Measuring the Sorting and Incentive Effects of Tournament Prizes

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Abstract

Tournaments where ordinal position determines rewards are an important component of our economy. By studying sporting tournaments, we hope to shed light on the nature of other more economically significant tournaments where data may be less readily available. We separately quantify the sorting and incentive effects of tournament prizes by employing a two-part model which we apply to a unique data set of road running race results. We use concepts from spatial econometrics to estimate different competitive effects and we present a counterfactual example of how a hypothetical change in prizes would be predicted to change race participation and speed.

Keywords: Tournaments, Sorting, Incentives, Prizes

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1 Introduction

Tournaments are characterized by rewards that are determined by ordinal position; in tournaments it is relative output and not absolute output that determines payment. Competitions of this type appear often in the economy. Examples include such diverse situations as promotions in corporations and universities, patent races, political elections and sporting events. In this paper we study sporting tournaments where relatively rich data can be found. By doing so we hope to shed light on the nature of other more economically significant tournaments where data may be less readily available.

In general, tournaments are used as mechanisms to induce desired behavior by the participants. For example, promotion based on relative performance may serve to motivate employees to work hard, and prizes in sporting events based on competitors' finishing positions may motivate participants to train hard prior to a event and to exert effort in the event itself. In tournaments prize values and structure are chosen by the tournament organizers in light of the organizers' particular objectives.[2]

Although there are a wide range of questions that researchers have considered concerning tournament design and the behavior of tournament participants, as a practical matter researchers have most often considered "within" tournament issues rather than "between" tournament issues. The focus has usually been on the incentive effect of prizes on the behavior of participants assuming they are irrevocably committed to a particular tournament. In general, less attention has been given to the broader question of how

[2] It is worth noting that in otherwise similar contexts organizers could have differing objectives and this might imply different optimal prizes. In light of the possible variations in objectives, it would seem unlikely that there could ever be a universal "best" structure for tournament prizes.

participants sort into particular tournaments based on prizes and other tournament characteristics.

We propose a two-part model to examine the results from road running races that enables us to separately quantify the effect of prizes on race participation (the so called “sorting” effect) and on race effort (the so called “incentive” effect). In addition, in the equation used to estimate the speed of runners we apply a design that draws on ideas from spatial econometrics to account for the impact of global and local competitive effects. The data set we use is unique and, since unlike the data sets used in prior studies we are aware of it includes runners that did not win prizes, it is particularly well suited to the analysis we are undertaking.

To the best of our knowledge our method to separately model sorting and incentive effects is new and we are unaware of other researchers applying the techniques of spatial econometrics in this context. Using our methodology we demonstrate that in the races we examine sorting and incentive effects are present and we are able to quantify their impacts.

2 Relevant Literature

We summarize below three categories of tournament research that are relevant to our investigation: (a) theoretical work; (b) experimental studies; and, (c) sports related empirical studies. Other research topics that use ideas from tournaments include patent races, executive compensation and promotions in corporations and universities.

A. Theoretical

Contrary to the impression that might be acquired by a casual reader of the empirical literature regarding tournaments, there is no single complete theory that can directly be applied to most real life tournaments observed in the economy. Instead a number of special theories with simplifying assumptions offer clues regarding the expected behavior of participants.

In Lazear and Rosen (1981) the authors show that a tournament can theoretically be superior to hourly wages by proving that, for risk neutral workers with uniform abilities, optimally structured tournaments yield results identical to piece rate pay and, if workers are risk averse, tournaments can be more efficient. The authors point out that they are unable to completely characterize the conditions under which piece rates dominate rank-order tournaments and vice versa. However, they do provide examples and observe that persons with more endowed income and smaller absolute risk aversion are more likely to prefer contests and those with low levels of endowed wealth and larger absolute risk aversion are more likely to prefer piece rates. The authors assume that the cost of effort is increasing and convex in effort. The authors argue that in the case of workers of heterogeneous ability it would be necessary to use credentials or other mechanisms to sort individuals into to the "right" contests if contests are to be efficient in the sense of participants choosing socially optimal levels of investment.

Krishna and Morgan (1998) consider the optimal design of tournaments for homogenous workers. Unlike many other papers that further develop the ideas from Lazear and

Rosen's pioneering paper, Krishna and Morgan focus on tournament design rather than comparing tournaments, piece rate pay or combinations of the two. The authors show that, regardless of risk preferences, winner-take-all tournaments are optimal for up to three competing homogeneous workers. In this context optimal means the prize structure in a tournament that awards prizes based on rank position that induces the greatest total effort. In the case of four workers, the optimal structure of awards depends on the risk preferences of the participants. In the case of risk neutrality, winner-take-all tournaments are again optimal. If workers are risk averse, the optimal tournament pays prizes to the winner and the runner-up. The authors assume that the cost of effort is increasing and convex in effort.

In Moldovanu and Sela (2001) the authors consider risk neutral heterogeneous participants that differ in their cost of effort. In this context the authors show that winner-take-all tournaments are optimal when the cost of effort is linear or concave in effort while it may be optimal to have more than one prize if the cost of effort is convex in effort. The authors assume that ability is private information with abilities drawn independently from a known distribution. As in Krishna and Morgan (1998) optimal means the prize structure that induces the greatest total effort. Moldovanu and Sela's assumption of privately informed heterogeneous participants and a deterministic relationship between effort and output contrasts with the papers discussed earlier where agents are identical and observed output is a stochastic function of unobserved effort.

In Szymanksi and Valletti (2004) the authors show that if output is a stochastic function of effort a second prize may be optimal if the contestants differ enough in ability. The authors show that in a three-person contest with one strong competitor and two equally weak competitors a second prize can be optimal from the point of view of eliciting maximum effort from every contestant. In this model whether or not a second prize is optimal depends on the difference in the cost of effort of the weak and strong players. The authors assume that the cost of effort is increasing and linear in effort and that participants are risk neutral.

Having surveyed some of the relevant theoretical literature, it is perhaps useful to assess how close the available models come to describing road running races. Table 1 lists in the left-hand column what appear to us to be the most salient characteristics of road running races. The remaining columns indicate whether or not the models developed in the theoretical papers reviewed in this section incorporate the relevant feature. As the reader can observe, there simply is no model that reasonably corresponds to road running races.[3] A similar gap between theory and the outside world exists with most other real tournaments in the economy; usually there is no appropriate theoretical model to be tested by empirical economists.

[3] Of course there are many other interesting theoretical papers discussing one feature or another of tournaments. Although we have not presented a full review of all these papers here we believe that our conclusion would hold even if this exercise was to be completed.

Table 1

Analysis of Selected Theoretical Models

	Lazear & Rosen	**Krishna & Morgan**	**Moldovanu & Sela**	**Szymanski & Valletti**
Large number of participants	No	No	No	No
Competition between tournaments not just within tournaments	No	No	No	No
Heterogeneous ability	No	No	Yes	Yes
Ability is public information	--	--	No	Yes
Risk neutral and risk averse participants permitted	Yes	Yes	No	No
Output is a stochastic function of effort	Yes	Yes	No	Yes
Flexibility in the tournament organizers' objectives	No	No	No	No

Although it is obviously simplistic and fails to reflect fully all the features of the models considered, Table 1 suggests two challenges. First, there is a challenge for theoretical economists to develop a more comprehensive theoretical framework. Second, since no directly relevant model exists, empirical economists need to develop a way to organize the data produced by real world tournaments in order to understand them better and to suggest likely areas for fruitful theoretical work. This paper attempts to make a contribution in the second of these categories.

B. Experimental

In light of the complexity of many tournaments compared to the available theoretical models, experiments may provide a useful link between theory and the real world tournaments.

Harbring and Irlenbusch (2003) investigate different tournament design alternatives along two dimensions; tournament size and prize structure. Participants have homogenous costs of effort and a predetermined number of prizes are awarded to those choosing the greatest effort. The authors find that average effort tends to increase and variability of effort tends to decrease with the number of prizes.

Orrison, Schotter and Weigelt (2004) use an experimental design in which the cost of effort is homogenous and output is a stochastic function of unobserved effort. Prizes are awarded to a predetermined number of those achieving the highest output. The authors found that behavior was invariant to tournament size (i.e., behavior is the same for tournaments with two players and one prize and say six players and three prizes). If the number of prizes was varied for a given number of participants, effort changed little although there was some evidence of effort declining if the number of prizes was very high.

It would seem likely that further areas of experimental research would be fruitful. In particular, investigating additional characteristics listed in Table 1 might assist in the development of a more comprehensive tournament theory.

C. Sports Related Empirical Studies

The economics of sports has received significant interest from researchers. Szymanski (2004) provides an extensive overview of sports economics. Below we briefly review two pairs of papers in this area that are relevant to tournaments. Ehrenberg and Bognanno (1990) and Orszag (1994) study professional golf competitions and Maloney and McCormick (2000) and Lynch and Zax (2000) study road running races.

Ehrenberg and Bognanno (1990) use data from professional golf tournaments in US in 1984. The authors' analysis is restricted to the top 160 money winners for which the average score on all rounds during the year is available. The authors use these average scores as a proxy for each player's ability. Heterogeneity in the prestige of tournaments is accounted for by using a dummy variable for major tournaments. Controlling for the tournaments' characteristics, an individual's ability and the ability of other players, ordinary least squares analysis is employed to estimate the effect of total prize money on an individual's score. The authors find a negative coefficient for total prize money, which is larger if the analysis is restricted to exempt players (exempt players correspond to those players that have been most successful recently).

Orszag (1994) generally follows the same methodology and uses data from professional golf tournaments in 1992. In contrast to Ehrenberg and Bognanno, Orszag finds the coefficient for total prizes to be insignificantly different from zero. Orszag advances a possible explanation for the difference in results by pointing out that the weather variable in Ehrenberg and Bognanno's study appears to be highly correlated with the total prize, suggesting it could be measured with error. Orszag's results imply that prizes in a particular tournament have no significant impact on a golfer's scores in that tournament. As the author states; "Perhaps golf is not the ideal example to study tournament theory, or perhaps tournament theory does not elicit the desired incentive results." Although Orszag does not put it in these terms, our instinctive assessment is that at this level of competition, prizes do provide incentives to practice and to train in the time prior to a tournament; however they do not in general change an individual's performance in a particular tournament. If a player has honed his skills through extensive training, trying harder than usual in a particular tournament simply does not reliably improve his score. If this hypothesis is correct, an increase in prizes should improve an individual's scores in all tournaments and not just the one that increased prizes. In this case, if scores are better on average at tournaments with large prizes it is because the prizes attract better players not because particular individuals perform better than usual.

Is the balance between the efficacies of training prior to a competition (which improves performance for a number of different tournaments) and effort in the competition itself different for road running races so that the incentive effect in a particular race is meaningful and can be measured? As is the case of golf, the conclusions to date that can be drawn from the literature are contradictory. In both Maloney and McCormick (2000) and Lynch and Zax (2000) the authors endeavor to disentangle the anticipated positive relation between prizes and performance into incentive and sorting effects.

Maloney and McCormick use data for runners who won prizes in races that took place in the southeastern US over the years 1987 to 1991. The theoretical model proposed as a starting point is Lazear and Rosen (1981) with two identical, risk-neutral competitors. The authors attribute the coefficient for the prize spread, defined as the difference between the prize won by each runner and the next lowest prize, to the incentive effect and the coefficient for the average prize to the sorting effect. On this basis the authors find both an incentive and a sorting effect.

Lynch and Zax use data from races organized in US and abroad in 1994. The authors show that without controlling for ability, runners seem to run faster if the prize difference is higher. In this context prize difference is defined to be the amount of prize money a runner would lose in a particular race if she finished one place below the position implied by her pre-race ranking rather than at the position implied by her pre-race ranking. The pre-race ranking was constructed using 1993 world road rankings. Once ability was taken into account using fixed effects or 1993 rankings, the coefficient for the prize difference that would measure the incentive effect is neither of the expected sign nor statistically significant for almost all distances analyzed. Since after taking ability into account apparent incentive effect seem to disappear, the authors conclude "that races with large prizes record faster times because they attract faster runners, not because they encourage all runners to run faster."

3 Econometric Model

In our model we assume runners make three decisions. First, runners decide whether to race in a particular period. Second, if they choose to race, runners determine in which of the available races to participate. Third, once they know the other race participants, runners choose the effort to exert in their chosen race. For simplicity we assume that: (1) the first decision is based on idiosyncratic factors that are independent from any of the factors that impact the second and third decisions and the runners performance in their chosen races; and, (2) the disturbance terms in the equations that describe the attractiveness of the races, the "sorting equation", (relevant for the second decision) and runners' performance, the "speed equation", (relevant to the third decision) are independent. Assumption (1) allows us to model just runners' second and third decisions. Assumption (2) simplifies the estimation of the sorting and speed equations.

As described in greater detail in the next section, in our analysis we look only at "top runners" which we define as male runners that finish in the top 30 in at least one race in our data set and in the top 150 in at least one additional race in the data set. We believe that these top runners are likely to be less subject to idiosyncratic reasons influencing race choice and race effort compared to less successful runners and limiting our attention to these runners reduces computational complexity.

For each race we identify up to two most closely competing races based on the date on which the races take place, the distance between race venues and the lengths of the races. In particular, the methodology to identify the most closely competing races used the following algorithm. First, only races occurring within a time period starting one weekend prior to the race of interest and ending one weekend after the race of interest were considered as candidate competing races.[4] Second, from the group of races occurring within the required time period we retained as candidate competing races only those races for which the venue was within 1,000 miles of the race of interest. Third and finally, from the remaining candidate competing races the two most similar in length to the race of interest were chosen.[5] Applying this methodology to our data results in eight of the 71 races in our data set with top runner participation having no competing races,

[4] In the case of races that were half marathons or longer, this time period was extended to two weekends before the race of interest until two weekends after the race of interest. We believe that this approach is appropriate in light of the longer recovery time associated with races of this length.
[5] In Section 5 we describe two alternative methodologies we used for determining competing races in order to check the robustness of our results.

ten having only one competing race and the remaining 53 races having two competing races.

We assume that having decided to race in a particular period, a runner chooses between the race of interest and the most closely competing races only. If runner *i* has chosen to compete in a particular period, the relative attractiveness of race *k* is A_{ik}. We assume that A_{ik} is determined as shown in Equation 1.

$$A_{ik} = x_k \alpha + r_i x_k \beta + \varepsilon_{ik} \qquad (1)$$

In Equation 1 x_k is a vector characterizing the prizes in race *k* and will include variables such as the total value of all prizes offered and the Herfindahl index of the value of the prizes offered, r_i is runner *i*'s ranking and ε_{ik} is an error term that accounts for a runner's idiosyncratic race preferences (i.e., preferences unrelated to prizes and ranking). For computational convenience we will assume that the error term is i.i.d. extreme value. We assume that ranking is a measure of a runner's ability. The methodology we use to calculate each runner's ranking is based on a runner's success in beating other runners and is discussed in detail later in this section.

If a particular runner participates in race *k* rather than any of the most closely competing races which we designate $\tilde{k}$ then we know $A_{ik} \geq A_{i\tilde{k}} \ \forall \ \tilde{k}$. We define P_{ik} as the probability of observing runner *i* participating in race *k* rather than any of the most closely competing races. Since we are assuming ε_{ik} is i.i.d. extreme value we can use a logit model to calculate P_{ik} as shown in Equation 2. [6]

$$P_{ik} = \frac{e^{x_k \alpha + r_i x_k \beta}}{e^{x_k \alpha + r_i x_k \beta} + \sum_{\tilde{k}} e^{x_{\tilde{k}} \alpha + r_i x_{\tilde{k}} \beta}} \qquad (2)$$

Our estimates for the vectors α and β are the values that maximize the likelihood function shown in Equation 3; this is the sorting equation.

$$L = \sum_{k}^{\substack{\text{all} \\ \text{races}}} \sum_{i}^{\substack{\text{all runners} \\ \text{in race } k}} \log P_{ik} \qquad (3)$$

For estimation purposes we parameterize the relationship determining the average speed for runner *i* in race *k* as shown in Equation 4; this is the speed equation.

$$speed_{ik} = W_k a + X_k b + Y_{ik} c + Z_i d + e_{ik} \qquad (4)$$

In Equation 4, W_k represents intrinsic race characteristics and will include variables such as race prizes, distance, distance squared, the topography of the race course and the prevailing temperature on the race day. X_k represents global competitive intensity and will include variables relating to the group of top runners participating in the race such as the total number of top runners in the race. Y_{ik} represents local competitive intensity. Unlike the earlier terms, this effect is specific to a particular runner in the race of interest.

[6] As described in Appendix 1, in some cases individual runners participated in one of the two competing races that had been identified. In this situation P_{ik} is defined as the probability that the relative attractiveness of the race the runner chooses not to participate in is less than the other two. If *k* and *l* are the races in which the runner participates and *m* is the one he does not, P_{ik} can be written as:

$$P_{ik} = \frac{e^{-(x_m \alpha + r_i x_m \beta)}}{e^{-(x_k \alpha + r_i x_k \beta)} + e^{-(x_l \alpha + r_i x_l \beta)} + e^{-(x_m \alpha + r_i x_m \beta)}}$$

Using the terminology from spatial economics Y_{ik} is based on a weighting matrix that accounts for the difference in ranking between *i* and the other top runners participating in race *k*. Anselin (1999) and Anselin (2002) provide a review of the relevant theory and techniques concerning the application of spatial econometrics. Our two measures of competitive intensity are designed to capture the intuition that individual effort (and therefore race speeds) will be influenced by the size and quality of the overall field and will also be impacted by the presence of runners of a similar ability. Z_i includes personal characteristics of runner *i* such as ranking. The error term, e_{ik}, is intended to reflect all other factors impacting a runner's speed and is assumed to vary by runner and race. Since we are assuming that e_{ik} and ε_{ik} are independent, we use ordinary least squares to estimate the vectors *a, b, c* and *d*.

As discussed in Section 2, some prior researchers have used ranking information that is intended to reflect a runner's ability as an explanatory variable. In the case of road running, rankings are not available for all the runners that are of interest to us and therefore we are required to develop our own ranking system. For simplicity we assume that the relationship between rankings and the probability of one runner beating another does not depend on race characteristics.

Considering the top runners in our data set we have a total of 7,377 pairwise tournaments where we observe a top runner beating another in a race. Our goal is to assign a ranking to each individual that results in the fewest possible number of "wrong" predictions (i.e., the smallest number of cases where the lower ranked runner beats the higher ranked runner) and which assigns plausible rankings to groups of runners that never interact.

The algorithm we used to determine rankings had two parts. First, initial rankings were calculated based on each runner's won/lost count which is equal to the number of pairwise victories minus the number of pairwise losses for each runner. Using the initial rankings calculated in this way, the number of pairwise tournaments in which the lower ranked runner beat the higher ranked runner was calculated. Second, we tried to improve the initial rankings through an iterative search. One of the incorrectly forecasted pairwise tournaments was chosen at random and the rank of the loser reassigned to the winner and the ranks of the loser and all the runners with rankings between the original rankings of the winner and loser were reduced by one. Using these revised rankings, the number of pairwise tournaments in which a lower ranked runner beat a higher ranked runner was recalculated. If the number of incorrectly forecast pairwise tournaments was reduced the initial candidate rankings were replaced with the revised rankings. If the revised rankings did not reduce the number of forecasting errors the initial candidate rankings were not changed. Next another random draw from the incorrectly forecast pairwise tournaments based on the candidate ranking was taken and the process repeated. Using this methodology the initial proportion of incorrectly forecast pairwise tournaments was 13.7%. 100 iterative searches reduced this to 12.0%. 500 searches reduced it to 8.0%. 1,000 searches reduced it to 5.9% and 5,000 searches reduced it to 5.1%. We used the rankings produced after 5,000 iterative searches in our analysis.

Our methodology for determining rankings outlined above uses runners' observed finishing positions. If Equation 4 correctly specifies each runner's performance, runner i's speed and therefore finishing position in race k is influenced by e_{ik}. As a result e_{ik}

impacts runner *i*'s ranking. But runner *i*'s ranking is included in Y_{ik} and Z_i in Equation 4 and so Y_{ik} and Z_i may be correlated with e_{ik}. Therefore as we have specified Equation 4, Y_{ik} and Z_i may be endogenous and consequently the coefficients we estimate for Equation 4 could be biased. This potential problem is partially addressed by the fact that the ranking for runner *i* depends not just on e_{ik} but on the error terms for runner *i* in all the races he participates in our data set (and indirectly on the error terms for other runners).

4 Description of the Data

Our data set was collected based on the information about races that found in the *2002 Road Race Management Directory*. This booklet contains details about the most important road race competitions that took place in 2002. We selected only the competitions for men held in United States that awarded monetary prizes for which we had a complete and accurate list of prizes and results.[7] The results for each competition were downloaded from the competitions' official sites. Weather data were obtained from a meteorological specialist. The characterization of courses in terms of topography (hilly, downhill or flat) was based on the way in which the race managers advertised their race in the *2002 Road Race Management Directory*.

A subset of the data set is used in the analysis in this paper. As described in Section 3 we limit our analysis to what we term top runners defined as runners that finish in the top 30 in at least one race in our data set and in the top 150 in at least one additional race in our data set. This defines a revised data set consisting of 861 observations including 366 top

[7] As a consequence, all of the very large races such as the New York and Boston marathons are not part of our data set.

runners in a total of 71 races. We observe a maximum of 42 top runners in each race and a minimum of one top runner. The maximum number of appearances by a top runner is six and the minimum two.

Summary information regarding the 71 races included in our analyses is shown in Table 2. In addition, 13 of the races were classified as hilly, five as down hill, 33 as flat and 20 as topography unknown.

Table 2

Summary Information by Race

Variable	Minimum Value	Maximum Value	Mean Value
Total value of prizes ($000s)	0.05	30.25	4.68
Herfindahl Index of prize values	0.14	1.00	0.34
Race distance (kilometers)	1.6	80.5	19.4
Average temperature (° F)	35	84	62
Number of top runners	1	42	12.1

Additional information regarding the 861 observations included in our analyses is shown in Table 3.

Table 3

Summary Information by Observation

Variable	Minimum Value	Maximum Value	Mean Value
Top runners within 20 ranking places	0	13	2.6
Ranking	1	366	176

A particular advantage of our data set over those used in the analyses that dealt with foot races in the context of tournament theory reviewed in Section 2 is that it permits us to observe the effects of prizes on athletes who did not win prizes.

5 Results

In this section we discuss the results of estimating Equation 3 (the sorting equation) and Equation 4 (the speed equation) and the robustness checks we performed.

Equation 3: The Sorting Equation

As shown in Appendix 1, based on the algorithm we chose to identify competing races, out of the total of 861 observations, 760 were suitable for use in estimating the coefficients in Equation 3. We first report results for estimating Equation 3 using a vector of variables to characterize race prizes (x_k) that contains the total value of all prizes and the Herfindahl index of the prizes offered. We chose to specify x_k in this way in the belief that prospective race participants might respond to both the overall purse of prize money and the manner in which it was divided between different prizes. The total value of all prizes and the Herfindahl index of the prizes appears to be a parsimonious way to capture these two features of the prizes offered in a particular race. Table 4 indicates the magnitude, standard errors and p values for the coefficients estimated using maximum likelihood.

Table 4

Sorting Equation Results

Variable	Coefficient	Standard error	P value
Total value of prizes ($000s)	0.1039	0.0298	0.000
Herfindahl index of prize values	-1.085	2.419	0.654
Rank x Total value of prizes ($000s)	-0.0003315	0.0001506	0.028
Rank x Herfindahl index of prize values	0.006923	0.010904	0.525

As can be observed from Table 4, it appears that in our data the total value of prizes and the total value of prizes interacted with a runner's rank are relevant to a runner's choice of race. The coefficients for both of theses variables are statistically significant at the 5% level. Neither of the coefficients relating to the variables including the Herfindahl index of prize values are statistically significant. If the estimation of Equation 3 is repeated using x_k consisting of only total value of prizes the results are as shown in Table 5.

Table 5

Alternative Sorting Equation Results

Variable	Coefficient	Standard error	P value
Total value of prizes ($000s)	0.1101	0.0267	0.000
Rank x Total value of prizes ($000s)	-0.0003746	0.0001364	0.006

Now both of the relevant coefficients are statistically significant at the 1% level.

Based on the results of this second estimation, we can parameterize the attractiveness of race *k* to runner *i* as shown in Equation 5.

$$A_{ik} = 0.1101(\textit{total value of prizes}_k) - 0.0003746(\textit{rank}_i \text{ x } \textit{total value of prizes}_k) \quad (5)$$

As expected, for highly ranked runners (i.e., runners with a ranking number that is low), increasing the total value of prizes offered increases the attractiveness of the race. For less highly ranked runners this effect is more muted. Using the point estimates of our results, we can conclude that the runner ranked 294th is indifferent to total prizes when choosing between races and for runners that are ranked below 294th increasing total prizes actually reduces the attractiveness of a race.

Section 6 includes an analysis of the numerical implications of the coefficients just reviewed on the probability of particular runners defecting to a different race in the event of a hypothetical change in prizes.

We explored the robustness of our reported results for the sorting equation by estimating three additional model specifications. We considered two alternative methodologies for determining competing races and a specification that included the number of other top runners actually observed to have participated in each race as an explanatory variable.

Table 6 compares our base case methodology for determining competing races with the two alternative methodologies.

Table 6

Comparison of Methodologies for Determining Competing Races

	Base Case	**Alternative 1**	**Alternative 2**
1st selection criteria	Races occur +/- one weekend of race of interest (+/- two weekends for half marathons or longer)	Races occur +/- two weekends of race of interest (+/- three weekends for half marathons or longer)	Every race is assigned to one of 42 categories based on seven predetermined time periods, two predetermined geographical regions and three predetermined race length categories
2nd selection criteria	Race venue is within 1,000 miles of the race of interest	Race venue is within 800 miles of the race of interest	--
Final selection criteria	Choose up to two races most similar in length to the race of interest	Choose up to two races most similar in length to the race of interest	Choose up to two races from the same category as the race of interest based on the proximity of race venues

When the sorting equation was re estimated using the competing races produced by Alternative 1 and Alternative 2 summarized in Table 6, the coefficients for total value of prizes and rank x total value of prizes remained statistically significant at the 10% level and varied in value by less than one standard error from the values reported earlier using the base case methodology for identifying competing races. In the case of Alternative 1 the coefficients for the Herfindahl index of prize values and rank x Herfindahl index of prize values remained statistically insignificant at the 10% level. In the case of Alternative 2 these coefficients were statistically significant at the 10% level. The signs of the coefficients for the Herfindahl index of prize values and rank x Herfindahl index of prize values were unchanged by the choice of methodology used to determine competing races. If the number of top runners was included as an explanatory variable the coefficient values and p values changed only slightly with the coefficients for total value of prizes and rank x total value of prizes remaining statistically significant at the 10% level while the coefficients for the Herfindahl index of prize values and rank x Herfindahl index of prize values were not.

On balance the results of the robustness checks support our choice of model specification described earlier.

Equation 4: The Speed Equation

The estimated coefficients for Equation 4 are shown in Table 7. The dependent variable used in the regression is average speed measured in units of meters per second.

Table 7

Speed Equation Results

Variable	Coefficient	Standard error	P value
W_k (intrinsic race characteristics)			
Total value of prizes ($000s)	0.00452	0.00286	0.117
Herfindahl Index of prize values	-0.6479	0.2139	0.003
Race distance in km	-0.03850	0.00653	0.000
(Race distance in km)^2	0.0002657	0.0001037	0.013
Dummy for hilly course	-0.1697	0.0600	0.006
Dummy for down hill course	0.1117	0.0462	0.018
Dummy if course topography unknown	0.02330	0.0813	0.775
Average temperature	-0.01638	0.02230	0.465
(Average temperature)^2	0.001478	0.0001962	0.454
X_k (global competitive intensity)			
Number of top runners	0.003715	0.002164	0.090
Y_{ik} (local competitive intensity)			
Number of top runners within 20 ranking places	0.01877	0.01061	0.081
Z_i (personal characteristics of runner)			
Runner ranking	-0.003458	0.000135	0.000
Constant	6.516	0.637	0.000

The adjusted R squared for the regression is 0.83. Standard errors are adjusted for clustering.

The results for each component of the speed equation are discussed below.

W_k (intrinsic race characteristics): We find that prizes impact average speed, which is consistent with the direct incentive effect. Average speed increases with the total value of the prizes offered and as the Herfindahl Index of prize values decreases. These effects are modest in size. An increase in total prizes by $10,000 is predicted to increase average

speed by approximately 0.05 meters per second.[8] A decline in the Herfindahl Index of prize values by 0.25 (this could occur if a race organizer decided to double the total amount of prizes offered by awarding four equally sized prizes rather than two equally sized prizes) is predicted to increase average speed by approximately 0.16 meters per second. As expected, the magnitude and sign of the distance and distance squared variables indicate that average speed is predicted to fall with increasing distance with the rate of change declining as distance increases. For example, an increase in distance from 10 to 20 kilometers is estimated to reduce average speed by approximately 0.31 meters per second all other tings being equal while an increase in distance from 30 to 40 kilometers is estimated to reduce speed by only 0.20 meters per second. As indicated by the hilly course and down hill dummies, compared to a flat course a hilly venue reduces average speeds while a down hill course increases speeds. The dummy variable if course topography is unknown is not statistically significant. This dummy variable was employed if it was not possible using the information provided by race organizers to confidently categorize the race as hilly, downhill or flat. The fact that the coefficient is not statistically significant is consistent with the hypothesis that the races assigned to this category were not predominately of one type or another. The coefficients of the average temperature and average temperature squared variables are not statistically significant.

X_k *(global competitive intensity):* We used one variable in this category; the number of top runners participating in the race. The coefficient is statistically significant at the 10% level. As expected, as the number of top runners increases so does average speed. All

[8] As an illustration, in race 8 (which is discussed in the next section) the first top runner finished the 8 km course in 1,574 seconds. If this runner were to increase his speed by 0.05 meters per second his time would decline by 15 seconds; an improvement of 1%.

other things being equal the results imply that a race with 10 additional top runners will have speeds that are approximately 0.04 meters per second faster.

Y_{ik} (local competitive intensity): After some experimentation we chose a single measure of local competitive intensity which appeared to have the greatest explanatory power; the number of competitors within 20 ranking places of the runner of interest. The estimated coefficient, which is statistically significant at the 10% level, indicates that a single additional runner close in ranking to the runner of interest is expected to increase his average speed by approximately 0.02 meters per second.

Z_i (personal characteristics of runner): Ranking is the only variable in this category. As expected, higher ranked runners (i.e., those with lower ranking numbers) are predicted to run faster. All other things being equal the runner with ranking 1 is predicted to have a speed that is approximately 0.35 meters per second faster than the runner that is ranked 101. This coefficient is significant at the 0.1% level.

In order to investigate the robustness of our reported results, we estimated a number of alternative model specifications for the speed equation. First, we omitted runner rankings as an explanatory variable. This resulted in a substantial reduction in the reported adjusted R squared from 0.83 to 0.54. The reported coefficients for total value of prizes and Herfindahl Index of prizes retained the same signs, were statistically significant, and were greater in absolute value than in our earlier analysis. These results are consistent with our assumption that ability is an important determinant of speed and that our rankings are a good measure of ability. As expected, the omission of a measure of ability reduced the model's explanatory power and upwardly biased the absolute values of the coefficients for total value of prizes and Herfindahl Index of prizes since when rankings are omitted we fail to account for the fact that on average races with more attractive prizes include runners of higher ability. Second, we added two additional interaction variables to the model; runner ranking multiplied by total value of prizes and runner ranking multiplied by Herfindahl Index of prizes. Neither of the new variables was statistically significant. Third, we estimated the model with runner fixed effects. The coefficient of total value of prizes was statistically significant at the 10% level and was approximately unchanged in magnitude. The coefficient for the Herfindahl Index of prizes was no longer statistically significant at the 10% level.

Overall the three robustness checks we undertook provide some additional comfort that the version of the model we presented earlier in this section is reasonable.

6 Counter Factual Analysis

In order to illustrate the expected impact of changing race prizes we consider the predicted result of a hypothetical change in the prizes offered in race 8. This race was chosen since it has only one competing race (race 28) and has only a limited number of top runner participants (five). These characteristics make the arithmetic for calculating the expected consequence of changing prizes a little easier than it would be for other races. The principles that would need to be applied are the same in all situations including those where there are two competing races and where the number of top runners is large. Race 8 and race 28 are of the same distance (8 kilometers), occurred on the same day (June 16, 2002) and took place at relatively nearby venues (Boston in the case of race 8 and New York City in the case of race 28). The actual prizes for race 8 and the hypothetical revised prizes are shown in Table 8 along with the corresponding prize attributes that are relevant to Equation 2, and Equation 4.

Table 8

Actual and Hypothetical Prizes for Race 8

	First prize	Second prize	Third prize	Fourth to Eight prizes	Total prizes	Herfindahl Index
Actual	$150	$75	$50	$25	$400	0.2109
Hypothetical	$1,500	$750	$500	$250	$4,000	0.2109

Using the coefficients estimated earlier as shown in Equation 5 and employing Equation 2, we are able to calculate the individual probabilities of each of the 18 top runners who actually participated in race 28 of having chosen to participate in race 8 at the actual prizes and at the hypothetical prizes. If these probabilities are represented by p_a and p_h respectively then the probability of a runner participating in race 8 at the hypothetical prizes conditional on the fact that at the actual prizes he chose to participate in race 28 is the larger of $\frac{p_h - p_a}{1 - p_a}$ and zero. The first table in Appendix 2 shows these defection probabilities for each of the 18 top runners that participated in race 28. In a similar way for the five top runners that actually participated in race 8 we can calculate the probability of each runner choosing race 8 at the actual and hypothetical prizes. If as before these probabilities are represented by p_a and p_h respectively then the probability of a runner participating in race 8 at the hypothetical prizes conditional on the fact that he chose to participate in race 8 at the actual prizes is the smaller of $\frac{p_h}{p_a}$ and one. The second table in Appendix 2 shows these probabilities. In the case of the top runners originally in race 28, for eight runners the probability of defecting to race 8 exceeds 10%, for three runners the probability is between zero and 5% and for the remaining seven runners the probability is zero. In order to simplify the analysis only those runners with a 10% or greater chance of defection will be considered in the remainder of the discussion. In the case of the five runners originally in race 8, based on the hypothetical prizes three are estimated to remain in race 8 with certainty while two are estimated to defect to race 28 with a probability of less than 5%. For simplicity the possibility of defections away from race 8 will be ignored in the remainder of the analysis. Based on these simplifying assumptions the probability of at least one runner from race 28 joining race 8 if the hypothetical prizes were introduced is 0.7 and the expected number of runners defecting from race 28 to race 8 is 1.2. This is the sorting effect in action.

Now we turn to the incentive effect of the hypothetical revised prizes on the runners already committed to race 8. We call the predicted effect of the revised prizes before taking into account the impact of runners defecting into race 8 the "direct" incentive effect. We call the total effect of the hypothetical prizes when allowing for defections the "combined" incentive effect and the difference between these two effects the "indirect" incentive effect. As shown in Appendix 3, we estimate the combined incentive effect to be of the order of 0.5% of the runners' original times. Except in the case of the top ranked runner in race 8 the direct incentive effect represents the overwhelming majority of this predicted change in speed. For the top ranked runner in race 8 the indirect incentive effect is larger than is the case for the other runner since for this runner three of the runners that may defect from race 28 are within 20 ranking places and therefore the local competitive effect has a positive impact on the runner's predicted speed.

As we have chosen to model them, the direct incentive effect is common to all race participants while the indirect incentive effect can vary between individuals since the hypothetical change in prizes can result in changes in local competitive intensity that differs between runners of different rankings. In the particular example that has been worked out here the direct incentive effect is larger the indirect incentive effect**.** Of course, this may not be true for other races and other runners.

7 Conclusions

Tournaments of many types are important in the economy. In many cases data are difficult to collect and, as a result, to a large extent empirical analysis has not been available to support the development of theory. In this environment, sports tournaments may offer a fruitful area for study that complements research in areas of more direct interest but where data may be limited and of poor quality. In a sense sports tournaments provide an analytical bridge between laboratory experiments and economically important tournaments. In the case of laboratory experiments data are of high quality but the applicability to other economically important tournaments may be questioned. In the case of a direct study of say work place promotions, the applicability is clear while high quality data may be scarce. Sports tournaments stand between these two extremes and may play an important part in furthering understanding.

This paper’s contribution is fivefold. First, we elaborate a two-part model to separately quantify the sorting and incentive effects of tournament prizes. We believe that in this setting our approach is novel. Second we apply the model to a unique data set of road running race results. Since, unlike the data sets used in prior studies we are aware of, the one used here includes runners that did not win prizes, it allows us to observe the impact of prizes on the race choice and speed of all runners not just prize winners. Third, we use concepts from spatial econometrics to estimate effects that relate to the overall competitiveness of a race and the local impact of runners of similar rank to the runner of interest. Fourth, we demonstrate that in the races we examine both sorting and incentive effects are present. These conclusions are robust to a number of alternative specifications of the sorting and speed equations. Fifth we present a counterfactual example showing how a hypothetical change in prizes would be predicted to change race participation and speed.

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Appendix 1 – Data Availability to Estimate Equation 3

Category	Number of observations
A. No competing races in the defined time interval	101
B. One competing race	
1. runner participates in the competing race	0
2. runner does not participate	117
C. Two competing races	
1. runner participates in both competing races	0
2. runner participates in one	103
3. runner participates in neither	540
Total	861
Availability for estimating Equation 3	
Available (B2, C2 and C3)	760
Not available (A, B1 and C1)	101
Total	861

The table above indicates the number of observations that are available to estimate Equation 3 in light of the base case algorithm we chose to identify competing races.

Appendix 2 – Defection Probabilities

Ranking	Probability
24	0.1778
27	0.1759
57	0.1566
71	0.1476
86	0.1379
87	0.1372
118	0.1170
136	0.1052
227	0.0449
260	0.0228
261	0.0221
294	0.0000
295	0.0000
296	0.0000
297	0.0000
355	0.0000
360	0.0000
363	0.0000

The table above shows the probability of runners originally in race 28 choosing to defect to race 8 at the hypothetical prize levels.

Ranking	Probability
82	1.0000
282	1.0000
284	1.0000
310	0.9892
343	0.9670

The table above shows the probability of runners originally in race 8 choosing to remain in race 8 at the hypothetical prize levels.

Appendix 3 – Incentive Effects for Race 8 Participants

(Effects are measured in meters per second)

Ranking	Direct Effect	Indirect Effect	Combined Effect
82	0.01627	0.01223	0.02850
282	0.01627	0.00429	0.02056
284	0.01627	0.00429	0.02056
310	0.01627	0.00429	0.02056
343	0.01627	0.00429	0.02056

The table above shows the predicted impact of the hypothetical change in prizes on the speed of all the top runners already committed to race 8. Note that the runner ranked 82 was the first placed top runner with a time of 1,574 seconds. The race had a length of 8km and so this winning time corresponded to an average speed of 5.1 meters per second. For this runner the predicted combined incentive effect corresponds to an increase in speed of approximately 0.6%. The runner ranked 343 was the slowest of the top runners in this race with a time of 1,927 seconds. In this case the predicted combined incentive effect corresponds to an increase in speed of approximately 0.5%.