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Dual-Currency Economies as Multiple-Payment Systems

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Introduction

The phrase "multiple payment systems" typically brings to mind objects such as checks, credit cards, debit cards and, more recently, "smart" cards. However, many countries throughout history have used more than one currency at a time. In fact, although we tend to forget it, the use of multiple currencies as media of exchange in the United States was common into the 1930s. Until their abolition in 1935, privately issued banknotes were commonly used simultaneously with government-fiat and commodity-backed money. During its bimetallism period, the United States used two different government-issued commodity monies: gold and silver coins. More recently, we observe two-currency payment systems in developing and transitional economies, in which many modern payment systems are unavailable. Nevertheless, citizens may adopt a dual-payment system by using the dollar in addition to their own locally issued fiat currency as a medium of exchange, store of value, and unit of account. Indeed, this practice concerns the Federal Reserve System because a large amount of U.S. currency is being shipped overseas, partly to finance multiple-payment options (see Porter and Judson [1996]).

What is particularly fascinating is that people's use of a foreign currency in addition to their own government's fiat currency arises spontaneously (in response to market desires) rather than by government edict. Hence, governments' traditional reasons for circulating fiat currency (legal restrictions, use of fiat currency to discharge tax liabilities) fail to explain why a country's citizens would adopt fiat currency issued by a foreign sovereign as a medium of exchange. Consequently, to understand how a foreign currency comes to circulate in the domestic economy, one must model the private decision to accept foreign currency in exchange for goods and services. This is especially important for policymakers who wish to "drive out" a foreign currency or increase the acceptability of the domestic one. To do so, they must understand the foreign currency's fundamental benefits and costs.

In this article, we discuss recent research on dual-currency economies, focusing on monetary search models. Search models have special applicability to dual-currency economies because they explicitly model economic agents' decisions to accept a fiat currency in trade. They also yield insights concerning what the local government can do to alter the acceptability of the foreign relative to the domestic currency. The rest of this article is structured as follows: In section I, we present a simple one-country, two-currency search model of money to illustrate the acceptability of multiple currencies. Section II surveys the findings of dual-currency models from the search-theory literature. In section III, we use our simple model to study the policies used by one country, Ukraine, to increase the acceptability of a newly issued domestic currency and eliminate the acceptability of the dollar as a medium of exchange.

I. A Simple Dual-Currency Model

The fundamental purpose of search-theoretic models is to describe the trading frictions that produce the intrinsically useless object called money. The key friction in these models is the absence of a double coincidence of wants, which implies that bilateral trade (barter) is not possible and thus some payment system is needed before trade can occur. A dual-currency economy simply allows more than one object to serve as a medium of exchange. Several models of the type considered here appear in Aiyagari, Wallace, and Wright (1996), Li and Wright (1998) and Wallace (1998).

Preferences

Agents in this economy consume and produce goods and services but cannot produce their desired consumption good; hence, they must trade in order to consume. There are $k > 3$ types of goods in the economy distributed along the unit circle. Goods are divisible and nonstorable (services). An agent who produces good i desires good $i+1$ for consumption; consequently, a double coincidence of wants is not possible between a given pair of agents. The probability of a single coincidence of wants is given by $x = 1/k$. Agents receive utility from consumption $u(q)$, where q is the quantity of their consumption good and $u'(q) > 0$, $u''(q) < 0$ and $u(0) = 0$. Agents' disutility from production is given by $c(q)$, with $c'(q) > 0$, $c''(q) \geq 0$ and, $c(0) = 0$. It is also assumed that $u'(0) - c'(0) > 0$, and that there is some value of $\bar{q}$ such that $u(\bar{q}) - c(\bar{q}) = 0$. The number of agents in the economy is normalized to 1.

Agents do not trade in centralized markets but rather search for suitable trading partners. Individuals meet at random with probability α and meet only one person at a given point in time.[1] It is typically assumed that trading histories are private information, which makes trade credit impossible. Hence, some other payment mechanism is necessary for trade. That mechanism is money.

Money

Besides production goods, another type of good exists in this economy—money. While most search models study fiat currency, which is intrinsically worthless (that is, it provides no utility) but is costless to carry around, other models assume that money generates some basic cost (such as storage or transportation) or benefit (for example, aesthetic beauty), independent of its use as a medium of exchange. To capture all of these possibilities, we assume that a holder of currency i incurs a per-period holding cost (benefit) of $t_i > 0$ (< 0). For $t_i = 0$, the currency is truly a costless fiat currency.

Unlike a consumption good, money is durable. For analytical reasons, we assume that money is indivisible and agents can carry only one unit at a time.[2] We also assume that agents cannot produce until they have consumed.[3] As a result of this assumption, there are only two possible trading states for agents in this economy: They hold either 0 units of money (sellers) or 1 unit of money (buyers). Let M denote the proportion of agents in the economy who hold money, and $m_0 = 1 - M$ denote the proportion of sellers. Since we have two currencies, let m_1 denote the proportion of agents in the economy holding currency 1, and let m_2 denote the proportion holding currency 2. It then follows that $M = m_1 + m_2$ and that $m_0 + m_1 + m_2 = 1$.

1 This is modeled more formally and precisely by saying that meetings occur according to a Poisson process with arrival rate α.

2 This is typically done to avoid having to solve for the steady-state distribution of money holdings and trading prices.

3 This eliminates transactions in which money trades for money plus some goods. Aiyagari, Wallace, and Wright (1996) analyze a model in which these types are allowed.

Bargaining

We consider equilibria in which a money holder meets a seller who produces the money holder's preferred good. We assume that in such a meeting the buyer makes a take-it-or-leave-it offer which entails specifying an amount of the good for the unit of currency. The seller must decide whether to accept this offer or go on. This is true for either currency. Furthermore, if the offer is accepted, the money holder must judge it worthwhile to give up the unit of currency for the consumption good.

Returns to Search

Given the structure above, we can write down the steady-state returns to searching for both buyer and seller:

(1) $$rV_2 = \alpha x m_0 \Pi_0^2 \max[u(q_2) + V_0 - V_2, 0] - t_2,$$

(2) $$rV_1 = \alpha x m_0 \Pi_0^1 \max[u(q_1) + V_0 - V_1, 0] - t_1,$$

and

(3) $$rV_0 = \alpha x m_1 \max_{\pi_0^1} \pi_0^1[-c(q_1) + V_1 - V_0] + \alpha x m_2 \max_{\pi_0^2} \pi_0^2[-c(q_2) + V_2 - V_0],$$

where $i = 0$ denotes sellers, $i = 1$ denotes holders of currency 1, and $i = 2$ denotes holders of currency 2. V_i denotes the value function for a trader of type i and measures the expected present discounted value of utility from trading in the future, given that the current trading position is i. The parameter r is the real interest rate, q_i is the quantity of goods given up by a seller for currency i, and t_i is the per-period cost associated with holding currency i. In addition, the parameter Π_0^i, $i = 1, 2$ denotes the probability (as perceived by the buyer) that a seller will accept currency i in return for goods. The parameter π_0^i captures the seller's decision to accept or reject an offer to trade his good for currency i. If $\pi_0^i = 1$, the currency is accepted by the seller as payment; if $\pi_0^i = 0$, the seller chooses not to accept the currency. In a Nash equilibrium with identical sellers, $\pi_0^i = \Pi_0^i$, for $i = 1, 2$.

The right sides of equations (1) and (2) denote the expected return from trading. This is the probability of a buyer meeting a seller who has his desired consumption good, times the utility from consuming q_i, minus the cost of switching from a buyer to a seller ($V_0 - V_i$). If this payoff is positive or zero, the buyer makes the trade; if it is negative, he does not. No other match of agents yields a payoff, since money holders who meet money holders do not trade currencies and because trading currency 1 for currency 2 plus some goods is ruled out by assumption. For the sellers, equation (3) is the expected return from producing q_i for a unit of currency i and then reversing roles. If this payoff is positive or zero, the seller chooses $\pi_0^i = 1$; if the payoff is negative, the seller does not accept the currency and sets $\pi_0^i = 0$. If the seller is indifferent as to accepting the money or rejecting the offer of money for goods, $0 < \pi_0^i < 1$; in other words, the seller flips a coin to decide whether to accept the currency. In the latter case, we say that a currency is only *partially* acceptable in trade.

Given our bargaining assumptions, the buyer's offer leaves the seller indifferent as to accepting the offer or walking away. Since the payoff is non-negative, the seller accepts the offer.[4] Thus, the equilibrium offer satisfies

(4) $$V_1 = c(q_1) + V_0$$

and

(5) $$V_2 = c(q_2) + V_0.$$

Furthermore, for the offer to be acceptable to the buyer as well, it must be the case that

(6) $$u(q_1) + V_0 \geq V_1$$

and

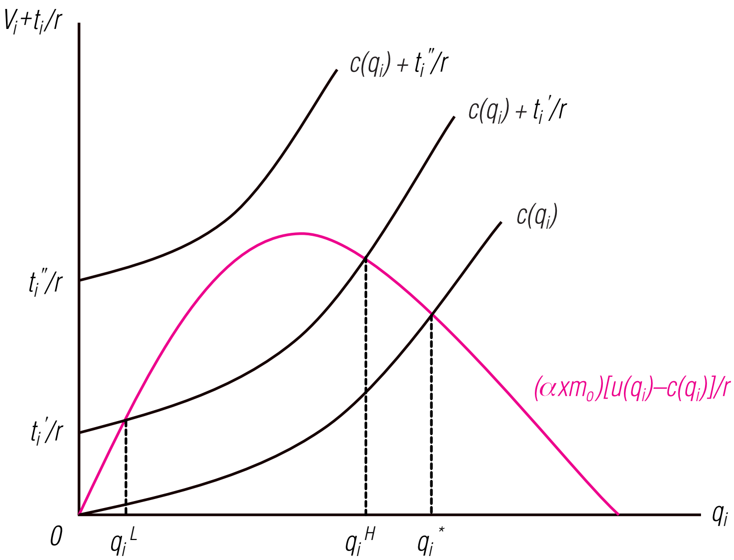
(7) $$u(q_2) + V_0 \geq V_2.$$

Hence, (6) and (7) are the buyer's incentive-compatibility constraints. Combining (4)–(7) implies that bargains acceptable to both sides must satisfy $u(q_i) - c(q_i) \geq 0$ for $i = 1, 2$.

4 In this case, the seller could choose any value of $0 \geq \pi_0^i \geq 1$, since he is indifferent. However, the buyer can always decide to accept a slightly smaller quantity of goods in return for the currency. This would make it rational for the seller to set $\pi_0^i = 1$, since he receives a positive surplus from trading.

FIGURE 1

Possible Monetary Equilibria



SOURCE: Authors' calculations.

A Dual-Currency Equilibrium

Consider the equilibrium in which both currencies are fully acceptable, $\pi_0^i = \Pi_0^i = 1$, for $i = 1, 2$. This corresponds to a dual-currency economy. Under full acceptability and buyer-take-all bargaining, equations (3)–(5) imply that $V_0 = 0$. In short, since buyers extract all the surplus from trade, the value of being a seller is zero.[5] All that remains is to determine the quantities q_1 and q_2 that can then be used to solve for V_1 and V_2 through equations (4)–(5). Substituting (4)–(5) into (1)–(2) for V_1 and V_2, with $V_0 = 0$, yields

$$(8) \quad c(q_2) + t_2/r = \alpha x m_0 [u(q_2) - c(q_2)]/r$$

and

$$(9) \quad c(q_1) + t_1/r = \alpha x m_0 [u(q_1) - c(q_1)]/r.$$

Equations (8)–(9) yield two independent equations in two unknowns, q_1 and q_2. If (8) and (9) have a solution, (q_1^*, q_2^*), in which the equations' right side is positive, then we have a steady-state equilibrium that satisfies the buyer's incentive-compatibility constraints.

Since (8) and (9) are functionally the same equation, we can illustrate the solution to both with figure 1. The right side of (8) and (9) is the expected present discounted value of the surplus from trading minus the holding cost and, given our assumptions on the utility and cost functions, is the hump-shaped curve in figure 1. The left side of (8) and (9) is an upward-sloping function in q with an intercept of t_i/r, which depends on the time cost of holding money of type i, t_i, normalized by the interest rate, r. For $t_1 = t_2 = 0$, it is relatively easy to show that a nonzero solution to (8) and (9) exists, is unique, and satisfies the buyer's incentive-compatibility constraints. The values of (q_1^*, q_2^*) that solve this equation will be a function of the parameters governing the trading environment and the preferences of the individual agent. The decision to accept a currency is endogenous and also a function of these fundamentals; we do not impose the currency's use from outside the model. Hence, we are able to derive the existence of a dual-currency equilibrium for an optimizing model of exchange in which the acceptability of the currencies is endogenously determined.

For the case of $t_1, t_2 > 0$, we will have either two solutions to each equation, (q_i^L, q_i^H), or no solution.[6] In this situation, holding money is costly, so a seller must be compensated for giving up his good to accept money and incur this holding cost. If the cost of holding money is sufficiently low, then there are a low and a high equilibrium. Again, if $t_1 = t_2$, the two solutions are the same for both currencies, since they are indistinguishable. As this cost increases, the low and high equilibrium values converge until, for sufficiently high costs, no equilibrium exists. In short, sellers demand such a high payoff to overcome the holding cost of money when becoming buyers that no trade exists, which is incentive-compatible for current buyers. Figure 1 shows how a single-currency equilibrium arises. If t_1 is sufficiently small and t_2 is sufficiently large, then private agents will use currency 1 but not currency 2 to trade, and vice versa. Thus, if the government can manipulate these two holding costs, it can alter the acceptability of each currency relative to the other.

5 If we assumed an alternative bargaining structure, such as Nash bargaining, V_0 would be greater than zero but it would be harder to derive the equilibrium quantities traded.

6 On the other hand, if holding a unit of money provides a net benefit in and of itself, $t_i < 0$, then if a monetary equilibrium exists, it will be unique. This would be shown in figure 1 by shifting $c(q_i)$ down to reflect a negative intercept. As a result, the two curves would intersect once in the positive quadrant, if at all.

We will return to this idea in section III to discuss a case in which one government accomplished this feat by altering the relative transaction costs to "de-dollarize" its economy.

Implications of the Model

Several important aspects of the model are worth mentioning. First, for $t_1, t_2 > 0$, there are multiple monetary equilibria. This common feature of search models illustrates the importance of focusing on the dynamics of the model to determine which equilibria are stable and which unstable. However, it is possible to Pareto-rank the two equilibria—q_i^H yields the higher equilibrium value of V_i. Since production is costly, sellers will be willing to incur the greater disutility of producing q_i^H only if the unit of currency they receive in return is more highly valued. As a result, unless $V_i(q^H) > V_i(q^L)$, it would not be rational to produce q^H rather than q^L.

Second, the currency values are independent of one another, that is, q_1 does not depend on q_2, and vice versa. In general, we would expect economic fundamentals that change the value of one currency to change the value of the other currency as well. For example, if the holding cost of one currency increases, we might expect to see currency substitution to occur as people move from the high-cost currency to the other one. Portfolio reallocation of this type does not happen here.

Third, the only difference between the two currencies' values results from the ad hoc cost/benefit ratio of holding the currencies. In the case where $t_1 = t_2$, it is also true that $q_1 = q_2$, since (8) and (9) are identical. This means that the two currencies are identical. If that is so, why use more than one? The use of two different currencies suggests that there is a fundamental difference between them; the model above suggests that the difference is related to the holding costs or benefits of each currency rather than the trading environment.

Fourth, currency trades do not occur in the model because of the one-unit-of-money inventory restriction.[7] Consequently, no nominal exchange rate exists in the model. Nevertheless, an implicit real exchange rate exists, given by the ratio of q_1/q_2. However, it will differ from 1 only if $t_1 \neq t_2$. In short, the real exchange rate depends on costs that are determined outside the model rather than on the trading environment or on preferences.

Fifth, the relative amounts of the two currencies in the economy, m_1 and m_2, do not influence the equilibrium quantities of goods. This is a result of the buyer-take-all assumption. The values m_1 and m_2 appear in the returns to search only for the seller, rV_0. But with the buyer-take-all condition, $V_0 = 0$ for all values of m_1 and m_2. Consequently, the actual stocks of each currency are irrelevant for determining their equilibrium values. This result would change if we adopted a more general bargaining structure such that both parties gain from trade.

Sixth, the closed-economy assumption used here is plausible for two domestic currencies. However, in most dual economies today, one of the currencies is foreign. Hence, the model ignores the fact that the foreign currency enters the economy in some fashion and also leaves the economy through purchases of imported goods. Therefore, the model is clearly missing an important feature of dual-currency economies, namely, trading interactions with other economies. To address this issue, one needs a two-country, two-currency model of money.

Finally, in the current set-up, it is difficult to see exactly how government policy can affect the equilibrium values of q_1 and q_2, since they depend only on the fundamentals of the trading environment and on preferences. Unless one argues (as we did above) that the government can alter the fundamental costs of holding the two currencies, a more elaborate formulation is needed to capture fully the government's role in the search model.

As the foregoing discussion points out, the simple model outlined above is lacking in many dimensions, despite its appealing features for endogenously determining the acceptability of fiat currencies. In the next section, we review the literature to show how the simple search model described here can be amended to include more interesting and realistic features of dual-currency economies.

7 There are equilibria in this model in which a unit of currency 1 trades for a unit of currency 2 plus some amount of goods and vice versa. However, these trades cannot occur if we assume that consumption must precede production, since they require the currency-2 money trader to produce twice before consuming.

II. Research on Dual-Currency Search Models

One-Country, Two-Currency Models

Indivisible Goods

In their first paper on search models of money, Kiyotaki and Wright (1989) look at the possibility of two commodity monies circulating in a closed economy. In this model, agents produce an indivisible good and then carry it with them in search of trading partners. As a result, there are real storage and transportation costs associated with goods production. Kiyotaki and Wright then ask whether one or more of the commodities will be accepted by traders who do not want the good for consumption purposes. They show that in certain equilibria, one of the goods uniquely serves as commodity money (the low-storage-cost good); in other equilibria, two of the goods circulate as money (the two lowest-cost goods). The authors explore whether a good that is useless to all traders but has zero storage costs can circulate as money—a true fiat currency. They show that there are equilibria in which both the commodity money and fiat money circulate. Thus, this was the first search model that looked at dual-currency issues.

In a 1993 paper, the same authors present a more elegant description of the indivisible-good-and-money search model and its solution. They explore the coexistence of two fiat currencies that have different dividends or storage costs, similar to t_1 and t_2 in the model described in section I. They show that there are equilibria in which both currencies circulate. Similar equilibria are derived in Aiyagari and Wallace (1992). One interesting feature of Kiyotaki and Wright's results is that even if the two currencies have identical storage costs ($t_1 = t_2$ in the model above), they may have different acceptabilities and thus different expected exchange values. Kiyotaki and Wright show that an equilibrium exists in which one currency is fully acceptable ($\Pi_0^1 = 1$) while the other is only partially acceptable ($0 < \Pi_0^2 < 1$). Since it is less widely accepted, the value of holding it is lower. This is an appealing finding for those of the developing and transitional economies where the foreign currency is not universally accepted in exchange. Thus, the dual-currency equilibria with different acceptabilities mimic a real-world feature of dual-currency economies.

Divisible Goods

Kiyotaki and Wright's finding that two identical currencies can have different trading values is puzzling, given our model showing that if $t_1 = t_2$, the two currencies will have the same equilibrium exchange value. What is the reason for these contradictory results? The different exchange values result from having a partially acceptable currency, which in turn results from the use of mixed strategies when goods are indivisible. Mixed strategies come into play when sellers are indifferent as to accepting or rejecting money in exchange for an indivisible unit of the good. Shi (1995) and Trejos and Wright (1995a) demonstrate that when goods are divisible, the buyer can always offer to take an infinitesimally smaller amount of the good to ensure that the seller is not indifferent and trade occurs. Since the seller will always accept such an offer, partial acceptability never occurs. Thus, while introduction of divisible goods into the standard search models was a great step forward in understanding exchange, it eliminated an empirically relevant equilibrium, namely, the partial acceptability of one of the currencies. Amending the search model to generate partial acceptability (when only some sellers accept the foreign currency all of the time) remains to be done.

Another feature of the equilibrium derived in the model in section I is that if $t_1 = t_2$, both currencies circulate at par. How can we generate different trading values of the currencies in the model above? Aiyagari, Wallace, and Wright (1996) show that there are more equilibria in our simple model than we discuss: An equilibrium exists in which one currency is perceived to have more value than the other. In pairings of currency-1 and currency-2 money traders, currencies are exchanged and the holder of the less valuable currency (the seller) also gives up some goods. To see this, suppose agents believe for some reason that currency 2 is more valuable than currency 1. One can think of these equilibria as ones in which the seller "gives change"—trades some of the good for the currency but also gives the buyer change (the seller's currency). This version of the model would alter equations (1)–(3) as follows (setting $V_0 = 0$ and $\Pi_0^1 = \Pi_0^2 = 1$, for $i = 1, 2$.):

(10) $$rV_2 = \alpha x m_0[u(q_2) - V_2] - t_2 + \alpha x m_1[u(q_{12}) + V_1 - V_2],$$

(11) $rV_1 = \alpha x m_0[u(q_1) - V_1] - t_1$

$+ \alpha x m_2[-c(q_{12}) + V_2 - V_1]$,

and

(12) $rV_0 = 0$,

where q_{12} is the quantity of goods given up by a currency 1 holder in addition to his unit of currency 1 in return for a unit of currency 2. Assuming that the currency 2 holder makes a take-it-or-leave-it offer to the currency 1 holder, the bargaining conditions for q_1, q_2, and q_{12} are

(13) $V_1 = c(q_1)$,

(14) $V_2 = c(q_2)$,

(15) $V_2 - V_1 = c(q_{12})$,

and

(16) $V_2 - V_1 \leq u(q_{12})$,

where (16) is the incentive-compatibility constraint for the currency 2 holder in a "making change" trade. Equations (15)–(16) imply $u(q_{12}) \geq c(q_{12})$. Substituting (13)–(15) into (10)–(11) and then subtracting (10) from (11), with $t_1 = t_2$, yields

(17) $c(q_2) + t_2/r = \{\alpha x m_0[u(q_2) - c(q_2)]$

$+ \alpha x m_1[u(q_{12}) - c(q_{12})]\}/r$,

(18) $c(q_1) + t_1/r = \alpha x m_0[u(q_1) - c(q_1)]/r$,

and

(19) $c(q_{12}) = \{\alpha x m_0[u(q_2) - u(q_1)]$

$+ \alpha x m_1[u(q_{12}) - c(q_{12})]\}/(r + \alpha x m_1)$.

Equation (18) is the same as equation (9), so the solution for q_1 is the same as before. However, it is clear from (17) that, since the last term must be positive for "making change" trades to occur, $q_2 > q_1$ if a monetary equilibrium exists. Finally, $q_2 > q_1$ implies that (19) yields a positive value for q_{12} (under appropriate parameter values).

The point of this example is that even though the two currencies are fundamentally the same, as long as traders believe that one of the two is more valuable, it will be in equilibrium, and currency 2 will trade for currency 1 if the currency 1 trader gives a "side payment" of q_{12}. One last point is that, in this example, the quantity q_2 coming out of (17) depends on q_{12}, while the value of q_{12} that solves (19) depends on both q_1 and q_2. Thus, unlike the equilibrium studied in section I, the values of the two currencies in this equilibrium are interdependent.

Shi (1995) proposes an alternative method for generating different trading values for fundamentally equivalent currencies. He assumes that different currencies are associated with different bargaining arrangements. For one currency, the bargaining rule is buyer take all, with the seller getting zero surplus from the trade; the other currency trades under a Nash bargaining rule in which the surplus of trade is split between the buyer and seller. In short, Shi assumes that the bargaining conditions in (4)–(5) now look like

(4′) $V_1 = c(q_1) + V_0$

and

(5′) $V_2 > c(q_2) + V_0$.

Under this formulation, sellers expect to receive a positive surplus if they trade for currency 2, but no surplus when they trade for currency 1. Shi then shows that both currencies can circulate in trade but with different trading values, since sellers view the two currencies differently. However, it is not clear why traders would adopt different bargaining strategies based solely on the currency's national origin.

Gresham's Law

The one-country, two-currency framework has also been used to study Gresham's Law, which posits that "bad money drives out good." Velde, Weber, and Wright (1999) use a search model to study this long-standing issue. Their framework features a commodity money that yields a dividend to its holder; good money generates a higher dividend than bad. In the model above, this would correspond to currency 1 being the good one and currency 2 being the bad one by setting t_1, $t_2 < 0$ and $t_1 < t_2$. Velde, Weber, and Wright assume that some sellers have imperfect information and cannot determine which currency they are trading for. This creates a "lemons" problem—uninformed sellers are not willing to produce a sufficient amount of the commodity for the good money, since they are afraid of getting the bad money in return. The authors show that under some parameters, holders of the good money will not trade with these uninformed sellers, who undervalue the good currency. In this sense, Gresham's Law holds, since only the bad money circulates in trade.

Private versus Public Currency

As we mentioned earlier, the coexistence of privately issued bank notes and government-issued currency ("outside" money) has a long history in the United States. However, in the typical search model, no individual can unilaterally issue his own commodity-backed currency, since it would have to be redeemable at some point—an impossibility if all trading histories are private information. Cavalcanti and Wallace (1999) loosen this assumption and allow the trading histories of a subset of agents to be public information. These agents can then effectively function as banks and thus issue commodity-backed banknotes. Calvalcanti and Wallace show that banknotes and government-issued currency can coexist if the supply of outside money is sufficiently scarce.

Government Policy

Although the government is implicitly present in the search models as the creator of fiat currency, a prototypical search model lacks an active government and so has very little analysis of government policy.[8] Incorporating government into search models typically means assuming that the government is a subset of agents in the economy who adopt various strategies for trading when matched with private agents. With regard to government policy in dual-currency economies, Curtis and Waller (2000) argue that a policy in developing and transitional economies commonly makes the foreign currency illegal for internal trade. To give this illegality any meaning, however, the government must enforce the policy. Curtis and Waller adopt Li's (1995) approach, assuming that when government agents meet private agents who hold foreign currency 1, they either confiscate the currency or impose a fine.[9] Li assumes that a proportion g of the agents in the economy are government agents, with $g = g_0 + g_1$, where g_0 is the proportion of government agents without a unit of currency 1 and g_1 is the proportion of government agents holding a unit of the foreign currency 1. Upon meeting a holder of currency 1, government agents without a unit of currency confiscate the currency and use it to buy goods from sellers according to a take-it-or-leave-it offer. In addition, the government imposes a fine on using the foreign currency which corresponds to having $t_1 > 0$ and $t_2 = 0$. Under this set of assumptions, the returns to search equations (1) and (2) become

$$(20)\quad rV_2 = \alpha x m_0 [u(q_2) - V_2]$$

and

$$(21)\quad rV_1 = \alpha x m_0 [u(q_1) - V_1] - g_0 [V_1 - V_0 - t_1].$$

The second term in (21) is the expected cost to a holder of currency 1 of having the currency confiscated by the government agent and paying the fine. Curtis and Waller show that in some variants of the model, increased enforcement of currency restrictions lowers the trading value of the foreign currency and can drive it out of the economy while strengthening the value of the domestic currency. In figure 1, this would correspond to increasing t_1 to the point where currency 1 is not accepted.

Li and Wright (1998) take a different approach to studying policy. They too assume that government agents produce goods for money, but rather than accepting a currency according to optimizing behavior, they base acceptance on an exogenously determined trading rule. They examine a dual-currency economy to see whether a government strategy of "accept domestic currency, reject foreign currency" can drive the foreign currency out of circulation. The model presented in section I can be amended to illustrate this argument. Let currency 1 be the foreign currency. Let g be the proportion of government agents in the economy, with $g = g_0 + g_2$, where g_0 is the proportion of government sellers in the economy and g_2 is the proportion of government agents holding a unit of the domestic currency 2. The adding-up constraint requires that $1 = m_0 + m_1 + m_2 + g$, which implies $m_0 = 1 - m_1 - m_2 - g$. Government buyers make take-it-or-leave-it offers to sellers and accept such offers of currency 2 but not currency 1. The returns to search for holders of currency 1 and currency 2,

8 Ritter (1995) explicitly models the government as a subset of private agents who get together and issue currency and adopt the strategy that they will always accept the currency in trade. However, it is hard to distinguish a government from a private bank in his model. Dual currencies would reflect currencies issued by competing private banks or by competing governments, either state or local.

9 In single-currency models, confiscation by the government is considered equivalent to an inflation tax.

given earlier in equations (1) and (2), are now given by

$$(20') \quad rV_2 = \alpha x m_0 [u(q_2) - V_2] - t_2 + \alpha x g_0 [u(q_{g2}) - V_2]$$

and

$$(21') \quad rV_1 = \alpha x\,(1 - m_1 - m_2 - g)[\,u\,(q_1) - V_1] - t_1,$$

where q_{g2} is the quantity of goods a government agent gives up for a unit of currency 2. From these two equations we see that as g increases, all else being equal, the value of holding currency 1 falls, while the corresponding increase in g increases V_2 through the increase in g_0. Thus, Li and Wright show that if government is a large enough subset of the population, its transaction strategy will succeed in driving the foreign currency out of circulation.

Velde, Weber, and Wright (1999) adopt a different approach to modeling government policy in their study of Gresham's Law. They have the government adopt a debasement policy whereby private agents can bring in the high-value commodity money and convert it to the low-value commodity money. The private agent gets some of the surplus commodity from reminting for consumption purposes, and the government gets some of the commodity as seigniorage revenue. For certain parameterizations of their model, all holders of the good money will choose to remint their coins; thus, government seigniorage policy is capable of driving out one of the currencies.

Multiple Money Holdings

All the models described so far share one key assumption—the restriction that agents cannot hold more than one unit of money. Allowing agents to hold more implies that the proportion of agents holding a certain quantity of money is not constant, since people buy their way into and sell their way out of a level of money holdings. Permitting agents to hold multiple units requires solving for a steady-state distribution of money holdings in addition to all the quantities traded between a large (possibly infinite) number of traders who enter into bargaining with differing levels of money holdings. While research has begun to move in this direction for one-country, one-currency models (see Molico [1998], Camera and Corbae [1999], and Green and Zhou [1998]), very little has been done for *two*-currency, multiple-money holding search models. An exception is the work of Craig and Waller (1999), which examines how agents choose to hold *portfolios* of currencies and how the government's "inflation tax" policies affect the values of these portfolios. That model merges the inflation-tax model of Li (1995) with the multiple-units-of-money model of Camera and Corbae (1999). Although we do obtain some analytical results, in general the model must be solved using numerical methods. We find equilibria that mimic the simple model above: If the currencies are fundamentally equivalent (no inflation tax), then similar portfolios will have similar value in trade. Furthermore, when the currencies are fundamentally different because of their inflation-tax risk, we find parameterizations in which currency trades for currency plus goods in equilibrium (the Aiyagari, Wallace, and Wright [1996] result for portfolios or currencies). We also find parameterizations in which currency trades for currency. This latter result is interesting in that currency-for-currency trades occur when a single coincidence of wants does not arise; hence, there are pure financial trades in the model. Also, the existence of currency trades creates an explicit nominal and real exchange rate.[10] A typical dual-currency search model has an implicit endogenous real exchange rate but no endogenous nominal exchange rate.

Two-Country, Two-Currency Models

Until now, all of the models discussed were closed-economy models with multiple media of exchange. In addition to two-currency, one-country models, a fair amount of research has tried to capture the open-economy aspects of dual-currency models.

The earliest two-country, two-currency search model that we know of is Matsuyama, Kiyotaki, and Matsui (1993). Their paper uses the simple indivisible-commodity, indivisible-money model of Kiyotaki and Wright (1993), but designates agents as coming from different countries. These agents are randomly paired with agents from their own country and the other country and then decide to trade or not. A key issue is whether one or both currencies will be acceptable in international pairings of

10 There is actually a distribution of nominal and real exchange rates, since individual pairs of traders can specify different quantities of goods and currencies to be exchanged, depending on the portfolio composition of the buyer and seller. This is equivalent to the price distributions obtained by Camera and Corbae (1999).

traders. Furthermore, the authors consider conditions under which the foreign currency will be used as a medium of exchange between two domestic traders. In this model, no currency exchange occurs, despite its international flavor; due to the indivisibilities of goods and money, the implied real exchange rate is 1.

Zhou (1997) amends the Matsuyama, Kiyotaki, and Matsui model to generate currency exchange. He assumes that some home agents desire home goods produced by home agents, while others desire goods produced by foreign agents. However, agents' preferences are subject to random shocks, which means they may switch from home goods to foreign goods or vice versa. In this set-up, home-goods producers who prefer to consume other home goods never accept foreign currency from foreign buyers. However, home-goods producers who want to consume foreign goods will accept foreign currency to pay for that consumption. Consequently, at any point in time, there are buyers of country 1 with country-2 currency and vice versa. At each point, some of these traders receive a shock that reverses their consumption preferences so that they now want to consume home goods. But home-goods sellers will not accept the foreign currency and the buyer does not want to use it to buy foreign goods. Consequently, buyers are stuck with the foreign currency unless they are paired with a foreign agent who is holding the home currency and has also experienced a preference reversal. As a result, in each period there is some currency exchange between country-1 buyers holding currency 2 and country-2 buyers holding currency 1. Although Zhou generates currency exchange in this model, the nominal exchange is always 1:1 because of the restriction that agents can hold only one unit of either currency.

Trejos and Wright (1995b, 1996) extend the model of Matsuyama, Kiyotaki, and Matsui by allowing for divisible goods. They derive conditions under which a) both currencies are national; b) there is one national and one international currency; and c) both currencies are international. Furthermore, they show that the currencies will have different trade values depending on several factors such as the probability of meeting someone from one's own versus the other country, the relative quantities of the two currencies in circulation, and whether the transaction occurs between two traders from the same country or traders from two different countries. They show that if the economies are symmetric and both currencies circulate internationally, then they have equal trading value. This is similar to the findings of the model in section I. Trejos and Wright also incorporate different government policies following the method of Li (1995) to see whether wildly different government inflation policies can either drive out the foreign currency for internal trades or cause it to be used for internal trade. In this set-up, each government's agents confiscate their own currency from traders they meet, regardless of their nationalities.

III. A Case Study of Government Policy in a Dual-Currency Regime

In this section, we describe a government's confrontation with the problem of making its new fiat currency, the *hryvna*, acceptable in a dollarized economy. The government also wanted to de-dollarize the economy in order to secure more seigniorage revenues. The country is Ukraine, which introduced the hryvna as part of its 1994 currency reform.

After the ruble zone collapsed, Ukraine issued its own currency, which was a coupon (although it behaved exactly like explicit fiat currency). Seigniorage considerations led to a rapid increase in the issue of these coupons, which produced hyperinflation of 10,000 percent in 1994. The result was massive currency substitution by Ukrainian citizens and the dollarization of the nation's economy. After stabilizing inflation by restraining the issuance of coupons, the government faced the challenge of issuing a new fiat currency—the hryvna—and inducing Ukrainian citizens to use it rather than dollars. Made cautious by the previous hyperinflation, however, citizens seemed reluctant to give up their dollars for hryvnas.

This situation can be described by our model in section I. Let currency 1 be the dollar and currency 2 be the hryvna. We showed that if t_1 was sufficiently low and t_2 sufficiently high, the dollar would be acceptable in trade but the hryvna would not be. This is, in some sense, similar to the situation that confronted Ukraine's central bank when it introduced its new currency.

Given the analysis in our model, how could the central bank reverse this situation? It had to figure out a way to lower the holding cost of the hryvna and raise that of the dollar as a medium of exchange. Raising the costs of using the dollar was easy—make it illegal and enforce the laws strictly enough to drive it out of the economy. This is the result Curtis and

Waller (2000) report for currency restrictions. How could the government lower the cost of using the hryvna? Since it had generated hyperinflation very recently, its promises to keep inflation low were probably not credible. With this in mind, we can think of the holding cost as the utility loss arising from the risk of devaluation through hyperinflation. Since the threat of another hyperinflation was very real, sellers demanded such a high premium to accept the local currency that using it as a medium of exchange was not worthwhile. Unless the holding cost of the hryvna fell, the launch of the new currency would fail.

The government needed a commitment device to lower the cost of using the hryvna yet make it easy to switch back to dollars should inflation get out of control. The government decided to make it very easy to obtain a license to set up a currency-exchange booth. As a result, booths sprouted up all over (particularly in Kiev), with three dramatic effects on citizens' willingness to hold the local currency. The proliferation of exchange points made it easy to exchange the currency in a hurry; this minimized the nonpecuniary "shoe-leather" costs of converting the hryvna into dollars, thereby increasing the hryvna's liquidity. Second, competition among the multitude of currency exchanges lowered the buy–sell spread almost to zero, which made the pecuniary costs of currency conversion almost nil.[11] Finally, since all currency exchanges posted the exchange rate, they acted as an information loudspeaker regarding the behavior of current monetary policy. Simply by glancing in the windows as one walked around the city, it was very easy to see if the hryvna was depreciating as a result of loose monetary policy. In short, the presence of competitive currency exchanges dramatically lowered the cost of holding the new currency. By recognizing how the fundamentals of trading affected the acceptability of currencies, the Ukrainian government was able to launch a new currency and significantly reduce the dollarization of its economy.

IV. Concluding Thoughts

The purpose of this article is to examine how modern monetary theory aids our understanding of an old and venerable multiple-payments system—the dual-currency economy. Dual-currency economies persist today as a way to avoid devaluation of domestic currencies, unstable banking systems, and government restrictions on trade using other means of payment. Monetary search models are very useful for studying how currency acceptability arises endogenously in economies that lack more sophisticated payment systems. While search models' basic assumptions may not be consistent with modern financial systems, they provide fairly good descriptions of transitional and developing economies. In particular, the economies of the former Soviet Union are well described by the basic assumption of the search models—an absence of credit, a lack of smoothly functioning banking systems, reliance on currency as the sole medium of exchange, and primitive trading environments. Thus, the application of dual-currency search models to these economies should yield interesting case studies of monetary theory and will offer potentially helpful policy prescriptions for the beleaguered governments of these countries.

■ **11** In June 1998, the buy–sell spread in downtown Kiev was ½ cent per dollar exchanged, or 50 cents per $100. There were no fixed commissions on exchanges.

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New Results on the Rationality of Survey Measures of Exchange-Rate Expectations

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Introduction

Analyses of economic and financial developments often rely on propositions about the rationality of market participants. Particularly in financial markets, where information is widely and readily disseminated, it is commonly presumed that economic agents have "rational expectations" about the future course of events. This assumption can have powerful implications for the efficacy of certain government policies.

Questioning the rationality of foreign-exchange market participants is particularly tempting, in light of the widely acknowledged poor performance of many economists' models of exchange-rate determination. Models that view exchange rates as equal to the expected present discounted value of future "fundamentals" (for instance, monetary policy, fiscal policy, and trade flows) and assume that participants have rational expectations about future fundamentals have done poorly in predicting exchange-rate movements. Although this latter finding has stimulated a wide body of research, the validity of the *rational expectations hypothesis* remains unresolved.

An anomaly in international finance related to the rational expectations hypothesis is the forward discount puzzle, in which the forward foreign exchange rate predicts the wrong direction of movement for the future spot rate. Most verifications of this puzzle presume rational expectations and, thus, are worth reconsidering if the hypothesis is to be rejected. Lewis (1995) summarizes work surrounding this puzzle. Baillie and Bollerslev (1997) examine earlier findings, suggesting that with more recent data the puzzle no longer appears.

Dominguez and Frankel (1993) illustrate how crucial the rational expectations assumption might be in analyzing the impact of government policy. They conclude that central banks' foreign-exchange intervention had a significant impact on risk premia in currency markets during 1982–88. However, their finding hinges on survey data on exchange-rate expectations. When rational expectations are imposed, intervention is seen to be ineffective.

In this article, I extend previous research on the rationality of survey measures of expectations for foreign exchange rates. Section I reviews the literature, highlighting several interpretations of rationality. Section II provides some motivation for my choice of econometric tests. Section III presents summary information about the data and then the results of the main tests of interest. Finally, I summarize what has been gained from the exercise and what might be suggested for future research.

I. Related Literature

Econometric analyses of survey data have typically focused on the rationality of participants' expectations. The most familiar interpretation of rationality is expressed in terms of expectations representing unbiased forecasts of the actual future outcome. However, several related questions might be of interest. For example, one might question whether expectations incorporate—or react to—news of fundamentals. In addition, rationality might imply a specific relationship between short-run and long-run expectations. These questions suggest the value of studying alternative mechanisms through which expectations are formed, and they are closely related to the issue of whether a risk premium exists in foreign exchange markets.[1]

In his survey of surveys, Takagi (1991) notes three characteristics of survey data on expectations of future exchange rates. First, the dispersion of expectations tends to increase with the forecast horizon, an outcome that may be related to group effects.[2] Interestingly, Ito (1993) finds that Japanese exporters had expectations of greater yen depreciation, while Japanese importers had exactly the opposite expectations.[3] Second, expected changes in exchange rates tend to underpredict the actual extent of exchange-rate movements, implying that much of actual exchange-rate movements are unexpected. The third characteristic is referred to as "twist"—that is, longer-run expectations tend to reverse the direction of short-run expectations. For example, an appreciation would tend to be followed by an expectation of further depreciation, but an expectation of further appreciation in the more distant future.

The "Unbiasedness" Interpretation of Rationality

This paper will focus on the most familiar interpretation of rationality—that survey measures are unbiased forecasts of actual future outcomes. Dominguez (1986) tested this hypothesis by regressing actual depreciation on expected depreciation for the 1983–85 period using data from both Money Market Services and the Japanese Center for International Finance. Her results strongly reject the "unbiasedness" hypothesis with one-week, one-month, and three-month data. Ito (1993) also rejected the hypothesis for the 1985–87 period, at least for longer-run horizons. Cavaglia, Verschoor, and Wolff (1993) confirm this finding with EMS exchange rates, as well as with exchange rates against the U.S. dollar for 1986–90.[4] Beng and Siong (1993) also reject unbiasedness for the Singapore currency against the dollar for 1984–91 for all forecast horizons. However, Liu and Maddala (1992), using cointegration techniques, cannot reject the rational expectations hypothesis for the 1984–89 period.

Orthogonality

The second most familiar interpretation of the rational expectations hypothesis is that expectations incorporate all available information. Takagi (1991) summarizes examinations of this "orthogonality" hypothesis. Generally, Dominguez (1986), Froot and Frankel (1989), Ito (1993), MacDonald and Torrance (1989), Cavaglia, Verschoor, and Wolff (1993), and Beng and Siong (1993) find that the survey data do not fully incorporate all available information.

Long Run versus Short Run

Findings of a twist (short-run expectations show bandwagon effects, but long-run expectations are stabilizing) motivate an examination of the connection between the short run and long run. Froot and Ito (1989) propose a defini-

■ **1** The use of survey data to extract risk premia or to study the mechanisms through which expectations might be formed will not be discussed here, except to note that several of the articles surveyed discuss the issues. Among those that use survey data to extract risk premia are Frankel and Froot (1987a) and Dutt and Ghosh (1995). Among studies of mechanisms, see Frankel and Froot (1987a, 1987b).

■ **2** Cavaglia, Verschoor, and Wolff (1993) find that the mean expected depreciation tends to fall with the forecast horizon, as does the variance of forecast errors. This is contrary to the findings of other researchers.

■ **3** Ito (1993) is the only study to use panel data and to examine heterogeneity among survey respondents. He also finds individual as well as industry effects. As Ito points out, if all relevant information about exchange rates is public, the finding of heterogeneity implies a rejection of rationality.

■ **4** These authors remind us that the "peso problem"—wherein market participants allow for a small probability of a large change in the future exchange rate—can explain an ex post finding of bias even if expectations are formed rationally.

tion of consistency. However, any definition of consistency requires the mechanism through which expectations are formed to be specified. Their evidence is mixed with results that differ with horizon. Ito (1993) confirms the existence of twist and also rejects consistency. However, in their study of Singapore's currency, Beng and Siong (1993) find no evidence of twist.

Expectations Formation

Takagi (1991) summarizes studies of the mechanisms of expectations formation by Frankel and Froot (1987a,b), the Bank of Japan (1989), and Froot and Frankel (1990). *Extrapolative expectations* means that the expected currency movement is related to the most recent movement. Generally, examination of this mechanism leads to the conclusion that bandwagon effects are present in the short run, but effects of the opposite sign are present for longer horizons. The effects are usually stabilizing. With *adaptive expectations*, the expected movement represents an average of the actual current and the expected current. The results of Frankel and Froot (1987a, b) are inconclusive regarding the validity of this mechanism. The length of the horizon seems to matter, and findings are not inconsistent with an unanticipated appreciation leading to an expected depreciation in the long run. *Regressive expectations* are said to exist when the actual exchange rate is expected to move toward an equilibrium rate. The results of Frankel and Froot (1987a, b), the Bank of Japan (1989), and Froot and Frankel (1990) point to the conclusion that expectations can be destabilizing in the short run, moving away from equilibrium, whereas the opposite effect occurs for longer horizons. Beng and Siong (1993) examine the same three mechanisms and find stabilizing extrapolative expectations but no stabilizing adaptive mechanism operative; in terms of the regressive mechanism, both short-run and long-run expectations move backward toward an equilibrium value. Ito (1994) finds that, despite the presence of mean reversion in the actual exchange-rate series for the yen, such reversion was not captured in a six-month horizon for expectations.

Chartists and Fundamentalists

Froot and Frankel (1990) and Allen and Taylor (1990) have suggested that the differences between short-run and long-run expectations might be related to the types of forecasting techniques employed. In particular, it is reasonable to speculate that short-run forecasts are derived from "chartists," or technical analysis, while longer-run forecasts are based on models of fundamentals. This possibility has been largely unexplored (as of Takagi [1991]). Hung (1997) relates the activities of chartists to the conduct of U.S. central bank foreign-exchange intervention policy. Bhattacharya and Weller (1997) explore possible implications of asymmetric information for intervention.

II. Test Specifications

In this article I test the unbiasedness hypothesis using Money Market Services data for 1989–97 on the deutsche mark/dollar (DM/\$) exchange rate. My results will update Liu and Maddala's (1992) analysis of 1984–89. In order to properly compare our results with those previously published, I will review the progression of econometric techniques that have been utilized in this area.

At first one might be tempted to estimate the equation

$$(1) \qquad S_{t+k} = \alpha + \beta S^{e}_{t,t+k} + e_{t+k},$$

where S_{t+k} is the actual future exchange rate and $S^{e}_{t,t+k}$ is the expectation at time t of the future exchange rate at $t+k$.[5] The unbiasedness hypothesis stipulates that $\alpha = 0$ and $\beta = 1$. In addition, we might look at the orthogonality condition, regressing $(S_{t+k} - S^{e}_{t,t+k})$ on information available at t and testing for coefficients equal to zero. Or, we might test to see if forecast errors were serially uncorrelated. If the latter is not the case, it would imply that the forecast could be improved by considering past errors. However, as a wide body of research has discussed, if S_{t+k} and $S^{e}_{t,t+k}$ are nonstationary and follow unit-root processes, conventional t tests will be incorrect. To avoid this, some have suggested estimating equation (2)

$$(2) \qquad S_{t+k} - S_t = \alpha + \beta (S^{e}_{t,t+k} - S_t) + e_{t+k},$$

and testing $\alpha = 0$ and $\beta = 1$. Although the left side of equation (2) is stationary, if both components have unit roots it is not clear that the same can be said for the right side. By analogy

■ 5 For the time being, I ignore the serial correlation issues that arise when k is not equal to one when the sampling frequency is one.

FIGURE 1

One-Week-Ahead Exchange Rate

SOURCE: Author's calculations.

FIGURE 2

One-Month-Ahead Exchange Rate

SOURCE: Author's calculations.

to the arguments of Liu and Maddala, the right side can be written as $(S^e_{t,t+k} - S^e_{t-1,t+k-1}) + (S^e_{t-1,t+k-1} - S_t)$; the second term is stationary only if $\alpha = 0$ and $\beta = 1$. Another possibility is to estimate equation (3),

$$(3) \quad S_{t+k} - S_{t+k-1} = \alpha + \beta(S^e_{t,t+k} - S^e_{t-1,t+k-1}) + e_{t+k},$$

which omits $\upsilon(S^e_{t-1,t+k-1} - S_t)$ from the right side.

I will follow the more direct approach suggested by Liu and Maddala. First, note that if S_{t+k} follows a random walk, so should its rational forecast; thus, the two time series should be cointegrated with a factor of 1 and random residuals. However, standard cointegration tests allow estimation of the cointegrating factor and do not require random-error terms. Rather than estimate equation (1) as the cointegrating equation, we first test whether S_{t+k} and $S^e_{t,t+k}$ are unit-root processes, and then test for the stationarity of $\mu_t = (S_{t+k} - S^e_{t,t+k})$. The second step restricts $\alpha = 0$ and $\beta = 1$, so that it can be referred to as a restricted cointegration test. As Liu and Maddala state, if μ_t is stationary, then S_{t+k} and $S^e_{t,t+k}$ are cointegrated with a factor of 1 because the cointegrating factor is unique when it exists.

Testing for Unit Roots

The most widely used unit-root tests for a variable y_t rely on equations of the form

$$(4) \quad \Delta y_t = \delta_0 + \delta\, t + (\alpha - 1) y_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta y_{t-j} + e_t$$

and focus on the coefficient associated with y_{t-1}. A key consideration is how many lags of the left-side variable to include. Liu and Maddala follow a procedure suggested by Schwert (1989). Ng and Perron (1995) describe two popular procedures, the Akaike information criterion and a criterion suggested by Schwartz, both of which minimize an objective function of the form

$$(5) \quad I_k = \log\, \hat{\sigma}^2_k + k\, C_T / T,$$

where σ^2 is the maximum-likelihood estimate of the variance, T is the number of observations, and k is the number of right-side variables. For the Akaike information criterion, $C_T = 2$, and for the Schwartz criterion, $C_T = \log(T)$. Ng and

TABLE 1

Unit-Root Tests on Logarithms of Future and Expected Future Deutsche Mark/Dollar Exchange Rates

Statistics	Weekly		Monthly	
	Actual	Survey	Actual	Survey
τ^{τ} (LR)[a]	–2.429	–2.117	–2.430	–2.288
τ^{τ} (BIC)[b]	–1.887	–1.935	–1.858	–1.719
τ^{μ} (LR)[a]	–2.615	–2.357	–2.612	–2.470
τ^{μ} (BIC)[b]	–2.205	–2.252	–2.125	–2.011
ZA^{τ} (LR)[a]	–9.799	–11.065	–9.694	–9.389
ZA^{τ} (BIC)[b]	–8.173	–12.547	–7.995	–7.557
ZA^{μ} (LR)[a]	–10.115	–11.459	–9.948	–9.952
ZA^{μ} (BIC)[b]	–8.683	–12.686	–8.441	–8.340
DF-GLS$^{\tau}$(LR)[a]	–2.653	–2.331	–2.577	–2.430
DF-GLS$^{\tau}$(BIC)[c]	–2.122	–2.545	–2.064	–2.036
DF-GLS$^{\mu}$ (LR)[a]	–2.652	–2.336	–2.570	–2.427
DF-GLS$^{\mu}$(BIC)[b]	–2.251	–2.323	–2.104	–2.090

a. Lag lengths by column (left to right) are 8, 4, 8, and 3.

b. Lag lengths by column (left to right) are 0, 1, 0, and 0.

c. Lag lengths by column (left to right) are 1, 0, 0, and 0.

NOTE: DF tests use the t-statistic for β from estimation of the equation:

$$\text{(A1)} \quad \Delta \log y_{t+1} = \alpha + \beta \log y_t + \gamma t + \sum_{i=1}^{p} \Theta_i \Delta \log y_{t+1-i} + e_{t+1},$$

τ^{τ} and τ^{μ} include and exclude the linear time trend, respectively.

ZA (usually written as Z_a) statistics are based on the same equation without lagged changes, though a choice of lag length is needed to estimate an analogue to the covariance matrix.

The τ and μ superscripts have the same significance as above.

DF-GLS$^{\mu}$ and DF-GLS$^{\tau}$, respectively, exclude and include a linear trend from the first-stage regression (as described in the text) and then estimate equation (A1) without constant, trend, or lagged changes.

LR and BIC indicate the criteria by which lag length was chosen. Critical values for τ^{τ}, ZA^{τ}, and ZA^{μ} can be found in MacKinnon (1994). For DF-GLS$^{\mu}$, the 5 percent critical value for a large number of observations is –2.89. The DF-GLS$^{\mu}$ statistic has the same distribution as τ^{μ}.

SOURCE: Author's calculations.

Perron (1995) also propose starting with a maximum value for k and decreasing the number of lags until the coefficients on the last n lagged terms are significant, but not if the total number of lags were decreased again by 1. When n=1, this procedure finds the lag length k where the t-statistic on the $k+1^{\text{th}}$ lag is not significant but the t-statistic on the k^{th} lag is significant. In his survey of the area, Stock (1994) suggests using the sequential procedure of Ng and Perron (1995), denoted LR, as well as the Schwartz criterion with the lag lengths constrained to between three and eight.

The unit-root tests also differ in their treatment of the "nuisance parameters" in equation (4), the constant and linear trend t. Stock (1994) evaluates the various tests and suggests using the DF-GLS test subsequently presented in detail in Elliott, Rothenberg, and Stock (1996). The DF-GLS requires two steps. First, let $z_t = (1, t)$. Then, assuming that the process for y is AR(1) with a coefficient $\alpha = 1 + c/T$, estimate $\hat{\beta}$ by regressing $[y_1, y_2(1-\alpha L), \ldots y_T(1-\alpha L)]$ onto $[z_1, z_2(1-\alpha L), \ldots z_T(1-\alpha L)]$ and then creating $y^d = y - z_t'\hat{\beta}$. Second, estimate equation (4) without the constant or the trend. The t-statistic on lagged y is the DF-GLS$^{\tau}$ statistic with critical values given in Elliott, Rothenberg, and Stock (1996). The DF-GLS$^{\mu}$ statistic omits t from the first stage. Its distribution is that of the more familiar Dickey–Fuller τ^{μ} statistic. The constant c is set equal to –7.0 for the no-trend case and –13.5 with trend. For comparison with Liu and Maddala, we present results of the Dickey–Fuller tests and the Phillips–Perron tests.[6] All are presented with both constant but no trend (μ) or constant and trend (τ).

III. Data and Results

We analyze data provided by Money Market Services for the deutsche mark/dollar exchange rate from January 6, 1989, through October 24, 1997.[7] The survey data represent the medians of the forecasts of the respondents for the one-week and one-month horizons (figures 1 and 2). The frequency of the data is weekly, and one month should be interpreted as corresponding to four weeks.

Table 1 indicates the unit-root test statistics for log (future DM/$) and the survey median of expectations of the future DM/$. Despite the wide variation in chosen lag lengths, in no case do we reject the null hypothesis of a unit root for either weekly or monthly forecast horizons at the familiar 5 percent level of confidence.

At the top of table 3 we also include Q-statistics similar to those presented by Liu and Maddala and which allow us to assess the extent of serial correlation present in residuals for four, eight, and 12 lags. We do this for the first differences of the individual series. Our inability to reject the null hypothesis of no serial correlation supports a conclusion of rationality.

■ 6 The Phillips–Perron tests omit the lagged terms but still require a choice of lag length.

■ 7 Our monthly series ends at October 3, 1997.

TABLE 2

Restricted Cointegration Test on Logarithms of Future and Expected Future Deutsche Mark/Dollar Exchange Rates: Unit-Root Tests on log (S_{t+1}) – log ($S^e_{t,t+1}$)

Statistics	Weekly	Monthly
τ^τ (LR)[a]	–7.255	–7.247
τ^τ (BIC)[b]	–18.979	–7.247
τ^μ (LR)[a]	–18.972	–7.244
τ^μ (BIC)[b]	–7.213	–7.244
ZA^τ (LR)[a]	–396.780	–103.862
ZA^τ (BIC)[b]	–404.361	–103.862
ZA^μ (LR)[a]	–398.312	–103.796
ZA^μ (BIC)[b]	–404.273	–103.796
DF-GLS$^\tau$(LR)[c]	–6.147	–5.432
DF-GLS$^\tau$(BIC)[b]	–18.322	–7.214
DF-GLS$^\mu$ (LR)[c]	–4.783	–5.315
DF-GLS$^\mu$ (BIC)[b]	–17.002	–7.081

a. Lag lengths by column (left to right) are 9 and 6.
b. Lag lengths by column (left to right) are 0 and 6.
c. Lag lengths by column (left to right) are 9 and 10.
NOTE: LR and BIC indicate the criteria by which lag length was chosen.
SOURCE: Author's calculations.

TABLE 3

***Q*-Statistics for Test of Serial Correlation**

Series	$Q(4)$	$Q(8)$	$Q(12)$
(1–L)* (weekly, actual)	2.1	7.3	11.2
(1–L)* (weekly, survey)	11.5	12.9	17.2
(1–L)* (monthly, actual)	2.3	7.4	11.2
(1–L)* (monthly, survey)	9.4	10.9	16.8
Forecast error, one-week forecast	6.2	10.7	16.5
Forecast error, one-month forecast[a]	7.6	13.5	20.8

a. Analysis of residual from MA(3) estimation of the weekly series of one-month-ahead forecasts.
NOTE: All $Q(k)$ statistics are distributed as x^2 (k).
SOURCE: Author's calculations.

Table 2 lists the restricted cointegration test statistics for weekly and monthly horizons. Because the test statistics are less than the critical values for the 5 percent level in all instances, we reject the null hypothesis of no cointegration. Table 3 also presents Q-statistics for the forecast errors. However, since the frequency of the data is weekly, a weekly series on the one-month forecast error will display serial correlation even if the forecasts are rational. In fact, this series will have a third-order moving-average error process (MA[3]).[8] In this case, we analyze the residual from estimating this series as an MA(3) process. We find no evidence of serial correlation for either of the two forecast errors.

IV. Interpretation of Results

Using more recent data and newer techniques, we confirm the findings of Liu and Maddala in favor of the rationality of Money Market Services survey forecasts. Although this would seem to support the rational expectations hypothesis, several notes of caution are in order. First, we would not necessarily confirm other implications of the rational expectations hypothesis, such as the orthogonality of forecast errors with respect to publicly available information. Second, it is not clear how median expectations are linked to the marginal prices (exchange rates) observed in the marketplace. This is one reason why economists have tended to downplay survey measures.

Nonetheless, analyses of these data bear directly on the issue of whether a risk premium exists in foreign-exchange markets and whether policy changes are anticipated correctly. Consequently, these findings are relevant to analysis of U.S. central bank intervention or other macroeconomic policies in the 1990s. Future research might fruitfully explore these possibilities.

8 We could follow Baillie and Bollerslev (1990) and impose the three moving-average parameters which would be implied by the assumption that the weekly process follows a random walk.

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The Fiscal Theory of the Price Level

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Introduction

A traditional function of the central bank is to control the price level. This function is a natural implication of economic theory: The celebrated quantity theory of money can be summarized in Milton Friedman's dictum that "inflation is always and everywhere a *monetary* phenomenon." As reviewed in Robert Lucas' Nobel lecture (1996), there is a wealth of empirical evidence linking price movements to movements in the money stock.

This traditional analysis has been challenged by the *fiscal theory of the price level (FT),* which maintains that the price level is determined by the budgetary policies of the fiscal authority. This attack on the conventional position has come in two parts, *weak-form FT* and *strong-form FT.*

Weak-form FT begins with an obvious link between monetary and fiscal policy. Since seignorage (revenue from money creation) is a possible revenue source, long-run monetary and fiscal policy are jointly determined by fiscal budget constraints. Whether monetary or fiscal policy determines prices involves an assumption about which policymaker will move first, the central bank or the fiscal authority. Weak-form FT assumes that the fiscal authority moves first by committing to a path for primary budget surpluses/deficits, forcing the monetary authority to generate the seignorage needed to maintain solvency. Sargent (1986) describes this as a "game of chicken."

If both the monetary and the fiscal authority refuse to generate the needed seignorage, then the nation's debt-to-GDP ratio will grow at an unsustainable rate. This implies ever-increasing real rates of interest on government debt, as the market demands larger and larger default premiums. This process cannot continue: One of the two players, the fiscal authority or the central bank, must alter its behavior. Weak-form FT assumes the central bank will respond and generate the seignorage needed to avoid default. Using the game-of-chicken analogy, weak-form FT assumes that the monetary authority loses and is forced to "blink."

This version of the fiscal theory predicts that fiscal policy determines future inflation as well. Although this is true, it does so only by determining future money growth. The traditional version of the FT, therefore, is not at odds with the quantity theory, in the sense that prices are still driven by current or future money growth.

Sargent and Wallace's (1981) celebrated example, in which tight money today increases the price level, occurs because future money growth—and hence future inflation—increases. The theory simply posits that the ultimate driver of the money supply is the fiscal authority. In other words, fiscal policy is exogenous, while money supply movements are endogenous.

More recently, a stronger version of the fiscal theory has been posited. Strong-form FT maintains that fiscal policy determines future inflation, but independent of future money growth. Unlike the weak theory, where inflation is still (ultimately) a monetary phenomenon, strong-form FT maintains that fiscal policy affects the price level and the path of inflation independent of monetary policy changes.

This new version of the fiscal theory is possible because, in a wide variety of monetary models, the initial price level is not pinned down; different initial price levels are consistent with different paths for future inflation. In contrast, prices are uniquely determined in the weak form of the FT analyzed by Sargent and Wallace. Strong-form FT assumes that the fiscal budget constraint, and thus fiscal policy, pins down the initial price level. Without this constraint, the initial price level may be indeterminate, even if the money supply is given exogenously—that is, even if the monetary authority moves first by committing to a path for the money stock. This is in sharp contrast with weak-form FT, in which the money supply is endogenous in order to satisfy the government's budget constraint. Strong-form FT assumes that both fiscal and monetary policy are given exogenously and that prices adjust to ensure solvency. In this game of chicken, neither player blinks.

This article begins with a discussion of weak-form FT by reviewing basic budgetary arithmetic and its implications for monetary policy. In particular, the "unpleasant arithmetic" of Sargent and Wallace (1981) is presented. This paper is a natural place to begin, as it provides a powerful demonstration of the restrictions that the government budget may place on monetary policy. Section I analyzes a partial-equilibrium model where real cash balances immediately jump to their steady state—that is, equilibria in which the level of real cash balances remains constant. Section II broadens the analysis to a more fully specified general-equilibrium model, allowing for consideration of equilibria where the level of real balances varies with time and for consideration of strong-form FT. Section III extends the discussion to models in which the central bank targets the interest rate and in which the money supply is endogenous, asking whether this case is an example of weak- or strong-form FT. Section IV presents our conclusions.

I. Weak-Form FT: A Partial-Equilibrium Analysis

This section will present some basic results of the budgetary linkages between monetary and fiscal policy. For illustrative purposes, we assume that the real rate of interest (denoted by r) and the real level of output (normalized to one) are constant. We also assume a form for money demand instead of deducing it from a more completely specified environment. These partial-equilibrium simplifications limit our discussion to steady-state equilibria, where real cash balances immediately jump to their steady-state (constant) level and remain there forever. Since money growth is equal to the inflation rate in such an equilibrium, this partial-equilibrium model can give rise only to weak-form FT. In later sections we extend this analysis to a general-equilibrium model, where this is not necessarily true, so that either weak- or strong-form FT can arise.

Equilibrium is defined by two conditions, fiscal budgetary balance and money-market equilibrium. Money-market equilibrium (real money supply = real money demand) is defined by

$$(1) \qquad M_0/P_0 = f(R),$$

where money demand (f) is a function of the nominal interest rate ($R = r + \pi$) and π is the inflation rate. Money demand is a function of inflation only because the real interest rate and output are both assumed to be constant. M_0 is the nominal money stock during the first period of the model, and P_0 is the corresponding nominal price level.

Fiscal budget balance is given by

$$(2) \qquad D + S(\pi) = B_0/P_0,$$

where $S(\pi)$ ($S'(\pi) > 0$) denotes the present value of seignorage, and D is the present value of future primary budget surpluses (negative values represent deficits). Annual real seignorage from a constant money growth rate of g (and thus a constant inflation rate of $\pi = g$) is $\pi f(R)$. The present discounted value of seignorage then, is $S = \pi f(\pi)/r$. The accumulated real value of government debt due at time zero, denoted by B_0/P_0, must equal the present value of future primary budget surpluses plus revenues from seignorage.

Total government liabilities are defined as the sum of money (the liability of the central bank) and government bonds (the liability of the Treasury). We assume that the initial level of total government liabilities, $\bar{H} = M_0 + B_0$, is fixed. The ratio $M_0/\bar{H}$ is the fraction of total liabilities that are monetary. This fraction changes via open-market operations by swapping (newly printed) money, M_0, for government debt, B_0, holding $\bar{H}$ fixed. Rewriting equation (2) by substituting out B_0 gives

$$(3) \qquad S(\pi) + \frac{M_0}{P_0} + D = \frac{\bar{H}}{P_0}.$$

Notice that there are two forms of seignorage in this model. One comes from future money growth, $S(\pi)$. The other comes from movements in the current money stock, M_0/P_0. Open-market purchases swap B_0 for M_0 and thus lower the nominal (and real) value of government debt.

Solving for P in equation (1) and substituting into equation (3), we have

$$(4) \qquad S(\pi) + D = \frac{(\bar{H} - M_0)}{M_0} f(R).$$

Assuming that S is increasing (that is, we are on the "correct" side of the Laffer curve) and f is decreasing (money demand slopes down), then for a given D and $\bar{H}/M_0$, there is at most one inflation rate (future money growth), π, that satisfies equation (4).

To close the model, we must define monetary and fiscal policy. A policy is defined by choosing two of the following variables: π, D, or $\bar{H}/M_0$. The third variable is determined endogenously to satisfy equation (4).

Weak-form FT assumes *fiscal dominance,* which is defined in the following way: The fiscal authority commits to D, thus forcing the central bank to choose either current (initial) M_0 or future inflation, π, to satisfy equation (4). The central bank can react to a change in fiscal policy by changing either M_0 or π.

If future inflation is held constant, a decrease in D (that is, an increase in the deficit) necessitates increasing the current money stock, M_0 (and hence P_0), lowering the real value of government debt outstanding. If money is held constant, then the monetary authority must react by increasing future inflation. A decrease in D must result in either a one-time increase in money, M_0, and hence P_0 (a one-time jump in inflation), or an increase in future (sustained) inflation, π. We define fiscal dominance as weak-form FT because the price level is still determined by current or future money supply movements. The central bank, however, is driven by the fiscal authority. In terms of the game of chicken, the central bank is forced to blink; that is, the money supply is dictated by fiscal policy and is, therefore, endogenous.

Fiscal dominance is the assumption made by Sargent and Wallace (1981) in their classic paper, "Some Unpleasant Monetarist Arithmetic." They assume that "the path of [primary surpluses, D] is given and does not depend on current or future monetary policies. This assumption is ... about the behavior of the monetary and fiscal authorities and the 'game' that they are playing. Since the monetary authorities affect the extent to which seignorage is exploited as a revenue source, monetary and fiscal policies have to be coordinated. The question is which authority 'moves first' … who imposes discipline on whom?"[1]

The arithmetic implied by this game of chicken is unpleasant. Tight money today (a low M_0 and more debt, B_0) necessitates loose money (a high π) in the future to pay off the debt. Equivalently, low seignorage today (low M_0) implies high seignorage (high π) tomorrow. An even more unpleasant possibility of weak-form FT is that tight money today could increase *today's* price level. This would occur if money demand were significantly elastic, as higher inflation and, in turn, a higher nominal interest rate lowered real money demand and increased the price level. Solving equation (3) for P_0 yields

$$P_0 = \frac{\bar{H}}{S(\pi) + f(R) + D}.$$

The effect of an increase in future inflation on current prices, $dP_0/d\pi$, depends on the relative magnitude of decreased nominal money (thus lowering prices) versus the decline in real money caused by higher future inflation. Using the fact that $S = \pi f(R)/r$ and $R = r + \pi$, we have

$$(5) \qquad P_0 = \frac{r\bar{H}}{Rf(R) + rD}$$

and

$$\frac{dP_0}{d\pi} = (\eta - 1)P_0 \frac{M_0}{r\bar{H}},$$

where $\eta = -Rf'(R)/f$ is the interest elasticity of money demand.

Notice that $dP_0/d\pi$ has the same sign as $(\eta - 1)$. If money demand is sufficiently elastic (greater than one), then low money supply

1 Sargent (1986), pp. 170–71.

(M_0) today (implying a high level of inflation tomorrow) implies a high price level (P_0) today. The intuition is as follows: Low M_0 tends to lower P_0. But the resulting higher inflation, π, tends to lower real money demand, driving P_0 upward. This second effect overwhelms the first, if and only if the interest rate elasticity of money demand is greater than unity. Empirical estimates of η, however, are uniformly less than one. Thus Sargent and Wallace's unpleasant possibility that tight money leads to higher current prices (a low M_0 leads to a high P_0) is probably only a theoretical curiosity.

The polar opposite of the assumption of fiscal dominance is the assumption of *monetary dominance*. In this case, the central bank commits to π and $\bar{H}/M_0$. The fiscal authority must then choose D to satisfy equation (4)—that is, the fiscal authority is assumed to blink. Since the central bank chooses π and $\bar{H}/M_0$, it also determines P_0. Monetary dominance is the typical assumption in most theoretical monetary models and is not an example of the FT. For example, a standard simplifying assumption in many monetary models is that $B_0 = 0$, implying that D is endogenous and given by $S(\pi) = -D$. In this game of chicken, the monetary authority moves first and the fiscal authority blinks.

II. A General-Equilibrium Model

In this section, we examine the more general case in which the level of real cash balances is not necessarily at the steady state. Here we ask, is it possible for *neither* the monetary nor the fiscal authority to blink? We refer to this case as strong-form FT because movements in inflation do not result from money growth.

To explore this possibility, we consider the simple case where money supply growth is constant. This is an example of monetary dominance in the sense that the central bank moves first. In the previous section, this implied that fiscal policy would be endogenous and dictated by the government budget constraint. Is this still the case? Looking at fixed-money-growth equilibria is useful, since changes in inflation will, by definition, not be driven by money supply changes. To explore these possibilities we require a dynamic (general equilibrium) counterpart to the money-demand equation (1).

The economy consists of infinitely lived households with preferences over consumption and real balances given by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, M_t/P_t),$$

where β is a constant rate of time discount and c_t and M_t/P_t denote consumption and real money balances, respectively. This money-in-the-utility-function framework is quite general and stands as a proxy for the transactions-facilitation role of money.

We assume that preferences are separable and given by

$$U(c_t, m_t) \equiv V(c_t) + \frac{m_t^{1-\varepsilon}}{1-\varepsilon}.$$

The (absolute value of the) interest elasticity of the implied money-demand curve is equal to $\eta = 1/\varepsilon$. This assumption of separability is not as odd as it may seem. In a model with endogenous production, Carlstrom and Fuerst (1999) demonstrate that the model behaves as if utility were separable in consumption and real balances.

The household's intertemporal budget constraint is given by

$$M_{t+1} = M_t + X_t + B_{t-1}(1 + R_{t-1}) - B_t - P_t c_t + P_t y_t,$$

where M_t denotes money balances at the beginning of time t; X_t denotes a monetary transfer from the government (inclusive of lump-sum tax payments); B_{t-1} denotes bond-holdings acquired in period $t-1$; R_{t-1} denotes the nominal interest rate from $t-1$ to t; and the endowment is normalized such that $v'(y) = 1$. As preferences are separable, the constant level of income implies that the real rate of interest is constant at $r = (1/\beta) - 1$. Notice that the bond choice at time t, B_t, determines the amount of cash the household has available for the next period's purchases (M_{t+1}).

The Euler equations that define equilibrium are given by

$$(6) \qquad U_c(t)/P_t = (1 + R_t)\beta\, U_c(t+1)/P_{t+1}$$

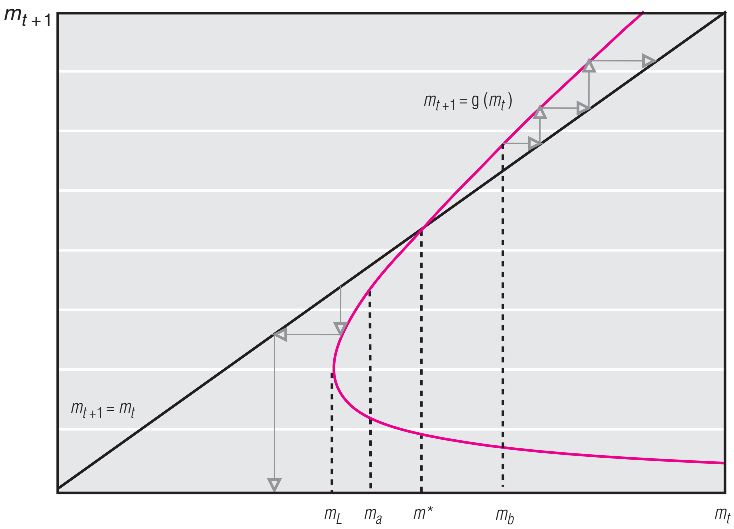
and

$$(7) \qquad \frac{U_c(t)}{P_t} = \beta\left[\frac{U_c(t+1) + U_m(t+1)}{P_{t+1}}\right].$$

Equation (6) arises from optimal bond choice and is the standard Fisherian decomposition of the nominal interest rate into the real interest rate and an inflation premium $[(1 + R) = (1 + \pi)(1 + r)$, or $(R \approx r + \pi)]$. Equation (7) is the choice of next period's money

FIGURE 1

Dynamics of Real Money Balances: $\varepsilon \geq 1$



SOURCE: Authors.

balances (M_{t+1}). Holding on to one dollar today comes at the loss of current consumption (the LHS of equation [7]), but provides for consumption and transaction services next period (the RHS of equation [7]). Combining equations (6) and (7) yields a demand-for-money equation:

$$(8) \qquad U_m(t+1)/U_c(t+1) = R_t.$$

By inverting equation (8) to express m as a function of R, we have the dynamic counterpart of equation (1). Money demand in $t+1$ is a function of the interest rate between t and $t+1$.

To focus on strong-form FT, suppose that the central bank fixes the current money stock at M_0 and the gross money supply growth rate at $G \geq 1$. In fixing the monetary rule in this way, we are assuming monetary dominance, in that money growth is exogenous and will not be deviated from for fiscal reasons. The key question now becomes: Is the path of the price level determined by this exogenous monetary policy? If not, then we have a case of strong-form FT.

Replacing R_t as defined by equation (6) in equation (8) implies that money-market equilibrium is given by

$$(9) \qquad m_{t+1}^{1-\varepsilon} + m_{t+1} = (G/\beta)m_t,$$

where $m_t = M_t/P_t$.[2] Our analysis will examine numerous real balance paths that satisfy equation (9) but are economically meaningful in that real balances remain positive. A steady-state solution is one in which $m_t = m^* \geq 0$ for all t. There is one positive steady state given by

$$(10) \qquad m^* = \left(\frac{\beta}{G-\beta}\right)^{1/\varepsilon}.$$

From equation (9) it is clear that if $\varepsilon < 1$, there is also another steady state in which $m_t = 0$ for all t. This is an equilibrium in which money is not valued. In contrast, if $\varepsilon \geq 1$, then equation (10) describes the only non-negative (and thus permissible) steady state. Therefore, we have two cases.

Case 1, $\varepsilon \geq 1$

This section examines the case where $\varepsilon \geq 1$. Equation (8) implies that the interest elasticity of money demand ($\eta = 1/\varepsilon$) is less than one. In this case, we will show that the general-equilibrium model collapses down to the partial-equilibrium model of the previous section, and thus cannot deliver strong-form FT; equivalently, the assumption of monetary dominance (constant money growth) implies that the fiscal authority must be passive.

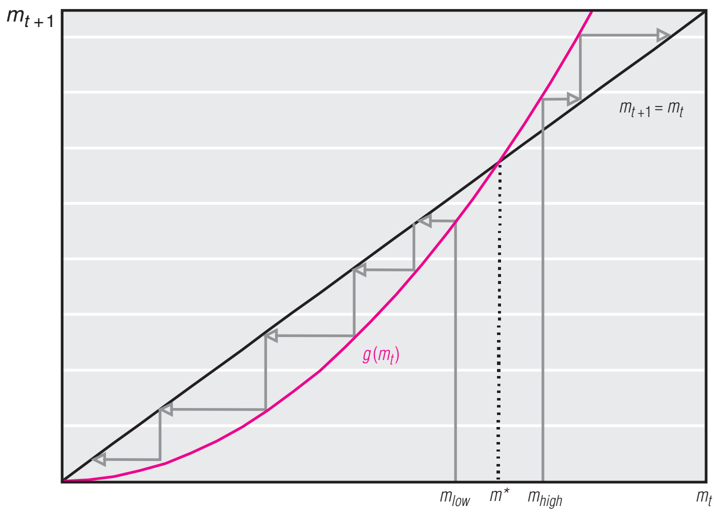
Figure 1 graphs m_{t+1} as a function of m_t [$m_{t+1} = g(m_t)$] to illustrate these dynamics. The arrows indicate how m_t evolves over time. Since there is a unique positive steady state, m^*, paths that begin below m^* (say m_a) imply that real balances become complex-valued in finite time and are thus nonsensical.[3] Real money balances starting to the right (say m_b) explode, eventually violating the transversality condition and thus do not satisfy the necessary conditions for an optimum (see appendix A). Hence, as long as money demand is not too elastic ($\eta = 1/\varepsilon \leq 1$), the current price level is uniquely determined and real balances must jump immediately to the steady state m^*.[4] Thus, since monetary policy was given (fixed G and M_0),

■ 2 $m_{t+1}^{-\varepsilon} = \dfrac{P_{t+1}}{\beta P_t} - 1 = \dfrac{Gm_t}{\beta m_{t+1}} - 1.$

■ 3 There are no real solutions to (9) when money balances are to the left of m_L on the graph. The solutions are then all complex which have no economic meaning. This occurs irrespective of whether you take the upper or lower part of the "C" in figure 1. We thank Larry Christiano and Terry Fitzgerald for pointing out an error in the earlier working paper version of this figure.

FIGURE 2

Dynamics of Real Money Balances: $\varepsilon < 1$



SOURCE: Authors.

it is dominant, and fiscal policy must adjust to ensure budgetary solvency (equation [2]). This general-equilibrium example is thus identical to the steady-state example of monetary dominance in the previous section.

Case 2, $\varepsilon < 1$: Strong-Form FT

Suppose instead that $\varepsilon < 1$, so that money-demand elasticity is greater than one ($\eta > 1$). In this case there are two steady-state solutions (equation [10] and $m^* = 0$).

Notice that the only stationary equilibrium with valued money is that in which real balances (and prices) immediately jump to the positive steady state, m^*. If we restrict the analysis to stationary equilibria, because M_0 is given exogenously the fiscal authority must move to maintain fiscal solvency. This once again corresponds exactly to the monetary dominance results in the previous section.

While one can argue that nonstationary equilibria can be ruled out on empirical grounds, there is nothing in the model (if $\eta > 1$) to rule out these nonstationary paths. Figure 2 illustrates the model's dynamics.[5] Unless current real-money balances are given by m^* (that is, M_0/P^*), real balances will either explode or implode over time. To the right of m^*, all paths have real balances exploding as the price level approaches zero (self-fulfilling hyperdeflations). As before, these paths are not equilibria because they violate the household's transversality condition (see appendix A). To the left of m^*, all paths are self-fulfilling hyperinflations: The real value of the money stock goes to zero in the limit. These equilibria cannot be ruled out a priori, since they also converge to a steady state in which money is not valued. Thus, there is an infinite number of equilibria, each indexed by the current price level, P_0. Any initial price level $P_0 > P^*$ is an equilibrium.

Returning to our game-of-chicken analogy, we ask the simple question of whether *anyone* has to blink. As we showed in section I, if the fiscal authority commits to a primary surplus path and the monetary authority commits to a seignorage path (that is, if both agents refuse to blink), then the fiscal solvency condition will be violated. Someone has to move. However, this is not necessarily true in the general equilibrium case, since (with $\varepsilon < 1$) there exist non-steady-state equilibria in which the current price level is free. If both parties refuse to move, then the initial price level will immediately jump to a level satisfying the government's budget constraint.

To see the effect of fiscal policy on the price path, consider the case where there is no money growth ($G = 1$). With no future seignorage revenues, equation (2) gives $P_0 = B_0/D$. Thus, fiscal policy determines the current price level and (from equation [9]) the path of prices. A higher D implies a lower P_0, and vice versa. Despite the exogeneity of monetary policy, fiscal policy maintains a great deal of autonomy, restrained only by the requirement that $m \leq m^*$, so that $D \leq (B_0/M_0) \bullet m^*$. If D exceeded this latter value, there would be no equilibrium if both parties refused to move.

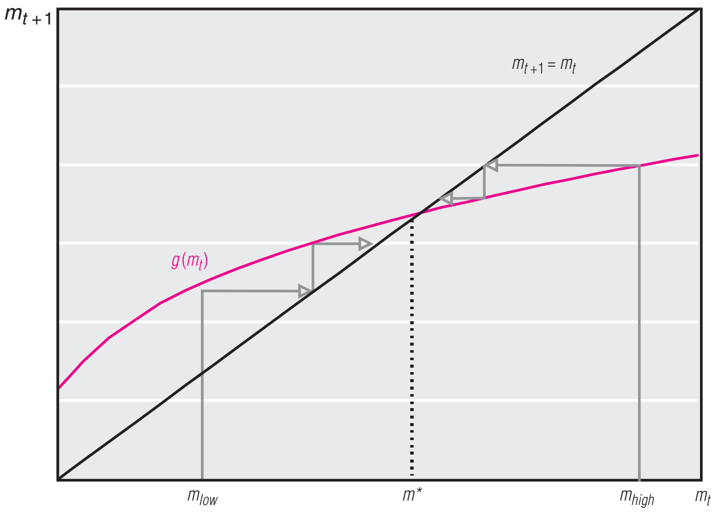
Referring to figure 2, the nonstationary equilibrium paths (where $m < m^*$) have prices rising and inflation increasing. Since the money-demand relationship still holds, the only way for current prices to rise is for the nominal interest rate and inflation to increase (remem-

4 In a private communication, Larry Christiano and Terry Fitzgerald note that if ε is sufficiently large (money demand is sufficiently interest inelastic), then the C-shape in figure 1 is shifted up so that the lower branch cuts the 45-degree line from above (this arises if $\varepsilon > G/(G-\beta)$). In this case, both branches of the g-mapping are relevant so that for a given m_t there is more than one possible m_{t+1}. The strong-form FT will be of no help in eliminating this type of multiplicity. As for empirical relevance, for $\beta = 0.99$, and $G = 1.02$, these pathologies arise only if $\varepsilon > 34$, an interest elasticity less than 0.029!

5 This case is examined in McCallum (1998).

FIGURE 3

Possible Dynamics of Real Money Balances with Nonseparable Preferences



SOURCE: Authors.

ber that money growth is constant). A change in fiscal policy (D) changes current prices by changing the path of future inflation. For example, an increase in the present discounted value of future surpluses (D) lowers current prices and future inflation.

This is a version of strong-form FT; fiscal policy affects the price path even though it has no effect on current or future money growth (nor on real output or the real rate of interest, both of which are assumed to be constant). This strong-form FT occurs because both monetary and fiscal policy are acting in a dominant fashion; in other words, neither party blinks. This is an intriguing possibility—namely, that fiscal policy can influence the price-level path independent of movements in the money stock. But the analysis has two peculiar but interrelated characteristics. First, the model exhibits self-fulfilling hyperinflations. Although this is an interesting theoretical possibility, there is scant empirical evidence for such phenomena.[6] Second, these hyperinflationary paths and the possibility of strong-form FT assume an implausibly high interest elasticity ($\eta = 1/\varepsilon > 1$). We know of no empirical estimates this high.

Nonseparable Preferences: Strong-Form FT

These peculiarities are not robust. For example, following Matsuyama (1990, 1991), suppose we relax the separability assumption on preferences.[7] In this case, it is possible to get the strong-form FT without an implausibly high interest elasticity of money demand *and* without nonstationary (exploding) price paths. The nonseparable counterpart to equation (9) is

$$(11)\quad \frac{G}{\beta} m_t U_c(m_t) = m_{t+1}[U_c(m_{t+1}) + U_m(m_{t+1})].$$

Since consumption is assumed, constant marginal utility is expressed as a function of real cash balances only.

As before, there exists a unique positive steady state. But unlike figure 1, which shows that the economy would immediately jump to this steady state, prices in this example will not necessarily immediately jump to P^*. A sufficient condition for this to occur—that is, for the existence of multiple stationary equilibria—is that the mapping of $m_{t+1} = g(m_t)$ cross the 45-degree line from above, or $0 < g'(m_{ss}) < 1$. Figure 3 shows such a case. The analysis resembles the earlier example where $\varepsilon < 1$ (figure 2), except that *all* initial real balances starting away from the steady state converge to m^* and thus do not have the counterfactual implication that prices will explode over time. Before, real balances beginning to the left of m^* converged to another steady state where money had no value.

Unlike this earlier nonstationary example, there are *no restrictions* on the initial stock of real money: Because these stationary paths converge to the steady state, the transversality condition is never violated. What is the initial level of real balances? On a theoretical level, it is the level of real money chosen at the beginning of time; however, the initial price level is chosen every period. If initial real balances are not determined at the beginning of time, then real balances every period are also undetermined. This leads to what economists call

■ 6 At a theoretical level, these hyperinflationary equilibria could be ruled out by a government promise to guarantee a lower bound on the real value of the currency by backing it with an arbitrarily small (but positive) real asset. Obstfeld and Rogoff (1983) make this point.

■ 7 In addition to considering nonseparability, Matsuyama (1990, 1991) uses a different timing convention. In the model of this article, beginning-of-period cash balances enter into the current-utility functional. In contrast, Matsuyama assumes that end-of-period balances enter into the current-utility functional. See Carlstrom and Fuerst (1999) for a discussion of these issues.

sunspot equilibria, in that purely extraneous information leads to a shift in public beliefs and thus affects the model's equilibrium. The hallmark of sunspot equilibria is the presence of self-fulfilling behavior.

Returning to the details of the nonseparable case, appendix B shows that equation (11) looks like figure 3 [$0 < g'(m_{ss}) < 1$] if and only if

$$(12) \qquad \frac{mU_{cm}}{U_c} < -1,$$

where this ratio is evaluated at the unique positive steady state. This is the ratio of elasticity of the marginal utility of consumption to the level of real balances.

To understand why sunspot equilibria (or self-fulfilling prophecies) are present in this economy under condition (12), let us walk through a simple example. Suppose there is a "sunspot event" at time t (an event independent of market fundamentals) that leads households to increase their holdings of real cash balances by 1 percent (P_t falls by 1 percent). For this sunspot movement to be stationary, the economy must move back toward the steady state: $m_t > m_{t+1} > m_{ss}$. For $m_{t+1} > m_{ss}$, the nominal rate at time t must be below the steady state (recall equation [8]). For real balances to deteriorate between t and $t+1$ ($m_t > m_{t+1}$), however, the inflation rate must be above the steady state.

But how can the nominal rate be below the steady state, while the inflation rate is above the steady state? If and only if the real rate of interest is sufficiently below the steady state. The logic is as follows: Because the real interest rate (r) is the ratio of the marginal utility of consumption today divided by the marginal utility of consumption tomorrow, r falls by more than the increase in expected inflation; thus, the increase in real money balances significantly decreases the marginal utility of consumption today. This is exactly the restriction in equation (12).

As in the previous section, the only way to escape this indeterminacy is for both the monetary and fiscal authorities to be completely unconcerned with balancing the government's books, in which case the strong-form FT provides the additional restriction needed to uniquely determine equilibrium. By pinning down initial real balances, it essentially eliminates the possibility of sunspot equilibria. Now the only "sunspots" that can change current prices and future inflation are changes in the primary budget surplus, D.[8]

This stationary example, however, makes two unusual assumptions. First, the elasticity in equation (12) is negative—additional cash balances *lower* the marginal utility of consumption. Second, this response (in absolute value) is quite large, *greater than one*. Both of these assumptions are problematic, especially given the results of Carlstrom and Fuerst (1999) that in a production economy with elastic labor, the model economy acts as if $U_{cm} = 0$.[9]

These examples assume that money growth is constant. In this case, it is hard to obtain price-level indeterminacy. What about other monetary rules? For instance, it has long been recognized that interest rate targeting leaves the initial value of nominal money (and prices) free. Section III demonstrates this and asks whether this is still an example of strong-form FT.

III. Endogenous Money: The Case of a Fixed Interest Rate

Most central banks conduct policy by way of directives for the nominal rate of interest. Such a policy implies that the money supply and seignorage are endogenous, opening up some interesting possibilities for the fiscal theory of the price level. By committing to an interest rate peg regardless of fiscal concerns, the central bank is acting in what seems to be a dominant fashion. But since the money supply and seignorage are endogenous, the monetary authority moves last and the fiscal authority maintains a great deal of discretion. On the surface, it is unclear whether monetary policy is acting in a dominant fashion.

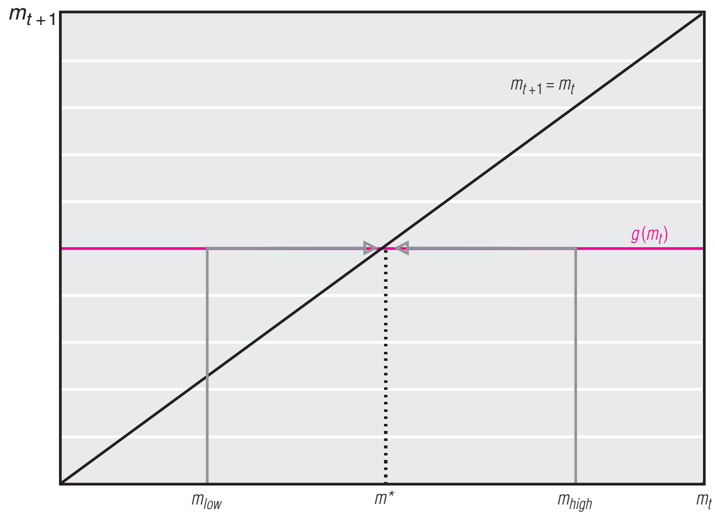
If we return to the game-of-chicken analogy, then in the case of an interest rate target there is another player in the game—the general public. Under such a monetary policy, the central bank agrees to engage in open-market operations to maintain the targeted rate, that is, buy and sell bonds at the request of the public. Thus, the public becomes an important player in the game. So the critical question is, who constrains whom? Does the fiscal authority con-

■ 8 The path of future surpluses (D) may not be completely free. There may be levels of D for which there is no initial price level and no subsequent path of prices that satisfy the fiscal budget contraint. D is not actually a sunspot since it is a market fundamental.

■ 9 There is another example where a constant-money-growth rule may lead to stationary indeterminacy: If the relative coefficient of risk aversion is greater than two (not implausible), then a cash-in-advance (CIA) economy with production implies indeterminacy. (See Carlstrom and Fuerst [1999].) This example suffers because it is extremely sensitive to the CIA assumption and does not arise in a money-in-the-utility-function framework, as assumed above.

FIGURE 4

Dynamics of Real Money Balances with an Interest Rate Peg



SOURCE: Authors.

strain the behavior of the general public? Or does the general public constrain the behavior of the fiscal authority?

This section explores these issues first in the steady-state model of section I, and then in the general-equilibrium model of section II. For simplicity, we will restrict the analysis to a fixed interest rate target (an interest rate peg).

The Steady-State Model

In the steady-state model, the central bank maintains a constant nominal interest rate by picking π, but then allows M_0 to be endogenous. The pegged nominal rate determines the level of real balances in equation (1). Combining equations (1) and (3), we have

$$(13) \qquad S(\pi) + D + f(\bar{R}) = \frac{\bar{H}}{P_0}.$$

Given that the real rate is fixed, a nominal interest rate peg also determines the inflation rate, π. What, then, determines the price level?

One can think of the monetary authority choosing M, the fiscal authority choosing D, and the public choosing real balances and, hence, P. By the definition of an interest rate peg, the central bank moves last since they chose money (endogenously) to ensure that the interest rate remains constant. Given this assumption, there are two cases to consider: the fiscal authority moves first, or the public moves first.

If the fiscal authority moves first, then D is exogenous. Since π and $\bar{H}$ are also given, equation (13) determines P_0 as a function of D: a low D implies a high P_0, and vice versa. Since M_0/P_0 is already determined by equation (3), we can also think of P_0 as a function of M_0. Returning to the game-of-chicken analogy, the general public blinks. Woodford (1994) uses this assumption to eliminate the price-level indeterminacy of operating under an interest rate peg. Notice that the situation resembles weak-form FT since fiscal policy, D, affects prices because it also affects the money supply, M_0.

If the public chooses first, then the fiscal authority must adjust D to satisfy fiscal balance —that is, the fiscal authority blinks. In this case, movements in the public's behavior (different choices for P_0 and M_0) translate directly into price movements. This creates self-fulfilling behavior, or sunspot equilibria: If the public expects a high price level and demands a high level of money balances to satisfy their transactions needs, then the money supply rises and generates the high price level they anticipate. This set of assumptions produces the standard nominal indeterminacy of operating under an interest rate peg: The current money stock (M_0) is free and so is the current price level (P_0).

A General-Equilibrium Model

Now let us consider the effect of an interest rate peg on the general-equilibrium model. With a constant level of consumption, the Fisher equation (6) implies that this corresponds to targeting the inflation rate at some rate π. The counterpart to equation (9) is

$$(14) \qquad m_{t+1}^{-\varepsilon} = \{[(1+\pi)/\beta] - 1\}.$$

Figure 4 graphs m_{t+1} as a function of m_t, a special case of figure 3. Here the initial m is free, but the economy immediately jumps to the steady state given in equation (10).[10]

10 To illustrate how the non-uniqueness of the initial price level leads to sunspot equilibria, note that with uncertainty, equation (13) becomes $E_t(m_{t+1}^{-\varepsilon}) = \{[(1+\pi)/\beta] - 1\}$. A quadratic approximation implies that real money balances will be given by $m_t = m^* + v_t$. There are no restrictions on the shock term v_t which, in principle, can be governed by sunspots.

Let us refer to this initial period as period 0. From equation (13), using the definitions of $\bar{H}$ and $f(R)$, the fiscal solvency constraint is given by

$$(15) \qquad \frac{M_1 - M_0}{P_0} + \frac{S(\pi)}{1+r} + D = \frac{B_0}{P_0}$$

(remember that because the real rate is constant, inflation is also constant).

Since the first bond market does not open until the end of period 0, M_0 and B_0 (since $M_0 + B_0 = \bar{H}$) are given by history. As for M_1, we have that $M_1 = f(\pi)P_1 = f(\pi)(1 + \pi)P_0$. There are only two free variables in equation (15), D and M_1.

The situation is symmetric with the steady-state model.[11] The public chooses M_1 in the bond market at the end of period 0. The central bank agrees to exchange money for bonds at the rate desired by the public. We have the same case as before. If the fiscal authority commits to a D path, then the only equilibrium choice for private agents is given by equation (15). If instead the general public moves first, then the fiscal authority must adjust D to satisfy equation (15). In this latter case, once again we have the possibility of sunspot equilibria.

Before closing, we should ask the question: Are these endogenous money cases examples of strong-form or weak-form FT? On one level they appear to satisfy the criteria for strong-form FT in that the fiscal authority is acting in a dominant fashion, as is the central bank since it chooses its goal (for example, an interest rate peg) regardless of fiscal concerns. But at a deeper level, they are really examples of weak-form FT. The monetary authority is not truly dominant because money supply and seignorage are endogenous. That is, if the fiscal authority chooses a different fiscal stance (D), then the monetary authority must change the money supply to ensure the interest rate target is still satisfied. The monetary authority moves last and, in essence, is the one that always blinks, as occurs under weak-form FT. Perhaps more importantly, it is only an example of weak-form FT since the fiscal authority only affects the price level by altering the endogenous supply of money.

IV. Conclusion

This article began with the observation that the implications of weak-form FT on monetary policy are not controversial. If the central bank is passive and the fiscal authority is dominant, then fiscal policy has an enormous influence on the price level. But this traditional form of the FT is also consistent with Friedman's dictum, since fiscal policy affects prices and inflation only through its effect on money.

Recently a much stronger version of this theory has been presented. There are two possibilities in the more recent versions of the fiscal theory of the price level: (1) strong-form FT, in which fiscal policy affects the price level independent of the money supply process, and (2) the case of interest rate targeting, in which the money supply is endogenous.

The strong-form FT, in which *both* the fiscal and monetary authorities move first (neither blinks), relies on large elasticities and thus is little more than an intellectual curiosity. It is difficult to take these examples too seriously.

As for interest rate targeting, our conclusion is more circumspect. This is actually *not* strong-form FT because movements in prices are still governed by movements in money.

This does not imply, however, that the FT has no important implications for monetary policy. There is a long line of research suggesting that interest rate targeting is indeed beneficial. A classic criticism of such a policy, though, is that the endogeneity of the money supply makes the price level unstable. In models with nominal rigidities, this also makes output unstable. FT advocates argue that this is not the case: If the fiscal authority commits to a budgetary path, then the general public must adjust its behavior to ensure equilibrium, and this restriction pins down the price level. If we accept such an argument, then the case for interest rate targeting is greatly strengthened. The government's budget greatly reduces these sunspot equilibria—only changes in D are sunspot equilibria in the sense that they can cause a one-time jump in the price level. But if the more appropriate way to view this game of chicken is to assume that the fiscal authority always moves last, then interest rate targeting remains problematic because it can result in instability.

■ **11** The only difference between the steady-state and the general-equilibrium models is that, in the latter, the timing assumption (the bond market opens at the end of the period) transforms the nominal indeterminacy in the steady-state model into a real indeterminacy (of real balances) in the general-equilibrium model.

Appendix A

This appendix demonstrates that if $G > 1$, then hyperdeflations do not satisfy the household's transversality condition. The transversality condition is given by

$$\text{(A1)} \quad \lim_{t \to \infty} \beta^t m_t = 0.$$

This requires that real balances grow at a rate less than $1/\beta$. Rewriting equation (9), we have

$$\text{(A2)} \quad m_{t+1} = \left[\frac{G}{(m_{t+1}^{-\varepsilon} + 1)}\right]\left(\frac{1}{\beta}\right)m_t.$$

Since real balances are exploding along a hyperdeflation, the bracketed term in equation (A2) is growing. Since $G > 1$, this term will eventually exceed one. Therefore, real balances will grow at a rate exceeding $1/\beta$ and will violate the transversality condition.

Appendix B

This appendix provides details of the case with nonseparable preferences (section II). From equation (7), the fundamental equation of the model is given by

$$\text{(B1)} \quad \frac{G}{\beta} m_t U_c(m_t) = m_{t+1}[U_c(m_{t+1}) + U_m(m_{t+1})].$$

Expressing m_{t+1} as a function of m_t, $m_{t+1} = g(m_t)$, and then totally differentiating equation (B1), yields

$$\text{(B2)} \quad g'(m_{ss}) = \left[1 - \frac{R/(1+R)}{\eta(1 + mU_{cm}/U_c)}\right]^{-1},$$

where $\eta > 0$ is the interest elasticity of money demand. A necessary and sufficient condition for $0 < g'(m_{ss}) < 1$ (so that we have a mapping as in figure 3), is for $(1 + mU_{cm}/U_c) < 0$. This is just the condition in equation (12).

There are, of course, other possibilities. If $(1 + mU_{cm}/U_c) > 0$, then there are two cases. If η is sufficiently small,

$$\text{(B3)} \quad \eta < \left(\frac{R}{1+R}\right)\left(\frac{1}{1 + mU_{cm}/U_c}\right),$$

then $g'(m_{ss}) < 0$. This tends to produce oscillatory behavior. Remarkably, Matsuyama (1991) demonstrates that if $g'(m_{ss})$ is sufficiently negative, then there are chaotic dynamics.

In the more likely case that

$$\text{(B4)} \quad \eta > \left(\frac{R}{1+R}\right)\left(\frac{1}{1 + mU_{cm}/U_c}\right),$$

then $g'(m_{ss}) > 1$, and we are back to a model similar to that in figure 1 or figure 2.

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