &\to \varphi(\beta)\big[P_n^A - \big(1 - P_n^A\big)\big] + \big(1 - \varphi(\beta)\big)\big[\big(1 - P_n^B\big) - P_n^B\big] \\ &\to \varphi(\beta) + \big(1 - \varphi(\beta)\big)\big[1 - 2P_n^B\big].\end{aligned}$$

Thus, $\Phi_n(1, q_\beta^n, \beta) = 0 \Rightarrow \varphi(\beta) + (1 - \varphi(\beta))[1 - 2P_n^B] \to 0$, which rearranged gives the result. The proof of part (ii) is analogous. □

Proof of Theorem 5. Part (i). Unordered equilibria are dominated by the uninformative voting equilibria, for which $P_n(1,0) = \pi < 1$ and $P_n(0,1) = 1 - \pi < 1$. Informative voting gives $P_n(1,1) \to 1$ as $n \to \infty$ by the law of large numbers. The randomness of mixing equilibria implies that outcomes do not always reflect the majority of private signals and outcomes are informationally dominated by $\sigma = \{1,1\}$; more specifically, in mixing equilibria such that $q_\beta \in (0,1)$ or $q_\alpha \in (0,1)$, we have from Lemma 6 that as $n \to \infty$,

$$\begin{aligned}P_n(1, q_\beta) &\to \pi.1 + (1 - \pi).\frac{1}{2(1 - \varphi(\beta))} < 1, \\ P_n(q_\alpha, 1) &\to \pi.\frac{1}{2\varphi(\alpha)} + (1 - \pi).1 < 1,\end{aligned}$$

as $0 < \varphi(\beta) < \frac{1}{2} < \varphi(\alpha) < 1$.

Part (ii). Denote the expected fraction of voters in the majority by $M_n(\sigma)$ for strategy σ, and the same measure conditional on candidate J being better by $M_n^J(\sigma)$. I begin by showing that mixed equilibria are utilitarian dominated by informative voting. For $\{1, q_\beta\}$, Lemma 6 implies that $M_n^B(1, q_\beta) \to \frac{1}{2}$ as $n \to \infty$ (if the expected vote share differed from $\frac{1}{2}$ then the law of large numbers implies one candidate would win almost surely, which is ruled out by the lemma). This requires $\frac{1}{2} = q_\beta p$, giving:

$$\begin{aligned}M_n(1, q_\beta) &\to \pi\left[p + \left(1 - \frac{1}{2p}\right)(1 - p)\right] + (1 - \pi)\frac{1}{2} \\ &= \frac{2\pi p - \pi + p}{2p}.\end{aligned}$$

In contrast, $M_n(1,1) \to p$, and:

$$M_n(1,q_\beta) - M_n(1,1) \to \frac{2\pi p - \pi + p}{2p} - p$$
$$\to -\frac{1}{2}(2p-1)\frac{p-\pi}{p} < 0,$$

as $p > \frac{1}{2}$ and $p > \pi$. Thus, $M_n(1,1) > M_n(1,q_\beta)$ for all $q_\beta < 1$ as $n \to \infty$, and, combined with Part (i), we have $E_n(1,1) > E_n(1,q_\beta)$. Mixed equilibria of the form $\{q_\alpha, 1\}$ are similarly dominated.

It is obvious that unordered equilibria are also utilitarian dominated. That leaves informative and uninformative voting equilibria. We have $E_n(1,1) = 1 + k.p$, $E_n(1,0) = \pi + k$, and $E_n(0,1) = 1 - \pi + k$. The result follows from simple algebra for when each candidate is the slight front-runner. □

Proof of Theorem 6. From Theorem 1 the possible symmetric equilibria as $n \to \infty$ are of the form $\{1,0\}, \{0,1\}$, and $\{q_\alpha, 1\}$ for $q_\alpha \in (0,1)$. $P_n(1,0) = \pi > 1 - \pi = P_n(0,1)$, and from Lemma 6,

$$P_n(q_\alpha, 1) \to \pi\frac{1}{2\varphi(\alpha)} + (1-\pi)$$
$$\to \frac{1}{2}\left(1 + \frac{1-\pi}{p}\right)$$

by substituting for $\varphi(\alpha)$. Comparing: $P_n(1,0) > P_n(q_\alpha, 1)$ requires

$$\pi > \frac{1}{2}\left(1 + \frac{1-\pi}{p}\right)$$
$$\pi > \frac{1+p}{1+2p}.$$

As $\frac{\mathrm{d}}{\mathrm{d}p}(\frac{1+p}{1+2p}) = \frac{-1}{(1+2p)^2} < 0$ and $p = \frac{1+p}{1+2p} \Rightarrow p = \frac{1}{\sqrt{2}}$, the inequality holds for all $p \geqslant \frac{1}{\sqrt{2}}$ as $\pi > p$, proving the first case. The second case follows from reversing the inequalities, $P_n(q_\alpha, 1) < P_n(1,0) \Rightarrow \pi < \frac{1+p}{1+2p}$, and the third case is given by the cut-point.

The utilitarian efficiency of uninformative voting is $E_n(1,0) = \pi + k$, implying that $\{1,0\}$ is utilitarian optimal whenever it is informationally optimal. It remains to compare $\{1,0\}$ with the mixed equilibrium for when the mixed equilibrium is informationally optimal. Following the proof of Theorem 5,

$$M_n(q_\alpha, 1) \to \pi\frac{1}{2} + (1-\pi)\left[p + \left(1 - \frac{1}{2p}\right)(1-p)\right] = \frac{3p - 2\pi p + \pi - 1}{2p},$$

as $n \to \infty$. Thus, $E_n(1,0) \geqslant E_n(q_\alpha, 1)$ requires that $k \geqslant \hat{k} = \frac{(1-\pi)-p(2\pi-1)}{(1-\pi)+p(2\pi-1)}$. To show $\hat{k} > 0$, note that the denominator is positive and the numerator is decreasing in p, as $\frac{1}{2} < p < \pi < 1$. Solving for $\hat{k} = 0$ implies $p = \frac{1}{\sqrt{2}}$, and so for the domain under consideration (of case ii), $\hat{k} > 0$ whenever $\{q_\alpha, 1\}$ is informationally optimal. □

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