A re-examination of the predictability of the yield spread for real economic activity*

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Abstract

This paper revisits the yield spread's usefulness for predicting future real GDP growth. We show that the contribution of the spread can be decomposed into the effect of expected future changes in short rates and the effect of the term premium. We find that both factors are relevant for predicting real GDP growth but the respective contributions differ. We account for part of the contribution of the term premium using a logarithmic term structure model with a time-varying variance of the inflation forecast error. We find that the variance of inflation is a significant component of the term premium and part of the reason that the term premium helps predict real GDP growth.

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1. Introduction

A large literature has examined variables that help predict the business cycle. Interest rates and interest rate spreads—that is, differences between interest rates on alternative financial assets—have attracted considerable attention from market analysts, policy-makers and academic economists. Stock and Watson (1989) found that two interest rate spreads—the difference between the 6-month commercial paper rate and 6-month Treasury bill rate, and the difference between the 10-year and 1-year Treasury bond rates—were important to include in their newly constructed index of leading economic indicators. Since then, various authors have investigated a variety of alternative interest rates and spreads.[1]

The usefulness of the yield spread between long- and short-term interest rates for forecasting future economic activity has been particularly well-established. Harvey (1988) has shown that there is information about future consumption growth in the real term structure. Estrella and Hardouvelis (1991) documented that the yield spread between the 10-year Treasury bond rate and the 3-month Treasury bill rate is a useful predictor of future growth in output, consumption and investment, and the probability of a recession. Plosser and Rouwenhorst (1994) examined the information contained in the term structure about future real economic growth in three industrialized countries. They found that the term structure has significant predictive power for long-term economic growth and showed that the term structure contains information about future real activity that is independent from information about current or future monetary policy. Haubrich and Dombrosky (1996) found that the yield spread is an excellent predictor of four-quarter economic growth but its predictive content has changed over time. Estrella and Mishkin (1997) confirmed that the basic results of Estrella and Hardouvelis (1991) continue to hold in a number of European countries as well as in the United States. Dueker (1997) has shown that the yield spread among leading indicators is a relatively good recession predictor. Estrella and Mishkin

(1998) found the spread to be the best out-of-sample predictor of the probability of a recession occurring in the next four quarters. Dotsey (1998) has thoroughly investigated the forecasting properties of the yield spread for economic activity. He concluded that the spread contains useful information beyond that contained in past economic activity or past monetary policy, although over more recent periods the spread has not been nearly as informative as it has been in the past. Many other papers also have demonstrated the predictive power of the spread for future real economic activity.[2]

Why does the yield spread help forecast the business cycle? While a large literature provides evidence on the usefulness of the yield spread as a predictor of real economic activity, few studies have addressed this question. In particular, even though several researchers have observed that the time-varying term premium is a significant component of the yield spread, no one has yet proposed a way of separately measuring the role of the term premium itself in accounting for the spread's usefulness in forecasting. [3]

The paper begins with a review of the forecasting usefulness of the spread and notes how it can be decomposed into separate contributions of expected changes in interest rates and the term premium. Specifically, we attempt to answer the following question: given that the short rate rises relative to the long rate prior to a recession, to what extent is this because future short rates are rationally expected to fall (simple expectations hypothesis), and to what extent is it because the forecastable excess yield from holding long-term bonds (term premium) has fallen (which must be either a risk premium or a liquidity premium)? We find that both factors make statistically important contributions. We then go on to investigate why the term premium may be playing a useful role.

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[1] See for example Bernanke (1990), Friedman and Kuttner (1993), Stock and Watson (1999).

[2] These papers include Hu (1993), Davis and Henry (1994), Bernard and Gerlach (1996), Davis and Fagan (1997), Bonser-Neal and Morley (1997), Kozicki (1997), Smets and Tsatsaronis (1997), and Atta-Mensah and Tkacz (1998), among others.

Several studies, including Fama (1990), Mishkin (1990 and 1991), Jorion and Mishkin (1991), Abken (1993), Frankel and Lown (1994), Gerlach (1995), and Kozicki (1997), have reported that the yield spread helps predict inflation at moderate to long horizons. Clearly the expectation of market participants about future inflation is reflected in the term structure of interest rates. Furthermore, the volatility of inflation might be priced in the term structure of interest rates as a term premium or risk premium. Time-variation in the variance of an inflation forecast error is a measurable component of the term premium and one can investigate the role of this component in the overall findings.[4]

We develop a theoretical model of the role of the variance of the inflation forecast error in the term premium, and estimate its effect using the GARCH-X model developed in Brenner, Harjes, and Kroner (1996) to estimate the conditional variance of inflation forecast error.

Our basic findings can be summarized as follows. First, we confirm the conclusion of earlier studies that the yield spread between the 10-year Treasury bond rate and the 3-month Treasury bill rate has information for future real GDP growth and provides additional information beyond that contained in measures of monetary policy or oil price changes. Second, both expectations of future short-term interest rates and a time-varying term premium help account for the spread's usefulness in predicting real GDP growth up to 8 quarters ahead. The contributions are similar at short horizons but the effect of expected future short rates is much more important than the term premium for predicting GDP more than 2 years ahead. Third, inflation volatility is a statistically significant factor in the contribution of the term premium.

The plan of the paper is as follows. Several hypotheses of the predictive power of the yield spread for real GDP growth are discussed and the empirical findings are presented in Section 2.

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[3] See Kim (1998) for discussion of the time-varying term premium in the term structure of interest rates.
[4] See also Benninga and Protopapadakis (1983) and Sarte (1998).

Section 3 develops a theoretical model for measuring the importance of the variability of inflation. Conclusions are offered in Section 4.

2. Hypotheses of the Predictive Power of the Spread and Empirical Findings

In this section, we review several hypotheses on the relationship between the yield spread and future economic activity in terms of the expectation effect and the term premium effect. We then replicate earlier findings on the information content of the spread for future real GDP growth. Finally, we show how to decompose the contribution of the spread into the effect of the future expected short-term interest rates and the effect of the time-varying term premium.

2.1 Hypotheses of the predictive power of the spread for real economic activity

In general, the yield spread reflects the financial market's expectation of the future short rate and the term premium. The relationship between the yield spread and future economic activity could be explained either in terms of the spread's role as a signal of the future expected short rate (the expectation effect) or as a signal of the change in the term premium (the term premium effect).

Suppose that the Fed adopts a contractionary monetary policy. In this case, market participants expect that tight monetary policy will *temporarily* raise short-term interest rates. If the current short-term interest is higher than the expected future short-term rate, this means that the long-term rate should rise less than the short-term rate according to the expectations hypothesis. Thus, the yield spread will be flattened. The monetary contraction will eventually also reduce spending in interest sensitive sectors of the economy, causing economic growth slow. Conversely, easy monetary policy would result in a high yield spread, which would signal higher future real economic growth. According to this scenario, the positive correlation between the spread and future economic growth results from the expectations hypothesis of the term structure and the temporary influence of monetary policy.

Alternatively, market expectations of future economic growth may be reflected in the spread through the expected future change in the short-term rate. If market participants anticipate an economic boom and future higher rates of return to investment, then expected future short rates exceed the current short rate and the yield on long-term bonds should rise relative to short-term yields according to the expectations hypothesis.

Both of these interpretations of the yield spread's usefulness for forecasting real output operate through the spread's role as a signal of future expected short rates. However, the spread also contains a term premium, which represents the added yield that must be offered by long-term bonds to compensate for the risk of changes in short-term rates or a liquidity premium reflecting investors' preference for short-term securities. Suppose, for example, that inflation risk increases in an economic boom. Then long rates might rise relative to short rates, not because future short rates are expected to rise, but because the term premium itself has risen. The yield spread might help forecast future GDP because it is responding to investor's forecasts of future inflation variability.

To investigate these possibilities, it would be useful to be able to decompose the spread's forecasting contribution into an expectations effect and a term premium effect, to see which mechanism accounts for the historical correlation.

2.2 Empirical findings

A. The predictability of real economic activity using the yield spread

Our study uses the 10-year T-bond rate, 3-month T-bill rate, and real GDP from 1953:II to 1998:II. The source of interest rates is the Statistical Release H.15 of the Federal Reserve Board of

Governors, while real GDP is taken from the DRI Economic Database (formerly Citibase Economic Database).[5]

Figure 1 displays (1) the yield spread between the discount equivalent yield on the 10-year Treasury bond and the 3-month Treasury bill and (2) the annualized rate of growth of real GDP over the next 4 quarters. The NBER recession dates are shaded in. On several occasions prior to historical recessions, short rates rose above prevailing long rates, a phenomenon known as an inverted yield curve. The figure illustrates episodes when the gap between two interest rates became negative. The yield curve has flattened or become inverted prior to all seven recessions. The extent to which the yield curve is tilted away from its normal slope has been identified by many researchers as a valuable indicator of recessions. Of course, the yield curve does not have to become inverted to signal that recession is imminent; it may simply flatten relative to normal.

Many previous studies, such as Estrella and Hardouvelis (1991), Estrella and Mishkin (1997), Haubrich and Dombrosky (1996), Bonser-Neal and Morley (1997), Kozicki (1997) and Dotsey (1998), used the following regression to examine the predictability of the yield spread for real activity:

$$y_t^k = a_0 + a_1 Spread_t + e_t ,$$

(2.1)

$$y_t^k = (400/k)*(\ln Y_{t+k} - \ln Y_t) ,$$

$$Spread_t = i_t^n - i_t^1 ,$$

where Y_{t+k} is real GDP in quarter $t+k$, y_t^k is the annualized real GDP growth over the next k quarters, and i_t^n , i_t^1 are the 10-year Treasury bond rate and the 3-month Treasury bill rate at time *t*. Table 1 shows the results of the estimation of equation (2.1) using OLS. These estimates are

[5] The monthly average interest rate series were converted to quarterly by using average-of-period values.

qualitatively similar to previous ones; the OLS estimates confirm that the yield spread helps predict real GDP growth up to 12 quarters ahead.

B. The role of other variables

Following Haubrich and Dombrosky (1996), Bonser-Neal and Morley (1997), Kozicki (1997), and Dotsey (1998) we also estimated the following equation:

$$y_t^k = b_0 + b_1 Spread_t + b_2 y_{t-1}^1 + b_3 y_{t-2}^1 + b_4 y_{t-3}^1 + b_5 y_{t-4}^1 + e_t$$

(2.2)

where y_{t-i}^1 is real GDP growth beginning in quarter *t-i* over one quarter. Because current and lagged real rates of growth of real GDP may be useful for forecasting future GDP, these real growth rates are included in the estimated equation (2.2).

Table 2 shows the estimation results for equation (2.2). Again these results are qualitatively similar to previous studies. The values of the estimated coefficient on the spread are slightly smaller than the estimated coefficients without including lagged real GDP growth, but remain statistically significant at conventional levels up to 12 quarters ahead. Thus, the yield spread provides additional information beyond that contained in current and lagged growth rates.[6] One interesting result is that the statistical significance of the estimated coefficient on the spread shows a similar pattern with that of the estimated coefficient on the spread without lagged real GDP growth as explanatory variables.

Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994), Estrella and Mishkin (1997), and Dotsey (1998) have investigated whether the yield spread has additional information beyond that contained in monetary policy. The following regression allows us to take a look at

[6] Dotsey (1998) shows that the information content of the spread differs across sample periods and the spread does not appear to be statistically significant over the most sample period of 1985:I to 1997:IV.

whether there is predictive power of the yield spread over and above that provided by other variables that reflect the stance of monetary policy:

$$y_t^k = b_0 + b_1 Spread_t + b_2 X_t + e_t$$

(2.3)

where X_t is the contemporaneous measure of monetary policy. Following Plosser and Rouwenhorst (1994), and Estrella and Mishkin (1997), we introduce two monetary policy variables, the Federal funds rate and a monetary aggregate. The source of Federal funds rate and narrow (M1) and broad (M2) monetary aggregates is the Statistical Release H.15 and H.6 of the Federal Reserve Board of Governors.[7] The estimate results are shown the Table 3A and Table 3B.

Even though we include the change in the Federal funds rate as the monetary policy variable on the prediction of real economic activity, the coefficient on the spread remains statistically significant at the 1% level up to 8 quarters ahead. One point to make is that the coefficient on the change in the Federal funds rate 1 quarter ahead is statistically significant and positive but the coefficients from 8 quarters ahead are statistically significant and negative. The positive value of the coefficient on the change in Federal funds rate might imply that the behavior of the Federal funds rate might be endogenous to economic state. That is, the Fed might try to raise the Federal funds rate to hold down the inflation pressure in an expansionary period. In the monetary aggregate case, the coefficient on the spread is also statistically significant at the 1% level up to 8 quarters ahead regardless of M1 or M2. These results confirm the previous ones and thus the yield spread provides additional information beyond that contained in monetary policy.

Smets and Tsatsaronis (1997) also state that the predictive content of the term spread is not time-invariant.

[7] The monthly average Federal funds rate series and monthly monetary aggregate were also converted to quarterly by using average-of-period values. Data for Federal funds rates are from July, 1954 to June, 1998 and data for the monetary aggregates are from January, 1959 to June, 1998 due to data availability.

Hamilton (1983) showed a strong relation between oil price changes and GNP growth, though recent evidence surveyed in Hamilton (1999) suggests that the relation is highly nonlinear. We examine whether the yield spread has additional information beyond that contained in his measure of current and lagged oil price changes:

$$y_t^k = b_0 + b_1 Spread_t + b_2 y_{t-1}^1 + b_3 y_{t-2}^1 + b_4 y_{t-3}^1 + b_5 y_{t-4}^1 \\ + f_1 o_t + f_2 o_{t-1} + f_3 o_{t-2} + f_4 o_{t-3} + e_t$$

(2.4)

where o_t is the oil variable reflecting oil price changes at time *t*. Two oil variables are used. The first is the percent quarterly logarithmic growth rate of the nominal crude oil producer price index. This oil variable doesn't catch the nonlinear relation between oil price changes and GDP growth. The second is Hamilton (1996)'s measure of the net oil price increase and is defined as the amount by which oil prices in quarter *t* exceed their peak value over the previous 4 quarters; if they do not exceed the previous peak, then the value is taken to be zero. This measure is the nonlinear transformation data of the oil price changes.

Table 4A and 4B show the estimation results for equation (2.4). The coefficient on the yield spread still remains statistically significant at the 1% level up to 8 quarters ahead even when we include the nonlinear transformation data of the oil price change on the prediction of real GDP growth. Thus, the yield spread provides additional information beyond that contained in oil price changes. In the case of the percent change in the nominal price of crude petroleum, the oil price change helps predict real GDP growth only one quarter ahead. The net oil price increase, however, helps predict real GDP up to 16 quarters ahead and thus, the predictability of oil variable remains quite strong even though the predictability of the spread disappears.

C. A decomposition of why the yield spread helps forecast.

As before, let i_t^n, i_t^1 denote the *n*-period interest rate (long-term rate) and one-period interest rate (short-term rate) respectively. Consider the following definition of the time-varying term premium TP_t:

$$i_t^n = \frac{1}{n}\sum_{j=0}^{n-1} E_t i_{t+j}^1 + TP_t ,$$

(2.5)

where $E_t(i_{t+j}^1)$ denotes the market's expectation at time *t* of the value of i_{t+j}^1. The term premium TP_t could be viewed, for example, as the sum of a liquidity premium (h_t) and risk premium (q_t) : $TP_t = h_t + q_t$; see Kim (1998). Equation (2.5) can alternatively be written

$$i_t^n - i_t^1 = \left(\frac{1}{n}\sum_{j=0}^{n-1} E_t i_{t+j}^1 - i_t^1 \right) + TP_t .$$

(2.6)

Equation (2.6) implies that the spread can be decomposed into two terms. The first term on the right-hand side of equation (2.6) is the difference between the future expected short-term interest rates over the next *n* periods and the current rate. The second term is the time-varying term premium. Thus, if the spread predicts U.S. recessions, it could either be because (1) a *temporarily* high short-term rate suggests a coming recession, or (2) a fall in the premium on long-term bonds relative to short-term bonds suggests an economic recession. Given that the short rate rises relative to the long rate prior to a recession, to what extent is this because future short rates are rationally expected to fall (the simple expectations hypothesis), and to what extent is it because the forecastable excess yield from holding long-term bonds has fallen (which must be a risk premium or a liquidity premium)? We now show how this question can be answered from the data.

Notice that the spread can be written

$$i_t^n - i_t^1 = \left(\frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^1 - i_t^1 \right) + \left(i_t^n - \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^1 \right).$$

(2.7)

Substituting equation (2.7) into (2.1),

$$y_t^k = a_0 + a_1 \left(\frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^1 - i_t^1 \right) + a_1 \left(i_t^n - \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^1 \right) + e_t \, .$$

(2.8)

Expression (2.8) decomposes the contribution of the spread into the effect of expected future changes in short rates $\left(\frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^1 - i_t^1 \right)$ and the effect of the term premium $\left(TP_t = i_t^n - \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^1 \right)$. A generalization of equation (2.8) would allow these two components to have different implications for future GDP:

$$y_t^k = g_0 + g_1 \left(\frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^1 - i_t^1 \right) + g_2 \left(i_t^n - \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^1 \right) + e_t \, .$$

(2.9)

Let v_{t+n} denote the error in forecasting future short-term rates:

$$v_{t+n} = \frac{1}{n} \sum_{j=0}^{n-1} i_{t+j}^1 - \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^1 \, .$$

Then (2.9) can be written

$$y_t^k = g_0 + g_1 \left(\frac{1}{n} \sum_{j=0}^{n-1} i_{t+j}^1 - i_t^1 \right) + g_2 \left(i_t^n - \frac{1}{n} \sum_{j=0}^{n-1} i_{t+j}^1 \right) + u_t$$

(2.10)

where $u_t = e_t - g_1 v_{t+n} + g_2 v_{t+n}$. Under rational expectations, the error term u_t should be uncorrelated with any variable known at time *t*. Thus, (2.10) can be estimated using instrumental variable estimation with variables such as the current interest rates, current spread or current inflation as the instruments.

Table 5 shows the estimation results for equation (2.10). The results are very interesting. The estimated coefficient on the future expected short-term interest rate change over *n* periods is statistically significant up to 12 quarters ahead at the 5% level and the coefficient on the term premium is statistically significant at the 1% level up to 8 quarters ahead. Thus, both the expected change of the short-term rate over *n* periods (the simple expectations hypothesis) and the time-varying term premium help predict real GDP growth up to 8 quarters ahead. Which factor contributes more to predicting real GDP growth? The results of a Wald test of the null hypothesis that the coefficient on the expected change of short-term rates over *n*-periods is equal to that of the term premium are shown in the fifth column of Table 5. Even though the estimated coefficients are similar, the null hypothesis is rejected in all cases where both estimated coefficients are statistically significant. The contribution of the future expected change of short-term rates to prediction of real GDP growth is statistically significantly bigger than that of the term premium. One of the key reasons that a positive yield spread predicts faster real GDP growth is that a positive spread implies higher future short-term interest rates.

The signs of the estimated coefficients are all positive from 1 to 8 quarters ahead. One possible reason for the significant values for g_1 is that increases in inflation expectations lead to both higher expected short rates and faster output growth. Another possibility is that the short-term real interest rate is procyclical owing to procyclical variation in investment demand. If investors forecast a recession, they would then also predict falling short-term rates, accounting for a positive correlation between y_t^k and $\left(\frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j}^1 - i_t^1 \right)$.

Another interesting result is that the term premium helps predict real GDP growth up to 8 quarters ahead and the sign of estimated coefficient on the term premium is positive. This implies that there exists a positive relationship between the business cycle and term premium. Why is an increase in term premium associated with an increase in future real GDP growth?

Suppose that the economy is in an expansionary period in which the market not only expects an increase in inflation but may also perceive an increase in the variance of inflation (i.e. inflation volatility). If inflation is not only high but also volatile, people don't want to hold relatively long-term bonds. So, the volatility of inflation will be priced in the term structure of interest rates as a positive term premium. On the other hand, in a recessionary period, people expect that there will be a decrease in inflation and the volatility of inflation will be low. So, people won't require as high a term premium. Hence, changes in inflation volatility could account for both the change in the nominal interest rates and the term premium. This could account for the positive relationship between the business cycle and the term premium.

The following section develops a theoretical model for investigating the relationship between the term structure of interest rates and the volatility of the inflation.

3. A model of the inflation variance term premium

3.1 A reformulation of the expectations hypothesis

Suppose that investors are risk neutral and seek to maximize expected wealth as of period $(t+n)$. Then market equilibrium requires the expected yield from rolling over n 1-period bonds to be the same as that from an n-period bond:

$$E_t(1+i_t^1)(1+i_{t+1}^1)\cdots(1+i_{t+n-1}^1)=(1+i_t^n)^n .$$

(3.1)

Suppose that the nominal 1-period interest rate evolves according to

$$\ln(1+i_t^1) = c + f\ln(1+i_{t-1}^1) + e_t,\ e_t \mid \Omega_{t-1} \sim iid\ \ N(0, s_{t|t-1}^2)$$

(3.2)

where $s_{t|t}^2$ denotes $E(e_t^2 \mid \Omega_t)$ and Ω_t is the information available at date b . If the real interest rate were constant, then $s_{t|t-1}^2$ would denote the conditional variance of the inflation forecast error and f the autocorrelation of inflation. Since $\ln(1+i_t^1)$ is known, the distribution of $\ln(1+i_{t+1}^1)$ conditional on information at time t can be written

$$\ln(1+i_{t+1}^1) \mid \Omega_t \sim N(c + f\ln(1+i_t^1), s_{t+1|t}^2)\ .$$

(3.3)

Similarly, we can find the joint distribution for $\ln(1+i_{t+1}^1), \ln(1+i_{t+2}^1)$ as follows;

$$\begin{bmatrix} \ln(1+i_{t+1}^i) \\ \ln(1+i_{t+2}^i) \end{bmatrix} \mid \Omega_t \sim N\left(\begin{bmatrix} c + f\ln(1+i_t^1) \\ (1+f)c + f^2\ln(1+i_t^1) \end{bmatrix}, \begin{bmatrix} s_{t+1|t}^2 & f s_{t+1|t}^2 \\ f s_{t+1|t}^2 & f^2 s_{t+1|t}^2 + s_{t+2|t}^2 \end{bmatrix} \right).$$

(3.4)

Note that $\ln(1+i_{t+1}^1)(1+i_{t+2}^1) = \ln(1+i_{t+1}^1) + \ln(1+i_{t+2}^1)$ and

$$\ln(1+i_{t+1}^1) + \ln(1+i_{t+2}^1) | \Omega_t \sim N((2+f)c + f(1+f)\ln(1+i_t^1), (1+f)^2 s_{t+1|t}^2 + s_{t+2|t}^2)\ .$$

Using the moments of the lognormal distribution, we obtain

$$E_t(1+i_{t+1}^1)(1+i_{t+2}^1) = \exp\{(2+f)c + f(1+f)\ln(1+i_t^1) + \frac{1}{2}[(1+f)^2 s_{t+1|t}^2 + s_{t+2|t}^2]\}$$

$$E_t(1+i_{t+1}^1)(1+i_{t+2}^1) = (1+i_t^1)^{f(1+f)} \exp\{(2+f)c + \frac{1}{2}[(1+f)^2 s_{t+1|t}^2 + s_{t+2|t}^2]\}\ .$$

(3.5)

Note that

$$E_t(1+i_t^1)(1+i_{t+1}^1)(1+i_{t+2}^1) = (1+i_t^1)E_t(1+i_{t+1}^1)(1+i_{t+2}^1)\ .$$

(3.6)

So, from equation (3.5) and (3.6), we have

$$E_t(1+i_t^1)(1+i_{t+1}^1)(1+i_{t+2}^1) = (1+i_t^1)^{1+f(1+f)}\exp\{(2+f)c + \frac{1}{2}[(1+f)^2 s_{t+1|t}^{2} + s_{t+2|t}^{2}]\}.$$

(3.7)

Taking the log of equation (3.7) and using the following relationships,

$$E_t \ln(1+i_{t+1}^1) = c + f\ln(1+i_t^1),\ E_t \ln(1+i_{t+2}^1) = (1+f)c + f^2\ln(1+i_t^1),$$

we have

$$\begin{aligned}&\ln E_t(1+i_t^1)(1+i_{t+1}^1)(1+i_{t+2}^1)\\ &= \ln(1+i_t^1) + E_t\ln(1+i_{t+1}^1) + E_t\ln(1+i_{t+2}^1) + \frac{1}{2}[(1+f)^2 s_{t+1|t}^{2} + s_{t+2|t}^{2}]\end{aligned}.$$

(3.8)

Taking the log of equation (3.1) for *n*=3 and combining it with equation (3.8) results in the following equation:

$$\begin{aligned}&\ln(1+i_t^3)\\ &= \frac{1}{3}[\ln(1+i_t^1) + E_t\ln(1+i_{t+1}^1) + E_t\ln(1+i_{t+2}^1)] + \frac{1}{6}[(1+f)^2 s_{t+1|t}^{2} + s_{t+2|t}^{2}]\end{aligned}. \qquad (3.9)$$

Equation (3.9) implies a logarithmic expectations hypothesis with time-varying term premium. The first term on the right-hand side of the equation is the weighted average of the expected logarithmic returns of the 1-period bond over the 3 periods and the second is a time-varying term premium. This term premium mainly comes from the variance of the inflation forecast error. Hence, even though investors are risk neutral, the simple linear expectations hypothesis does not hold.[8]

We can extend this framework to the case of an *n*-period bond. Our empirical analysis uses the 10-year T-bond rate and the 3-month T-bill rate. For the 3-month Treasury bill rate and 10-year Treasury bond rate (*n*=40), we have

[8] See Benninga and Protopapadakis (1983) and Sarte (1998) for other examples of this point.

$$\ln(1+i_t^n) = \frac{1}{n}[\ln(1+i_t^1) + E_t \ln(1+i_{t+1}^1) + \cdots + E_t \ln(1+i_{t+n-1}^1)]$$
$$+ \frac{1}{2n}[s^2_{t+n-1|t} + (1+f)^2 s^2_{t+n-2|t} + \cdots + (1+f+f^2+\cdots+f^{n-2})^2 s^2_{t+1|t}] .$$

(3.10)

3.2 Comparison between the logarithmic and linear term structure models

First we reproduce the results of the previous section using the logarithmic specification in place of the linear specification used earlier. We define the logarithmic term premium as

$$LTP_t = \ln(1+i_t^n) - \frac{1}{n}[\ln(1+i_t^1) + E_t \ln(1+i_{t+1}^1) + \cdots + E_t \ln(1+i_{t+n-1}^1)] .$$

(3.11)

Figure 2 plots both ex-post values for TP_t and LTP_t from 1953:II to 1988:III. The processes are very similar. The mean and variance of the term premium are 0.683 and 5.431 and the mean and variance of the logarithmic term premium are 0.635 and 4.563. We further estimated the following equation

$$y_t^k = g_0 + g_1\left(\frac{1}{n}\sum_{j=0}^{n-1}\ln(1+i_{t+j}^1) - \ln(1+i_t^1)\right) + g_2\left(\ln(1+i_t^n) - \frac{1}{n}\sum_{j=0}^{n-1}\ln(1+i_{t+j}^1)\right) + e_t \qquad (3.12)$$

by instrumental variables with a constant and current interest rates as instruments.

Table 6 shows the estimation results. These results are quite similar with the results in Table 5. All values of estimated coefficients are pretty close to those of estimated coefficients of the normal term structure model. Both coefficients on the expected change of short-term rate and logarithmic term premium are positive and statistically significant at the 1% level up to 8 quarters ahead. The test results for the null hypothesis that the value of the coefficient on the expected change of short-term rate is equal to that of coefficient on the logarithmic term premium are very similar with those of the normal term structure model. Hence, the logarithmic specification has virtually the same predictability for future real GDP growth.

3.3 Accounting for the size of the term premium.

Our logarithmic term structure model, equation (3.10), implies that the variance of the inflation forecast error could account for the coefficient g_2 on the logarithmic term premium in equation (3.12). Given a model for the conditional variance of the nominal interest rate, we can forecast the conditional variance given information at time t and thus, we can estimate the logarithmic term premium using a GARCH-type model.[9] Here we employ the model of Brenner, Harjes, and Kroner (1996) which treats volatility (variance) as a function of both the interest rate level and unexpected interest rate shocks:

$$\begin{aligned} &\ln(1+i_t^1) = c + f\ln(1+i_{t-1}^1) + e_t, e_t \mid \Omega_{t-1} \sim N(0, s_{t|t-1}^2) \\ &s_{t|t-1}^2 = w + a e_{t-1}^2 + b s_{t-1|t-2}^2 + g\ln(1+i_{t-1}^1) \end{aligned}.$$

(3.13)

Estimation results are as follows:

$$\begin{aligned} &\ln(1+i_t^1) = \underset{(0.00060)}{0.00265} + \underset{(0.01302)}{0.95191}\ln(1+i_{t-1}^1) + e_t, e_t \mid \Omega_{t-1} \sim N(0, s_{t|t-1}^2), \qquad \bar{R}^2 = 0.927 \end{aligned}$$

(3.14)

$$s_{t|t-1}^2 = \underset{(1.18E-06)}{-3.87E-06} + \underset{(0.1608)}{0.3475} e_{t-1}^2 + \underset{(0.1346)}{0.34912} s_{t-1|t-2}^2 + \underset{(6.55E-05)}{0.00031}\ln(1+i_{t-1}^1)$$

(3.15)

where the figures in parentheses are the standard error of coefficients. Following the method in Hamilton (1994, p. 666) and using equation (3.15), we can forecast $\hat{s}_{t+1|t}^2, \hat{s}_{t+2|t}^2, \cdots, \hat{s}_{t+n-1|t}^2$ in the future given information at time *t*. Using equation (3.10), (3.14) and the forecasting result, we can then estimate the logarithmic term premium for the 10-year T-bond rate and 3-month T-bill rate. The estimated logarithmic term premium at time *t*, which we denote q_t , is given by

$$q_t = \frac{1}{80}[\hat{s}^2_{t+n-1|t} + (1+\hat{f})^2 \hat{s}^2_{t+n-2|t} + \cdots + (1+\hat{f}+\hat{f}^2+\cdots+\hat{f}^{n-2})^2 \hat{s}^2_{t+1|t}]$$

(3.16)

where $\hat{s}^2_{t+n|t}$ is the forecast of the conditional variance of the inflation forecast error in the future *n-1* period given information at time *t*. We calculated the resulting estimated logarithmic term premium from 1953:III to 1988:IV. The sample average of the estimated logarithmic term premium under time-varying inflation variance is 0.35776 (35.8 basis points)[10]. This represents more than half the average value of the ex-post term premium or ex-post logarithmic term premium, which suggests that the estimated logarithmic term premium derived from the predicted variance of inflation forecast error might be a significant component in the term premium.

3.4 The relation between the logarithmic term premium and future real GDP growth

We now consider whether the estimated logarithmic term premium helps predict real GDP growth by estimating the following regression:

$$y_t^k = g_0 + g_1\left(\frac{1}{n}\sum_{j=0}^{n-1}\ln(1+i^1_{t+j}) - \ln(1+i^1_t)\right) + g_2\left(\ln(1+i^n_t) - \frac{1}{n}\sum_{j=0}^{n-1}\ln(1+i^1_{t+j})\right) + g_3 q_t + e_t \quad (3.17)$$

where q_t is the estimated logarithmic term premium at time *t* from equation (3.16). We estimated equation (3.17) using as instruments q_t, i^1_t, i^n_t and a constant. The result of this instrumental variable estimation is shown in the Table 7.

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[9] Bollerslev, Chou and Kroner (1992) and Bollerslev, Engle and Nelson (1994) provide excellent reviews for this class of models.

The coefficient on the estimated logarithmic term premium is statistically significant at the 5% level for 3- and 4-quarter-ahead-forecasts. In addition, the sign of the coefficient on the estimated logarithmic term premium is positive as expected in the theoretical model. The value of the coefficient, however is much bigger than expected. The coefficient on the expected change of the short-term rate is larger and the coefficient on the ex-post logarithmic term premium is smaller when including the estimated logarithmic term premium than when it is excluded. However, the coefficient on the ex-post logarithmic term premium is still statistically significant at the 1% level up to 8 quarters ahead. This implies that variation of the inflation forecast error variance is one reason, but not the major reason, why the term premium helps predict future GDP growth.

The interpretation suggested by these findings is as follows. Given the positive value of g in (3.13), an increase in inflation makes it more difficult to forecast short-term interest rates. From (3.8) (or, more generally, from Jensen's inequality), increased variability of i_{t+j}^{1} increases the expected return from rolling over 1-period bonds. Hence, from (3.10), for given expectation of $E_t \ln(1+i_{t+j}^{1})$, increased inflation variability should translate into a higher value for $\ln(1+i_t^n)$ relative to $E_t \ln(1+i_{t+j}^{1})$ and thus a higher value for the spread $\ln(1+i_t^n)-\ln(1+i_t^1)$. If inflation variability increases prior to an economic expansion, a positive correlation between the term premium component of the spread and future GDP growth could then be explained. Nevertheless, such an effect is only a small part of the story, as Table 7 reveals that most of the term premium effect remains to be explained.

4. Conclusion

[10] This compares with an estimated logarithmic term premium of 0.470 (47 basis points) under the assumption that the variance of the inflation forecast error is constant over time.

We have confirmed earlier results on the usefulness of the spread between long-term and short-term interest rates for forecasting GDP growth. We have shown how to decompose this effect into an expectations effect and a term premium effect. Both effects are statistically significant—a forecast of falling short-term interest rates is associated with a forecast of slower GDP growth, and an increase in the expected future return for 1-period bonds relative to an *n*-period bond is also associated with a forecast of slower GDP growth—though the first effect (the expectations effect) is slightly more important quantitatively and statistically.

We proposed a simple model for interpreting the second effect (the term premium effect) based on time-variation in the variance of short-term interest rates. The model attributes part of the term premium to the difference between a simple average of future short-term rates and the actual return from rolling over short-term bonds. From Jensen's inequality, the expected return from rolling over short-term bonds exceeds a simple average of expected future short rates, and gives a reason why the long rate should exceed a simple average of expected future rates, with the excess increasing when the variance of short rates increases. We calibrated this effect using Brenner, Harjes, and Kroner's (1996) GARCH(1,1)-X model, and found it makes a statistically significant contribution—a lower forecast error variance for short-term rates (which could account for a decrease in the *n*-period rate relative to an average of expected future 1-period rates) makes a statistically significant contribution to predicting slower GDP growth. However, this can only be a small part of the measured term premium effect, most of which must therefore be attributed to something else.

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Table 1. Predicting future real GDP growth using the yield spread

$y_t^k = a_0 + a_1 Spread_t + e_t$

k (quarters ahead)	$\hat{a}_0$	$\hat{a}_1$	$\bar{R}^2$
1	1.818***	0.940***	0.066
	(0.626)	(0.342)	
2	1.714***	1.029***	0.123
	(0.631)	(0.349)	
3	1.753***	1.011***	0.156
	(0.605)	(0.336)	
4	1.813***	0.979***	0.183
	(0.587)	(0.325)	
5	1.929***	0.902***	0.192
	(0.567)	(0.305)	
6	2.063***	0.806***	0.188
	(0.544)	(0.279)	
7	2.185***	0.714***	0.176
	(0.521)	(0.255)	
8	2.318***	0.609***	0.149
	(0.501)	(0.231)	
12	2.721***	0.283	0.049
	(0.436)	(0.171)	
16	2.953***	0.108	0.006
	(0.348)	(0.115)	

Note: a. In parentheses are Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors corrected with twelve lags.

b. *** denotes statistically significant at the 1% level in a two-tailed test.

Table 2. Predicting future real GDP growth using the yield spread and lagged real GDP growth

$$y_t^k = b_0 + b_1 Spread_t + b_2 y_{t-1}^1 + b_3 y_{t-2}^1 + b_4 y_{t-3}^1 + b_5 y_{t-4}^1 + e_t$$

k (quarters ahead)	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$	$\hat{b}_4$	$\hat{b}_5$	$\bar{R}^2$
1	1.281**	0.763***	0.264***	0.077	0.003	-0.064	0.15
	(0.534)	(0.241)	(0.074)	(0.076)	(0.076)	(0.072)	
2	1.611***	0.863***	0.193***	0.050	-0.007	-0.105	0.21
	(0.423)	(0.191)	(0.058)	(0.060)	(0.060)	(0.057)	
3	1.848***	0.876***	0.141***	0.028	-0.054	-0.070	0.21
	(0.586)	(0.254)	(0.057)	(0.050)	(0.044)	(0.044)	
4	1.997***	0.880***	0.100**	-0.015	-0.037	-0.056	0.22
	(0.333)	(0.151)	(0.046)	(0.048)	(0.047)	(0.045)	
5	2.188***	0.835***	0.046	-0.010	-0.027	-0.064	0.22
	(0.304)	(0.137)	(0.042)	(0.043)	(0.043)	(0.041)	
6	2.313***	0.757***	0.031	-0.010	-0.041	-0.045	0.21
	(0.278)	(0.126)	(0.038)	(0.040)	(0.039)	(0.038)	
7	2.401***	0.684***	0.022	-0.027	-0.032	-0.028	0.19
	(0.259)	(0.117)	(0.036)	(0.037)	(0.037)	(0.035)	
8	2.521***	0.595***	0.002	-0.024	-0.019	-0.025	0.16
	(0.245)	(0.111)	(0.034)	(0.035)	(0.035)	(0.033)	
12	2.974***	0.258***	0.004	-0.017	-0.024	-0.035	0.07
	(0.205)	(0.092)	(0.028)	(0.029)	(0.029)	(0.028)	
16	3.141***	0.094	-0.001	-0.016	-0.013	-0.020	0.02
	(0.176)	(0.080)	(0.024)	(0.025)	(0.025)	(0.024)	

Note: In parentheses are standard errors for the OLS estimator and *** and ** denote statistically significant at the 1% and 5% level in a two tailed test respectively.

Table 3A. Predicting future real GDP growth using the spread and change in the Federal funds rate

$$y_t^k = b_0 + b_1 Spread_t + b_2 \Delta ffr_t + e_t$$

k (quarters ahead)	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$	$\bar{R}^2$
1	1.642***	1.129***	0.711**	0.094
	(0.724)	(0.424)	(0.301)	
2	1.807***	0.999***	0.024	0.125
	(0.679)	(0.378)	(0.225)	
3	1.821***	0.973***	-0.016	0.159
	(0.662)	(0.367)	(0.214)	
4	1.892***	0.912***	-0.127	0.191
	(0.645)	(0.349)	(0.209)	
5	2.029***	0.807**	-0.244	0.207
	(0.617)	(0.319)	(0.195)	
6	2.162***	0.707**	-0.284	0.208
	(0.587)	(0.288)	(0.193)	
7	2.288***	0.611**	-0.316	0.203
	(0.556)	(0.259)	(0.180)	
8	2.444***	0.495**	-0.357**	0.187
	(0.528)	(0.232)	(0.175)	
12	2.853***	0.184	-0.303**	0.091
	(0.448)	(0.169)	(0.121)	
16	3.091***	0.026	-0.245***	0.046
	(0.347)	(0.112)	(0.095)	

Notes: a. Δffr_t is the one-quarter change in the quarterly average Federal funds rate at time *t*.

b. In parentheses are Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors corrected with twelve lags.

c. *** and ** denote significantly different from zero at the 1% and 5% level in a two-tailed test respectively.

Table 3B. Predicting real GDP growth using the spread and the growth of monetary aggregates

$$y_t^k = b_0 + b_1 Spread_t + b_2 X_t + e_t$$

k (quarter)	M1				M2			
	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$	$\overline{R}^2$	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$	$\overline{R}^2$
1	1.823**	0.706**	0.063	0.069	0.148	0.739***	0.288***	0.140
	(0.727)	(0.358)	(0.050)		(0.718)	(0.263)	(0.054)	
2	1.788**	0.845**	0.035	0.128	0.156	0.847***	0.267***	0.233
	(0.728)	(0.359)	(0.029)		(0.703)	(0.285)	(0.050)	
3	1.911***	0.875***	0.004	0.156	0.409	0.847***	0.230***	0.260
	(0.688)	(0.337)	(0.026)		(0.650)	(0.274)	(0.049)	
4	2.005***	0.890***	-0.019	0.188	0.567	0.839***	0.207***	0.287
	(0.666)	(0.322)	(0.021)		(0.613)	(0.271)	(0.045)	
5	2.079***	0.842***	-0.020	0.196	0.751	0.793***	0.189***	0.292
	(0.629)	(0.309)	(0.020)		(0.562)	(0.261)	(0.042)	
6	2.187***	0.769***	-0.020	0.190	0.995	0.722***	0.168***	0.277
	(0.617)	(0.283)	(0.019)		(0.559)	(0.245)	(0.039)	
7	2.326***	0.695***	-0.025	0.178	1.275**	0.646***	0.143***	0.248
	(0.602)	(0.258)	(0.020)		(0.563)	(0.230)	(0.038)	
8	2.503***	0.608***	-0.033	0.155	1.548***	0.551**	0.123***	0.207
	(0.578)	(0.236)	(0.021)		(0.553)	(0.216)	(0.039)	
12	2.897***	0.299	-0.029	0.059	2.352***	0.258	0.063	0.071
	(0.525)	(0.183)	(0.024)		(0.532)	(0.178)	(0.043)	
16	3.143***	0.122	-0.030	0.022	2.806***	0.079	0.030	0.009
	(0.444)	(0.124)	(0.023)		(0.469)	(0.126)	(0.039)	

Notes: a. M1 and M2 are the annualized one-quarter growth rates of seasonally non-adjusted narrow (M1) and broad (M2) monetary aggregate respectively.

b. In parentheses are Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors corrected with twelve lags.

c. *** and ** denote significantly different from zero at the 1% and 5% level in a two-tailed test respectively.

Table 4A. The predictability of the yield spread, lagged economic growth and oil price changes for real economic activity

$$y_t^k = b_0 + b_1 Spread_t + b_2 y_{t-1}^1 + b_3 y_{t-2}^1 + b_4 y_{t-3}^1 + b_5 y_{t-4}^1 + f_1 o_t + f_2 o_{t-1} + f_3 o_{t-2} + f_4 o_{t-3} + e_t$$

(2.4)

k	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$	$\hat{b}_4$	$\hat{b}_5$	$\hat{f}_1$	$\hat{f}_2$	$\hat{f}_3$	$\hat{f}_4$	$\bar{R}^2$
1	1.588***	0.661**	0.252***	0.072	-0.006	-0.072	-0.002	-0.009	-0.013	-0.060**	0.178
	(0.574)	(0.257)	(0.073)	(0.076)	(0.075)	(0.072)	(0.027)	(0.028)	(0.027)	(0.027)	
2	1.976***	0.721***	0.183***	0.044	-0.017	-0.110	-0.011	-0.013	-0.035	-0.037	0.229
	(0.457)	(0.205)	(0.058)	(0.060)	(0.059)	(0.057)	(0.022)	(0.022)	(0.022)	(0.022)	
3	2.233***	0.718***	0.132***	0.023	-0.059	-0.078	-0.015	-0.027	-0.029	-0.026	0.237
	(0.404)	(0.180)	(0.051)	(0.053)	(0.052)	(0.050)	(0.019)	(0.019)	(0.019)	(0.019)	
4	2.386***	0.721***	0.089**	-0.016	-0.043	-0.068	-0.025	-0.024	-0.022	-0.025	0.246
	(0.363)	(0.162)	(0.045)	(0.047)	(0.047)	(0.045)	(0.017)	(0.017)	(0.017)	(0.017)	
5	2.512***	0.699***	0.038	-0.010	-0.032	-0.073	-0.025	-0.017	-0.023	-0.015	0.239
	(0.332)	(0.148)	(0.042)	(0.043)	(0.043)	(0.041)	(0.016)	(0.016)	(0.016)	(0.016)	
6	2.578***	0.646***	0.024	-0.011	-0.043	-0.054	-0.018	-0.018	-0.015	-0.013	0.229
	(0.305)	(0.136)	(0.038)	(0.039)	(0.039)	(0.038)	(0.015)	(0.015)	(0.015)	(0.015)	
7	2.647***	0.582***	0.015	-0.026	-0.037	-0.036	-0.019	-0.014	-0.013	-0.014	0.214
	(0.283)	(0.126)	(0.035)	(0.037)	(0.037)	(0.035)	(0.014)	(0.014)	(0.014)	(0.014)	
8	2.765***	0.493***	-0.004	-0.025	-0.024	-0.032	-0.016	-0.014	-0.016	-0.015	0.184
	(0.269)	(0.120)	(0.034)	(0.035)	(0.035)	(0.033)	(0.013)	(0.013)	(0.013)	(0.013)	
12	3.199***	0.161	-0.002	-0.017	-0.028	-0.042	-0.018	-0.014	-0.014	-0.009	0.105
	(0.223)	(0.099)	(0.028)	(0.029)	(0.029)	(0.027)	(0.011)	(0.011)	(0.011)	(0.011)	
16	3.292***	0.024	-0.005	-0.015	-0.015	-0.025	-0.016	-0.010	-0.008	-0.002	0.044
	(0.193)	(0.087)	(0.024)	(0.025)	(0.025)	(0.024)	(0.009)	(0.010)	(0.010)	(0.009)	

Notes: a. o_t is the percent change in the nominal price of crude petroleum at time *t*.

b. In parentheses are standard errors for the OLS estimator and ***and ** denote significantly different from zero at the 1% and 5% level in a two-tailed test respectively.

Table 4B. The predictability of the yield spread, lagged economic growth and nonlinear transformation data of oil price changes constructed by Hamilton (1996, 1999b)

$$y_t^k = b_0 + b_1 Spread_t + b_2 y_{t-1}^1 + b_3 y_{t-2}^1 + b_4 y_{t-3}^1 + b_5 y_{t-4}^1$$
$$+ f_1 o_t^* + f_2 o_{t-1}^* + f_3 o_{t-2}^* + f_4 o_{t-3}^* + e_t$$

k	$\hat{b}_0$	$\hat{b}_1$	$\hat{b}_2$	$\hat{b}_3$	$\hat{b}_4$	$\hat{b}_5$	$\hat{f}_1$	$\hat{f}_2$	$\hat{f}_3$	$\hat{f}_4$	$\bar{R}^2$
1	2.864***	0.491	0.194***	0.048	-0.038	-0.102	-0.069	-0.040	-0.073	-0.163***	0.229
	(0.673)	(0.253)	(0.073)	(0.074)	(0.074)	(0.071)	(0.053)	(0.054)	(0.052)	(0.052)	
2	3.339***	0.520***	0.123**	0.016	-0.044	-0.143***	-0.075	-0.070	-0.117***	-0.110***	0.316
	(0.518)	(0.195)	(0.056)	(0.057)	(0.057)	(0.054)	(0.041)	(0.041)	(0.040)	(0.040)	
3	3.569***	0.488***	0.077	0.001	-0.081	-0.114**	-0.094***	-0.107***	-0.096***	-0.063	0.344
	(0.448)	(0.169)	(0.048)	(0.049)	(0.049)	(0.047)	(0.035)	(0.036)	(0.035)	(0.035)	
4	3.635***	0.506***	0.038	-0.032	-0.065	-0.105**	-0.118***	-0.094***	-0.058	-0.069**	0.365
	(0.398)	(0.150)	(0.043)	(0.044)	(0.043)	(0.042)	(0.031)	(0.032)	(0.031)	(0.031)	
5	3.626***	0.504***	-0.007	-0.024	-0.055	-0.104***	-0.112***	-0.068**	-0.061**	-0.057**	0.355
	(0.365)	(0.137)	(0.039)	(0.040)	(0.040)	(0.038)	(0.029)	(0.029)	(0.028)	(0.029)	
6	3.541***	0.471***	-0.015	-0.024	-0.061	-0.082**	-0.088***	-0.070**	-0.051	-0.045	0.331
	(0.339)	(0.127)	(0.036)	(0.037)	(0.037)	(0.036)	(0.027)	(0.027)	(0.027)	(0.027)	
7	3.554***	0.422***	-0.023	-0.038	-0.055	-0.062	-0.086***	-0.060**	-0.039	-0.057**	0.320
	(0.317)	(0.119)	(0.034)	(0.035)	(0.034)	(0.033)	(0.025)	(0.026)	(0.025)	(0.025)	
8	3.616***	0.346***	-0.040	-0.038	-0.043	-0.055	-0.078***	-0.050**	-0.050**	-0.052**	0.291
	(0.301)	(0.113)	(0.032)	(0.033)	(0.033)	(0.031)	(0.024)	(0.024)	(0.024)	(0.024)	
12	3.835***	0.048	-0.027	-0.024	-0.041	-0.061**	-0.071***	-0.048**	-0.031	-0.027	0.204
	(0.255)	(0.095)	(0.027)	(0.028)	(0.027)	(0.026)	(0.020)	(0.020)	(0.020)	(0.020)	
16	3.768***	-0.066	-0.023	-0.020	-0.024	-0.039	-0.057***	-0.037**	-0.021	-0.012	0.128
	(0.223)	(0.083)	(0.024)	(0.024)	(0.024)	(0.023)	(0.018)	(0.018)	(0.017)	(0.017)	

Notes: a. o_t^* is the amount by which oil prices in quarter *t* exceed their peak value over the previous 12 months; if they do not exceed the previous peak then the oil variable is taken to be zero. This oil variable is the nonlinear transformation of oil price changes and Hamilton (1996 and 1999)'s measure of the net oil price increase.

b. In parentheses are standard errors for the OLS estimator and *** and ** denote significantly different from zero at the 1% and 5% level in a two-tailed test respectively.

Table 5. Predicting future real GDP growth using the decomposition of the yield spread

$y_t^k = g_0 + g_1\left(\frac{1}{n}\sum_{j=0}^{n-1} i_{t+j}^1 - i_t^1\right) + g_2\left(i_t^n - \frac{1}{n}\sum_{j=0}^{n-1} i_{t+j}^1\right) + e_t$ with a constant, i_t^n, i_t^1 as instruments

k (quarter)	$\hat{g}_0$	$\hat{g}_1$	$\hat{g}_2$	Test: c_m^2 $H_0 : g_1 = g_2$
1	1.685***	1.614***	1.074***	8.689***
	(0.654)	(0.327)	(0.331)	
2	1.583***	1.740***	1.163***	11.096***
	(0.604)	(0.291)	(0.305)	
3	1.660***	1.691***	1.128***	11.125***
	(0.577)	(0.291)	(0.301)	
4	1.745***	1.626***	1.082***	10.539***
	(0.535)	(0.262)	(0.285)	
5	1.892***	1.495***	0.981***	9.373***
	(0.484)	(0.211)	(0.263)	
6	2.063***	1.327***	0.865***	7.411***
	(0.443)	(0.167)	(0.242)	
7	2.211***	1.172***	0.762***	5.581**
	(0.407)	(0.127)	(0.220)	
8	2.362***	1.004***	0.645***	4.073**
	(0.386)	(0.106)	(0.202)	
12	2.809***	0.484***	0.273	1.313
	(0.375)	(0.172)	(0.180)	
16	3.056***	0.219	0.057	0.898
	(0.307)	(0.148)	(0.170)	

Note: a. In parentheses are Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors corrected with twelve lags.

b. *** and ** denote statistically significant at the 1% and 5% level in a two tailed test respectively.

c. The figures in the fifth column are test statistics of c_1^2 with one restriction. *** and ** indicate rejection of the null hypothesis that the value of estimated coefficient of the future expected change of the short-term rate is equal to that of the term premium at the 1% and 5% level respectively.

Table 6. Predicting future real GDP growth using the decomposition of the logarithmic yield spread

$$y_t^k = g_0 + g_1\left(\frac{1}{n}\sum_{j=0}^{n-1}\ln(1+i_{t+j}^1) - \ln(1+i_t^1)\right) + g_2\left(\ln(1+i_t^n) - \frac{1}{n}\sum_{j=0}^{n-1}\ln(1+i_{t+j}^1)\right) + e_t$$

with a constant, i_t^n, i_t^1 as instruments

k (quarter)	$\hat{g}_0$	$\hat{g}_1$	$\hat{g}_2$	Test: c_m^2 $H_0: g_1 = g_2$
1	1.641**	1.742***	1.181***	8.164***
	(0.652)	(0.339)	(0.358)	
2	1.535**	1.879***	1.278***	10.590***
	(0.601)	(0.300)	(0.331)	
3	1.613***	1.825***	1.240***	10.809***
	(0.573)	(0.300)	(0.326)	
4	1.700***	1.756***	1.189***	10.271***
	(0.531)	(0.269)	(0.309)	
5	1.851***	1.614***	1.079***	9.052***
	(0.481)	(0.217)	(0.286)	
6	2.027***	1.433***	0.952***	7.074***
	(0.440)	(0.172)	(0.264)	
7	2.178***	1.265***	0.838***	5.278**
	(0.406)	(0.131)	(0.241)	
8	2.334***	1.084***	0.710***	3.834
	(0.386)	(0.112)	(0.221)	
12	2.795***	0.523***	0.301	1.254
	(0.375)	(0.186)	(0.197)	
16	3.050***	0.237	0.064	0.880
	(0.307)	(0.160)	(0.185)	

Note: a. In parentheses are Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors corrected with twelve lags.

b. *** and ** denote statistically significant at 1% and 5% level, respectively.

c. The figures in the fifth column are test statistics of c_1^2 with one restriction. ***, and ** indicate rejection of the null hypothesis that the value of estimated coefficient of the future expected change of the short-term rate is equal to that of the term premium at the 1% and 5% level respectively.

Table 7. The prediction of the estimated logarithmic term premium for future real GDP growth

$$y_t^k = g_0 + g_1\left(\frac{1}{n}\sum_{j=0}^{n-1}\ln(1+i_{t+j}^1) - \ln(1+i_t^1)\right) + g_2\left(\ln(1+i_t^n) - \frac{1}{n}\sum_{j=0}^{n-1}\ln(1+i_{t+j}^1)\right) + g_3 q_t + e_t$$

with a constant, $q_t, i_t^1, and\ \ i_t^n$ as instruments

k (quarter)	$\hat{g}_0$	$\hat{g}_1$	$\hat{g}_2$	$\hat{g}_3$
1	-0.708	2.206***	1.165***	6.058
	(2.115)	(0.504)	(0.407)	(4.660)
2	-0.909	2.367***	1.254***	6.348*
	(1.736)	(0.476)	(0.405)	(3.424)
3	-1.686	2.475***	1.221***	8.495**
	(1.712)	(0.479)	(0.432)	(3.387)
4	-0.708	2.232***	1.171***	6.222**
	(1.570)	(0.431)	(0.379)	(3.075)
5	0.082	1.965***	1.065***	4.573*
	(1.379)	(0.357)	(0.334)	(2.723)
6	0.999	1.637***	0.942***	2.662
	(1.027)	(0.264)	(0.289)	(1.882)
7	1.830**	1.335***	0.834***	0.907
	(0.916)	(0.213)	(0.247)	(1.704)
8	2.295***	1.093***	0.708***	0.110
	(0.842)	(0.180)	(0.222)	(1.579)
12	2.577***	0.567**	0.299	0.570
	(0.821)	(0.271)	(0.202)	(1.447)
16	2.290***	0.386	0.062	1.946
	(0.866)	(0.268)	(0.213)	(1.802)

Note: a. In parentheses are Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors corrected with twelve lags.

b. ***, **, and * denote significantly different from zero at the 1%, 5% and 10% level in a two-tailed test respectively.

Figure 1 The four-quarter growth rate of real GDP and the yield spread

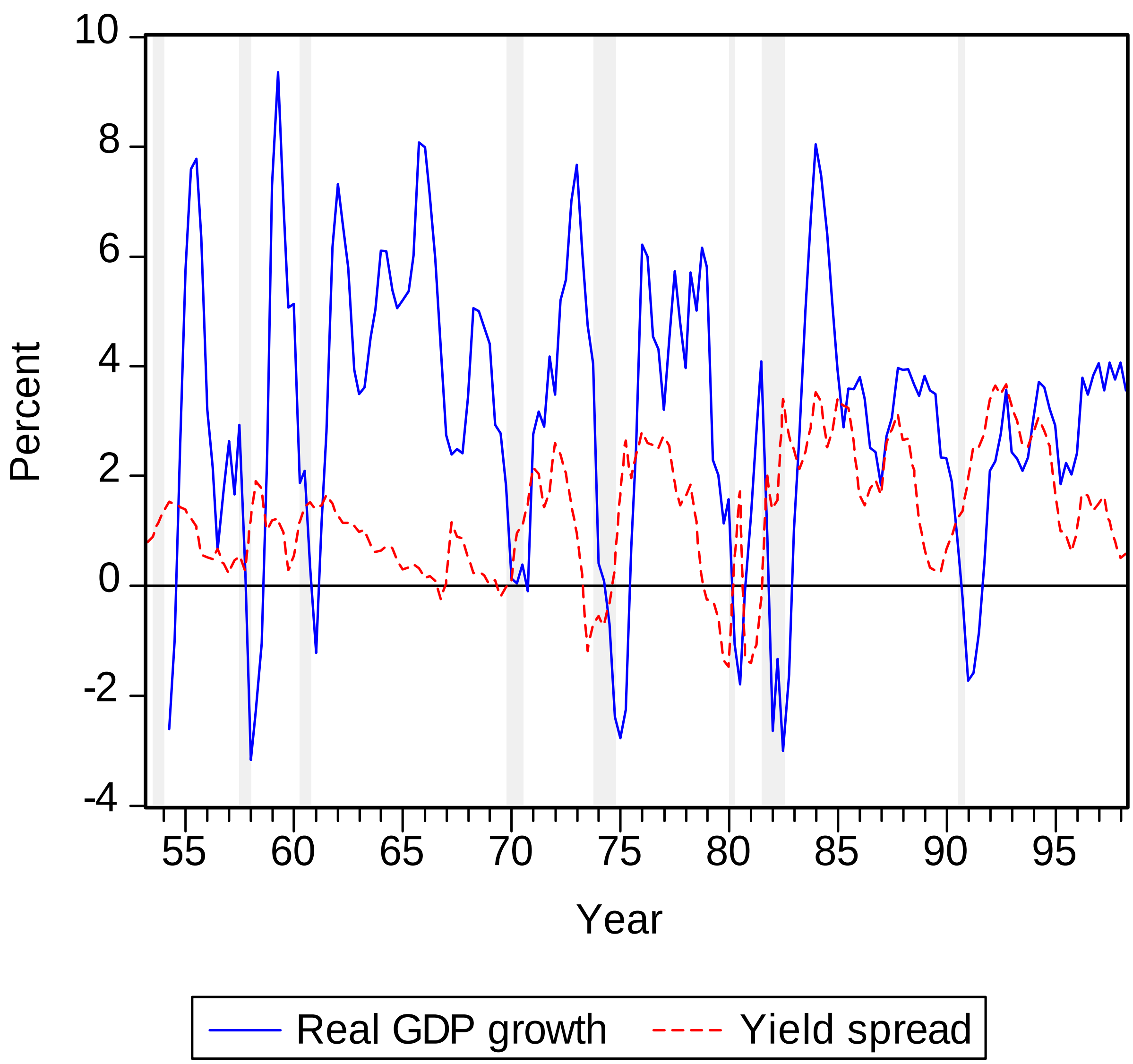

Figure 2 The ex-post term premium and logarithmic term premium