AIRCRAFT POSITIONING WITH INS/GNSS INTEGRATED SYSTEM

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The paper describes methods of INS/GNSS integration and rules of designing Kalman filters. An example of INS/GPS positioning system is presented. The system employs a Complementary Linearized Kalman Filter, estimating INS position, velocity and attitude errors, as well as GPS clock bias and drift. Chosen simulation results, comparing accuracy of INS, GPS and INS/GPS are included.

Keywords: INS, GNSS, GPS, Kalman filter, integrated positioning system

1. INTRODUCTION

Contemporary positioning systems are expected to provide for precise and timely information about the parameters of motion of the vehicle, such as position, velocity, acceleration and attitude. In less demanding applications, a single navigational device may prove sufficient for this purpose. However, there exist applications, e.g. aircraft positioning, where a single device is not capable of fulfilling stringent requirements with respect to accuracy, reliability, continuity and integrity of positioning. In such cases, the optimal from a technical and economic point of view positioning system is composed of several navigational devices. Their data are jointly processed via a dedicated navigation filter [1, 3].

In aircraft positioning and navigation, INS/GNSS integrated systems are frequently applied [3, 8]. Integration of Inertial Navigation Systems (INS) and Global Navigation Satellite System (GNSS) receivers via the Kalman filter presents one of the best achievements in positioning and navigation technology and one of the most successful applications of the Kalman filter [1, 2]. As advantages and disadvantages of INS and GNSS are to large extent complementary, their appropriate integration can eliminate drawbacks of both systems and make optimal use of their strengths.

The main reasons for INS/GNSS integration result from a fact that an integrated INS/GNSS positioning system can achieve significantly better accuracy, reliability, continuity and integrity than INS or GNSS receiver alone. This is due to the synergy of both systems. The fundamental part of INS is the Inertial Measurement Unit (IMU), composed of three orthogonal accelerometers and three orthogonal gyroscopes (gyros) of known relative orientation [5, 8]. The accelerometers provide for information on linear displacement of the vehicle, whereas the gyros provide for information about its angular displacement and with known initial attitude enable continuous calculations of the attitude of vehicle. Velocity and position of the vehicle are obtained through single and double integration of accelerations from accelerometers. Prior to integration, the accelerations have to be transformed from the body frame of reference to the navigation frame of reference and the integrations have to be initialized with the known starting velocity and position of the vehicle.

INS is the only navigational instrument providing for the complete set of information on position, velocity and angular orientation. Its further advantages include availability of output data at a very high rate, making navigation almost continuous, autonomy, i.e. lack of dependence on external facilities, and consequently immunity to intentional or unintentional jamming, as well as immediate response to rapid manoeuvres of the vehicle. The main drawback of INS, its unbound increase of errors resulting in poor long-term accuracy, is the main reason for development of integrated systems like INS/GNSS.

GNSS receivers ensure worldwide, full-time and all-weather navigation, the accuracy of which does not depend on the time of operation. A short-term accuracy of a navigation GNSS receiver, however, may be not good enough for demanding applications. Serious drawbacks of GNSS receivers include immunity to intentional or unintentional jamming, possibility of signal outages caused e.g. by antennae shading, and lack of attitude information in navigation GNSS receivers. As can be seen, combining both subsystems into one integrated positioning system can result in its superior parameters in comparison to the parameters of its elements.

2. METHODS AND ALGORITMS IN INS/GNSS INTEGRATION

Choice of an appropriate structure of INS/GNSS system is vital for making optimal use of the strengths of the system components and reducing or eliminating their disadvantages. There exist two general methods of integration that can be assumed when designing INS/GNSS systems [1]. The first group of methods is called integration according to scheme of filtration and the second group of methods is referred to as integration according to scheme of compensation. The systems integrated according to scheme of compensation can be designed as systems with feed-forward or feed-backward correction. In all mentioned types of integrated systems, centralized or decentralized data processing algorithms can be employed.

2.1. ALGORITMS OF NAVIGATION DATA PROCESSING

Diverse types of navigation data processing algorithms have been proposed in literature [1-5]. Nowadays, Kalman filters (KF) are commonly used in integrated positioning and navigation systems. The standard Kalman filter is the optimal algorithm of navigation data processing for a system with linear dynamics and observation models, under an assumption of Gaussian process disturbances, Gaussian measurement noises and Gaussian initial state vector. However, there exist numerous modifications of the standard Kalman filter, dealing with its limitations and extending its use to a wider class of applications.

Especially important for positioning and navigation applications is dealing with a common problem of nonlinearities in models of positioning and navigation systems. Traditionally, linearized Kalman filters (LKF) and extended Kalman filters (EKF) have been used in such cases [1-3]. Their applicability, however, is limited to systems with small nonlinearities. In last decade, unscented Kalman filters (UKF) and particle filters (PF) [4] have been proposed and successfully implemented in systems with large nonlinearities. Their performance in this class of systems occurs to be significantly better than that of LKF or EKF. The focus of this paper, however, is on use of the linearized Kalman filtering algorithm for INS/GNSS integration, and this type of filter have been applied in the designed and presented in the paper INS/GNSS system. LKF can be used in this application, because the model of system is only slightly nonlinear.

The standard Kalman filter is based on linear state-space model of the system. A discrete linear state-space model of dynamic system is given via a pair of equations [1], the first of which is called a dynamics model, and the second one is an observation (measurement) model:

$$\mathbf{x}(k+1) = \mathbf{\Phi}(k+1,k)\mathbf{x}(k) + \mathbf{w}(k) \tag{1}$$

$$\mathbf{z}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{v}(k) \tag{2}$$

where: $\mathbf{x}$ - state vector to be estimated via KF,

$\mathbf{w}$ - vector of random process disturbances,

$\mathbf{\Phi}$ - state transition matrix,

$\mathbf{z}$ - measurement vector,

$\mathbf{v}$ - vector of measurement noises,

$\mathbf{H}$ - observation (measurement) matrix,

k - index of discrete time.

KF realizes on alternate basis a prediction (time update) and a correction (measurement update). The previously formulated dynamics model is used for prediction and the measurement model is used for correction of the state vector. The simplified operation of the Kalman filter is explained in Fig. 1.

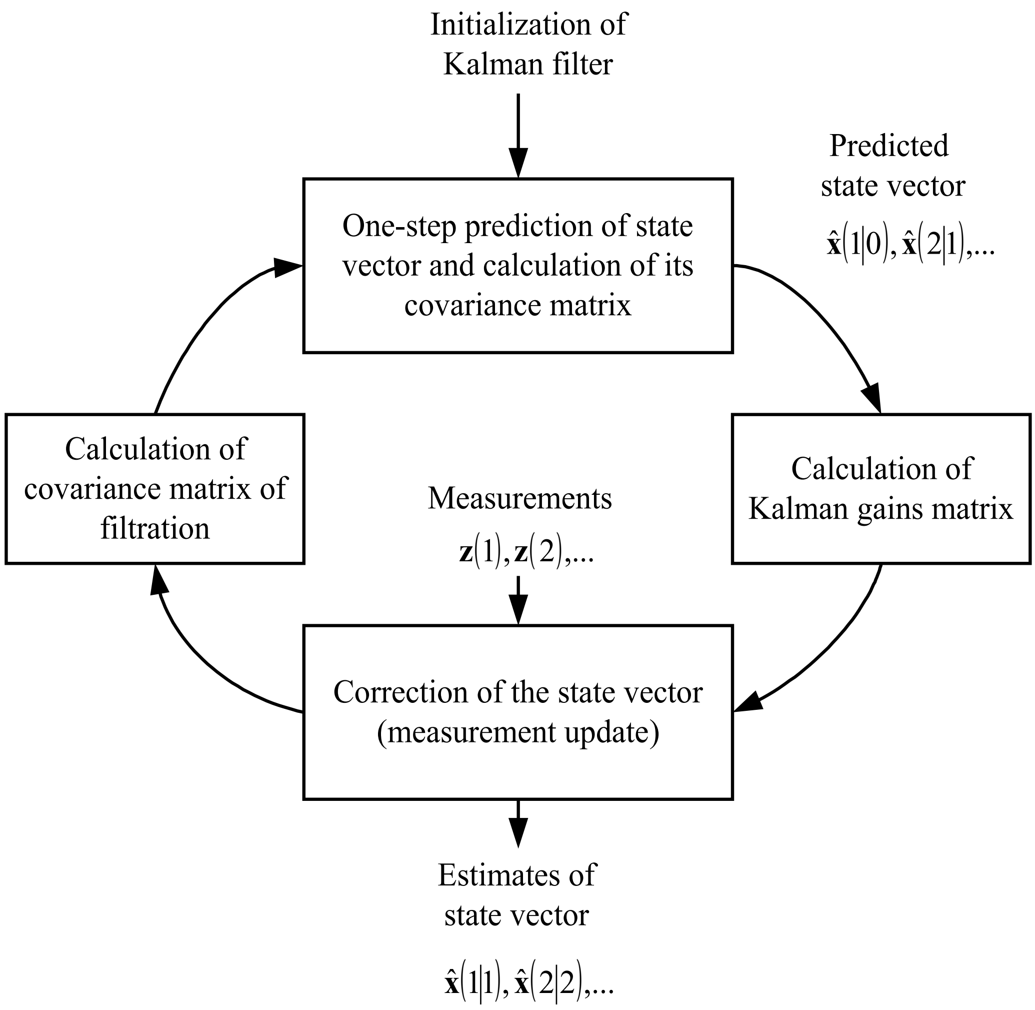

Fig. 1. Kalman filtering algorithm

If the system is nonlinear, standard KF cannot be used for its state estimation. A discrete nonlinear state-space model of dynamic system with additive noises is given via the following pair of equations [4]:

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), k] + \mathbf{w}(k) \tag{3}$$

$$\mathbf{z}(k) = \mathbf{h}[\mathbf{x}(k), k] + \mathbf{v}(k) \tag{4}$$

where **f** and **h** represent nonlinear vector functions. As will be shown later on, the designed INS/GNSS system has a linear dynamics model and a nonlinear observation model, but due to the linearization of observation model in LKF it can be modelled similarly to linear systems.

2.2. INTEGRATION ACCORDING TO SCHEME OF FILTRATION

Integration according to scheme of filtration consists in direct processing of measurements of navigation devices and estimating the navigation elements of the vehicle, i.e. position, velocity, attitude, etc. This is so-called full-states formulation, the full states being position, velocity and attitude. A block scheme of such INS/GNSS system is presented in Fig. 2.

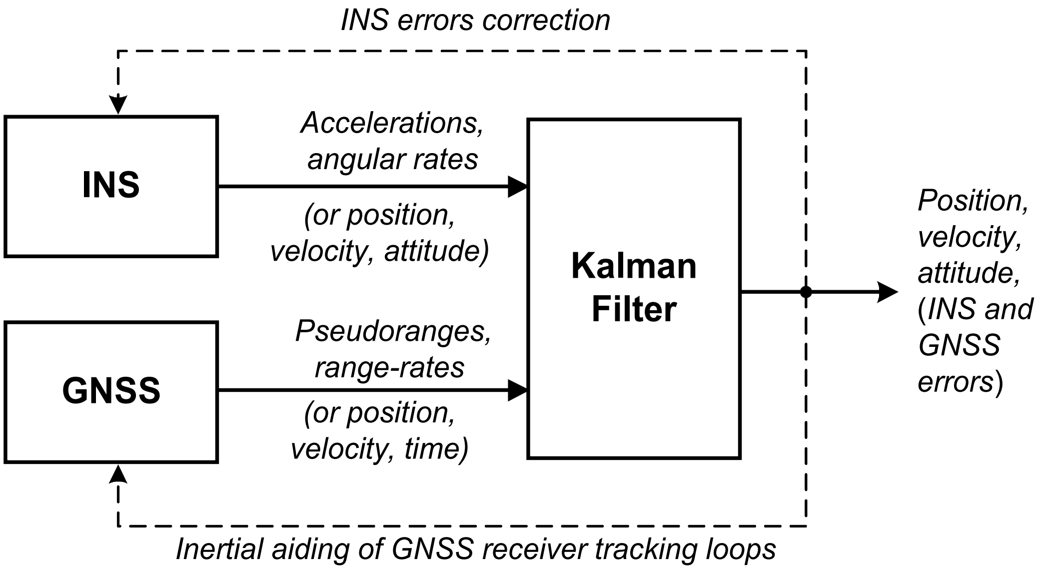

Fig. 2. INS/GNSS system integrated according to schemeof filtration

The level of integration of the above system depends on whether INS and GNSS receiver are treated as systems or as sensors and on the level of their coupling. If they are treated as systems, providing pre-processed data like position, velocity, attitude or time and all data buses are unidirectional, the integrated system is usually referred to as uncoupled. Such systems are the easiest to design, but do not exploit full possibilities of integration.

Introducing inertial aiding of GNSS receiver's code-tracking loops as shown in Fig. 2 (lower dashed line) allows the effective bandwidth of these loops to be reduced and consequently it improves availability and accuracy of the navigation solution [7]. It also improves the ability of the receiver to track signals in a noisy environment or in case of jamming. Adding additional feedback of INS errors estimated in the Kalman filter (higher dashed line) helps to hold INS errors small enough, to maintain the adequacy of the model of these errors embodied in the Kalman filter. The system with one or both above mentioned feedback paths is often referred to as the loosely coupled one. Loosely coupled systems have several advantages. They aresimple to design and well suited to retro-fit applications. They also provide redundancy of navigation data, because apart from integrated solution they may also output GNSS standalone and INS standalone solutions. This allows crosschecking of various outputs and use of INS or GNSS standalone solution in case of failure of the other subsystem or integrating filter.

Pre-processing of GNSS data in loosely coupled systems is usually realised via an additional Kalman filter in the GNSS receiver. Both Kalman filters working in such configuration constitute a so-called cascaded Kalman filter. Cascaded Kalman filters are suboptimal and usually less accurate then the centralized algorithms. Moreover, the principle of operation of the cascaded Kalman filter may lead to stability problems [3].

To avoid the above mentioned problems, INS/GNSS systems can operate with only one Kalman filter, combining functions of both filters of the loosely coupled system. This type of integration is referred to as the tightly coupled INS/GNSS system. In such a system, INS and GNSS receiver are treated as sensors providing unfiltered data such as pseudoranges, range rates, accelerations and angular rates. This complicates the system but provides several important advantages. In a tightly coupled INS/GNSS positioning system there is no problem of processing already pre-processed GNSS data and difficulties with cascaded Kalman filter can be avoided. An additional advantage of tightly coupled systems consists in a possibility of processing even a single available GNSS observable. In a loosely coupled system, GNSS solution can be used only when the receiver is able to obtain a position fix. This however can be done normally only with visibility of at least four GNSS satellites. Thus, tightly coupled systems perform better than the loosely coupled ones in poor satellites' visibility conditions.

2.3. INTEGRATION ACCORDING TO SCHEME OF COMPENSATION

In integration according to scheme of compensation, the measurement vector is composed of differences between data of the so-called reference navigation device and data of additional, correcting devices. Thus, the measurement vector contains combinations of errors of individual devices. The Kalman filter in scheme of compensation estimates so-called error states, which represent errors of the reference device. For this reason this integration principle is often referred to as an error-state formulation. Similarly to the system integrated according to scheme of filtration, systems integrated according to scheme of compensation can be uncoupled, loosely or tightly coupled. Configurations of such INS/GNSS systems are presented in Fig. 3 and Fig. 4.

The system shown in Fig. 3 is uncoupled if there is no feedback to GNSS and INS, and it becomes loosely coupled otherwise. The tightly coupled INS/GNSS system integrated according to scheme of compensation presented in Fig. 4, differs from the system from Fig. 3, because the measurements processed via the Kalman filter represent differences between ranges and relative velocities user-satellite from INS and unprocessed GNSS observables, i.e.

pseudoranges and range rates. The Kalman filter in this type of system is a centralized algorithm in contrast to a cascaded Kalman filter used in the loosely coupled system.

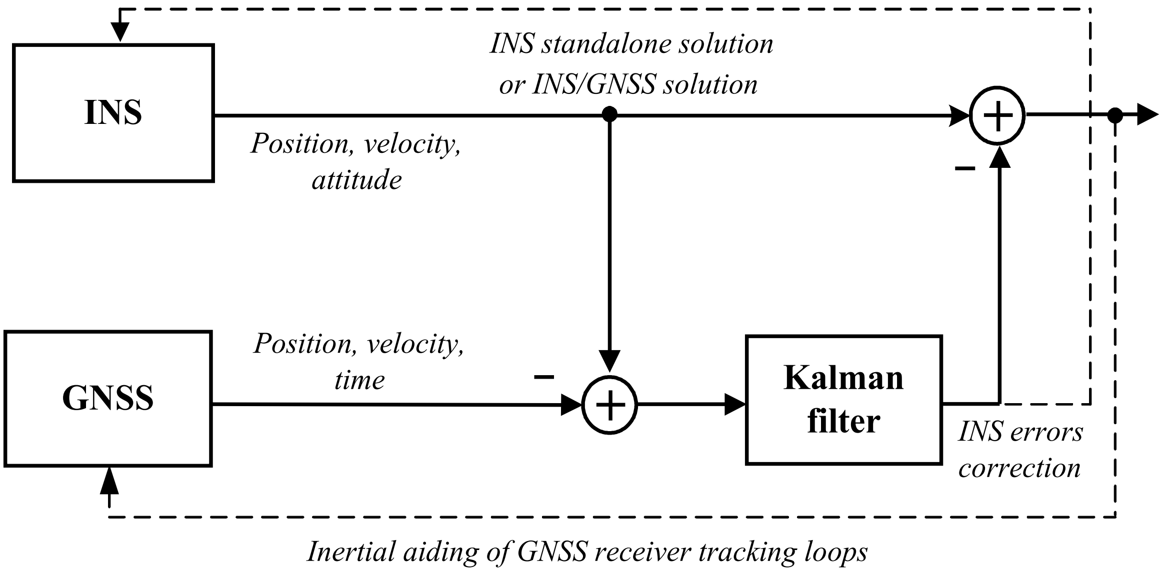

Fig. 3. Uncoupled or loosely coupled INS/GNSS system integrated according to scheme of compensation

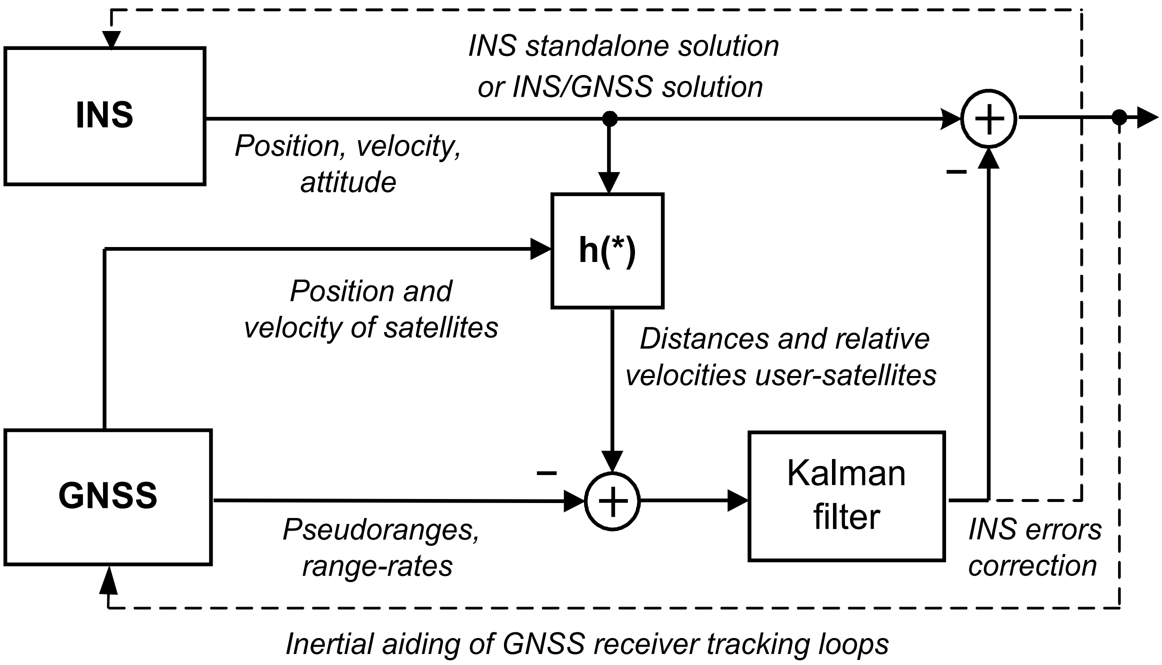

Fig. 4. Tightly coupled INS/GNSS system integrated according to scheme of compensation

In both systems presented in Fig. 3 and Fig. 4, INS can be corrected externally or internally. In systems working in open-loop configuration, i.e. with feed-forward correction, increasing INS errors are estimated by KF and subtracted from INS outputs outside of the inertial system. In INS/GNSS systems working in closed-loop configuration, i.e. with feed-backward correction, the estimates of INS errors are introduced to the inertial system itself and correct its position, velocity and attitude internally. Therefore, in this configuration, INS output becomes also the output of the whole integrated system.

One should be aware, however, that the terms like uncoupled, loosely coupled or tightly coupled INS/GNSS systems are not clearly defined due to many different types of inertial systems and GNSS receivers, as well as due to many possible ways of their combining into one system. There exist diverse variants of integration architectures in each of the above mentioned group of systems [8]. In literature one can also find systems referred to as deep or ultra-tightly coupled ones, which combine the GNSS signal tracking function and the INS/GNSS integration into a single algorithm [7, 8]. The system for aircraft positioning presented in this paper belongs of the group of tightly coupled INS/GNSS systems, integrated according to scheme of compensation and working in open-loop configuration.

3. INS/GPS SYSTEM MODELLING

Currently the only fully operational GNSS system is Global Positioning System (GPS) [7]. There exist also a Russian system GLONASS, but it does not have enough satellites in constellation to declare its operational capability. GLONASS signals can be used for positioning, but the availability and accuracy of positioning will be degraded in comparison to the parameters expected of the full-constellation system. The European GNSS system Galileo is under construction now. Currently it undergoes In-Orbit Validation Phase and it is far from being ready for positioning applications. There are also Space-Based Augmenting Systems (SBAS) which emit signals similar to GNSS from geostationary satellites. These systems include WAAS (US), EGNOS (Europe) and MSAS (Japan) but their signals can be used only together with GNSS signals to improve accuracy, availability and integrity of positioning. For the above stated reasons the INS/GNSS systems of today utilize GPS or integrated GPS/GLONASS receivers, usually with an option of accepting SBAS signals. Therefore, in the design of integrated positioning system for aircraft, a GPS receiver has been applied, as GPS is the only self-contained representative of GNSS today. The presented in the paper example of INS/GNSS system is composed of a strapdown INS and a navigation GPS receiver and it will be further referred to as INS/GPS.

3.1. DATA FLOWS IN INS/GPS SYSTEM

The structure of designed integrated positioning system is shown in Fig. 5. The system belongs to a category of tightly coupled INS/GPS systems as it uses only one centralized KF and treats INS and GPS as sensors rather then systems. The INS/GPS system works with feed-forward correction, i.e. there is no feedback to INS and its errors are corrected externally. This is a more flexible approach than use of feed-backward correction, because it makes the system less dependent on the type of INS and does not require INS to accept correcting signals. The system may contain INS aiding of GPS tracking loops, but as it does not influence the design of Kalman filter, it is not shown in Fig. 5.

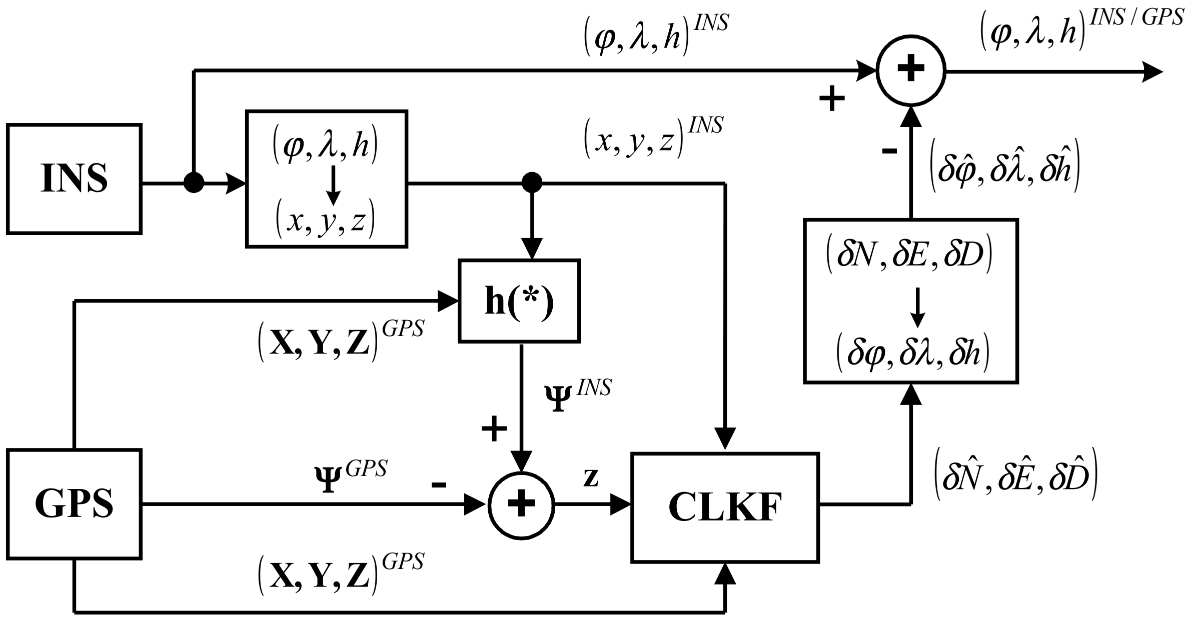

Fig. 5. Structure of INS/GPS integrated positioning system

For joint processing of navigation data from INS and GPS receiver, a Complementary Linearized Kalman Filter (CLKF) has been applied. The CLKF processes differences between INS calculated user-satellite ranges $\mathbf{\Psi}^{INS}$ and GPS measured pseudoranges $\mathbf{\Psi}^{GPS}$. INS ranges and GPS pseudoranges are composed of true values of distances user-satellites and errors of the respective devices. In subtraction before CLKF, total ranges user-satellites are eliminated from the measurement vector **z** and the Kalman filter receives only differences between INS and GPS errors. Fortunately, most of the INS and GPS errors components have entirely different statistical properties, i.e. they are of complementary nature. The CLKF exploits complementarities of INS and GPS errors, and it is able to separate them from each other. Hence, the name of the filter contains the word "complementary".

The processed measurement vector **z** is of variable size, depending on the number *m* of tracked GPS satellites. Positioning errors estimated by the Kalman filter are used as INS corrections in a feed-forward correction loop. In turn, INS-derived user position $(x, y, z)^{INS}$ is used as a reference trajectory in linearization of CLKF. The block **h**(*) represents operation of user-satellite range calculation on the basis of INS-derived user position and GPS satellites positions $(\mathbf{X}, \mathbf{Y}, \mathbf{Z})^{GPS}$ from GPS receiver.

It should be noted that user and satellites positions are expressed in Earth-centred Earth-fixed (ECEF) Cartesian frame of reference WGS-84 [7]. The coordinates in ECEF can be given as a triple (*x*, *y*, *z*) or alternatively as geographic latitude, longitude and height above the Earth (φ, λ, h). INS errors, however, are naturally modelled in navigation frame of reference NED, which is user-centred and moving along with the user. The axes of NED frame of reference point at the North, East and Down directions. Both frames of reference and their relationship are shown in Fig. 6.

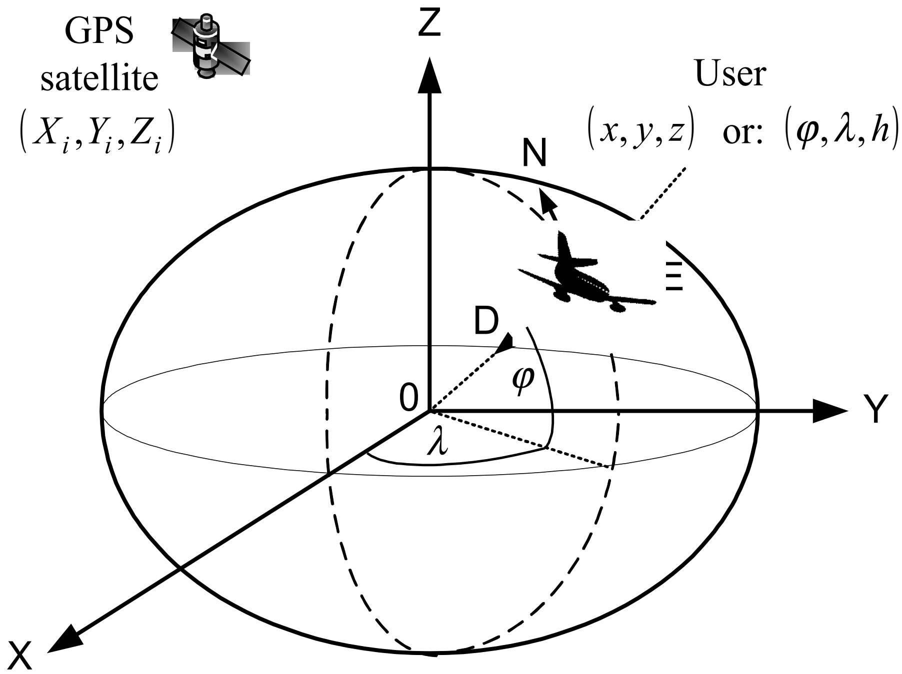

Fig. 6. ECEF frame of reference XYZ and local horizontal frame of reference NED

The INS/GPS system contains also two blocks responsible for transformation of data between frames of reference. The necessary transformations include calculation of ECEF Cartesian coordinates (*x*, *y*, *z*) from (φ, λ, h) and calculation of latitude, longitude and height errors on the basis of estimated INS positioning errors, which are output from the Kalman filter in NED frame of reference.

3.2. DYNAMICS MODEL

The dynamics model of designed INS/GPS system describes time propagation of INS errors and GPS clock errors. Detailed INS errors models can be very complicated and may contain even several tens of states [5, 8]. Application of such models in integrated systems, however, would not be reasonable in most cases. It should be noted that some of the states are observable only conditionally, e.g. during manoeuvres of aircraft, and only in high quality inertial systems [6]. Moreover, elimination of chosen states does not conspicuously affect the accuracy of INS/GPS system, but it significantly reduces requirements with respect to the processing power of navigation processor. Simple 9-state model of INS errors [1] can be used in practice, and in this design it has been further simplified to 8-state model. In spite of significant simplifications, the assumed INS errors model is still applicable, especially for aircraft that do not perform rapid turns around its pitch and roll axis.

The GPS receiver clock errors model describes a 2-state random process [1, 3]. It contains bias and drift, which represent range and velocity equivalents of the discrepancy between relatively inaccurate GPS receiver clock phase and frequency and the phase and frequency of accurate atomic clocks on GPS satellites, which are synchronized to the GPS time scale. The receiver clock bias and drift must be estimated along with other variables and their model should be used to augment the INS errors model. Thus, the augmented model of INS/GPS system contains 10 states, 8 for INS and 2 for GPS receiver clock. The assumed differential equations describing propagation of the states are as follows [1, 5]:

- North channel INS errors:

$$\begin{cases} \delta\dot{N} = \delta v_N \\ \delta\dot{v}_N = g\phi_E + u_{vN} \\ \dot{\phi}_E = -\frac{1}{R}\delta v_N + u_{\phi E} \end{cases} \tag{5}$$

- East channel INS errors:

$$\begin{cases} \delta\dot{E} = \delta v_E \\ \delta\dot{v}_E = -g\phi_N + u_{vE} \\ \dot{\phi}_N = \frac{1}{R}\delta v_E + u_{\phi N} \end{cases} \tag{6}$$

- Down channel INS errors:

$$\begin{cases} \delta\dot{D} = \delta v_D \\ \delta\dot{v}_D = u_{vD} \end{cases} \tag{7}$$

- GPS clock errors:

$$\begin{cases} \dot{b} = d + u_b \\ \dot{d} = u_d \end{cases} \tag{8}$$

where: δN - INS position error along the North axis [m],

δv_N - INS velocity error along the North axis [m/s],

ϕ_E - INS attitude error around the East axis [rad],

δE - INS position error along the East axis [m],

δv_E - INS velocity error along the East axis [m/s],

ϕ_N - INS attitude error around the North axis [rad],

δD - INS position error along the Down axis [m],

δv_D - INS velocity error along the Down axis [m/s],

b - GPS receiver clock bias [m],

d - GPS receiver clock drift [m/s],

$u_{v_N}, u_{\phi_E}, u_{v_E}, u_{\phi_N}, u_{v_D}, u_b, u_d$ - discrete random process disturbances,

g - gravity acceleration,

R - Earth's radius (spherical model).

The above differential equations can be combined to form a continuous state-space model of dynamics of INS/GPS system:

$$\dot{\mathbf{x}}(t) = \mathbf{F}(t)\mathbf{x}(t) + \mathbf{G}(t)\mathbf{u}(t) \tag{9}$$

where: $\mathbf{x}$ - state vector,

$\mathbf{u}$ - vector of continuous random process disturbances,

$\mathbf{F}$ - fundamental matrix of the system,

$\mathbf{G}$ - matrix of continuous process disturbances.

The above vectors and matrices are as follows:

$$\mathbf{x} = [\delta N \quad \delta v_N \quad \phi_E \quad \delta E \quad \delta v_E \quad \phi_N \quad \delta D \quad \delta v_D \quad b \quad d]^{\mathrm{T}} \tag{10}$$

$$\mathbf{u} = [u_{vN} \quad u_{\phi E} \quad u_{vE} \quad u_{\phi N} \quad u_{vD} \quad u_b \quad u_d]^{\mathrm{T}} \tag{11}$$

$$\mathbf{F} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{R} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

To formulate a discrete model of the system, necessary for KF implementation, the continuous dynamics model has to be converted into its discrete counterpart described by the equation (1). The conversion is laborious, but straightforward [1], and consists in calculation of the transition matrix $\mathbf{\Phi}$ and the covariance matrix $\mathbf{Q}$ of discrete random process disturbances $\mathbf{w}$ which is defined as follows:

$$E\left|\mathbf{w}(k)\mathbf{w}^{\mathrm{T}}(l)\right| = \mathbf{Q}(k) \cdot \delta(k,l) \tag{13}$$

where $\delta(k,l)$ is Kronecker delta function. The obtained transition matrix is as follows:

$$\mathbf{\Phi} = L^{-1}\left[(s\mathbf{I}-\mathbf{F})^{-1}\right] = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & gT & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{T}{R} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & T & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -gT & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{T}{R} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

where $L^{-1}(*)$ represents inverse Laplace transform, **I** is an identity matrix, and s is the Laplace variable. The **Q** matrix has been calculated with use of transfer function method [1] and it is a block-diagonal matrix of the following form:

$$\mathbf{Q}(k) = \mathbf{Q} = \begin{bmatrix} \mathbf{Q_N} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q_E} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Q_D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{Q_C} \end{bmatrix} \tag{15}$$

$$\mathbf{Q_N} = \begin{bmatrix} \frac{T^3 S_{vN}}{3} + \frac{g^2 T^5 S_{\phi E}}{20} & \frac{T^2 S_{vN}}{2} + \frac{g^2 T^4 S_{\phi E}}{8} & -\frac{T^3 S_{vN}}{3R} + \frac{g T^3 S_{\phi E}}{6} \\ \frac{T^2 S_{vN}}{2} + \frac{g^2 T^4 S_{\phi E}}{8} & T S_{vN} + \frac{g^2 T^3}{3} S_{\phi E} & \frac{g T^2}{2} S_{\phi E} - \frac{T^2}{2R} S_{vN} \\ -\frac{T^3 S_{vN}}{3R} + \frac{g T^3 S_{\phi E}}{6} & \frac{g T^2}{2} S_{\phi E} - \frac{T^2}{2R} S_{vN} & \frac{T^3}{3R^2} S_{vN} + T S_{\phi E} \end{bmatrix} \tag{16}$$

$$\mathbf{Q_E} = \begin{bmatrix} \frac{T^3 S_{vE}}{3} + \frac{g^2 T^5 S_{\varphi N}}{20} & \frac{T^2 S_{vE}}{2} + \frac{g^2 T^4 S_{\varphi N}}{8} & \frac{T^3 S_{vE}}{3R} - \frac{g T^3 S_{\varphi N}}{6} \\ \frac{T^2 S_{vE}}{2} + \frac{g^2 T^4 S_{\varphi N}}{8} & T S_{vE} + \frac{g^2 T^3}{3} S_{\varphi N} & -\frac{g T^2}{2} S_{\varphi N} + \frac{T^2}{2R} S_{vE} \\ \frac{T^3 S_{vE}}{3R} - \frac{g T^3 S_{\varphi N}}{6} & -\frac{g T^2}{2} S_{\varphi N} + \frac{T^2}{2R} S_{vE} & \frac{T^3}{3R^2} S_{vE} + T S_{\varphi N} \end{bmatrix} \tag{17}$$

$$\mathbf{Q_D} = \begin{bmatrix} \frac{T^3 S_{vD}}{3} & \frac{T^2 S_{vD}}{2} \\ \frac{T^2 S_{vD}}{2} & T S_{vD} \end{bmatrix} \tag{18}$$

$$\mathbf{Q_C} = \begin{bmatrix} TS_b + \frac{T^3 S_d}{3} & \frac{T^2 S_d}{2} \\ \frac{T^2 S_d}{2} & TS_d \end{bmatrix} \tag{19}$$

In the above equations $S_{vN}, S_{\phi E}, S_{vE}, S_{\phi N}, S_{vD}, S_b, S_d$ represent power spectral densities of Gaussian white noises, being components of the vector of continuous random process disturbances **u**, and T is sampling interval of the discrete model.

3.3. OBSERVATION MODEL

Contemporary GPS receivers usually form observables of pseudoranges and delta ranges (also referred to as range rates or Doppler), and sometimes also accumulated delta ranges (also referred to as carrier phase or integrated Doppler). These observables, along with positions and velocities of GPS visible satellites, calculated with data extracted from GPS navigation messages, are used in the receiver to solve for the user position and velocity [7]. For simplicity, the designed system INS/GPS utilizes only GPS pseudoranges, but it can be easily extended to process also GPS range rates.

The measurement vector **z** presented to the Kalman filter is composed of differences between calculated user-satellite ranges and measured GPS pseudoranges:

$$\mathbf{z} = \mathbf{\Psi}^{INS} - \mathbf{\Psi}^{GPS} = [\Delta\Psi_1 \quad \Delta\Psi_2 \quad \dots \quad \Delta\Psi_m]^{\mathrm{T}} \tag{20}$$

$$\Delta\Psi_i = \Psi_i^{INS} - \Psi_i^{GPS} \tag{21}$$

$$\begin{aligned} \Psi_i^{INS} &= \sqrt{\left(X_i - x^{INS}\right)^2 + \left(Y_i - y^{INS}\right)^2 + \left(Z_i - z^{INS}\right)^2} = \\ &= \sqrt{\left[X_i - (x + \delta x)\right]^2 + \left[Y_i - (y + \delta y)\right]^2 + \left[Z_i - (z + \delta z)\right]^2} \approx \\ &\approx \Psi_i + \frac{\partial \Psi_i}{\partial x}\delta x + \frac{\partial \Psi_i}{\partial y}\delta y + \frac{\partial \Psi_i}{\partial z}\delta z \end{aligned} \tag{22}$$

$$\Psi_i^{GPS} = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} + b + v_i = \Psi_i + b + v_i \tag{23}$$

where: Ψ_i - true user-satellite range for i-th satellite,

Ψ_i^{INS} - INS reckoned user-satellite range for i-th satellite,

Ψ_i^{GPS} - GPS pseudorange observable for i-th satellite,

X_i, Y_i, Z_i - i-th satellite position,

x, y, z - true user position,

x^{INS}, y^{INS}, z^{INS} - user position from INS,

b - GPS receiver's clock bias,

v_i - pseudorange measurement error.

Taking into account the above relationships the following equation can be written down:

$$\begin{bmatrix} \Delta\Psi_1 \\ \Delta\Psi_2 \\ \vdots \\ \Delta\Psi_m \end{bmatrix} = \begin{bmatrix} \frac{\partial\Psi_1}{\partial x} & \frac{\partial\Psi_1}{\partial y} & \frac{\partial\Psi_1}{\partial z} \\ \frac{\partial\Psi_2}{\partial x} & \frac{\partial\Psi_2}{\partial y} & \frac{\partial\Psi_2}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial\Psi_m}{\partial x} & \frac{\partial\Psi_m}{\partial y} & \frac{\partial\Psi_m}{\partial z} \end{bmatrix} \cdot \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} - \begin{bmatrix} b \\ b \\ \vdots \\ b \end{bmatrix} + \begin{bmatrix} -v_1 \\ -v_2 \\ \vdots \\ -v_m \end{bmatrix} = \mathbf{H1} \cdot \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} - \begin{bmatrix} b \\ b \\ \vdots \\ b \end{bmatrix} + \begin{bmatrix} -v_1 \\ -v_2 \\ \vdots \\ -v_m \end{bmatrix} \tag{24}$$

The values of partial derivatives in the above equations have to be calculated around nominal trajectory of the flight of aircraft, which in INS/GPS system can be obtained from INS before or after correction. The former method has been assumed in the designed INS/GPS system. Even though INS position diverges from the true trajectory of the aircraft, due to increasing INS errors, the impact of these errors on the values of derivatives will not be significant for typical times of flight of the aircraft. The partial derivatives represent direction cosines of angles under which the *i*-th satellite is visible from the user location. The satellites orbits have the altitude of 20,162.61 km above the Earth's equatorial radius, thus a user location error of several hundreds meters or even several kilometers does not change the values of direction cosines significantly.

Due to the large distances from aircraft to satellite, the INS/GPS system contains only small nonlinearities and originally nonlinear problem of estimation in this system can be easily linearized. Linearized observation model can be derived from the equation (24). As the INS position errors in this equation are expressed in ECEF frame of reference and the elements of state vector are expressed in NED frame, transformation of coordinates has to be introduced. This can be realized by multiplying INS position errors from the state vector by the NED to ECEF coordinate transformation matrix $\mathbf{C}_\mathbf{n}^\mathbf{e}$, as shown in the equation (25).

$$\begin{bmatrix} \Delta\Psi_1 \\ \Delta\Psi_2 \\ \vdots \\ \Delta\Psi_m \end{bmatrix} = \mathbf{H1} \cdot \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} - \begin{bmatrix} b \\ b \\ \vdots \\ b \end{bmatrix} + \begin{bmatrix} -v_1 \\ -v_2 \\ \vdots \\ -v_m \end{bmatrix} = \mathbf{H1} \cdot \mathbf{C}_\mathbf{n}^\mathbf{e} \cdot \begin{bmatrix} \delta N \\ \delta E \\ \delta D \end{bmatrix} - \begin{bmatrix} b \\ b \\ \vdots \\ b \end{bmatrix} + \begin{bmatrix} -v_1 \\ -v_2 \\ \vdots \\ -v_m \end{bmatrix} = \mathbf{H2} \cdot \begin{bmatrix} \delta N \\ \delta E \\ \delta D \end{bmatrix} - \begin{bmatrix} b \\ b \\ \vdots \\ b \end{bmatrix} + \begin{bmatrix} -v_1 \\ -v_2 \\ \vdots \\ -v_m \end{bmatrix} \tag{25}$$

Finally, the following linearized observation model can be formulated:

$$\mathbf{z}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{v}(k) \tag{26}$$

where the observation matrix **H** has been received by inserting into it appropriate columns of **H2** matrix and columns with all zeros and with all minus ones:

$$\mathbf{H} = [\mathbf{H2}(1) \quad \mathbf{0}_{m\times1} \quad \mathbf{0}_{m\times1} \quad \mathbf{H2}(2) \quad \mathbf{0}_{m\times1} \quad \mathbf{0}_{m\times1} \quad \mathbf{H2}(3) \quad \mathbf{0}_{m\times1} \quad -\mathbf{1}_{m\times1} \quad \mathbf{0}_{m\times1}] \tag{27}$$

In the above equation $\mathbf{H2}(i)$ means the i-th column of **H2** matrix, whereas $\mathbf{0}_{m\times1}$ and $\mathbf{1}_{m\times1}$ represent column vectors with m zeros or ones.

The GPS pseudorange measurement errors for all visible and tracked satellites are components of the measurement noise vector $\mathbf{v} = [-v_1 \quad -v_2 \quad \ldots \quad -v_m]^{\mathrm{T}}$. They are assumed to be Gaussian zero-mean white noises of variances σ_Ψ^2. To complete the observation model of INS/GPS system, the measurement error covariance matrix **R** has to be determined. The matrix **R** is diagonal and contains variances of all pseudorange measurements:

$$\mathbf{R} = \begin{bmatrix} \sigma_\Psi^2 & 0 & \cdots & 0 \\ 0 & \sigma_\Psi^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_\Psi^2 \end{bmatrix} \tag{28}$$

3.4. LINEARIZED KALMAN FILTER

Having formulated the dynamic model of the aircraft positioning system INS/GPS, the algorithm of navigation data processing for this system can be designed. As it has already been mentioned, a Complementary Linearized Kalman Filter has been applied for this purpose. The CLKF equations can be formulated as follows:

$$\hat{\mathbf{x}}(k+1|k) = \mathbf{\Phi} \cdot \hat{\mathbf{x}}(k|k) \tag{29}$$

$$\mathbf{P}(k+1|k) = \mathbf{\Phi} \cdot \mathbf{P}(k|k) \cdot \mathbf{\Phi}^{\mathrm{T}} + \mathbf{Q} \tag{30}$$

$$\frac{\partial \Psi_i}{\partial x} = \frac{-\left(X_i - x^{INS}\right)}{\sqrt{\left(X_i - x^{INS}\right)^2 + \left(Y_i - y^{INS}\right)^2 + \left(Z_i - z^{INS}\right)^2}} \quad \text{for} \quad i = 1 \ldots m \tag{31}$$

$$\frac{\partial \Psi_i}{\partial y} = \frac{-\left(Y_i - y^{INS}\right)}{\sqrt{\left(X_i - x^{INS}\right)^2 + \left(Y_i - y^{INS}\right)^2 + \left(Z_i - z^{INS}\right)^2}} \quad \text{for} \quad i = 1 \ldots m \tag{32}$$

$$\frac{\partial \Psi_i}{\partial z} = \frac{-\left(Z_i - z^{INS}\right)}{\sqrt{\left(X_i - x^{INS}\right)^2 + \left(Y_i - y^{INS}\right)^2 + \left(Z_i - z^{INS}\right)^2}} \quad \text{for} \quad i = 1 \ldots m \tag{33}$$

$$\mathbf{H1} = \begin{bmatrix} \frac{\partial \Psi_1}{\partial x} & \frac{\partial \Psi_1}{\partial y} & \frac{\partial \Psi_1}{\partial z} \\ \frac{\partial \Psi_2}{\partial x} & \frac{\partial \Psi_2}{\partial y} & \frac{\partial \Psi_2}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial \Psi_m}{\partial x} & \frac{\partial \Psi_m}{\partial y} & \frac{\partial \Psi_m}{\partial z} \end{bmatrix} \tag{34}$$

$$\mathbf{C}_\mathbf{n}^\mathbf{e} = \begin{bmatrix} -\sin\varphi\cos\lambda & -\sin\lambda & -\cos\varphi\cos\lambda \\ -\sin\varphi\sin\lambda & \cos\lambda & -\cos\varphi\sin\lambda \\ \cos\varphi & 0 & -\sin\varphi \end{bmatrix} \tag{35}$$

$$\mathbf{H2} = \mathbf{H1} \cdot \mathbf{C}_\mathbf{n}^\mathbf{e} \tag{36}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H2}(1) & \mathbf{0}_{m\times1} & \mathbf{0}_{m\times1} & \mathbf{H2}(2) & \mathbf{0}_{m\times1} & \mathbf{0}_{m\times1} & \mathbf{H2}(3) & \mathbf{0}_{m\times1} & -\mathbf{1}_{m\times1} & \mathbf{0}_{m\times1} \end{bmatrix} \tag{37}$$

$$\boldsymbol{\rho}(k+1) = \mathbf{z}(k+1) - \mathbf{H} \cdot \hat{\mathbf{x}}(k+1|k) \tag{38}$$

$$\mathbf{R}_\mathbf{e}(k+1) = \mathbf{H} \cdot \mathbf{P}(k+1|k) \cdot \mathbf{H}^\mathrm{T} + \mathbf{R} \tag{39}$$

$$\mathbf{K}(k+1) = \mathbf{P}(k+1|k) \cdot \mathbf{H}^\mathrm{T} \cdot \mathbf{R}_\mathbf{e}^{-1}(k+1) \tag{40}$$

$$\hat{\mathbf{x}}(k+1|k+1) = \hat{\mathbf{x}}(k+1|k) + \mathbf{K}(k+1) \cdot \boldsymbol{\rho}(k+1) \tag{41}$$

$$\mathbf{P}(k+1|k+1) = \mathbf{P}(k|k+1) - \mathbf{K}(k+1) \cdot \mathbf{H} \cdot \mathbf{P}(k|k+1) \tag{42}$$

where: $\hat{\mathbf{x}}(k|k)$ - estimated state vector at a time k - after the measurement update,

$\hat{\mathbf{x}}(k+1|k+1)$ - estimated state vector at a time k+1 - after the measurement update,

$\hat{\mathbf{x}}(k+1|k)$ - estimated state vector at a time k+1 - before the measurement update,

$\boldsymbol{\rho}(k+1)$ - residuals vector at a time k+1,

$\mathbf{P}(k|k)$ - covariance matrix of filtering errors at a time k,

$\mathbf{P}(k+1|k+1)$ - covariance matrix of filtering errors at a time k+1,

$\mathbf{P}(k+1|k)$ - covariance matrix of prediction errors at a time k+1,

$\mathbf{R}_\mathbf{e}(k+1)$ - covariance matrix of innovations at a time k+1,

$\mathbf{K}(k+1)$ - Kalman gains matrix at a time k+1,

$\mathbf{Q}$ - covariance matrix of discrete random process disturbances,

$\mathbf{R}$ - covariance matrix of measurement errors,

$\mathbf{C}_\mathbf{n}^\mathbf{e}$ - NED to ECEF coordinate transformation matrix,

X_i, Y_i, Z_i - i-th satellite position,

x^{INS}, y^{INS}, z^{INS} - user position from INS.

4. SIMULATION RESULTS

The designed integrated INS/GPS positioning system for aircraft has been tested via computer simulations. The simulations have required generation of trajectory of aircraft and trajectories of all visible GPS satellites. Next, user-satellites ranges have been calculated. In parallel, INS and GPS errors (pseudorange noises and clock bias) have been generated and added to error-free INS position and GPS user-satellites ranges. GPS errors included most types of the errors that are present in real GPS observables, i.e. thermal noise, tropospheric and ionospheric delays, as well as multipath. The thermal noise is uncorrelated in time and the rest of the errors are time-correlated. The simulated INS and GPS data have been jointly processed via the designed CLKF. Chosen simulation results comparing accuracy of INS, GPS and INS/GPS system are presented in Fig. 7-9.

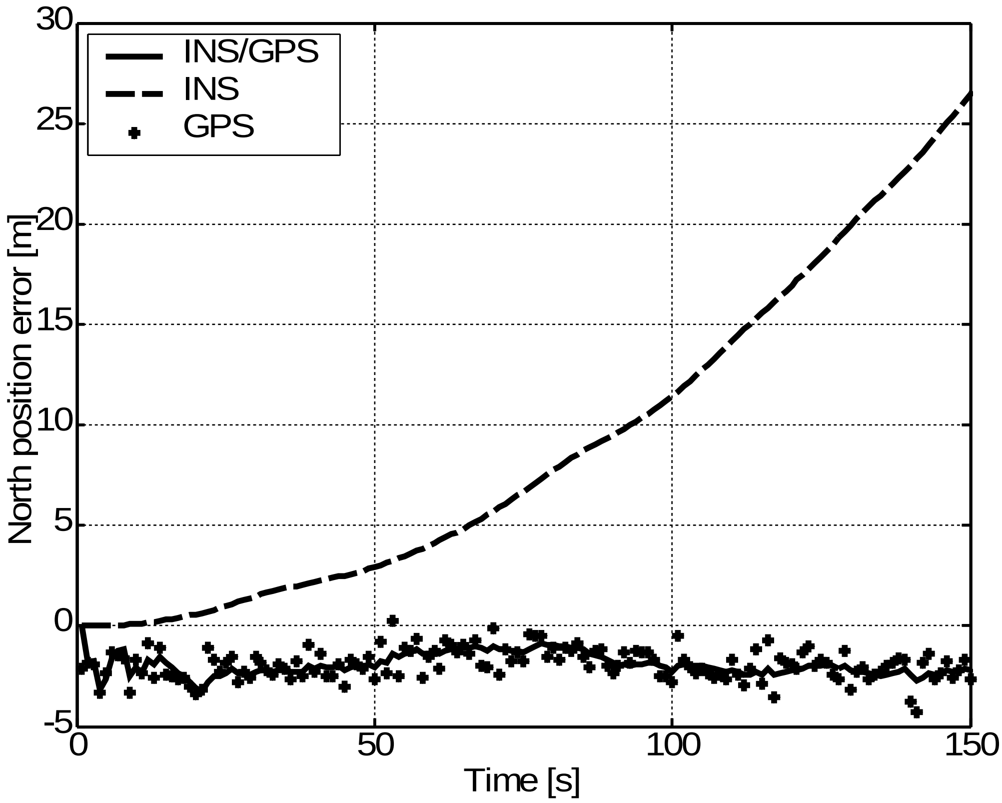

Fig. 7. Comparison of INS, GPS and INS/GPS positioning errors – North direction

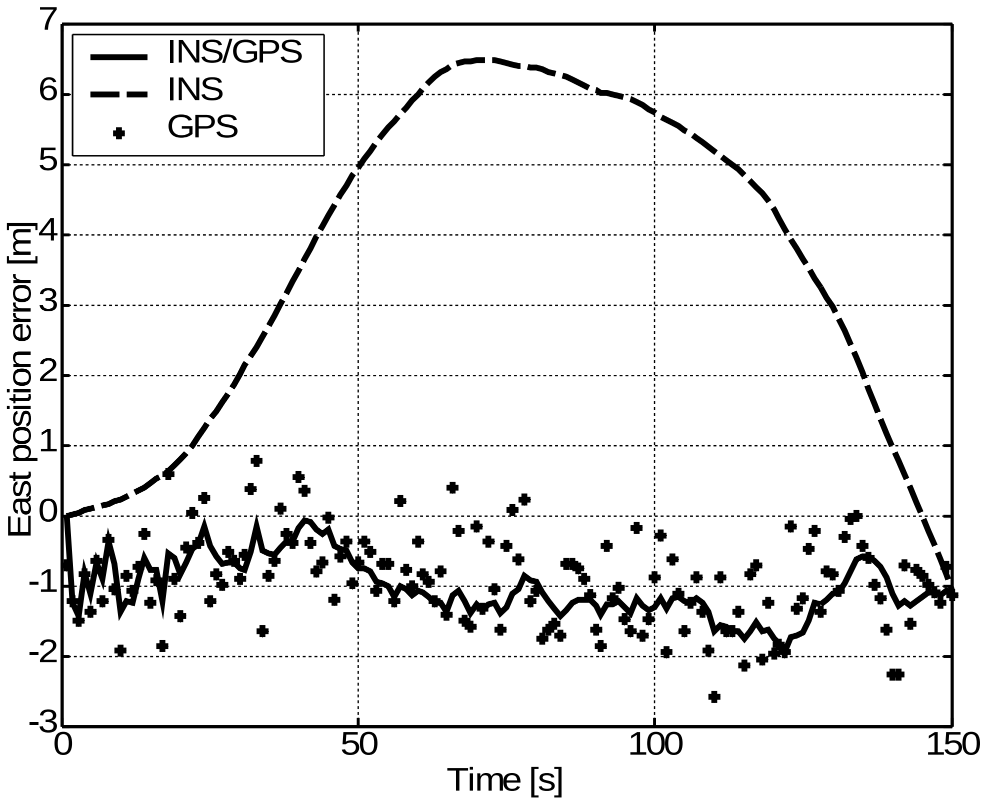

Fig. 8. Comparison of INS, GPS and INS/GPS positioning errors – East direction

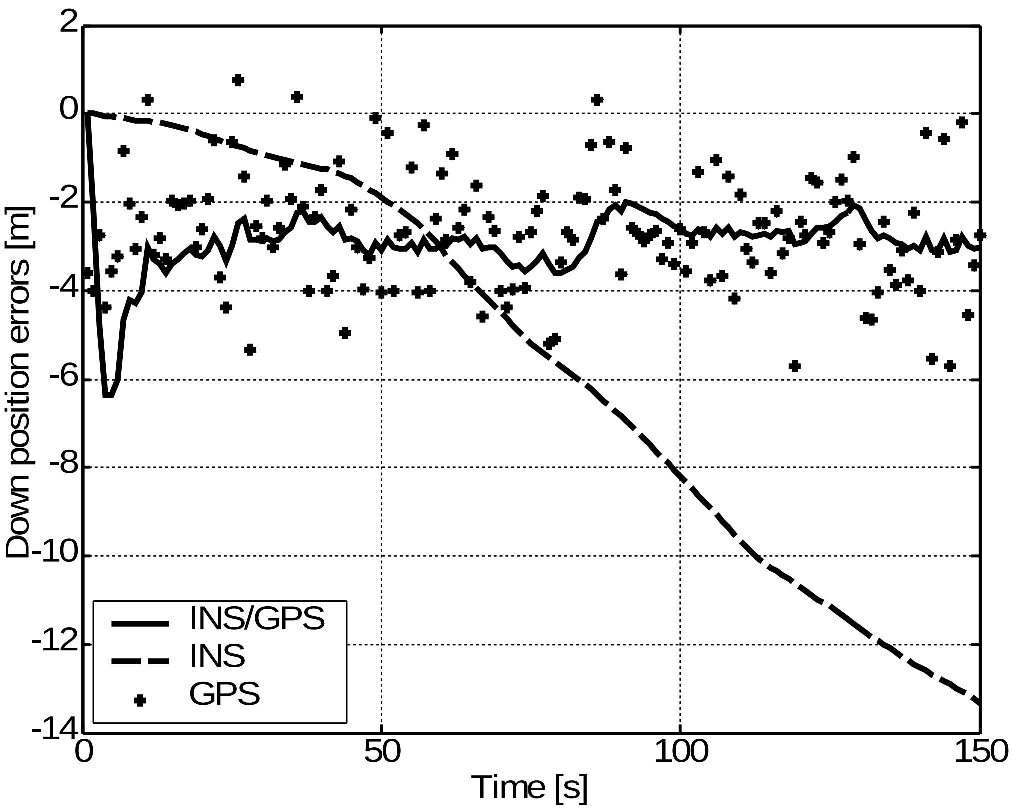

Fig. 9. Comparison of INS, GPS and INS/GPS positioning errors – Down direction

5. CONCLUSION

On the basis of the above simulation results several conclusions can be drawn. It is conspicuous that INS/GPS system is more accurate than any of its subsystems alone. The positioning errors of INS, increasing with the time of operation, have been eliminated and the GPS receiver uncorrelated positioning errors have been significantly reduced. Improvement of INS/GPS positioning accuracy in comparison to the accuracy of GPS depends however on the characteristics of GPS errors. Best results are achieved when the correlatedGPS errors are small in comparison to the uncorrelated ones. The obtained results demonstrate that the designed CLKF algorithm has been properly designed and it can be successfully applied in simple INS/GPS integrated aircraft positioning system. Extension of this algorithm will be a subject of further research and it will go in two main directions. Firstly, a more complex dynamics model of INS errors will be implemented, which is expected to improve the accuracy of system and make the algorithm applicable also for highly maneuvering aircraft. Secondly, the filter will be modified to accept GPS range rates apart from pseudoranges. Such a modification will also improve the accuracyof the INS/GPS system.

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