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Asymmetric velocity and acceleration profiles of human arm movements

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Summary. Displacement, velocity, acceleration and jerk (change of acceleration with time) were analyzed for arm flexion movement over a wide range of movement amplitudes and speeds. Relative time to peak velocity or relative duration of acceleration, k, was approximately 0.5 for the movements with intermediate speed (about 0.5 s in movement time), i.e., symmetric velocity and acceleration profiles. For the slow and ballistic movements, k shifted towards values below and above 0.5, respectively creating asymmetric profiles. Consistent k-dependence of movement time, peak velocity, maximum acceleration and maximum deceleration were observed. "Jerk cost", the square of the magnitude of jerk integrated over the entire movement, was calculated for each movement. A dynamic optimization technique to minimize jerk cost under the constraint on jerk input was applied to interpret the results, assuming that a major goal of skilled movements was to produce optimally smooth movements. The constrained minimum-jerk model explained speed-dependent asymmetry of the velocity and acceleration profiles. Jerk cost consumed by the movements with intermediate speed approximately satisfied minimum-cost criterion predicted by the model but was higher than the criterion for slow and ballistic movements. The results suggested that optimality criteria other than jerk cost also should be considered to predict movement profiles over the entire range of speeds.

Key words: Velocity profile – Acceleration profile – Jerk – Dynamic optimization – Human arm movement

Introduction

An action can be performed using many different movements, i.e., motor equivalence or uniqueness of movement (Smyth and Wing 1984). One could hold, for instance, a cup of coffee on a table and then move it towards one's lips smoothly or clumsily. Skilled movements, on the other hand, are developed through training and practice to achieve remarkable consistency as an action over individual movements or different situations. Physically, this may be understood as the selection of a class of movement trajectories from the infinite set of possible ones for a given motor act. Then, what principles function to select specified trajectories for skilled movements?

Recently, a method which promises to specify the organization principle of skilled movements has been proposed, i.e., an application of the dynamic optimization technique (Hogan 1984; Nelson 1983). Nelson (1983), for instance, suggested that skilled movements satisfy an "economical principle" in which certain "costs" associated with the muscular exertion in the movement are minimized. He examined several performance objectives minimizing such measures of physical costs as energy and jerk for a linear second-order motor system. "Jerk cost" is expressed here by $1/2 \int_0^T \dot{a}^2(t)dt$, where $\dot{a}(t)$ is the rate of change of acceleration, jerk, and T the movement time. The minimum-jerk solution of the optimal control problem is one of the most appealing, because jerk cost is a measure of smoothness of movements which is one of the characteristics of skilled movements, and furthermore, movement trajectories minimizing jerk cost, namely the optimally smooth ones, are predicted to have a simple form described by a 5th order polynomial. Hogan (1984) formulated and solved the minimum-jerk control problem for point-to-point forearm movements predicting movement trajectories with bell-shaped symmetric profiles of velocity. He then demonstrated that the solution could yield close agreement with observed pointing movements of the arm with moderate movement amplitude and speed (Bizzi et al. 1982).

After these theoretical works by Nelson (1983) and Hogan (1984), Flash and Hogan (1985) and Edelman and Flash (1987) applied the minimum-jerk model to multi-joint arm movements and handwriting, respectively. However, the applicability of the optimization technique to minimize jerk cost to human skilled movements including simple forearm movements has not been fully examined over a wide range of movement amplitudes and speeds, although theoretical predictions, such as the existence of an "invariant relationship" between maximum and average velocities have been used by several authors for kinematic analysis of articulator and arm movements (Munhall et al. 1985; Ostry et al. 1987; Soechting 1984). This paper examines displacement, velocity, acceleration and jerk for a simple pointing movement involving arm flexion and gives a theoretical analysis based upon a constrained minimum-jerk solution of the optimal control problem.

Methods

Subjects

Four male subjects aged from 31 to 49 years participated in the study.

Apparatus

The subject was seated with his right forearm on a light, horizontally rotating handle (moment of inertia 0.024 kg $\cdot$ m^2) of a specially designed arm-rotator. The axis of the elbow joint was aligned with the pivot of the handle. The shoulder was at 90° flexion, 20° horizontal abduction and the elbow at 30° flexion at the start of the task. The subject held a vertical rod attached to the handle and flexed his arm from the fixed starting point at rest to one of the visual targets. The targets were colored vertical rods 9 mm in diameter and 29 mm in height located circularly in the plane of the apparatus. Angular displacement was measured by a goniometer aligned with the pivot of the handle and stored in a digital computer (NEC PC9800) via an A/D converter with sampling frequency of 1 kHz. The start-signal of the movement triggered the A/D converter.

Procedure

The subject was requested to flex his forearm as smoothly as possible to one of the 5 targets located at 20, 30, 40, 50 and 60 deg from the starting position. Accuracy of targeting was not the primary constraint in this experiment. The five movement amplitudes of targeting were assigned at random within each of three speed conditions; S: slow but not so slow as to exceed 1.5 s in movement time, M: with intermediate speed, i.e., faster than S but not so fast as B, and B: ballistic, with speed as fast as possible. In condition S, trials over 1.5 s in movement time were discarded from the data. Ten trials at each movement amplitude under each speed condition were performed. The subject was requested to reach the target using one continuous stroke without intermittent voluntary correction. The experiment consisted of three sessions on different days; for every subject the movements under speed conditions S were performed first, followed by M and then B.

Data analysis

Natural cubic spline functions were fitted to the raw data of the angular displacement. The interval between the knots for the spline approximation was 60 ms for the speed conditions S and M, and 20 ms for B. The average absolute error of the spline fit was less than 0.05 deg. By differentiating the spline functions angular velocity, acceleration and jerk (time derivative of acceleration) were obtained.

Movement time (T) was defined as an interval from the start-signal to the point where the velocity curve first crossed the zero-line. Movement amplitude (D) was an angular displacement at time T. Instantaneous peak velocity Vmax and its time of occurrence were obtained by using the velocity curve. Maximum, minimum and final accelerations were also found on the acceleration curve. "Jerk cost" within movement time defined as $1/2 \int_0^T \dot{a}^2(t)dt$ was calculated by differentiating the acceleration curve a(t) and then accumulating its square from the start up to movement time. Since the spline functions were cubic polynomial and thus jerk $\dot{a}(t)$ has a constant value in each interval between neighboring knots, jerk cost might include considerable error of approximation. To evaluate this possible error, the cost was also calculated for several trials under each speed condition using acceleration data which were obtained directly from a uniaxial accelerometer attached on a distal end of the handle of the arm-rotator and processed by the spline fit. Jerk costs calculated using both methods showed good agreement and no consistent deviations were found between them. The variables defined above were averaged across ten trials at each amplitude under each speed condition, and thus 15 mean values for each variable were obtained for each subject.

Results

Asymmetry of velocity profile

In Table 1 several kinematic variables averaged for four subjects are listed at each condition of movement amplitude and speed; movement amplitude (D), movement time (T), maximum velocity (Vmax), relative time to maximum velocity (k). Table 1 also includes the ratio of Vmax to mean velocity, c = Vmax / (D/T). In Table 2 maximum, minimum and final accelerations relative to the mean (D / T^2), i.e., Amax, Amin and Afin, respectively, are listed. Movement time (T) and maximum velocity (Vmax) differentiated correspondingly to the speed conditions as instructed by the experimenter and changed systematically with movement amplitude (D) within each speed condition.

There was a large amount of variability in k within each speed condition. Figure 1 displays the distribution of k for all data across subjects and movement amplitudes. Since some slow movements

Table 1. Mean and SD of movement amplitude (D, degrees), movement time (T, ms), peak velocity (Vmax, deg/s), relative time to peak velocity (k) and c = Vmax / (D/T) at each target position and speed condition: S: slow, M: intermediate, B: ballistic movements

Target	D	T	Vmax	k	c
S					
20	19.9	874.2	43.6	0.451	1.86
	0.9	149.0	9.4	0.084	0.19
30	30.1	978.7	57.7	0.435	1.80
	0.7	173.6	14.0	0.082	0.13
40	39.8	1067.5	67.8	0.428	1.77
	0.8	160.4	12.9	0.077	0.15
50	49.8	1142.8	81.9	0.420	1.83
	0.9	169.8	15.8	0.087	0.14
60	60.0	1222.3	93.4	0.390	1.89
	1.0	90.5	12.4	0.073	0.17
M					
20	20.4	567.1	74.1	0.464	1.98
	1.2	122.9	16.4	0.074	0.22
30	30.4	624.4	99.5	0.469	1.95
	1.0	122.5	24.8	0.060	0.14
40	40.5	635.2	128.4	0.443	1.94
	1.2	119.2	26.1	0.040	0.13
50	50.4	688.3	143.6	0.429	1.91
	1.2	115.3	25.4	0.047	0.14
60	60.4	699.6	172.5	0.439	1.94
	1.3	124.6	32.2	0.057	0.13
B					
20	22.5	184.8	253.6	0.548	2.01
	2.1	33.0	57.8	0.062	0.16
30	32.0	201.1	343.4	0.556	2.07
	2.4	43.6	82.1	0.060	0.13
40	42.2	222.4	417.0	0.531	2.11
	2.4	49.5	85.3	0.079	0.16
50	52.7	236.0	476.7	0.531	2.05
	2.6	52.7	94.6	0.071	0.16
60	62.0	247.2	540.3	0.529	2.09
	2.3	51.1	95.0	0.070	0.16

Table 2. Mean and SD of average acceleration ($A = D/T^2$, deg/s^2), maximum (Amax), minimum (Amin) and final (Afin) accelerations relative to A at each target position and speed condition (S: slow, M: intermediate, B: ballistic)

Target	A	Amax	Amin	Afin
S				
20	28.7	8.12	−6.71	−3.43
	11.1	1.09	1.02	0.57
30	35.1	8.54	−6.38	−3.56
	12.2	1.32	0.52	1.18
40	37.7	8.75	−6.25	−2.96
	9.4	1.56	0.34	0.59
50	41.6	8.59	−6.40	−2.63
	12.2	1.22	0.67	0.59
60	40.9	9.27	−6.57	−2.85
	4.7	1.17	0.85	0.30
M				
20	72.0	7.90	−6.38	−2.98
	25.9	0.20	1.00	0.90
30	87.8	7.97	−6.05	−2.44
	33.4	0.59	0.35	0.66
40	111.2	7.86	−5.61	−2.16
	39.5	0.53	0.56	0.28
50	115.6	7.92	−5.66	−2.06
	30.8	0.32	0.49	0.69
60	135.2	7.96	−5.77	−2.15
	45.0	0.92	0.51	0.41
B				
20	720.8	6.79	−6.94	−4.15
	228.6	0.82	1.15	1.80
30	914.4	7.00	−7.02	−4.08
	431.4	1.24	1.24	2.23
40	985.5	7.65	−7.13	−3.49
	434.3	1.41	1.31	2.04
50	1098.4	7.21	−6.76	−3.32
	475.6	0.98	1.34	1.13
60	1151.1	7.60	−7.02	−3.32
	480.7	1.23	1.64	1.12

which lasted over one second did not have unimodal velocity profiles, a wide variation of k in speed condition S might be predictable. However, in M and B where the velocity profiles were mostly unimodal, k also had a similar wide variability as in S. The ranges of k were 0.329–0.517, 0.411–0.502 and 0.457–0.634 for speed conditions S, M and B, respectively. Figure 1 indicates that the velocity exhibited a symmetric, bell-shaped profile (k = 0.5) only occasionally and that the asymmetry of the velocity profile, k, was dependent on speed of movement; under speed conditions S and M, k tended to be below 0.5 but it increased with speed and the direction of asymmetry reversed at very high speeds, B. Two-way ANOVA (movement amplitude × speed) for k gave a significant main effect only for speed. $F(2,45) = 32.82, p < 0.01$. For all 60 measures taken across subjects and conditions, k showed a significant negative correlation with movement time ($r = -0.775$, $p < 0.01$). By a linear regression equation, $k = 0.561 - 0.141T$ (s), k was estimated to equal 0.5 at T = 0.433 s.

"Invariant", Vmax / (D / T)

Several researchers demonstrated that maximum velocity relative to mean velocity, c = Vmax / (D / T), was kept invariant in arm movements (Soechting 1984) and articulator movements in speech (Munhall et al. 1985). The ratio was close to 1.88 which is predicted by Hogan's model (1984). In our present experiment, c varied to a limited extent within each speed condition but increased with speed as seen in Table 1. ANOVA indicated that the main effect of speed was significant, $F(2,45) = 35.05, p < 0.01$, but

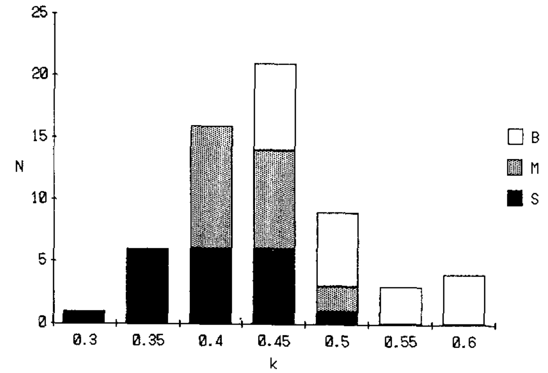

Fig. 1. Distribution of relative time to peak velocity, k, for 60 pooled data. On ordinate 0.3 means 0.3 ≤ k < 0.35 and so on

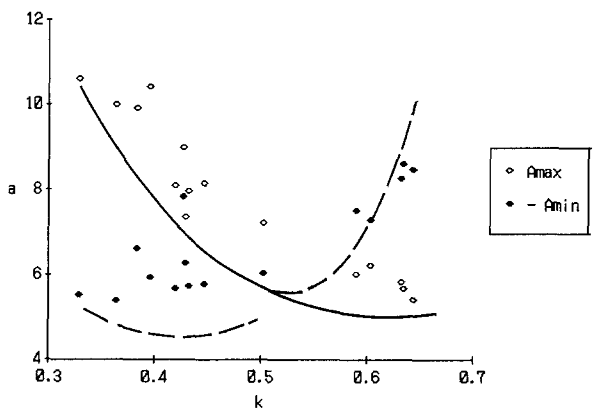

Fig. 2. Peak acceleraration (Amax) and deceleration (Amin) normalized by the mean (D/T^2) are plotted against relative time to peak velocity, k, for one subject. Solid and dashed lines are Amax and – Amin, respectively, calculated by the constrained minimum-jerk model

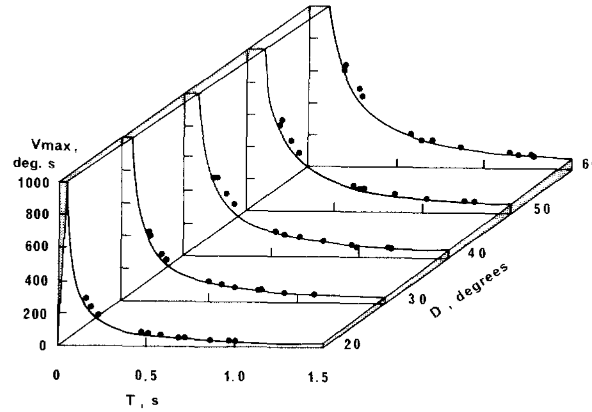

Fig. 3. Peak velocity or "effort cost" (Vmax) as a function of movement time (T) at each movement amplitude (D). Solid lines show the minimum-cost criterion Vmax = 1.88 D/T predicted by the unconstrained minimum-jerk model

the effect of amplitude and the interaction were not. For all data across subjects and conditions, c correlated significantly with movement time (r = –701, $p < 0.01$). From a linear regression between c and T, T was 0.914 s at c = 1.88. There was a weak correlation (r = 0.575, $p < 0.01$) between c and k for all 60 data.

Peak accelerations

Acceleration generally exhibited an asymmetric profile in which it reached a positive peak during the first accelerative phase, turned to a decelerative phase at time k × T and attained a minimum during this phase. Final acceleration at time T was always negative and smaller in magnitude than Amin. Peak acceleration ranged in absolute magnitude from 2.32 rad / s^2 (tangentially about 0.09 G) to 279.6 rad / s^2 (11.4 G) and naturally increased with movement amplitude and speed. Relative accelerations (Amax, Amin and Afin), on the other hand, depended on speed differently. The main effect of speed was significant for them (ANOVA, $F(2,45) = 9.020$, 6.424 and 6.697, $p < 0.01$, for Amax, Amin and Afin, respectively), but can be seen in Table 2, Amax decreased with speed whereas |Amin| and |Afin| increased in the ballistic condition. The main effect of amplitude and the interaction were not significant.

Rather, relative peak accelerations seem to relate directly to k, the velocity profile asymmetry. Figure 2 demonstrates for one subject that Amax and Amin changed as a function of k. Amax decreased while |Amin| remained approximately constant at k < 0.5, increased with k at k > ·0.5, and the order of magnitude between Amax and Amin reversed at about k = 0.53. In short, relative peak acceleration and deceleration were both related to accelerative / decelerative timing, k; the acceleration profiles also changed depending on movement speed.

Cost

When the velocity profile is unimodel, Nelson (1983) predicted that the peak velocity Vmax is numerically equal to the "impulse cost" of the movement, i.e. 1/2 $\int_0^T |a(t)|dt$ where a(t) is acceleration. Nelson paraphrased this as "effort cost". In Fig. 3 Vmax is displayed as a function of movement time at each movement amplitude for all data from the present experiment. Figure 3 shows that the cost did not increase so much in a wide range of movement times for the trials under the speed conditions S and M, while it did increase steeply in the ballistic condition,

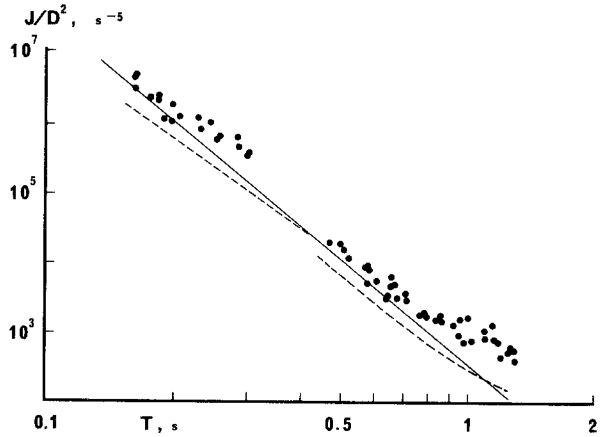

Fig. 4. "Jerk cost" per (movement amplitude)^2 as a function of movement time (T). Solid line is the minimum-cost criterion $J/D^2 = 360\ T^{-5}$ predicted by the unconstrained minimum-jerk model. Dashed lines were calculated by the constrained minimum-jerk model

B. Figure 4 shows experimentally obtained jerk cost per (movement amplitude)^2. As with "effort cost", jerk cost was about 100 times more for the ballistic condition than for the other speed conditions, S and M.

Impulse variability

Although exact targeting was not a primary concern in the present study, several results suggest an "impulse-variability model" (Schmidt et al. 1979) which has been the problem at issue up to the present. The peak height and duration of the accelerative impulse, i.e., Amax and k, were not mutually independent in contrast to the fundamental hypothesis of the model. For the pooled data the standard deviation of the movement amplitude, SD(D), correlated with impulse (i.e., Vmax) variability (SD(Vmax), $r = 0.838$) and with mean velocity (D/T, $r = 0.850$) as pedicted by the model. SD(D), however, correlated as well with standard deviation of mean velocity (SD(D/T), $r = 0.898$) as predicted by the invariant relation Vmax = c (D/T), hence, SD(Vmax) α SD(D/T).

Minimum-jerk model

Unconstrained minimum-jerk solution

Hogan (1984) developed a model for minimum-jerk movement control, assuming that human skilled movements with moderate speed and amplitude would realize optimally smooth movements. Minimizing the cost function $J = 1/2 \int_0^T \dot{a}(t)^2\, dt$, "jerk-cost", he predicted the minimum-jerk movement trajectory to be a fifth-order polynomial in time and determined it for point-to-point forearm movements under the boundary conditions as follows:

$$x(0) = 0 \quad x(T) = D$$
$$\dot{x}(0) = 0 \quad \dot{x}(T) = 0$$
$$\ddot{x}(0) = 0 \quad \ddot{x}(T) = 0 \qquad (1),$$

where x(t), T and D are a movement trajectory, movement time and amplitude, respectively (see Appendix). This solution gives the constant movement parameters, namely, $k = 0.5$, Vmax/Vmean = 1.88 and Amax = –Amin = 5.77. Hogan's solution, however, cannot simulate the movements in the present study over the entire range of speeds where k as well as Amax and Amin changed consistently with speed. Thus, we tried another solution to simulate our movements by exchanging one of the boundary conditions in (1), namely $\ddot{x}(T) = 0$, with a more realistic one, $\ddot{x}(T)$ = Afin (final acceleration Afin < 0) or with its equivalent. $\ddot{x}(kT) = 0$. The alternative, however, only gave movements with velocity-profiles of $k > 0.5$ under the constraint Afin < 0; i.e., no directional reversal of asymmetry emerged.

Constrained minimum-jerk solution

There may be another way to utilize the optimal control model with minimum-jerk, namely to solve the model under certain constraints on the control input, jerk. In Fig. 5a, the jerk profile is illustrated for the unconstrained, symmetric solution. Two types of constraints on this jerk input may produce improved simulation; jerk is constrained (i) during a starting phase of the movement and (ii) through intermediate and final phases. These two types of constraint are illustrated schematically in Fig. 5b and c together with the velocity and acceleration profiles expected under each constraint. Two types of the solutions of the optimal control problem with minimum-jerk cost under these constraints are briefly described below (see Appendix).

(i) At the start of the movement, positive jerk is constrained not to exceed a constant $c_1(60D/T^3)$, $0 < c_1 < 1$ (Fig. 5b). Due to a final velocity condition $V(T') = 0$, where T' is movement time in this case, accelerative and decelerative impulses (areas enclosed by a(t) and time-axis) must be equal. Movement time T' is shortened compared to T of the unconstrained solution, and thus velocity peak will be relatively delayed compared to the symmetric profile. In fact, k increased approximately linearly

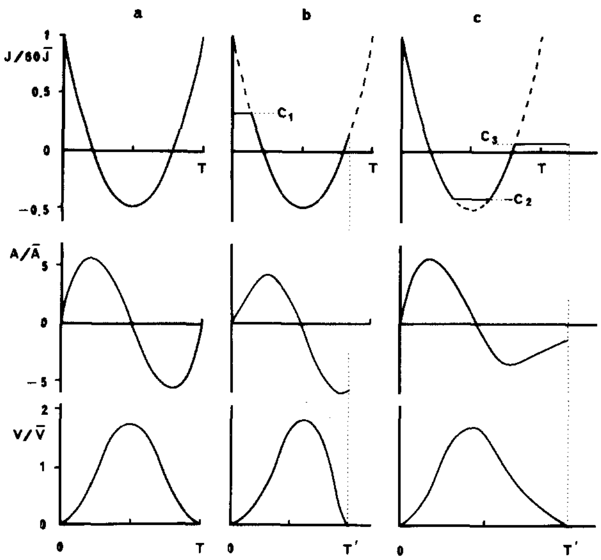

Fig. 5a–c. Jerk, acceleration and velocity profiles normalized by the means for the unconstrained (**a**) and constrained (**b, c**) minimum-jerk movements

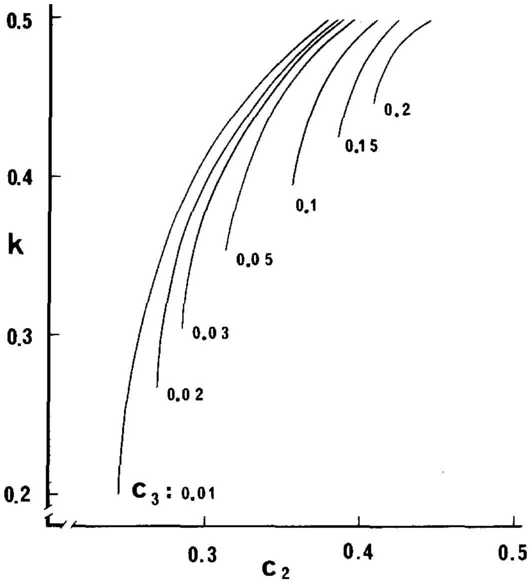

Fig. 6. Relation of relative time to peak velocity (k) to parameters c_2 and c_3 of the constrained minimum-jerk movements (Fig. 5c)

from 0.5 to 0.65 with c_1 from 1 to 0.25. (ii) During the intermediate phase of the movement, negative jerk is constrained under a constant $-c_2(60D/T^3)$, $(0 < c_2 < 0.5)$ and during the final phase positive jerk is under $c_3(60D/T^3)$ $(0 < c_3 < 1$, Fig. 5c). Movement time will be prolonged in this solution and k will be relatively advanced compared to the unconstrained solution. Under the boundary condition V(T') = 0, c_2 and c_3 were possible within a narrow domain of (c_2, c_3). Figure 6 shows the predicted changes of k with (c_2, c_3) and k changes from 0.3 to 0.5, for instance, by varying c_2 from 0.32 to 0.39 under a constant $c_3 = 0.03$.

Simulation

The observed asymmetry of the velocity profile, $0.3 < k < 0.7$, could be obtained using the constrained minimum-jerk solutions as described above. Under the constraints, k increases with speed, thus, approaching 0.5, i.e., the symmetric velocity profile which is predicted by the model without constraints, and the direction of asymmetry reverses $(k > 0.5)$ with further increase of speed. The constrained minimum-jerk solution thus explains the speed-dependency of the velocity asymmetry observed in the present experiment; namely, the profile was approximately bell-shaped for the movements with intermediate speed (movement time about 0.5 s), while the velocity profile with a shorter or longer accelerative phase than the decelerative phase emerged at slow or very high speeds, respectively.

Calculated k-dependence of relative peak accelerations is depicted in Fig. 2 by a solid line for Amax and by a dashed line for Amin, superimposed on the observed data of one subject. Here k-dependence of Amax and Amin at $k < 0.5$ was calculated by varying c_2 from 0.3 to 0.4 at $c_3 = 0.03$. The model predicts (1) Amax decreases with k, hence with speed, while Amin increases in magnitude with k above 0.5; (2) below k = 0.5 Amax is greater in magnitude than Amin which is approximately constant; (3) Amin is greater in magnitude than Amax for k from 0.5 to 0.6 and Amin is equal to Afin beyond k = 0.6; (4) for k between 0.5 and 0.53 Amax and Amin are very similar in magnitude and minimum absolute acceleration is found at about k = 0.53. As seen in Fig. 2, the model explains the characteristic k-dependence of the experimentally obtained acceleration profile, though the quantitative agreement is not very good.

The unconstrained minimum-jerk solution predicts that minimum "impulse" or "effort" cost equals Vmax and that Vmax = 1.88(D/T). In Fig. 3 the relation Vmax = 1.88(D/T) is drawn at each D by a solid line. Figure 3 suggests that the actual movements realized "optimally easy" motion in the speed conditions M and S, but had a somewhat higher "effort cost" than the minimum criterion under the ballistic condition. In Fig. 4 the relation between minimum-jerk cost and movement time that was predicted by the unconstrained and constrained solutions are shown superimposed on the experimental data. As seen in Fig. 4, the actual movements involved somewhat higher cost than the minimum jerk-cost criterion and the relation between the cost and movement time (T) appears to deviate slightly from the "minimum cost α T^{-5}" law (Appendix) predicted by the unconstrained minimum-jerk model. The constrained solution of the model did not improve these discrepancies. However, the jerk cost consumed by the movements with intermediate speed at about T = 0.5 s, and also with very high speed at T < 0.2 s, tended to satisfy the minimum-cost criterion.

Discussion

The present study examined a discrete arm movement over a wide range of speeds including trials near the limits of neuromuscular performance (speed conditon B or S). Near both limits of speed, the profiles of velocity and acceleration deviated substantially from the symmetric form. The symmetric velocity and acceleration profiles were observed only at the intermediate speed, movement time T = 433 ms at k = 0.5. The movements of this speed are comparable with those of monkeys (T = 692 ms at D = 60°, Bizzi et al. 1982), to which the minimum-jerk movement with symmetric velocity profile was fit by Hogan (1984).

Asymmetry of the movement profile should not be attributed to certain "noise" in exerting muscular force, since an index of asymmetry, k, correlated consistently with speed, on the one hand, and with acceleration on the other hand. Speed-dependent asymmetries of the velocity profile similar to ours have been shown by several researchers using simple arm movements (Beggs and Howarth 1972; Moore and Marteniuk 1986; Ostry et al. 1987; Zelaznik et al. 1986). Ostry et al. (1987), among others, provided data showing the directional reversal of the velocity profile asymmetry with speed. Bullock and Grossberg (1988) thus inferred that the direction of asymmetry reverses over a range of movement speeds for planned arm movements and explained this type of speed-dependent asymmetry using their model of neural dynamics for arm movements. They used velocity profile asymmetry as an argument against models based on optimization theory, because the latter, such as formulated by Hogan (1984), cannot predict it.

The preceding section of this paper interprets the movement with the asymmetric profile within the framework of minimum-jerk control but puts constraints on the jerk input; jerk was limited in magnitude during the starting phase for the ballistic movement (Fig. 5b), but limited during intermediate and final phases for slow movement (Fig. 5c). Note that the constraint on jerk during the intermediate phase, c_2 in Fig. 5c, was very weak when compared with c_3 during the final phase, namely $c_3/c_2 < 0.1$ (see Fig. 6). Consequently, the constraint on jerk for the slow movement could be regarded as imposed essentially during the final phase. This contrained-jerk model yielded a qualitative explanation of observed asymmetry of the velocity and acceleration profiles and its directional reversal as described in the preceding section. Bullock and Grossberg (1988) also predicted the same type of asymmetry as ours, though they could not verify their theoretical results quantitatively by experiments partly because of the lack of available data. Their model is constructed on a fundamentally different basis from optimization theory; it needs no explicit preprogramming of movement kinematics. Optimization theory, on the other hand, is one of those models which assume internal representation of a generalized motor program for skilled movements thereby generating each of the desired actions (Schmidt et al. 1979). In the unconstrained minimum-jerk model by Hogan (1984), for instance, movement kinematics are computed from the optimally smooth trajectories (A-1 in Appendix) by specifying movement time and amplitude. The constrained jerk model in the present study also assumed optimally smooth trajectories with the addition of speed-dependent constraints on jerk input. Further work would be necessary for quantitative explanation of the origin of the movement profile asymmetry.

Jerk cost consumed during the movements tended to be higher than the minimum-cost criterion predicted by the unconstrained minimum-jerk model (Nelson 1983). Note that in discrete movements like those used here, velocity rises from zero at the start of the movement and returns to zero at the target position, thereby requiring relatively large jerk costs during the starting and final phases which are not necessary for continuous repetitive movements. Nelson (1983) demonstrated that the minimum-jerk cost of a periodic movement is ⅙th of the cost for single movement, and in fact the cost measured in periodic flexion-extension of the elbow joint satisfied the minimum-cost criterion over a wide range of movement speeds from 1.0 to 5.9 Hz (Nagasaki, unpublished data). Our constrained jerk model assumed that the ballistic and slow movements were controlled so as to reduce the abrupt change in acceleration at the start and the end of the discrete movements. The actual movements, however, did not satisfy the minimum-cost criterion thereby introduced.

It appears consequently, that a single cost function for the optimality criterion is not adequate to predict the kinematics of a simple arm movement over the entire velocity range. "Effort cost" (Nelson 1983) in our movements with intermediate and slow speeds showed close agreement with minimum-effort criterion (Fig. 3), but the ballistic movements did not. Jerk cost for the ballistic and slow movements also deviated from optimality. Only the movements with moderate speed, movement time about 0.5 s, achieved both effort and jerk optimization, and further they exhibited nearly symmetric movement profiles as required by the minimum-jerk criterion. Skilled motor acts in daily activities may use moderate speed movements most frequently, which, our present study suggests, are the most effective in terms of smoothness. It would be necessary to examine cost functions other than jerk or effort for ballistic or slow movements.

Appendix

The minimum-jerk solution of the optimal control problem is written as follows for the boundary conditions (1) (Hogan 1984):

$$\begin{aligned} j(t) &= 60(D/T^3)[6(t/T)^2-6(t/T)+1] \\ a(t) &= 60(D/T^2)[2(t/T)^3-3(t/T)^2+(t/T)] \\ v(t) &= 30(D/T)[(t/T)^4-2(t/T)^3+(t/T)^2] \\ x(t) &= D[6(t/T)^5-15(t/T)^4+10(t/T)^3] \end{aligned} \quad \text{(A-1)}$$

where j(t), a(t), v(t) and x(t) are jerk, acceleration, velocity and angular displacement, respectively (Fig. 5a). Minimum jerk-cost is

$$\text{cost} = 1/2 \int_0^T j(t)^2\,dt = 360(D^2/T^5) \quad \text{(A-2)}$$

When jerk in (A-1) is constrained under $c_1(60D/T^3)$ during $0 < t < T_1$ as shown in Fig. 5b, the minimum-jerk solution is obtained by the following algorithm;

jerk:

$$\begin{aligned} J(t) &= c_1 && 0 < t < T_1 \\ &= j(t) && T_1 < t < T' \end{aligned}$$

acceleration:

$$\begin{aligned} A(t) &= c_1 t && 0 < t < T_1 \\ &= a(t) + K_1 && T_1 < t < T' \\ K_1 &= c_1 T_1 - a(T_1) \end{aligned}$$

velocity:

$$\begin{aligned} V(t) &= 1/2\, c_1 t^2 && 0 < t < T_1 \\ &= v(t) + K_1 t + K_2 && T_1 < t < T' \\ K_2 &= 1/2\, c_1 T_1^2 - v(T_1) - K_1 T_1 \end{aligned}$$

displacement:

$$\begin{aligned} X(t) &= 1/6\, c_1 t^3 && 0 < t < T^1 \\ &= x(t) + 1/2\, K_1 t^2 + k_2 t + k_3 && T_1 < t < T' \\ K_3 &= 1/6\, c_1 T_1^3 - x(T_1) - 1/2\, K_1 T_1^2 - K_2 T_1 \end{aligned} \quad \text{(A-3)}$$

where K_1, K_2 and K_3 are constants. Movement time T', movement amplitude D' and velocity asymmetry k in this solution can be determined by the conditions,

$$V(T') = 0, \qquad X(T') = D', \qquad A(kT') = 0 \quad \text{(A-4).}$$

Varying the parameter c_1 from 1 to 0.25, k changed from 0.5 to 0.65. Using the equations in (A-3) and (A-4) peak accelerations relative to their means, D'/T'^2, were obtained as a function of k (Fig. 2). Jerk-cost is written as follow;

$$\text{COST} = 1/2 \int_0^T J(t)^2\,dt \quad \text{(A-5).}$$

Using J(t) in (A-3), the minimum cost per deg^2 was calculated as a function of T', where T' was obtained by the experimentally determined linear regression, k = 0.561–0.141 T' (s). The relation between $COST/D'^2$ and T' is displayed in Fig. 4 by a dotted line. The simulated cost was a little smaller than that of the unconstrained solution, (A-2), at a given movement time between 0.185 to 1.215 s.

Another constrained minimum-jerk solution based upon Fig. 5c can be calculated using procedures similar to those described above.

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