A Quantitative Rat Race Theory of Labor Market Dynamics [*]

Andrew Glover

University of Texas at Austin

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Abstract

A fundamental puzzle for productivity driven business cycle theories is that labor productivity and hours are negatively correlated. I refine the empirical puzzle and provide a theory to account for it. I emphasize that hours per worker and individual hours relative to *usual* hours move counter to productivity. In the cross-section, I find that almost all workers increase their hours when productivity is low, except for low (residual) wage earners. Furthermore, low wage earners suffer larger wage losses when productivity falls. Based on these findings, I hypothesize that non-neutral movements in productivity exacerbate frictions due to adverse selection. I use recent advances in competitive search theory to imbed Akerlof's Rat Race into a Neo-Classical growth model with search frictions. Firms bundle high earnings with long hours in order to separate more able and willing workers from the less productive. When low productivity workers fall (relatively) farther behind, the firm requires longer hours from everyone else; perversely, many workers work harder even as the market value of their time falls. A calibrated version of the model is used to measure the aggregate volatility generated by productivity shocks when labor markets are burdened by adverse selection.

1 Introduction

This paper is a new attempt to reconcile productivity driven theories of the business cycle with the aggregate evidence against them. Specifically, I provide a theory to account for three empirical regularities in macroeconomic data that are puzzling for many business cycle models: 1) aggregate hours (but even more so, individual hours worked) and average labor productivity are negatively or weakly positively correlated, 2) the residual from a representative household's intratemporal trade off between hours and consumption (the "labor wedge" as in [5]) is strongly positively correlated with hours, and 3) total hours are much more volatile than labor productivity.

Traditional equilibrium theories of business cycles ([16]) generate tight predictions on the comovement between consumption, labor supply, output, and productivity. In these theories, the substitution effect on leisure dominates in response to a temporary rise in real wages; under complete markets the aggregate response to productivity mirrors the individual response to wages. One finds that 1) the response of hours to productivity is overwhelmingly positive, 2) the intratemporal optimality condition on labor supply holds with equality at all times and states of the world, and 3) a large fraction of output volatility is endogenous only if the elasticity of hours with respect to productivity is substantially larger than empirical micro estimates.

The key mechanism in this paper is that workers are heterogeneous in their productivity but hours are aggregated so that employers cannot observe the marginal product of any given worker. This creates an adverse selection problem in the labor market. Furthermore, good workers are assumed to receive lower disutility for every marginal hour worked, which allows employers to separate good and bad workers by bundling high paying jobs with long hours. In this environment, the productivity of bad workers and the labor supply options

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available to good workers are linked. When the productivity of bad workers falls, they are more willing to work long hours to escape their low paying jobs. In response, firms require longer hours from good workers even if their productivity is constant. Thus 1) aggregate hours can rise while average labor productivity falls (both because some workers are truly less productive and others are increasing hours under decreasing *average* returns), 2) hours are high when the intratemporal optimality condition indicates an implicit subsidy (the labor wedge is high), and 3) changes in productivity dispersion induce additional movements in hours beyond those due to TFP shocks.

In order to measure the quantitative importance of the theory, I embed Akerlof's Rat Race ([1]) into the labor market of a stochastic neo classical growth model. In order to avoid measurement issues involved with identifying bad workers in the data, I consider the limit of economies as the share of bad workers approaches zero. A well known pathology arises in adverse selection environments when there are few of the bad agents: competitive equilibria fail to exist (see [22]). I use competitive search theory (as in [11]) to overcome this problem; when workers and employers match bilaterally, market tightness provides an additional margin with which to sort workers by type. In this economy, competitive equilibria exist for any composition of the workforce. This is the first paper to capitalize on **(author?)** [11]'s work to study adverse selection in a dynamic business cycle model.

I measure the impact of adverse selection on a benchmark economy in which workers have complete insurance against employment and type risk and in which job posting costs are small enough to ignore unemployment. If the gap between good and bad workers is constant, then this economy mimics a standard frictionless RBC model: the correlation between hours and labor productivity is near 1, the labor wedge is constant, and the standard deviation of output is only 22% greater than that of TFP. [1] When the volatility of the productivity gap

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[1]I use a Frisch labor supply elasticity of $\epsilon = 0.43$. This is the value used for men in [14], and is within

between good and bad workers is chosen to match the volatility of the labor wedge relative to output, the correlation between hours and labor productivity falls to -0.23 and the labor wedge is positively correlated with hours. Furthermore, the standard deviation of output rises by 25% relative to the full info economy.

The paper proceeds as follows. After a survey of previous research on this topic, I review the aggregate data and emphasize that individual workers' hours respond counter to labor productivity, which is the margin most relevant for the theory. I then look at the cross-section to understand *who's* hours move counter to labor productivity and how do changes in productivity affect these individuals differently. While it is clearly impossible to construct the *unobservable* residual productivity distribution, this exercise is illustrative. Next, I outline a static version of the theory in which ex-ante homogeneous workers are insured against type and employment risk in a large family. I then embed this labor market into the growth model and quantitatively assess the impact of adverse selection on the macroeconomy; intitial findings indicate that the effect is large. I conclude with directions for future research.

1.1   Related Literature

Many previous researchers have studied the macroecomic puzzles considered in this paper, and it would be amiss to ignore these contributions. At the same time, my theory of macroeconomic labor dynamics rests on a particular friction at the microeconomic level, and so I also review the relevant literature on adverse selection in labor markets.

the bounds of micro estimates.

1.1.1 Aggregate Labor Dynamics

(author?) [8] emphasize that aggregate hours are perfectly positively correlated with labor productivity in vanilla RBC model, since movements in real wage are proportional to movements in labor productivity and the Frisch elasticity is positive. They explore the effects of adding a second shock, which they identify as government spending. Government spending doesn't translate one for one into consumption, so a rise in G has a negative wealth effect: both consumption and leisure fall in response. Their model can dampen the correlation between labor productivity and aggregate hours, but quantitatively cannot change the sign.

A similar line of attack is taken in [13], who explore the effect of indivisible labor, non-seperable leisure, and home production of the comovement between hours and labor productivity. They find that shocks to the value of home production are as successful as government spending. In that model, what matters for aggregate labor supply is not the productivity of time at work, but it's market value relative to value at home. If market productivity falls but home productivity falls even more, hours supplied to market activities can increase. A common criticism of this method is that it adds an unobservable sector to the economy (home production) which comoves negatively with the observable sectors (market production). The theory that I propose, while still relying on unobservable shocks to relative productivities between good and bad workers, generates cross-sectional implications that are absent in the home-production model.

(author?) [4] attempt to reconcile the comovement between labor productivity and hours in a way most similar to this paper. They consider an economy with heterogeneity, indivisible labor, and incomplete markets. The aggregate elasticity of labor supply is no longer dictated by the value at the micro level, but is instead determined by the fraction of the population who are inframarginal on supplying a fixed quantity of labor. Their model generates a very low positive correlation between hours and labor productivity, but is silent

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on the intensive margin. Interestingly, they show that an aggregate labor supply elasticity as measured by a representative agent cannot be taken as a structural parameter in an economy with nonconvexities and heterogeneity. My theory suggests that the nonconvexity in labor supply cannot be taken as a technological parameter in a heterogeneous agent economy with adverse selection.

(author?) [10] argues solutions that rely on multiple shocks are unsatisfying. He provides evidence from a structural VAR that hours respond negatively to an innovation in productivity. While this finding is contentious ((**author?**) [9] provide an overview of the debate), it leads **(author?)** [10] to propose a theory that relies on the interaction between price stickiness in the goods market and changes in productivity: if demand is fixed (because prices are fixed) and workers become more productive, then they must work fewer hours. Common criticisms of ad hoc stickiness aside, the model is less applicable when labor productivity falls. In such a situation, the goods market is exactly the same as when productivity rises, but now the assumption that firms must produce to meet demand is no longer innocuous. Firms would rather just produce at shortage levels without changing their workforce.

1.1.2   Micro Evidence

There is little direct evidence on workers' hours being distorted upward. **(author?)** [17] test whether the hours of some workers are distorted due to a rat race. They ask associates at two law firms which option they would prefer in addition to a 5% wage raise: decrease hours by 5% (earnings constant), keep hours constant (earnings rise 5%), or increase hours by 5% (earnings rise 10%). They find that two thirds of respondents would prefer to reduce hours and keep earnings constant, while only 12% would want to increase hours. Many associates seem to be working longer hours than would be implied by the first best (with

the caveat that the fractions didn't differ by income level and therefore respondents likely weren't reducing hours due to the income effect). Furthermore, partners were substantially more likely to support promotion for a hypothetical associate who worked long hours than one who requested to work short hours, keeping their contribution constant (20% vs 2% levels of very heavy support, respectively).

They also cite a statistic from the Canadian Labor Force Survey, which asked workers if they would reduce hours at the current wage if given the chance. Of the college educated group, about 26% said that they would. This is likely a lower bound on the fraction of workers who work in rat race conditions, since a worker who expects to have higher earnings due to long hours might make consumption commitments (such as mortgages or car payments) that prohibit reducing hours in the hypothetical situation presented in the survey.

The labor market is more likely afflicted by adverse selection if an individual worker's marginal product is difficult for the firm to observe. This is more likely to be the case if workers produce in groups, such as a team of engineers who spend time in brain storming sessions in the process of bringing a new good to market. **(author?)** [18] provide some evidence that such forms of production are prevalent in the modern US economy. They report that, between 1987 and 1999, over 80% of large firms typically had some production done by workers in teams. Further, the fraction of these firms that had more than 20% of their workers employed in teams was 61% in 1999 (up from 37% in 1987).

It is also widely known that the past thirty years have seen a large increase in residual wage inequality ([3]), which, under the theory of adverse selection, would generate long-run changes in the variance of labor supply and its covariance with wages. **(author?)** [15] show that the share of men working long work weeks (over 50 hours) rose by 6%-points between 1979-2006 and that most of the rise occured during the eighties when residual wage inequality was rising fastest. Furthermore, the rise was greatest in occupations and industries with 1)

the largest rise in residual wage inequality, 2) the smallest rise in average real wages, and 3) independently from rising returns to long hours at the individual level. All of these observations are consistent with the micro-economic environment used in this paper. [2]

2 Data

2.1 Aggregate Accounting

I organize the data around an aggregate production function of the form $Y = zF(K, H)$, where Y is real output, z is total factor productivity, K is the stock of capital, and H is total hours. Labor productivity is defined as $p = \frac{Y}{H}$. Many previous researchers have found a negative or weakly positive correlation between average labor productivity and aggregate hours, defined as total private real output divided by total private hours. For example, the data collected by (author?) [21] from 1947 to 2009Q3 exhibit a correlation between these series of -0.34 at the business cycle frequency.

I focus on the comovement between hours per worker and labor productivity. Again, using (author?) [21], the correlation is calculated as -0.43. Furthermore, the elasticity of total hours with respect to labor productivity is -0.83 and the elasticity of hours per worker is -0.42, hence half of the negative comovement between total hours and labor productivity is due to hours per worker. [3] This cannot be explained entirely by changes in composition, as the ratio of current *usual* hours also moves counter to labor productivity [4], with an elasticity of -0.24. [5] The negative correlation between hours and labor productivity is the reason

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[2](author?) [20] give a human capital accumulation theory of this phenomenon in the U.S. relative to Europe.

[3]The elasticities are $\eta_{H,p} = \text{corr}(H, p)\sigma_H/\sigma_p$ and $\eta_{h,p} = \frac{\text{corr}(h,p)\sigma_h}{\text{corr}(H,p)\sigma_H}\eta_{H,p} = 0.51\eta_{H,p}$.

[4]Data concerning usual hours are since 1979Q1

[5]Usual hours are below current hours on average, because the CPS asks about current hours during weeks without a holiday. This affects the level (the mean of the ratio is 1.01, but not the covariance with labor

that the labor wedge is procyclical [6] Detailed moments [7] can be found in Table 1.

Table 1: Second Moments at Business Cycle Frequencies, 1947-2009

	p	Y	h
	$\sigma_Y = 1.72\%$		
$\frac{\sigma_x}{\sigma_Y}$	0.60	1.0	0.45
$\sigma_{x,p}$	1.0	0.31	**-0.41**

2.1.1 Cross-Sectional Dynamics

I next turn to cross-sectional features of hours per worker and wages for guidance on whether the forces in the model seem relevant. I find that essentially all workers' hours move counter to average labor productivity, but that the magnitude diminishes at the lower tail of the residual wage distribution. To this end, I regress a worker's wage (defined in the CPS as their earnings divided by their usual hours) against available observables, including Mincerian terms, gender, race, and occupation. I then group them by residual deciles and create time series of each group's current relative to usual hours. Table 2 shows the elasticities of each of these time series with respect to average labor productivity. It is clear that all deciles comove negatively with labor productivity, but the lowest decile is smallest in magnitude and is statistically insignificant from zero at the 5% level.

I group workers by their residual because, ideally, I want to compare the comovement of labor productivity with the hours of workers with different *unobservable* productivities. Be-

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productivity. I thank Sam Shulholfer-Wohl for point this convention out to me.

[6] I use one minus **(author?)** [23]'s wedge, which is the residual from the intratemporal optimality condition of a stand-in household with preferences consistent with balanced-growth and constant Frisch elasticity in an economy with Cobb-Douglas production.

[7] All moments from deseasoned variables at quarterly frequency, detrended via hp-filter with smoothing parameter of 1600

cause the theory always yields a separating equilibrium, such an exercise is conceptually well defined. Practically speaking, however, it is impossible to control for every worker characteristic that employers observe. This means that any given grouping of workers based on measured wages likely contains some workers who differ along a characteristic that is observable in the market but not in the CPS data. If there is a distribution of private worker productivity within each of these observable classes, and the empirical group contains members of both classes, then some movements in hours will be due to optimal changes for undistorted workers and some will be due to increases/decreases in the distortion on (privately observed) high productivity workers within a given (publicly observed but unmeasured) class. Nonetheless, this is an illustrative exercise if those at the bottom of the observable residual distribution make up a disproportionate fraction of the lowest unobservable residual groups. This can be seen by considering the worker at the very bottom of the wage distribution: no matter what observable characteristics he shares with with another group of workers, he must be at the bottom of that group as well.

Table 2: Elasticities with Respect to Avg Labor Productivity, 1979-2009

	h_{0-10}	h_{10-20}	h_{20-30}	h_{30-40}	h_{40-50}	h_{50-60}	h_{60-70}	h_{70-80}	h_{80-90}	h_{90-100}
Elasticity	-0.15	-0.25*	-0.21*	-0.17	-0.32*	-0.19*	-0.30*	-0.39*	-0.45*	-0.39*

It can be seen that the hours of high residual wage earners are more negatively correlated (in terms of point estimates, and likelihood of being significantly non-zero) than are low residual wage earners. Given the likely noise previously mentioned, this is remarkable. With the measurement issues in mind, I focus on the difference between the lowest decile and everyone else in Table 3 since it is at the 10th percentile that statistical significance is lost. In addition, I compute the elasticity of each group's real wage with respect to aggregate

labor productivity and find support for modeling productivity shocks as non-neutral, with lower productivity workers affected by more than the average worker.

Table 3: Elasticities wrt Average Labor Productivity, 1979-2009

	h_L	h_H	w_L	w_H
Elasticity	-0.10	-0.44*	0.16	0.05

3 Model

3.1 Static Version

I start by considering a static model which is similar to the Rat Race example in [11], except that workers are ex-ante homogeneous and are members of a large family which insures them against type and employment risk. The family comprises a unit mass of workers. They wake up in the morning and τ_H of the members are good workers, while τ_L are bad. An hour from a good worker will generate more output than that of a bad worker. In addition, workers differ in their disutility of labor supply, with high productivity workers *less* averse to supplying a marginal hour than low productivity workers. At this point workers are heterogeneous and enter the labor market in which employers cannot observe their type. Workers direct their search to a job which specifies that they work h hours and receive w in earnings, which they find with some probability. Workers who find work return at the end of the day with their earnings at which point the head distributes the total earnings amongst family members.

There is an aggregate firm with production function $zF(H)$, where H is total *effective* hours used in production, and $F' > 0, F'' < 0, \frac{\partial}{\partial H}F(H)/H < 0$. An hour of the good worker's time translates to ζ_H hours in production, whereas an hour of the low productivity worker

is effectively $\zeta_l < \zeta_H$ hours. The aggregation of hours makes it impossible to infer any given worker's marginal contribution to output. The firm comprises a large group of risk neutral capitalists. Capitalists post vacancies at cost κ; these posts specify the hours and earnings paid to a worker who reports being type i. Profits are distributed to capitalists according to their marginal contribution to aggregate profits.

3.1.1 Time Zero Problem of the Family

Assuming that the labor market equilibrium allocation is given by earnings, hours, and job finding rates for each type $(w_i, h_i, \bar{f}_i)_{i=L,H}$, the head of the family chooses how to allocate consumption across workers to solve:

$$\max_{\{c_i^1, c_i^0\}_{i=L,h}} \sum_{i=L,H} \tau_i \left(\bar{f}_i(u(c_i^1) - v_i(h_i)) + (1 - \bar{f}_i)u(c_i^0) \right) \text{ s.t.}$$

$$\sum_i \tau_i \left(\bar{f}_i c_i^1 + (1 - \bar{f}_i)c_i^0 \right) = \tau_L \bar{f}_L w_L + \tau_H \bar{f}_H w_H$$

I will assume that u is strictly concave. The first order conditions from this problem ensure that the marginal utility of consumption is equalized across all workers. Thus, because of separability between consumption and labor supply, the actual consumption of each member is independent of employment or type.

In order to map the labor market for this economy into the language of **(author?)** [11] I will derive a worker's valuation of a job paying w and requiring h hours. Think of a positive mass, δ, of workers of type i, each taking the job (w, h). Family income is increased by $w\delta$, which is then split evenly amongst the unit mass of workers. The family utility gain is $u(c + w\delta) - u(c) \approx u'(c)w\delta$. The utility gain per worker taking the job is then just $u'(c)w$. This is also the marginal value of w for the worker taking the job. The family's disutility

of labor supply changes by $-v_i(h)\delta$, which is just $-v_i(h)$ per worker. Thus the worker's valuation of (w, h) (which agrees with the family's) is $u'(c)w - v_i(h)$. With this in mind, I review the environment of competitive search with adverse selection.

3.1.2 Competitive Search With Adverse Selection

Call a job $y_i = (h_i, w_i) \in Y$ the hours and earnings paid to a worker who claims to be type i, where $Y = [0, 1] \times \mathbb{R}_+$ is the feasible set of jobs. Define $\mathbf{y} = (y_L, y_H)$ to be a mechanism, which is a pair of jobs depending on the reported type. A worker who is type i and reports for a job y_j gets utility:

$$U_i(y_j) = u'(c)w - v_i(h) \tag{1}$$

Now consider a capitalist who posts a mechanism $\mathbf{y}$ and meets with a worker who claims to be type j but is in fact type i. The capitalist gets profits from this worker of:

$$\pi_i(y_j) = zF'(H)\zeta_i h_j - w_j \tag{2}$$

By the Revelation Principle, capitalists will only post mechanisms that are incentive compatible. Call this set of mechanisms $\mathbb{C} = \{\mathbf{y} \;:\; U_i(y_i) \geq U_i(y_j)\ \forall i, j\}$.

Capitalists post mechanisms in $\mathbb{C}$, to which workers apply and report their types. Since in equilibrium mechanisms will be separating, we can consider *contracts* instead of mechanisms. A contract is a mechanism that specifies the same job for any worker type, so contract y is just the mechanism $\mathbf{y} = (y, y)$. Let $\theta(y)$ be the market tightness (capitalist-worker ratio) for contract y. Then $f(\theta(y))$ is the probability that a worker who applies to y finds a job and $q(\theta(y)) = f(\theta(y))/\theta(y)$ is the probability that a capitalist posting y meets a worker. Let $\gamma_i(y)$ be the share of $i-$type workers out of all workers who apply to y. The expected utility

of a worker who applies to contract y is:

$$\hat{U}_i(y) = f(\theta(y))U_i(y) \tag{3}$$

and the expected profit of a capitalist who posts contract y is:

$$\hat{\pi}(y) = q(\theta(y)) \sum_i \gamma_i(y) \left(zF'(H)\zeta_i h_i - w_i\right) - \kappa \tag{4}$$

With this notation in hand, I define the labor market equilibrium as in [11].

Definition 1. *A competitive search equilibrium is a vector $\{\bar{U}_i\}_{i=L,H}$, a measure λ on Y with support $\bar{Y}$, a function θ on Y, and a function $\Gamma(y) = (\gamma_i(y))_{i=L,H}$ on Y satsifying:*

I. *Capitalists maximize profits and free entry: $\forall y \in Y$,*

$$\hat{\pi}(y) \leq 0$$

with equality if $y \in \bar{Y}$.

II. *Workers search is optimal and generates $\bar{U}_i$.*

$$\bar{U}_i = \max_{y' \in Y} \hat{U}_i(y')$$

with $\bar{U}_i = 0$ if $\bar{Y} = \emptyset$. For any $y \in Y$ and i, $\bar{U}_i \geq f(\theta(y))U_i(y)$ with equality if $\theta(C) < \infty$ and $\gamma_i(C) > 0$. Also, if $U_i(y) < 0$ then either $\theta(y) = \infty$ or $\gamma_i(y) = 0$.

III. *The market clears.*

$$\int_{\bar{Y}} \frac{\gamma_i(y)}{\theta(y)} d\lambda(\{y\}) \leq \tau_i$$

with equality if $\bar{U}_i > 0$.

The first part of the definition reflects optimizing behavior of capitalists and the competitiveness of the market, so that a given capitalist posts the contract that maximizes his profits but capitalists post a given contract until the cost of doing so κ is equal to the returns. The second condition says that workers maximize their utility when applying to contracts, and must be indifferent between any contracts to which they actually apply in equilibrium. The final condition says that the sum of type i applications over all possible contracts must equal the total number of workers of type i who are supposed to apply (τ_i).

It is important to note the restrictions placed on jobs *not* posted in equilibrium. Specifically, consider a the case where the fraction of bad workers approaches zero. Without competitive search, equilibrium doesn't exist. Any possible equilibrium must be separating. If it wasn't then a capitalist could offer a contract with slightly higher hours that would attract only good workers; underpaying them slightly gives positive profits. On the other hand, the equilibrium cannot be separating because a firm could post a pooling contract in which good worker's hours were not distorted and the (infinitesimally few) bad workers were subsidized. Because this contract would attract all workers, the average product would essentially be equal to the good worker's production. Since neither a separating nor a pooling contract can be an equilibrium, none exists.

With competitive search the separating equilibrium always exists. The key condition is that the share of workers applying to a contract y is zero if y gives less utility than their equilibrium contracts. Starting with the least cost separating contracts as a proposed equilibrium, consider a capitalist who posts a pooling equilibrium that lessens the hours distortion for good workers. It is no longer the case that the average product will calculated according to the population shares of each worker; instead, he must forecast what fraction of applicants for the job will be each type. Since a given bad worker gains more from applying to the pooling contract (he is cross-subsidized by the good workers), the applicant pool will

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be predominantly bad. Since good workers always have less to gain from applying to the pooling contract, there is always a separating equilibrium.

It is now possible to check that this economy satisfies the conditions required to appeal to **(author?)** [11]'s existence result in proposition 3 and their algorithm for computing equilibria.

Proposition 3.1. *The functions U_i and π_i satisfy:*

A1 *Monotinicity: For any y: $\pi_L(y) \leq \pi_H(y)$.*

A2 *Local Non-Satiation: Let $B_\delta(y) = \{y' : ||y - y'|| < \delta\}$. Then for any y with $w > 0$, and any $\delta > 0$, there is some $y' \in B_\delta(y)$ such that $\pi_H(y') > \pi_H(y)$ and $U_L(y') < U_L(y)$*

A3 *Sorting: For any $y >> 0$ and any $\delta > 0$, there is some $y' \in B_\delta((h', w'))$ such that:*

$$U_H(y') > U_H(y) \text{ and } U_L(y') < U_L(y)$$

The first condition is obviously satisfied, since having a low productivity worker work h hours and paying him w gives lower profits than if her productivity was instead high. The second condition holds since for any (h, w) with $w > 0$, it is possible to keep h the same and put $w' = w - \delta$, thereby reducing the utility for every worker and raising the capitalist's profits. Under the assumption that $v'_L(h) > v'_H(h)$ for all h, the final condition holds because a good worker is willing to increase her hours for a smaller increase in consumption than would a low productivity worker. Formally, consider taking some $\delta > 0$ and increasing h and w to:

$$h' = \frac{u'(c)}{v'_H(h)}\delta \text{ and } w' = w + \delta$$

Then the change in utility for the good worker is approximately zero (and could be made positive by increasing hours slightly less), whereas the change in utility for the bad worker

is:

$$\Delta U_L = u'(c)\delta \left(1 - \frac{v'_L(h)}{v'_H(h)}\right) < 0$$

Where the inequality is due to $v'_L(h) > v'_H(h)$. Thus the perturbed utility is lower for the low productivity worker.

The sorting condition **A3** is fundamental for the existence of seperating equilibria, and is why the assumption that $v'_L(h) > v'_H(h)$ is typically made. This is a strong assumption, and, while plausible, it is very hard to verify empirically. It is plausible due to two mechanisms through which high productivity workers would endogenously provide a marginal hour of labor for less marginal earnings. The first is learning by doing: even if productivity is initially the same, workers with lower disutility of labor supply will be more productive after a short time, since they will work longer hours and accumulate more human capital. Another mechanism is consumption commitments made before contracting with employers. If good workers know that they will have high earnings, then they may commit to payments on durable consumption goods before entering the labor force (car payments, mortgages, etc). If utility over these goods and flow consumption is separable, then good workers will have a higher marginal utility for every dollar in earnings than will bad workers (see [7] for how such commitments can be modelled). This would generate preferences over earnings and hours that satisfy assumption **A3**, even without ex-ante differences in preferences over consumption and hours. I leave these implementations for future research, and continue with complete markets and preference heterogeneity, which allows me to appeal to (**author?**) [11]'s existence result:

Proposition 3.2. *There exists a unique seperating labor market equilibrium and it can be computed by solving the programming problems below.*

For the low productivity worker:

$$\bar{U}_L = \max_{\theta,w,h} f(\theta)U_L(w,h) \tag{5}$$

$$s.t. \tag{6}$$

$$\kappa \leq q(\theta)\left(F'(H)\zeta_L h - w\right) \tag{7}$$

For the high productivity worker (taking $\bar{U}_L$ parametrically):

$$\bar{U}_H = \max_{\theta,w,h} f(\theta)U_H(w,h) \tag{8}$$

$$s.t. \tag{9}$$

$$\kappa \leq q(\theta)\left(F'(H)\zeta_H h - w\right) \tag{10}$$

$$\bar{U}_L \geq f(\theta)\left(u'(c)w_H - v_L(h_H)\right) \tag{11}$$

This result is powerful for two reasons. The first is that it makes computation of equilibria relatively easy. The second is more subtle: because it is impossible to identify the workers by type in the data, I want to look at the limit of economies as $\tau_L \to 0$. But this is exactly the situation in which equilibria do not exist without competitive search! Competitive search therefore guarantees well behaved equilibria for the exact parameterizations that allow me to overcome previously unsurmountable measurement issues.

3.1.3 Labor Market Characterization

The low productivity worker is entirely undistorted, and therefore behaves as in traditional theory. Specifically, her intratemporal Euler Equation is given by:

$$\frac{\phi_L^{-1}v'(h_L^*)}{u'(c^*)} = F'(H^*)\zeta_L \tag{12}$$

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Where a variable with $*$ superscript is the first best. If the incentive compatibility constraint doesn't bind, then the high productivity worker's hours also satisfy the standard intratemporal Euler Equation:

$$\frac{\phi_H^{-1} v'(h_H^*)}{u'(c^*)} = F'(H^*)\zeta_H \tag{13}$$

On the other hand, the good worker's hours are distorted upwards whenever the incentive compatibility constraint on the low productivity worker binds. This happens if:

$$f(\theta_H^*)\left(u'(c^*)w_H^* - \phi_L^{-1} v(h_H^*)\right) > \bar{U}_L$$

In this case, letting a hat variable denote the distorted adverse selection equilibrium value, the good worker's intratemporal condition becomes:

$$\frac{\phi_H^{-1} v'(\hat{h}_H)}{u'(\hat{c})} = \left(\frac{1 - \psi}{1 - \psi \frac{v'_L(\hat{h}_H)}{v'_H(\hat{h}_H)}}\right) F'(\hat{H})\zeta_H \tag{14}$$

This latter equation drives the results. The term ψ is the Lagrange multiplier on the incentive compatibility constraint, and if the constraint binds then the "wedge" in front of the good worker's marginal product is greater than one. When ζ_L falls, the constraint tightens and ψ rises, as does the wedge. This is the mechanism through which average labor productivity can fall, but some workers increase their hours. In addition, for typical parameterizations, hours per worker and vacancies for that worker move in the same direction. Thus, in response to a fall in the bad worker's productivity, the total hours of good workers (bodies time hours per body) can rise. In the extreme case, the fraction of bad workers can approach zero, so that *every* worker's hours rise, and since $F(H)$ is concave, labor productivity falls. This is

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summarized in the following proposition.

Proposition 3.3. *If the incentive compatibility constraint binds, then $\hat{h}_H > h_H^*$ and $\frac{\partial h_H}{\partial \zeta_L} < 0$.*

At this point the first order conditions on earnings and market tightness, along with the zero profit condition for the firms, give the remaining necessary conditions for equilibrium. Let $\eta_{x,y}$ be the elasticity of x with respect to y. In the appendix I show that, after algebraic manipulation, the conditions become:

$$\kappa = q(\hat{\theta}_i) \left(1 - \eta_{v,h_i}^{-1}\right) \eta_{f,\theta_i} \zeta_i F'(\hat{H}) h_i \tag{15}$$

$$\hat{w}_L = (1 - \eta_{f,\theta_L}) \zeta_L F'(\hat{H}) \hat{h}_L + \eta_{f,\theta_L} \frac{v_L(\hat{h}_L)}{u'(\hat{c})} \tag{16}$$

$$\hat{w}_H = (1 - \eta_{f,\theta_H}) \zeta_H F'(\hat{H}) \hat{h}_H + \eta_{f,\theta_H} \frac{v_H(\hat{h}_H)}{u'(\hat{c})} \left(\frac{1 - \psi \frac{v'_L(\hat{h}_H)}{v'(\hat{h}_L)}}{1 - \psi} \right) \zeta_H F'(\hat{H}) \hat{h}_H \tag{17}$$

The earnings equation for the bad worker should be familiar: earnings are a convex combination of the worker's marginal production and their utility cost (expressed in units of the good) from taking the job. The weights for the combination are given by the elasticity of the job finding function, due to competitive search. A similar result attains in full information models of job search in which earnings are determined via generalized Nash bargaining with weights that satisfy the Hosios condition (see [2], [6], [19]). Interestingly, the good workers' earnings take a similar form, except that they get η_{f,θ_H} times the distorted cost of their labor supply. Further manipulation yields a unified equation for earnings:

$$\hat{w}_i = \left(1 - \eta_{f,\theta_i} + \eta_{f,\theta_i}/\eta_{v,h_i}\right) \zeta_i F'(\hat{H}) \hat{h}_i \tag{18}$$

3.1.4 Equilibrium

At this point we can define the equilibrium of the static economy. It is:

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Definition 2. *A competitive equilibrium consists of:*

    i Labor market allocations: per capita $(w_L, h_L, \theta_L, \bar{f}_L, w_H, h_H, \theta_H, \bar{f}_H)$ *and aggregate,* H

    ii Consumption by type and job finding outcome, $(c_L^1, c_L^0, c_H^1, c_H^0)$

Such that:

    I Aggregate hours are consistent with per capita hours, ie $H = \tau_H f(\theta_H)\zeta_H h_H + \tau_L f(\theta_L)\zeta_L h_L$

    II Given the items in [i] (with $\bar{f}_i = f(\theta_i)$*), the consumption in [ii] solve the family's optimization problem. In effect:*

$$c_i^1 = c_i^0 = c = \tau_H \bar{f}_H w_H + \tau_L \bar{f}_L w_L, \ \forall i = L, H$$

    III Given the value of c, the items in [i] form a labor market equilibrium as previously defined.

Conditions under which the equilibrium exists are forthcoming in an online appendix. I note here that equilibria are readily computable for every parameterization that I have considered.

3.2 Dynamic Model

I now build the static model into the neo-classical growth model, period by period. Labor market equilibria are essentially identical to the static case, except that preference terms include expected future values.

3.3 Environment

Time is infinite and discrete. There is a unit mass of families, each with a unit mass of ex-ante identical workers, each of whom has one unit of time that can be used in production or as leisure. Workers have preferences over consumption and labor that are additively separable over time and states of the world with $U(c, h) = u(c) - v(h)$. Workers live forever and have lifetime expected utility given by $U_0 = \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t U(c_t, h_t)$.

At the beginning of each period, the aggregate state of the economy is known and is denoted $(\hat{z}, S)$ and will be described below. Each worker then draws a type $i \in L, H$ with probability τ_i, which determines her disutility of labor supply and the efficiency of her hours in production. I assume that more efficient workers find labor less displeasurable, so that $v'_L > v'_H$, but will eventually take the limit of economies as $\phi_L \to \phi_H$.

Search frictions keep some workers from immediately finding work in each period. Each family has n_t employed members at the beginning of a period. At the aggregate there are N_t employed workers. An employed worker's type is observed by firms, whereas an unemployed worker's is not. An employed worker can then choose to take a job immediately, whereas an unemployed worker must apply for a job specifying (h, w), which she finds with probability $f(\theta(h, w)) \leq 1$. I assume that search occurs at the beginning of the period, so that new matches produce immediately.

There is an aggregate firm with neoclassical production function $zF(K, H)$, where H is total *effective* hours used in production and K is rented capital. An hour of the good worker's time translates to $\zeta_H = 1$ hours in production, whereas an hour of the low productivity worker is effectively $\zeta_{L,t} < 1$ hours; hence the exogenous state is $\hat{z} = (z, \zeta_L)$. The firm rents capital on a spot market and thus chooses K to maximize $\Pi(H; \hat{z}) = zF(K, H) - (r + \delta)K$, where r is the rental rate and δ the depreciation rate. The firm comprises a large number of capitalists, each of whom is risk neutral and lives for one period. Capitalists can meet with

unemployed workers costlessly in spot markets, taking the linear wage w_i^* as given, or can post a contract for unemployed workers. The profit from an employed worker of type i is given by $\pi_i^n = \max_h z F_H(K,H)\zeta_i h - w_i^* h$.

3.4 Equilibrium

3.4.1 Family Programming Problem

There are three outcomes for a worker: those concerning his type, employment, and job finding outcome. The family enters the period with assets a and n workers who are employed. The head takes the outcomes from the labor market as given and decides how much to consume this period as well as how to allocate the consumption across workers. As before, consumption will be equated in equilibrium, and so I start with that result in the definition of the family's problem. At the time of entering the labor market, workers are heterogeneous with different (hidden) types and the competitive search market with adverse selection commences.

$$W(a,n,\hat{z},S) = \max_{c,a',h_L,h_H} u(c) - \tilde{v}(a,\hat{z},S) + \beta \mathbb{E} W(a',\hat{z}',S') \tag{19}$$

$$\text{s.t.} \tag{20}$$

$$c + a' \leq (1 + r(\hat{z},S))a + n\left(\tau_H w_H^*(\hat{z},S)h_H + \tau_L w_L^*(\hat{z},S)h_L\right)... \tag{21}$$

$$... + (1-n)\left(\tau_H \bar{f}_H(\hat{z},S)w_H(\hat{z},S) + \tau_L \bar{f}_L(\hat{z},S)w_L(\hat{z},S)\right) \tag{22}$$

$$n' = (1-s)\left(n + (1-n)\sum_i \tau_i \bar{f}_i(\hat{z},S)\right) \tag{23}$$

$$S' = G_S(\hat{z},S) \tag{24}$$

$$z' = \rho z + \nu_z' \tag{25}$$

$$\zeta_L' = G_\zeta(z',\zeta_L) + \nu_\zeta' \tag{26}$$

A standard stochastic process is assumed for aggregate TFP. The efficiency of a bad worker's hours, ζ_L, is stochastic and can depend on current z_t, whereas $\zeta_H = 1$ as a normalization. I will consider two specifications of G_ζ in the quantitative section; one in which ζ_L is a stochastic iid random variable in $(0, 1)$ and another where ζ_L is a determinitic function of z.

For workers who are employed to start the period, there is no assymetric information and so competition drives $w_i^*(z, S) = zF_H(K, H)\zeta_i$ and the marginal rate of substitution between consumption and leisure equal to $w_i^*(z, S)$. The second utility term is the weighted average of disutilities:

$$\tilde{v}(a, z, S) = N\left(\tau_H v_H(h_H^*) + \tau_L v_L(h_L^*)\right) + (1 - N)\left(\tau_H \bar{f}_H v_H(h_H) + \tau_L \bar{f}_L v_L(h_L)\right)$$

from which it is clear that there are utility gains from having an additional worker in the employed pool, since newly employed good workers work longer hours due to the Rat Race.

3.4.2 Labor Market Equilibrium

There is now a simple dynamic consideration for workers when searching for a job. The family's value tomorrow rises by the discounted expected value of having another worker employed. Let this value be denoted by $\Lambda(a, n, z, S) = \frac{\partial W(a,n,z,S)}{\partial n}$. An applciation of the envelope theorem give a recursion:

$$\Lambda(a, n, z, S) = \Omega_H + \Omega_L + \beta(1 - s)(1 - \tau_H \bar{f}_H - \tau_L \bar{f}_L)\mathbb{E}\Lambda(a', n', z', S') \tag{27}$$

$$\Omega_i = u'(c)\tau_i\left(w_i^* h_i^* - \bar{f}_i w_i^0 - (v_i(h_i^1) - \bar{f}_i v_i(h_i^0))\right) \tag{28}$$

Thus, letting $\Lambda(a', n', z', S') = \Lambda'$ and suppressing dependence on family and aggregate states, the value of a job y_j for a worker of type i is now:

$$U_i(y_j) = u'(c)w - v_i(h) + \beta(1 - s)\mathbb{E}\Lambda' \tag{29}$$

The definitions of the profit function and equilibrium are unchanged, and it is clear that the sorting condition **A3** is still satisfied. The only thing that changes are the programming problems solved in order to find the equilibria. For the low productivity worker:

$$\bar{U}_L = \max_{\theta, w, h} f(\theta) U_L(w, h) \tag{30}$$

$$\text{s.t.} \tag{31}$$

$$\kappa \le q(\theta) \left((1 - \alpha)\frac{Y}{H}\zeta_L h - w \right) \tag{32}$$

For the high productivity worker (taking $\bar{U}_L$ parametrically):

$$\bar{U}_H = \max_{\theta, w, h} f(\theta) U_H(w, h) \tag{33}$$

$$\text{s.t.} \tag{34}$$

$$\kappa \le q(\theta) \left((1 - \alpha)\frac{Y}{H}\zeta_H h - w \right) \tag{35}$$

$$\bar{U}_L \ge f(\theta) \left(u'(c)w_H - v_L(h_H) + \beta(1 - s)\mathbb{E}\Lambda' \right) \tag{36}$$

4   Calibration and Results

There are a number of parameters and choices that must be made. I parameterize the economy to admit a balanced growth path and retain a constant Frisch elasticity of labor

supply, thus:

$$U(c, h) = \log(c) - \phi^{-1} \frac{\epsilon}{1 + \epsilon} h^{\frac{1+\epsilon}{\epsilon}}$$

A period is set to a quarter. Production is Cobb-Douglas $F(K, H) = K^\alpha H^{1-\alpha}$ and the matching function is bounded above by 1, as in **(author?)** [12]: $f(\theta) = \frac{\theta}{(1+\theta^l)^{1/l}}$. The growth rate is set to $\gamma = 1.015^{1/4}$ to match historical data, the depreciation rate is then $\delta = 1.67\%$ to match the investment to capital ratio given γ. The discount rate is then set to $\beta = 0.993$ so that the quarterly output to capital ratio is $1/12$. The resulting steady state net interest rate is 1.08%.

4.1   Benchmark Model

As a benchmark, I want to use the theory to avoid measurement difficulties and to focus on the intensive margin, since it is the focus of my cross-sectional evidence. I therefore make the following restrictions:

1. The economy is homogenous for all practical purposes, ie $\tau_L \to 0$. Note that the distortion due to adverse selection still occurs so long as $\tau > 0$, since workers search after their types are realized. Since it is the good workers' hours that are distorted, and every worker is good, this increases the effect of adverse selection on aggregate hours.

2. Search is cheap enough that unemployment can be ignored, ie $\kappa \to 0$. With my specification of the matching function this ensures that $\theta_i \to \infty$ for $i = L, H$.

3. There is complete turnover of the labor market in every period, ie $s = 1$ so that $N_t \to 0$. This ensures that every worker is *newly* employed in every period, and therefore affected by adverse selection. In effect, this is a frictionless economy in which

adverse selection determines hours but workers get paid their marginal product.

4. The steady state value of $\zeta_L = 1$ and the stochastic process is $\zeta_{L,t} = e^{-\nu_{\zeta,t}^2}$. Thus in steady state there is only one type of worker and the hours allocation is first best. Along a sample path, bad workers are always less productive than good workers.

5. Finally, $\phi_L \to \phi_H$. In the theory, so long as there is an infinitesimal difference in the disutilities, the equilibrium is separating. This ensures that the incentive compatibility constraint always holds with equality, which allows me to study the log-linearized economy.

The resulting system of equations can be solved at the detrended steady state and then approximated in order to study the cyclical properties of the economy:

$$\gamma c_t^{-1} = \beta \mathbb{E}_t \{(1 + r_{t+1}) c_{t+1}^{-1}\} \tag{37}$$

$$\phi^{-1} h_{L,t}^{1/\epsilon} c_t = (1 - \alpha) \frac{Y_t}{H_t} \zeta_{L,t} \tag{38}$$

$$w_{L,t} = (1 - \alpha) \frac{Y_t}{H_t} \zeta_{L,t} h_{L,t} \tag{39}$$

$$c_t^{-1} w_{H,t} - \phi^{-1} v(h_{H,t}) = c_t^{-1} w_{L,t} - \phi^{-1} v(h_{L,t}) \tag{40}$$

$$w_{H,t} = (1 - \alpha) \frac{Y_t}{H_t} h_{H,t} \tag{41}$$

$$H_t = h_{H,t} \tag{42}$$

$$\hat{z}_t \alpha \left(\frac{K_t}{H_t}\right)^{\alpha - 1} = r_t + \delta \tag{43}$$

$$c_t = w_{H,t} + (1 + r_t) K_t - \gamma K_{t+1} \tag{44}$$

The key equation is number 40, which is the incentive compatibility condition for the low productivity worker. This gives the distortion on high productivity workers' hours. Notice that, since in steady state the workers have the same productivities there is no distortion.

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Also, since $s = 1$, the Λ terms vanish.

4.2 An Analytical Solution

The model is sufficiently simple that a well known property of the neo-classical growth model carries through: since utility is logarithmic over consumption and separable between consumption and hours, when capital depreciates fully every period ($\delta = 1$), there is a simple analytic solution. The proposition is as follows:

Proposition 4.1. *Under the restrictions of the benchmark model and $\delta = 1$, the dynamics of the economy are fully characterized by:*

$$K' = \alpha\beta z K^\alpha H^{1-\alpha} \tag{45}$$

$$\left(\frac{1-\alpha}{1-\alpha\beta}\right) H^{1+\epsilon} = \frac{\epsilon}{1+\epsilon}\phi^{-1}H^{\frac{(1+\epsilon)^2}{\epsilon}} + \left(\frac{1-\alpha}{1-\alpha\beta}\right)^{1+\epsilon}\phi^\epsilon\frac{\zeta^{1+\epsilon}}{1+\epsilon} \tag{46}$$

Furthermore, if $\epsilon = 1$ then:

$$H = \left(1 + (1-\zeta^2)^{1/2}\right)^{1/2}\left(\phi\frac{1-\alpha}{1-\alpha\beta}\right)^{1/2} \tag{47}$$

It is especially informative to consider the case when $\zeta = 1$, which I impose in steady state. In that case, equation 46 simplifies and hours are set to the full information optimum, so that $H = H^* = \left(\phi\frac{1-\alpha}{1-\alpha\beta}\right)^{\frac{\epsilon}{1+\epsilon}}$. With this in mind, implicit differentiation of equation 46 yields (letting $\lambda = \frac{1-\alpha}{1-\alpha\beta}$ for now):

$$\frac{\partial H}{\partial \zeta} = \left(\frac{\lambda^{1+\epsilon}\phi^\epsilon\zeta^\epsilon}{(1+\epsilon)H^\epsilon}\right)\left(\lambda - \phi^{-1}H^{\frac{1+\epsilon}{\epsilon}}\right)^{-1} \tag{48}$$

This derivative takes the same sign as the term $\lambda - \phi^{-1}H^{\frac{1+\epsilon}{\epsilon}}$. Since $H > H^* = (\phi\lambda)^{\frac{\epsilon}{1+\epsilon}}$

whenever $\zeta < 1$, this term is negative, and so hours rise when bad workers become less productive, as expected. Furthermore, when $\epsilon = 1$, equation 47 ensures that $H = \varphi(\zeta)H^* = \left(1 + (1 - \zeta^2)^{1/2}\right)^{1/2} H^*$. Here it is clear that the upward distortion on hours in never greater than $2^{1/2}$. What is important for the model's ability to generate large variation quantitatively, however, is the elasticity of the distortion near $\zeta = 1$. As can be seen in figure 1, the slope is greatest around this point.

Note also that, as can be seein in Figure 1, the Frisch elasticity of labor supply ϵ is not a particularly important parameter for determining the labor response to relative productivity shocks. Only when labor supply is very inelastic ($\epsilon = 0.1$) is the curve flat over most of the productivity gap region. Once the Frisch elasticity is above 0.5 (which is the realm of recent estimates), hours respond essentially identical to when $\epsilon = 1$. This is especially striking since TFP shocks do not affect labor supply in this example, regardless of the Frisch elasticity. This indicates that market structure and the source of shocks can drive aggregate dynamics largely independent of the micro parameters.

4.3   Quantitative Model

The value of ϕ is chosen so that the average labor supply is $1/3$ in steady state and $\epsilon = 0.48$. Higher values of ϵ generate greater volatility in both hours and output, but an interesting test of the model is whether adverse selection further amplifies or dampens these volatilities for a low value of the Frisch elasticity.

   The most important parameters are those that govern the steady state value of N, the fraction of bad workers (τ_L), and the relationship between $\hat{z}$ and ζ_L. My benchmark is to get rid of unemployment altogether by taking κ to be small, which also gets rid of l as a parameter. I then vary the fraction of bad workers and restrict the innovations to $\hat{z}$ and ζ_L to be uncorrelated. I choose the persistence and standard deviation of innovations to TFP

Table 4: Relative Volatilities

Variable	Data	Full Info	Adverse Selection
σ_Y	1.72	1.11	1.39
σ_H/σ_Y	0.82	0.24	0.90
$\sigma_{Y/H}/\sigma_Y$	0.60	0.76	0.67
$\sigma_H/\sigma_{Y/H}$	1.64	0.32	1.34
$\sigma_{\text{wedge}}/\sigma_Y$	0.92	0.35	0.92
σ_C/σ_Y	0.45	0.28	0.22
σ_I/σ_Y	2.41	3.08	3.02

to be $\rho = 0.95$ and $\sigma_\nu = 0.007$. The standard deviation of ν_ζ is unmeasurable, but, as is explained below, determines the standard deviation of the labor wedge. In the data, the wedge, when measured from the perspective of a representative household with $\epsilon = 1.5$, is 92% as volatile as output ([4]). This necessitates that I set $\sigma_{\nu_\zeta} = 0.018$.

4.3.1 Benchmark Results

In Table 4 I compare the volatilities of the data to the full information version of the benchmark model and to the adverse selection version. Relative to the full information version of the model, adverse selection can generate substantial additional volatility in hours, output, and the labor wedge without affecting consumption or investment very much. In some sense, this is to be expected because the full information economy is not affected by shocks to the bad worker's productivity (since they are negligible in the population), whereas these shocks do affect hours and output in the adverse selection economy via their distortion on the good workers. In essence, the adverse selection economy sometimes experiences changes in hours and output for no apparent reason, which increases the volatilities of these variables.

The more relevant moments to consider are the cross-correlations and elasticities of output, hours, the labor wedge and labor productivity. These moments from the data and each

Table 5: Correlations and Elasticities

Variable	Data	Full Info	Adverse Selection
$\rho_{H,Y/H}$	-0.43	0.98	-0.23
$\beta_{H,Y/H}$	-0.83	0.32	-0.31
$\rho_{Y,Y/H}$	0.31	1.0	0.46
$\beta_{Y,Y/H}$	0.51	1.32	0.69
$\rho_{\text{wedge},Y}$	0.55	-0.99	0.33
$\beta_{\text{wedge},Y}$	0.60	-0.33	0.36

economy are in Table 5. Here we see that the full information model does a very poor job of capturing the correlations and elasticities seen in the data. Labor productivity is strongly positively correlated with both output and hours and the elasticity of output with respect to labor productivity is nearly three times what is in the data. When the economy is burdened by adverse selection, however, there are times when hours and output rise due to a fall in productivity of bad workers. These expansions cause labor productivity to fall due to the diminishing marginal product of hours. From the perspective of a representative household, it appears that hours must get subsidized, which generates the positive correlation between the labor wedge and output.

4.3.2 Changing Population Share of Bad Workers

I now consider the effects of having the share of bad workers positive (100% is the benchmark, 0% is all bad workers). The informational friction per good worker is unchanged, but now bad workers contribute to aggregate hours. The resulting economy is essentially a convex combination of the adverse selection benchmark and a standard RBC economy. The volatilities for the adverse selection economy with all parameters the same as the benchmark except for different values of $\tau_H = 1 - \tau_L$ can be found in Table 6. As the economy fills with bad workers, things look more like a standard RBC world. The volatility of output and hours

Table 6: Relative Volatilities

Variable	Data	Share of Good Workers				
		100%	90%	50%	10%	0%
σ_Y	1.72	1.39	1.34	1.20	1.11	1.11
σ_H/σ_Y	0.82	0.90	0.85	0.57	0.27	0.25
$\sigma_{Y/H}/\sigma_Y$	0.60	0.67	0.68	0.73	0.76	0.75
$\sigma_H/\sigma_{Y/H}$	1.64	1.34	1.24	0.78	0.36	0.33
$\sigma_{\text{wedge}}/\sigma_Y$	0.92	0.92	0.87	0.61	0.37	0.35
σ_C/σ_Y	0.45	0.22	0.23	0.26	0.28	0.28
σ_I/σ_Y	2.41	3.02	3.03	3.05	3.07	3.07

fall because adverse selection generates additional fluctuations in the good workers' hours, but these make up a smaller fraction of the total. The wedge becomes less volatile since the classical intratemporal optimality condition is always satisfied for bad workers; in the limit, as they populate the entire economy, the wedge is only volatile due to misspecfication of the Frisch elasticity at the aggregate (1.5 vs 0.48 at the micro).

A similar pattern emerges for comovements and can be seen in Table 7. As bad workers make up the entirety of the population, the response of hours to productivity is overwhelmingly positive [8]. Since hours move strongly positively with productivity, the model overstates output's response to productivity relative to the data.

5 Another mechanism: Employment Fluctuations and Selection

Here is a simple example where I the limiting economy with a linear production function to demonstrate how adverse selection and asymmetric shocks can lead to changes in the labor

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[8]The correlation becomes positive with $\tau_H \approx 2/3$

Table 7: Correlations and Elasticities

| Variable | Data | Share of Good Workers | | | | |
		100%	90%	50%	10%	0%
$\rho_{H,Y/H}$	-0.43	-0.23	-0.16	0.18	0.87	0.98
$\beta_{H,Y/H}$	-0.83	-0.31	-0.20	0.14	0.31	0.32
$\rho_{Y,Y/H}$	0.31	0.46	0.54	0.54	0.99	1.0
$\beta_{Y,Y/H}$	0.51	0.69	0.79	1.14	1.31	1.32
$\rho_{\text{wedge},Y}$	0.55	0.33	0.25	-0.20	-0.91	-0.99
$\beta_{\text{wedge},Y}$	0.60	0.36	0.22	-0.12	-0.34	-0.35

force. Imagine now that there is an observable characteristic of workers in addition to the private heterogeneity. Let's call that unobservable ξ and assume that it is distributed according to a Pareto distribution with minimum value normalized to one and shape parameter λ. Further, let's assume that individual productivities are given by $\zeta_L(\xi) = z_t \xi - \psi_t$ and $\zeta_H(\xi) = z_t \xi + \psi_t$. Furthermore, we will assume that $\alpha = 0$ in the production function, so that output is simply the sum of effective labor supplies and will take the limit as the Frisch elasticity approaches zero so that there is no movement in the intensive margin. Under these assumptions, the equilibrium is given by a stochastic process of C, N, Y, ξ_t that satisfy:

$$N_t = \int_{\xi_t}^{\infty} \lambda \xi^{-(1+\lambda)} d\xi$$

$$C_t = Y_t = \int_{\xi_t}^{\infty} (z_t \xi + \psi_t) \lambda \xi^{-(1+\lambda)} d\xi$$

$$\xi_t = \psi_t z_t^{-1}$$

Where the value for ξ_t is determined by finding the value of ξ such that the lower productivity of that type contributes zero marginal product (every lower value of ξ would have low types who contribute negatively). [9] Now, we can just use properties of the Pareto Distribution to

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[9]The low types could instead have a positive marginal product if there was a cost to posting vacancies, and this is a model I am working on.

get:

$$N_t = \zeta^{-\lambda} = (\frac{z_t}{\psi_t})^{\lambda}$$

$$Y_t = N_t \mathbb{E}[\zeta z_t + \psi_t | \zeta \geq \psi_t z_t^{-1}] = \frac{2\lambda - 1}{\lambda - 1} z_t^{\lambda} \psi_t^{1-\lambda}$$

$$\frac{Y_t}{N_t} = \frac{2\lambda - 1}{\lambda - 1} \psi_t$$

So from this we can see that labor rises with z and falls with ψ, output moves in the same directions but is less sensitive to changes in ψ, and so labor productivity (here also measured TFP) is independent of z_t but positively related to ψ. Thus, shocks to the dispersion of productivity generate recessions with rising labor productivity. This would generate the same unconditional correlations as the main theory, but via a completely different mechanism. In the main part of this paper, an increase in productivity dispersion generates an increase in hours dispersion, which, with a diminishing marginal product of labor, means that labor productivity falls in such an expansion. Here, an increase in dispersion leads to a fall in employment for less productive workers and therefore a rise in labor productivity due to improved composition. Interestingly, given my assumption that the bad types are of measure zero, the productive capacity of the economy is invariant to changes in ψ, so that a recession in this world is a movement to the interior of the production possibilities frontier rather than an inward shift of the frontier.

6   Conclusion

This paper makes three contributions. Empirically, I use micro data to extend the hours/labor productivity comovement puzzle to the hours of individuals, both on average and across the wage distribution. Theoretically, I use the recent breakthrough of **(author?)** [11] in order to

embed adverse selection into the labor market of an equilibrium macro model. Competitive search is vital, as it allows me to study an economy in which there are essentially zero bad workers; this is exactly the situation in which equilibrium do not exist without competitive search, and the one of interest due to measurement problems. Finally, I use the model to measure the extent to which adverse selection can account for the observed comovements between labor market variables and labor productivity. I find that the benchmark model exhibits labor market dynamics that are very similar to the data.

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7 Appendix

Proof. Proof of proposition 4.1. Via substitution of the earnings functions and interest rate, it is clear that the budget constraint simplifies to $C + K' = Y$. Guessing that $K' = \alpha\beta Y$ gives $C = (1 - \alpha\beta)Y$. It is straight forward to verify the intertemporal Euler Equation at
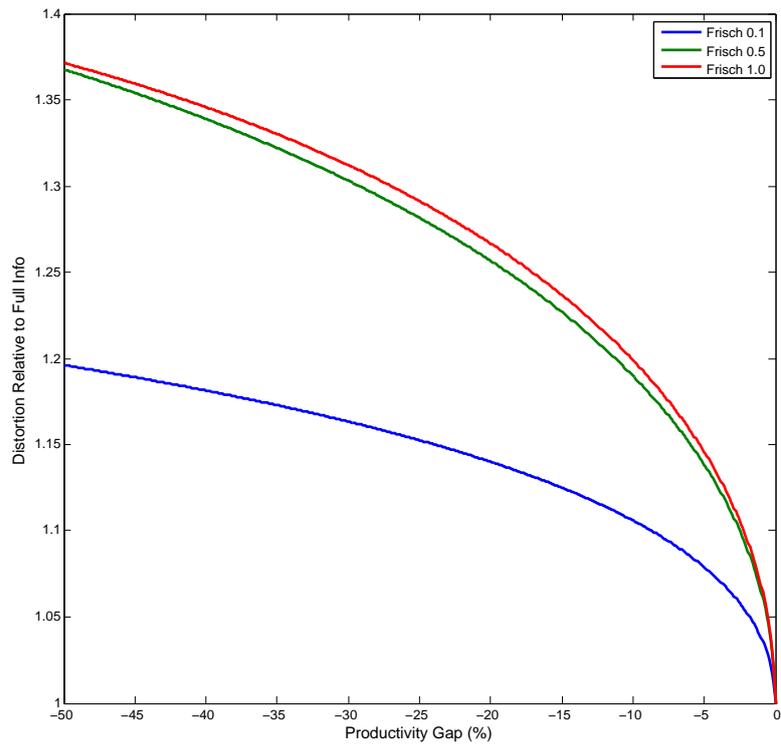
Figure 1: Distortion of Hours, Analytical Solution Case

this point, noting the importance of separability. Using this function for C in equation 38 gives the hours of the low productivity worker, $h_L = \left(\phi \frac{1-\alpha}{1-\alpha\beta}\right)^\epsilon \left(\frac{\zeta}{H}\right)^\epsilon$. Then using those hours in the incentive compatibility equation gives the equation to be solved for H, which after some simple manipulation yields equation 46.

Let $\lambda = \frac{1-\alpha}{1-\alpha\beta}$ for the time being. Then plugging $\epsilon = 1$ turns 46 into the quadratic form:

$$H^4 - 2\lambda\phi H^2 + (\lambda\zeta\phi)^2 = 0$$

which gives the solution:

$$H^2 = \lambda\phi \left(1 +/- (1-\zeta^2)^{1/2}\right)$$

Since H must be greater or equal to $H^* = (\phi\lambda)^{1/2}$, there is only one economically interesting root. Since hours must be positive, the solution is as given in the proposition. $\square$