

Self-Fulfilling Prophecies

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After a quarter century of unbroken development in the theory of allocation under uncertainty, it has become an obvious fact that randomness in endowments, preferences or technology will generally work its way to the prices and allocations which prevail in equilibrium. Is it true, as intuition may suggest in haste, that random prices necessarily reflect some intrinsic uncertainty in the structure of the economy, or can they arise, as some recent literature [3, 12, 14, 15] indicates, merely from extraneous, self-perpetuating beliefs that prices are stochastic?

The question is of interest for it raises the possibility that business cycles are set in motion by arbitrary shifts in any factor, however purely subjective, agents happen to deem relevant to economic activity: animal spirits, consumer sentiment or the prophecies of the Sibyl at Cumae may spark fluctuations in which prices change simply because they are expected to and price signals convey no structural information.

The evidence on the influence of subjective factors is ample and dates back several centuries¹; the Dutch "tulip mania," the South Sea bubble in England, and the collapse of the Mississippi Company in France are three well-documented cases of speculative price movements which historians consider unwarranted by "objective" conditions.

What follows is a demonstration that a kindred type of paradoxical behavior, which we name *extraneous uncertainty*, is both possible and "frequent" among rational expectations equilibria in an aggregative model of overlapping generations. In particular, if we constrain (the probability distribution of) the price level to clear markets, reproduce beliefs and, in

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¹ See, for example, [4, Chap. 10]. Although this influence has long been recognized in the oral tradition of economics [6, p. 204], it has not to my knowledge received much formal attention until Karl Shell's recent example in [12].

addition, to follow a simple two-state Markov process, then a significant fraction of the resulting equilibria suffer from extraneous uncertainty. We give some examples of self-fulfilling prophecies that correspond to a permanent “recession” and permanent “boom” in an economy entirely free from price rigidities.

This phenomenon appears to be robust to changes in preferences and in the store of value: it will exist, for instance, under a gross substitutes assumption or a replacement of fiat money with unconditional claims on productive assets. The effect of extraneous uncertainty disappears, however, when the horizon is truncated; in the stationary state this may occur as well if enough contingent claims markets are appended to the economy.

I. PERFECT FORESIGHT

The most convenient way to introduce the basic structure and fix notation is to review the dynamics of the perfect foresight case. We begin with the simplest possible overlapping-generations model²: Time extends from zero to infinity; at the beginning of each time period $t = 1, 2, \dots$, a fixed-size generation of identical individuals is born which lives for two periods, youth and old age. Consumption occurs only in old age, production takes place solely in youth. The very first generation, born at $t = 0$, is “old”: each member of it is endowed with one unit of fiat money. Each member of the generation currently young may use a constant-returns-to-scale technology to transform n units of his own leisure into $y \leq n$ units of a perishable good for which he has no immediate use. The young, endowed with one unit of divisible leisure each, are obviously motivated to trade goods for the fiat store of value which is held exclusively by the old.

For each generation, preferences over current work and future consumption are additive, viz.,

$$v_t(c_{t+1}, n_t) = u(c_{t+1}) - g(n_t), \tag{1}$$

where $u(\cdot)$ and $-g(\cdot)$ are smooth monotone, concave functions. Let p_t, y_t be the price level and goods supply per young person in period t . The pair of sequences $\{p_t^*\}_0^\infty \{y_t^*\}_0^\infty$ is an *equilibrium*, if:

(a) For each $t \geq 1$, y_t^* is indeed equal to the amount of goods supplied by each young person, given the price-level sequence $\{p_t^*\}_0^\infty$. In other words, y_t^* maximizes $\{u(c_{t+1}) - g(y_t)\}$ s.t. $c_{t+1} = p_t^* y_t / p_{t+1}^*$ and $0 \leq y_t \leq 1$;

(b) the initial condition y_0^* is feasible, i.e., lies in the interval $[0, 1]$; and

² More general treatments appear in [10, 11, 5]. Readers familiar with this literature may skim Section I to pick up the notation.

(c) the demand for nominal money balances per head equals the corresponding supply, i.e., $p_t^* y_t^* = 1$ for all t .

We begin our examination of perfect foresight dynamics quite informally. Let us define $s(p_t/p_{t+1})$ as the solution to the maximum problem in (a) above, i.e., as the amount of goods supplied by each young person in period t if current price is p_t and future price is p_{t+1} . Clearly $s(\cdot)$ is a single-valued function about which we cannot say much unless we are willing to restrict preferences somewhat. For instance, $s(\cdot)$ is monotone increasing (decreasing) if current leisure and future consumption are gross substitutes (gross complements) at all values of the price ratio p_t/p_{t+1} .

The market clears if the price sequence $\{p_t\}_0^\infty$ satisfies

$$s(p_t/p_{t+1}) = 1/p_t. \quad (1a)$$

This first-order difference equation, together with the price level p_0 in period zero, describes the evolution of equilibrium prices over time.

Equation (1a) clearly reveals that the production economy we are studying in this paper is very close to the simple pure exchange economy frequently studied in the overlapping-generations literature. Suppose, for instance, that each member of generations $t = 1, 2, \dots$, were endowed not with leisure but with positive amounts (e_1, e_2) of a perishable consumption good in youth and old age. Given this endowment vector, let $s(p_t/p_{t+1})$ be now what the young *save* at the price ratio p_t/p_{t+1} and assume that $s(p_t/p_{t+1}) > 0$, for all values of p_t/p_{t+1} . The dynamics of this exchange model is again given by Eq. (1a).

Returning to our production economy, we define a *stationary* equilibrium price sequence as any non-negative constant sequence $\{p^*\}_0^\infty$ satisfying Eq. (1a). Clearly, $p_t = \infty$ for all t is one such sequence and it supports an autarkic equilibrium with zero output and worthless money.

There is exactly *one* other stationary equilibrium price sequence with valuable money, i.e., such that $p^* < \infty$. This one is defined from

$$s(1) = 1/p^*; \quad (1b)$$

it is unique because $s(\cdot)$ is single-valued, i.e., only one value of goods supply corresponds to the maximal element of the budget set $\{y_t, c_{t+1} \mid 0 \leq c_{t+1} \leq y_t\}$.

The stability of this particular stationary state depends very much on preferences. Suppose, for instance, that consumption and leisure are gross substitutes and let $p_t > p^*$ for some t . Then $1/p_t < 1/p^*$, and equilibrium requires that $s(p_t/p_{t+1}) < s(1)$. Since $s(\cdot)$ is increasing, we have $p_t/p_{t+1} < 1$ or $p_{t+1} > p_t$, thus moving further away from p^* . Gross substitutability in our

simple perfect-forsight economy implies that the stationary state with finite price level is unstable.

More formally, if we define two functions

$$G(n) \equiv ng'(n), \quad U(c) \equiv cu'(c) \tag{2}$$

such that $G(n) \rightarrow 0$ as $n \rightarrow 0$, assume that $G(n) \rightarrow \infty$ as $n \rightarrow 1$, and solve the maximum problem in part (a) of the previous definition, we find that every equilibrium sequence satisfies three requirements: the initial condition $y_0 = y_0^*$; feasibility, i.e., $y_t \in [0, 1]$; and a "law of motion," viz.,

$$\begin{aligned} y_{t+1} &= 0, & \text{if } y_t &= 0, \\ U(y_{t+1}) &= G(y_t), & \text{if } y_t &> 0. \end{aligned} \tag{3}$$

We note that, in equilibrium, y_t equals the commodity price of money in period t ; Eq. (3) is merely Eq. (1a) with preferences made explicit.

For stationary equilibria, Eq. (3) becomes

$$U(y^{**}) = G(y^{**}). \tag{3'}$$

It is well known [2, 13] that these three requirements are not sufficient to determine the price level, p_t (equivalently, the commodity price of money) unless we know the initial price $y_0 = 1/p_0$. To each value of y_0 there corresponds typically a different equilibrium sequence $\{y_t^*\}_0^\infty$. To see this we graph Eq. (3) in Fig. 1 and confirm readily that there exist two steady states: one corresponding to zero price of money (at the origin), another to a positive price (at y^{**}). In panel III, for example, the stationary state S with positive price of money is stable and may be reached in infinite time from any $y_0 \in (0, 1)$. Hence, there is an infinity of equilibrium price sequences.

Initial conditions are, of course, arbitrary; our model provides the economy with no mechanism to choose among them, leaving the equilibrium price sequence indeterminate. Even if we somehow confined our attention to equilibria which are not Pareto-dominated by other equilibria,³ it is not obvious that we would be left with a unique solution since many of these multiple equilibria turn out to be Pareto-noncomparable. An illustration of this dilemma is in panel IV of Fig. 1, which corresponds to preferences $g(n) = n/k^2$, $u(c) = -c^{-1}$ for some constant $k \in (0, 1)$. Then $G(n) = n/k^2$, $U(c) = 1/c$, and (3) yields

$$y_{t+1} y_t = k^2. \tag{4}$$

³ The suggestion is discussed in [13, 1].

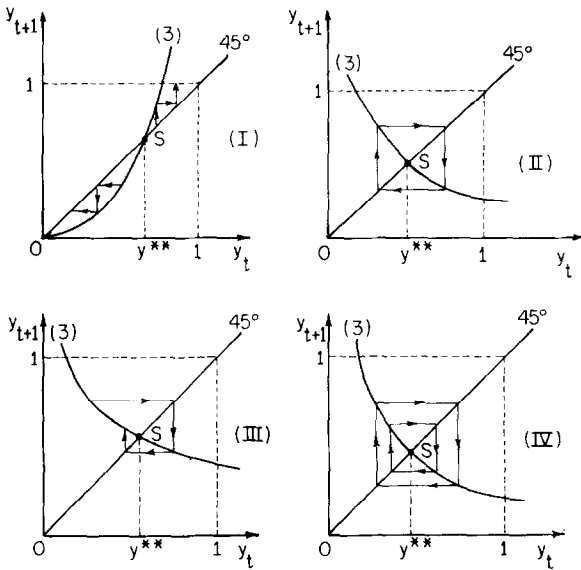


FIGURE 1

The non-trivial stationary solution is $y^{**} = k$ but there are others for which the economy cycles, e.g.,

$$y_t^* = a, \quad y_{t+1}^* = k^2/a, \tag{5}$$

for any constant $a \in [k^2, 1]$. The non-trivial stationary solution yields utility $-2/k$ for each generation whereas the cycling solution yields $(-2a/k^2, -2/a)$ to successive generations. For $a \neq k$, one sees easily that

$$\min(-2a/k^2, -2/a) < -2/k < \max(-2a/k^2, -2a) \tag{6}$$

and, hence, (5) cannot be compared with $y^{**} = k$.

To sum up: The non-uniqueness of the equilibrium price level in the overlapping generations model is a phenomenon which owes much to the infinite number of decision makers and dated commodities, and very little to properties like lack of gross substitutability which are crucial in a static general equilibrium context.⁴

What is less well understood is that extraneous uncertainty considerably

⁴ This point, also made by Shell *et al.* [13] and Calvo [2, Sect. II], is evident in panel I. Fig. 1. The price level sequence $\{p_t^*\}_0^\infty$ is generally indeterminate there *despite* the fact that the function U is increasing and, hence, current leisure and future consumption are gross substitutes.

enlarges the set of equilibrium prices that arise under perfect foresight, by adding in certain cases an infinity of self-replicating equilibria. Some of these are examined next.

II. REPLICATING EQUILIBRIA

We can now introduce extraneous uncertainty at little cost in additional notation. Denote by Ω_t the information set available to agents in period t ; this may include any historical element in the economy such as prices and quantities. Let ω_t be a typical element of Ω_t . By analogy with the preceding section, a pair of sequences of random variables $\{\tilde{p}_t\}_0^\infty \{\tilde{y}_t\}_0^\infty$ is an equilibrium if:

(a) For each ω_t and $t \geq 1$, $\tilde{y}_t = y(\omega_t)$ solves

$$\max\{E[u(c_{t+1}) | \Omega_t] - g(y_t)\} \quad \text{s.t. } c_{t+1} = \tilde{p}_t y_t / \tilde{p}_{t+1};$$

$$0 \leq \tilde{y}_t \leq 1;$$

(b) \tilde{y}_0 is a number in $[0, 1]$; and

(c) $\tilde{p}_t \tilde{y}_t = 1$ for all ω_t .

At the individual level, the probability distribution of \tilde{y}_t is derived from that of \tilde{p}_t and both are conditional on Ω_t . In the aggregate, the "law of motion" for the economy becomes

$$E\{U(y_{t+1}) | \Omega_t\} = G(y_t), \tag{7}$$

the solution being again an appropriate conditional probability distribution which confirms price expectations. This law, however, does not constrain sufficiently the solution for it says nothing whatever about the higher moments of the random variable $U(y_{t+1})$. Suppose, for example, that ϵ_t is any independent, identically distributed random variable with mean one, belonging to the information set Ω_t ; then the stochastic difference equation

$$U(y_{t+1}) = G(y_t) / \epsilon_{t+1} \tag{8}$$

"solves" Eq. (7) and corresponds to a rational expectations equilibrium if, given y_t , ϵ_{t+1} is defined so that $y_{t+1} \in [0, 1]$ with probability 1 for all t .

To avoid problems with feasibility and, at the same time, restrict somewhat the equilibrium price set, we limit ourselves to solutions with the Markov property, i.e., ones for which

$$\Omega_t = y_t. \tag{9}$$

In particular, suppose that, for all t , the price of money may attain at most two values, $y_t = \{y_1, y_2\}$, with the following stationary transition probability matrix:

$$\begin{array}{c|cc}
 & y_1 & y_2 \\
 \hline
 y_t & & \\
 \hline
 y_1 & q_1 & 1 - q_1 \\
 y_2 & 1 - q_2 & q_2
 \end{array} = T. \tag{10}$$

For those who like to think of price randomness as a “structural” phenomenon, this matrix simply reflects the stochastic properties of any variable deemed by public opinion to have a bearing on economic activity. Since there is myriad of such candidate variables, it makes some sense to treat the probabilities (q_1, q_2) as parameters in this economy—just as we do with prices.

A self-fulfilling equilibrium is now a set of four numbers (q_1, q_2, y_1, y_2) all lying in the interval $[0, 1]$ and satisfying Eq. (7), viz.

$$q_1 U(y_1) + (1 - q_1) U(y_2) = G(y_1), \tag{11a}$$

$$(1 - q_2) U(y_1) + q_2 U(y_2) = G(y_2). \tag{11b}$$

Since these are two equations in *four* unknowns,⁵ we should generally expect multiple equilibria. Some of them we already know: if the economy has a stationary state y^{**} under perfect foresight, that is, a solution to $G(y) = U(y)$, then every quadruple $(q_1, q_2, y^{**}, y^{**})$ such that $q_1 \in [0, 1]$ and $q_2 \in [0, 1]$ obviously solves (11a)–(11b) and is an equilibrium. Extraneous uncertainty, of course, involves *different* prices of money in the two states.

There are at least two special assumptions that will exclude extraneous uncertainty in this self-replicating example. The more obvious one is that consumption and leisure are gross substitutes, that is, U is an increasing function of y . Then, under perfect foresight, we recall that no equilibrium price sequence goes to the non-trivial stationary state unless it started there (see also Fig. 1, panel I). A similar problem appears under uncertainty: goods supply is increasing in the price level, a fact which is not consistent with market clearing, i.e., with

$$p_1 y_1 = p_2 y_2 = 1. \tag{12}$$

Formally, solve (11a)–(11b) for (q_1, q_2) to obtain

$$q_1 = [U(y_2) - G(y_1)]/[U(y_2) - U(y_1)], \tag{13a}$$

⁵ With N states if nature, we would have N equations in N^2 unknowns.

$$q_2 = [G(y_2) - U(y_1)]/[U(y_2) - U(y_1)]. \tag{13b}$$

Suppose, without any loss of generality, that $y_2 > y_1$, so that $U(y_2) > U(y_1)$. Then $q_1 < 1 (\Rightarrow) G(y_1) > U(y_1) (\Rightarrow) g'(y_1) > u'(y_1) (\Rightarrow) y_1 > y^{**}$. Similarly $q_2 < 1 (\Rightarrow) G(y_2) < U(y_2) (\Rightarrow) g'(y_2) < u'(y_2) (\Rightarrow) y_2 < y^{**}$. Hence $y_1 > y_2$, which contradicts the maintained assumption $y_2 > y_1$. The only solution is $y_1 = y_2 = y^{**}$.

The same situation arises under gross complementarity if $q_1 + q_2 \geq 1$. Then $y_2 > y_1$ implies $U(y_2) < U(y_1)$; from (13a) and (13b), however, $G(y_1) \geq G(y_2)$ if $q_1 \geq 1 - q_2$ and, hence, $y_1 \geq y_2$, which contradicts a maintained assumption, since G is by definition an increasing function. A symmetric contradiction obtains if the maintained assumption is $y_1 > y_2$.

For the remainder of this section we suppose that consumption and leisure are gross complements. If $y_2 > y_1$ then $U(y_2) < U(y_1)$, and q_1 is a probability if, and only if

$$U(y_2) \leq G(y_1) \leq U(y_1). \tag{13c}$$

Also, $q_2 \in [0, 1]$ if and only if

$$U(y_2) \leq G(y_2) \leq U(y_1). \tag{13d}$$

Note, however, that $G(y_1) \geq U(y_2) (\Rightarrow) G(y_2) \geq U(y_2)$ and $U(y_1) \geq G(y_2) (\Rightarrow) U(y_1) \geq G(y_1)$. Hence, with $y_2 > y_1$, q_1 and q_2 are probabilities if, and only if

$$G(y_1) \geq U(y_2); \quad U(y_1) \geq G(y_2). \tag{14a, b}$$

Then one easily verifies that $q_1 + q_2 < 1$. Similarly, with $y_2 < y_1$, the q 's are probabilities with sum no more than one if, and only if,

$$G(y_1) \leq U(y_2); \quad U(y_1) \leq G(y_2) \tag{14c, d}$$

The equations $G(y_1) = U(y_2)$ and $U(y_1) = G(y_2)$ are clearly symmetric in the plane around the straight line $y_1 = y_2$. In Fig. 2, then, inequalities (14a, b) are satisfied by all points lying simultaneously not below the line $U(y_2) = G(y_1)$, not above the line $U(y_1) = G(y_2)$ and above the 45° line; similarly, inequalities (14c, d) are satisfied by all points not above the line $U(y_2) = G(y_1)$, not below $U(y_1) = G(y_2)$ and below the 45° line. The set of points that satisfies all four inequalities (14a)–(14d) is shaded in Fig. 2. No equilibrium with extraneous uncertainty exists in panel I but there is an infinity of them in panel II. What is more important, the shaded area has significant size (i.e., is not a set of measure zero) relative to the set of all feasible allocations when q_1 and q_2 are both positive. Otherwise, some of the relations in (14) become strict equalities, the shaded area collapses to one of

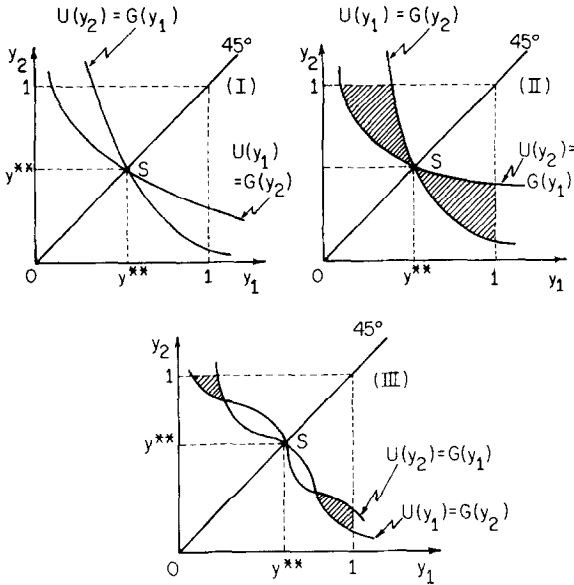


FIGURE 2

its boundaries, and the set of equilibria with extraneous uncertainty has measure zero.

A sufficient condition for panel II to obtain is obviously that the line $U(y_1) = G(y_2)$ be steeper than the line $U(y_2) = G(y_1)$ at point S , or

$$G'(y^{**})/U'(y^{**}) + 1 > 0, \tag{15}$$

i.e., that the “law of motion” in Eq. (3) yield a locally stable nontrivial stationary equilibrium in the perfect foresight case. Panel III in Fig. 2 shows that this condition is not necessary; it is violated at S , and yet extraneous uncertainty appears in the shaded areas near the NW and SE corners.

In summary, if $q_1 + q_2 < 1$, a set of sufficient (but not necessary) conditions that guarantee the existence of replicating equilibria with extraneous uncertainty under the stochastic structure postulated in (10) are: *gross complementarity* between consumption and leisure, and *local stability* of the non-trivial stationary state. Furthermore, if the transition probability matrix is ergodic, then the set of these equilibria has the same dimension as that of all feasible allocations.

III. ARE SELF-FULFILLING PROPHECIES LIKELY?

Readers who, for intuitive reasons of their own, remain unconvinced of the importance of self-fulfilling prophecies may regard the example in the

preceding section as an artifact of the imagination that is unlikely to occur in “practice.” Yet, under the assumptions made in the previous paragraph, extraneous uncertainty is not only possible but “probable” as well; for most configurations of the exogenous probabilities, q_1 and q_2 such that $q_1 + q_2 \leq 1$, there exists one stationary equilibrium and at least two other distinct equilibria such that $y_1 \neq y_2$. To see this, suppose $g'(1) = \infty$, $U(c) \rightarrow B \leq \infty$ as $c \rightarrow 0$, and define the function \hat{T} from

$$\hat{T}(q, y) \equiv (1 - q)^{-1} [G(y) - qU(y)] \tag{16}$$

for $q < 1$. Then \hat{T} is increasing in y , and Eqs. (11a)–(11b) reduce to

$$U(y_2) = \hat{T}(q_1, y_1); \quad U(y_1) = \hat{T}(q_2, y_2). \tag{17a, b}$$

To solve (17a, b) we require some additional information from (16), namely:

$$\hat{T}(q, y) \rightarrow \infty \quad \text{as } y \rightarrow 1; \tag{18a}$$

$$\hat{T}(q, y^{**}) = U(y^{**}), \quad \forall q; \tag{18b}$$

and

$$\hat{T} \geq 0 \quad \text{for } y \in [\hat{y}(q), 1]. \tag{18c}$$

Here the function $\hat{y}(q)$, defined as the solution to $g'(y) = qu'(y)$, is obviously increasing, because of the gross complements assumption, and such that

$$\hat{y}(1) = y^{**}. \tag{19}$$

From the information in (18a, b, c) we may now draw Eqs. (17a, b) in Fig. 3. By the local stability of the stationary equilibrium, the line $U(y_1) = \hat{T}(q_2, y_2)$ is steeper than $U(y_2) = \hat{T}(q_1, y_1)$ at point S . Hence, to each pair (q_1, q_2) of probabilities such that $q_1 \neq q_2$ corresponds the non-trivial stationary solution y^{**} and, in addition, at least two distinct equilibria with extraneous uncertainty: one at E_1 in which $y_1 < y^{**} < y_2$, another at E_2 such that $y_2 < y^{**} < y_1$. When $q_1 = q_2$, points E_1 and E_2 become symmetric about the 45° line and we have a single equilibrium other than S .

As (q_1, q_2) takes on values in the set $[0, 1] \times [0, 1]$, i.e., in the square area of Fig. 4, the two lines in Fig. 3 change position, their intersections yielding the set of all equilibrium prices of money. The shaded area in Fig. 4 corresponds to (q_1, q_2) values for which the stationary value y^{**} is the only equilibrium with positive price of money. Extraneous uncertainty will appear for (q_1, q_2) in the unshaded region and, outside the set of measure zero $Q = \{q_1, q_2 | q_1 \in [0, 1]; q_2 \in [0, 1]; q_1 + q_2 < 1; q_1 = q_2\}$,⁶ it will charac-

⁶ The set Q in Fig. 4 is merely the straight-line segment (OF) minus its end-point F .

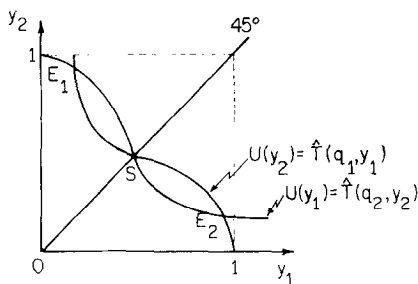


FIGURE 3

terize at least two-thirds of the equilibria in that region. In fact, if there are many intersections in Fig. 3 other than E_1, S, E_2 , then nearly *all* the equilibria which correspond to the unshaded area of Fig. 4 will suffer from extraneous uncertainty.

Averaging the two regions of that figure, then, it is clear in this example that self-fulfilling prophecies are at least “one-third” and less than “one-half” of all equilibria.

IV. PERMANENT RECESSIONS

The Markovian structure of the matrix T links the economy's present with its immediate past; in fact, it is fairly straightforward to verify by tedious computation that $q_1 + q_2 < 1$ implies a negative serial correlation in equilibrium output. To ascertain what the economy does over the long haul, one typically considers T^n , the n th power of the matrix T ; if T is ergodic, then as $n \rightarrow \infty$, T^n tends to some matrix⁷

$$\Pi = \begin{pmatrix} \pi_1 & \pi_2 \\ \pi_1 & \pi_2 \end{pmatrix}, \quad (16)$$

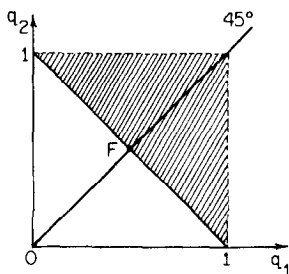


FIGURE 4

⁷ Cf. [9, p. 197–198].

which obviously satisfies $T \cdot \Pi = \Pi$. The numbers

$$\pi_1 = \frac{1 - q_2}{2 - q_1 - q_2}; \quad \pi_2 = 1 - \pi_1, \tag{17}$$

are steady-state probabilities that the economy will, in infinite time, find itself in states 1 or 2, irrespective of initial conditions.

The ratio of these probabilities,

$$\pi_1/\pi_2 = (1 - q_2)/(1 - q_1), \tag{18}$$

may attain values arbitrarily close to zero if $q_2 = 1 - 2\varepsilon$, $q_1 = \varepsilon$ and ε is a positive number sufficiently close to zero. Then the economy will either be in the stationary state or “settle” asymptotically on state 2, that is, on a nearly permanent, extreme level of economic activity—recessionary or boomlike.

Since we do not know how a particular equilibrium prevails when several are possible, we cannot be sure that, in some circumstances at least, an extreme equilibrium will obtain. But it is interesting to note that a nearly “permanent” low level of economic activity is consistent with a neoclassical model of equilibrium in which prices are flexible and expectations are rational.

V. ROBUSTNESS

We consider briefly here whether perturbations of the assumptions we employed in the extended example of Sections II–IV will alter the basic message, which is that infinitely many solutions exist with the extraneous uncertainty property.

(a) *Gross Complementarity*

Suppose that consumption and leisure are, instead, gross substitutes. In particular let $u(c) = c$ and $g(n) = n^2$ for $n \in [0, 1]$ so that the law of motion in Eq. (7) becomes

$$2y_t^2 = E(y_{t+1} | y_t) \tag{19}$$

with a unique, non-trivial stationary solution, $y^{**} = 1/2$. One verifies immediately that, for any initial $y_0 \in [0, 1/2]$, Eq. (19) is also solved by

$$\begin{aligned} y_{t+1} = z_{t+1} & \quad \text{w.p.} \quad q_{t+1} \equiv (1 - 4y_t^2)/(1 - 2z_{t+1}) \\ & = 1/2 \quad \text{w.p.} \quad 1 - q_{t+1}, \end{aligned} \tag{20}$$

where z_{t+1} is any arbitrary number in $[0, 2y_t^2]$. These solutions are infinitely

many, because z is picked arbitrarily each period from some interval; and they are all feasible since, for all t , $q_t \in [0, 1]$ and $y_t \in [0, 1]$ w.p. 1. Ergo, there are infinitely many equilibria.

For example, set $z_t = 0, \forall t$. Then for any $y_0 \in [0, 1/2]$ and $t \geq 1$, we have

$$\begin{aligned} y_t &= 0 && \text{w.p. } 1 - 4y_0^2 \\ &= 1/2 && \text{w.p. } 4y_0^2. \end{aligned} \tag{21}$$

In other words, starting from any point in $(0, 1/2]$, the economy will reach $y^{**} = 1/2$ with positive probability, an event which panel I in Fig. 1 shows to be impossible under perfect foresight.

(b) *The Role of Fiat Money*

Can we blame the inherent worthlessness of fiat money for the great multiplicity of equilibria which are consistent with self-fulfilling prophecies? If a productive asset like land were the sole store of value,⁸ then we know from Calvo [2, Sect. I] that an infinity of equilibrium price sequences are possible under perfect foresight, and we may guess that the situation does not change drastically if one injects prophecies into the economy.

This suspicion is borne out in the following model. Preferences are as in Eq. (1) and there is no money; land is transferred from the old to the young at the beginning of each period in return for consumption goods later in that period. Let N denote hours of work; Q = amount of land; output is $Y = F(Q, N)$ where F is smooth, concave, with constant returns-to-scale. Also, define $y \equiv Y/Q$; $n \equiv N/Q$; $f(z) \equiv F(1, z)$; p = price of land (including rental) in terms of current consumption. Endowments are $\bar{N} = 1$ for leisure and $\bar{Q} = 1$ for land. All variables are per member of the young generation, except \bar{Q} which applies to each member of the old generation.

To purchase one unit of land in period t , the young must work in return $n_t = h(p_t)$ hours, where $h = f^{-1}$ is the inverse production (e.g., labor requirements) function. Each unit of land will entitle the owner to consume p_{t+1} units of output in old age; to entertain the possibility of self-fulfilling prophesy we allow p_{t+1} to be stochastic in principle.

The young demand Q_t^* units of land, which is the solution to

$$\max_{Q_t > 0} E\{U(Q_t p_{t+1}) | \Omega_t\} - g[Q_t h(p_t)], \tag{22}$$

with Ω_t being the current information set. At equilibrium $Q_t^* = 1, \forall t$, provided $p_t > 0$. Hence, if one defines the increasing function $J(z) \equiv G[h(z)]$

⁸ Storage of goods over time does not destroy extraneous uncertainty unless the cost of carrying inventories forward is zero. With positive storage cost h per unit, it is possible (see Fig. 2) to pick equilibrium prices of money close enough to the stationary state so that the real rate of return on money always exceeds $-h$.

and goes through the same process as in the beginning of Section II, one finds that every interior rational expectations equilibrium satisfies

$$J(p_t) = E\{U(p_{t+1}) | \Omega_t\}. \tag{23}$$

This first-order difference equation is very similar to the one in Eq. (7), permitting again infinitely many stochastic equilibria.

I conclude that it is not fiat money in itself that sustains self-fulfilling prophecies in the economy at hand.

VI. THE ORIGIN OF EXTRANEOUS UNCERTAINTY

To isolate the factors responsible for self-perpetuating prophecies let us examine two changes in the basic model of Section II which are sufficient to do away with the influence of extraneous uncertainty. The first one is an announcement by some authority at $t = 0$ that exchange in period $T > 0$ will be permitted to occur only at some deterministic price p_T (or even at a random price \bar{p}_T , provided that is uncorrelated with p_{T-1}). Then, uncertainty unravels, for Eq. (7) says that, for $t < T$, all equilibrium prices will be deterministic.

By truncating the time horizon, this form of price controls yields finitely many decision makers and steers clear of the large equilibrium price sets which typically arise otherwise.⁹ Like many types of price controls, this one assumes the authorities know a lot: given T and the initial condition p_0 , the price p_T will support an equilibrium only if (p_0, p_T) are consistent with the law of motion

$$G(p_t^{-1}) = E\{U(p_{t+1}^{-1}) | p_t\} \quad \text{with} \quad p_t^{-1} \equiv 1/p_t, \tag{24}$$

that is, only if authorities have exact knowledge of the dynamic behavior of the economy.

A more important cause of self-fulfilling prophecies seems to be the shortage of claims markets. If the predictions of the Cumaean Sibyl affect allocations in Rome, can the Romans neutralize her interference by building up their financial system and, in particular, by storing value in claims which deliver consumption contingent on what the Sibyl says?

Let q_{ij} be the (stationary) probability that the future state will be j if the current state is i ($i, j = 1, 2$); π_j = money price of a claim delivering one unit of consumption in state j ; z_{ij} = number of claims on state j bought by the old if the past state is i ; w_{ij} = number of claims sold by young. Money serves no

⁹ See [8, pp. 111-112; 11].

function anymore, but it is useful to continue thinking of it as a medium of exchange and pretend that each old person has one unit of it.

The old choose z_{ij} by maximizing expected utility of consumption, i.e., solving

$$\begin{aligned} & \max q_{i1} u(z_{i1}) + q_{i2} u(z_{i2}) \\ \text{s.t. } & z_{i1} \geq 0, z_{i2} \geq 0; \pi_1 z_{i1} + \pi_2 z_{i2} \leq 1; i = 1, 2. \end{aligned} \quad (25a)$$

The young on the other hand, minimize the expected disutility of work needed to acquire any given money revenue, A , from the sale of claims. Thus the w_{ij} solve

$$\begin{aligned} & \min q_{i1} g(w_{i1}) + q_{i2} g(w_{i2}) \\ \text{s.t. } & w_{i1} \geq 0, w_{i2} \geq 0; \pi_1 w_{i1} + \pi_2 w_{i2} \geq A; i = 1, 2. \end{aligned} \quad (25b)$$

Furthermore, stationary equilibrium means $A = 1$ with probability one in the money market, and

$$z_{ij} = w_{ij}, \quad \forall (i, j), \quad (25c)$$

in the market for contingent claims.

For $i = 1, 2$, every interior equilibrium satisfies

$$\frac{\pi_1}{\pi_2} = \frac{q_{i1} u'(z_{i1})}{q_{i2} u'(z_{i2})} = \frac{q_{i1} g'(z_{i1})}{q_{i2} g'(z_{i2})}, \quad (26a)$$

which implies

$$g'(z_{i1})/u'(z_{i1}) = g'(z_{i2})/u'(z_{i2}). \quad (26b)$$

Hence, the solutions to (25a, b) satisfy

$$z_{i1}^* = z_{i2}^*, \quad i = 1, 2.$$

Call z_i^* the common value of z_{i1}^*, z_{i2}^* ; then, as $A = 1$ for every young person in every state, we have $(\pi_1 + \pi_2) z_1^* = (\pi_1 + \pi_2) z_2^* = 1 (\Rightarrow) z_1^* = z_2^*$. Consumption (more generally, economic activity) has become entirely independent of the state of nature.

The Sibyl of Cumae now stands thoroughly neutralized; but the vast Roman Empire is blessed with eight more Sibyls, a large number of oracles, religious seers and mystery cults. To render all of them ineffectual requires more claims markets than the Romans can reasonably hope to set up.

Even if we suppose momentarily that all requisite markets were costlessly available and open for business, there is no guarantee that they would rob the Sibyl of all power to influence economic events. The present section demonstrates this to occur in a *special case*, that is, when preferences are additive, beliefs are homogeneous and the probability distribution of future prices conditioned on current prices is stationary.

VII. CONCLUSIONS

The central message of this paper is that even perfectly well-behaved economies will typically admit rational expectations equilibria in which the expectations themselves spark fluctuations in the level of business activity. If many individuals naively believe that sunspots¹⁰ or some index of confidence are good predictors of future prices, then they may take actions which tend to bear out their beliefs.

Self-fulfilling prophecies are by their very nature a source of indeterminacy, augmenting appreciably the already very large number of perfect-foresight equilibria which typically emerges in monetary economies with infinitely many agents and commodities. In one fairly extended example (Sections II and III), self-fulfilling prophecies comprise between one third and one-half of all equilibria. Some of these resemble permanent "recessions" or "booms" and all of them are replicating Markov chains, that is, *perpetual* cycles ignited by expectations alone.

Phenomena of this sort will persist if an intrinsically valuable asset like land replaces fiat money as store of value, but *may* unravel if we set up markets for claims contingent on prophecies—whether they *do* unravel is an interesting topic for future research. As a practical matter, however, one ought to recognize that a vast number of financial markets would be needed to neutralize all subjective factors that individuals might consider influential in economic life.

Given some market incompleteness, what can be said in general about the solutions to Eq. (7)? In particular, under what conditions will extraneous uncertainty replicate itself in perpetuity (as it does in the example of Section II), vanish in finite time (as in Eq. (21)) or dissolve asymptotically?

These questions are outside the scope of the paper at hand but appear to be ones that we must face if we wish to characterize intelligently the myriad of equilibria which are consistent with the postulate of rational expectations.

¹⁰ Jevons [7, Chaps. VI–VII] attempted with some persistence to generalize and test William Herschel's hypothesis that sunspots influenced economic activity. Jevons' theory, however, is not psychological or expectational; it holds that solar activity affects climatic conditions and, through them, the aggregate production possibility frontier.

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