An Interference Temperature Constraints Model for Spectrum Access in Cognitive Radio Networks

Joe Bater[*,†], Hwee-Pink Tan[*,‡], Kenneth N Brown[*,†], Colman O'Sullivan[‡] and Linda Doyle[*,‡]

[*]Center for Telecommunications Value-chain Research

[†]Cork Constraint Computation Centre, University College Cork

[‡]Department of Electronic Engineering, Trinity College Dublin

Republic of Ireland.

Email: {j.bater@4c.ucc.ie, tanhp@tcd.ie, k.brown@4c.ucc.ie, cosull13@tcd.ie and ledoyle@tcd.ie}

Abstract

With the advent of cognitive radio technology, new paradigms for spectrum access can achieve near-optimal spectrum utilisation by letting each user sense and utilise available spectrum opportunistically while regulating the interference it imposes on other users through interference constraints. However, the most common forms of binary and transmitter-centric constraints are often inefficient since they only consider pair-wise sets of transmitters and prohibit any interference at receivers.

In line with the recently proposed interference temperature metric, we propose a non-binary receiver-centric constraints model that constraints whole subsets of transmitters and is easy to generate and test. By permitting interfering signals to be introduced, additional communication can be supported, leading to improved spectrum utilisation in cognitive radio networks compared with using transmitter-centric constraints. This is verified intuitively as well as through numerical experiments.

Our proposed constraints are currently being used to devise a co-operative negotiated etiquette for cognitive radios offering heterogeneous services in a wireless office networking scenario.[1]

[1]A preliminary version of this paper has been submitted to the IEEE CogNet 2007 Workshop.

I. INTRODUCTION

In today's wireless networks, a *command and control* approach to spectrum management is deployed, where fixed spectrum slices are licensed to each wireless service / technology. However, recent studies [1] have shown that spectrum utilisation is 6.5% (0.8%) and 78% (97%) of spectrum is unutilized in urban (rural) areas. This inefficient use of scarce wireless radio spectrum, along with a dramatic increase in spectrum access for mobile services, have been the driving forces towards new spectrum management paradigms [2].

In the *licensed* model, an exclusive-use license is assigned which may be traded in secondary markets. The licensee is responsible for making all substantive choices as to how the spectrum is used. In contrast, a *non-licensed* model that supports the *coexistence* of *primary* and *secondary* users is enabled by the advent of cognitive radios (CR) [3], [4]. While primary users have priority in spectrum access, secondary users can use available spectrum without interfering with primary users through opportunistic access (overlay) or low power spread-spectrum techniques (underlay). This results in efficient spectrum usage and simplifies deployment of new applications. One of the main challenges involved to achieve this is the development of spectrum sharing / access schemes. We refine the scope and define the following opportunistic spectrum access problem: *for a given spectrum availability (unused by primary users), how do we assign spectrum to secondary users to achieve maximum spectrum efficiency?*

However, the spectrum assignment problem certainly predates the advent of CR technology, e.g. [5]. There is a wealth of literature on solving the Frequency Assignment Problem (FAP) in cellular networks, key to which is in modelling this problem as generalised Graph Colouring (GC). The constraints used in FAP studies were largely *binary* (restricting the assignments of frequency on pairs of transmitters), and were usually derived from a *re-use distance*, or an estimation of the effect of interference on the cell's receivers from another potential interferer [6]. In [7], non-binary constraints were constructed which considered the effects of multiple sources of interference from across the network; these constraints placed restrictions on the simultaneous spectrum assignment for an arbitrary number of transmitters to ensure that the receivers in the cell all maintained communications of adequate quality.

In this paper, we refine these constraints and develop a framework for constraint-based approaches to spectrum access in CR networks in a non-licensed spectrum regime. In particular, we propose a non-binary receiver-centric constraint model in line with the recently proposed interference temperature metric. We demonstrate, through intuition as well as numerical results, that our proposed approach improves spectrum utilisation compared to traditional transmitter-centric approaches.

II. A Framework for Constraint-based Approaches to Spectrum Access

We consider a distributed deployment of *N* CR-enabled secondary users over a geographical region, self-configured to form an ad-hoc network and access (or share) spectrum in a *co-operative* manner. Without loss of generality, we assume that the spectrum band is divided into *M* discrete non-overlapping and non-interfering channels in the frequency domain, where $M << N$ is assumed and every channel is available to all users.

Each user has the ability to sense the radio environment and determine the available spectrum. Each user then *reconfigures* itself, e.g., in terms of transmission power [8], channel [9] or a combination of both [10] to maximize the spectrum utilisation while regulating the interference it imposes on other users. As is commonly adopted in the literature of dynamic spectrum access, we make the restrictive assumption that each user has access to a perfectly synchronized and dedicated (interference-free) Common Signalling Control Channel (CSCC) over which signalling messages for spectrum sensing and access are exchanged.

A. Problem Formulation

We assume that there are *p* transmitter/receiver pairs $\{t_i,\ r_i\}_{i=1:p}$, where each pair $(t_i,\ r_i)$ communicates in *unicast* mode over a *single* channel c. The transmission from t_i will be *detected* at r_i if the following condition holds:

$$P_{i,i,c} > NF_c, \tag{1}$$

where $P_{i,i,c}$ is the (desired) power (dB) received at r_i from t_i and NF_c is the noise floor (dB) at r_i corresponding to channel c. Given the radio propagation model and antenna gains, we can define an equivalent *detection* range, $r_d[t_i{,}c]$ for t_i in terms of $P_{i,i,c}$ and re-write Eq. (1) as follows:

$$Dist[t_i, r_i] < r_d[t_i, c],$$

where $Dist[a,\ b]$ is the *distance* between users a and b.

The quality of communication between t_i and r_i is *admissible* if the carrier-to-interference (C-$I_{i,c}$) ratio = $P_{i,i,c}$ - $\sum_{j \neq i} P_{j,i,c} - NF_c$ (dB) satisfies the following condition:

$$C{-}I_{i,c} \geq \theta_{i,c}, \tag{2}$$

where $\theta_{i,c}$ is the C-I *threshold* (dB) and $P_{j,i,c}$ is the (unwanted or interfering) power (dB) received at r_i from t_j, $j \neq i$. We have implicitly assumed a *co-channel* interference model, where adjacent channel signals are entirely rejected at each receiver.

Our objective is to determine the spectrum (channel) assignment c_p^T = $\{c_1, c_2, \cdots, c_p\}$ that requires the *minimum* number of channels to maintain admissible communication quality for all *p* pairs, where c_i is the channel assigned to communication pair i. For a given spectrum availability, a lower channel requirement for a given p implies that more channels will be available to support additional communication, i.e., better spectrum utilization.

B. Interference constraint models for spectrum access

The analysis of spectrum sharing / assignment techniques has been investigated through two major theoretical approaches. While some work uses optimization techniques to find the optimal strategies for spectrum sharing (e.g., [9], [11]), game theoretical analysis has also been used in this area (e.g., [8], [12], [10]). We adopt the former technique, and consider a *constraint programming* approach where spectrum is assigned by solving *interference constraints* T that are imposed on transmitters to meet various criteria (e.g., Eq. (1) and (2)).

A *constraint* consists of a *scope*, S, which is a subset of the variables in a problem; and a *relation*, R, which is a function or expression describing the simultaneously allowed (or disallowed) assignments of values to variables in the scope:

$$C = \langle S, R \rangle.$$

The relation can be expressed extensionally (i.e. as sets of values), or intensionally (i.e. a formulaic expression).

In the context of spectrum access, the scope of an interference constraint for a user is the set of transmitting users (potential interferers in its vicinity) that may potentially interfere with its ongoing communication with another user. The relation may specify, say, the channel separations required between the user and its potential interferers to maintain interference to within acceptable levels.

Constraints are generally described in terms of their *arity*, the number of variables in their scope. While *binary* constraints place restrictions on the simultaneously assign-able values to particular *pairs* of variables and are most common in the literature ([13] and [14] are good starting treatises on the subject), there is increasing interest in *non-binary* constraints which tackle larger subsets of variables in a particular problem than just two [15].

C. Binary Interference Constraints

Binary interference constraints place restrictions on which pairs of transmitters can transmit on the same channel, i.e., for each communication pair (t_i, r_i) in channel c, the constraints determine if t_j can also transmit in channel c. These constraints may be *transmitter-centric*, in that they are based on the detectability of signals at the receiver, or they may be *receiver-centric*, in that they are based on the quality of signals at the receiver.

1) Transmitter-centric Constraints: These are the simplest and most widely used interference constraints in the literature on spectrum access in CR networks [11], [9]. According to Eq. (1), the transmission from t_i (t_j) in channel c will (not) be detected at r_i if Dist[t_i, r_i] $<$ r_d[t_i, c] (Dist[t_j, r_i] $>$ d_s[t_j, c]). Accordingly, we can define a *re-use* distance between (t_i, t_j) given by d_r[t_i, t_j, c] = r_d[t_i,c] + r_d[t_j,c], within which the re-use of channel c is not permitted. The corresponding interference constraint is given by:

$$Dist[t_i, t_j] > d_r[t_i, t_j, c]. \tag{3}$$

2) Receiver-centric Constraints: Transmitter-centric interference constraints are *conservative* since they rule out any potential co-channel interferer. However, by monitoring $C\text{-}I_{i,c}$ at each receiver r_i, co-channel interference may actually be permitted as long as Eq. (2) is satisfied. The *interference temperature* metric recently proposed by the FCC [16] specifies the bound on the additional co-channel interference according to $\theta_{i,c}$, as illustrated in Fig. 1. Using Eq. (2), the interference temperature at r_i, $IT_{i,c}$ is given as follows:

$$IT_{i,c} = P_{i,i,c} - \theta_{i,c} - NF_c.$$

Accordingly, we can generate a binary constraint such that no transmitter t_j subjects r_i to interference beyond $IT_{i,c}$. In other words, if t_j is the only interferer, then it can share channel c with t_i as long as the following condition holds:

$$P_{j,i,c} < IT_{i,c}. \tag{4}$$

This constraint can be re-written in the form of Eq. (3) in terms of an equivalent *re-use* distance $d_r[t_i,\ t_j,\ c]$ = f($IT_{i,c}$).

By allowing additional interference at each receiver, additional communication links can be supported in its vicinity, giving rise to improved spectrum utilisation. While there is still controversy over its feasibility and usefulness, we attempt to demonstrate its merits in building receiver-centric constraint models for dynamic spectrum access.

3) Spectrum assignment using graph colouring model: By mapping each channel into a colour, binary interference constraints given in Eq. (3) can be abstracted into a graph colouring (GC) model [9], based on which channels (colours) can be assigned to transmitters in a CR network. We illustrate this using transmitter-centric constraints.

Let us consider a network with 3 transmitting users (nodes), $\{t_1,\ t_2,\ t_3\}$ sharing 3 channels, $\{A,\ B,\ C\}$ as shown in Fig. 2(a), where the detection range of each transmitter is given by the radius of the dotted circle around it. According to Eq. (3), transmitters t_2 and t_1 cannot use channel C simultaneously while t_1 and t_3 can.

The corresponding GC model is shown in Fig. 2(b). A label on edge $t_i - t_j$ indicates channel(s) unusable simultaneously by transmitters t_i and t_j. Accordingly, a feasible assignment is given by $\{A,\ B,\ C\}$.

D. Inadequacy of Binary Interference Constraints

Consider a multiple-interferer scenario depicted in Fig. 3, where r_1 is receiving a signal from t_1 in channel c while potentially being (co-channel) interfered by three other transmitters, t_2, t_3 and t_4 (i.e., $P_{j,1,c} \geq NF_c$, $2 \leq j \leq 4$). According to Section II-C, the scope of each transmitter- and receiver-centric constraint is depicted by the dotted circle in Fig. 3(a) and (b) respectively.

Assuming symmetric transmissions from the interferers, the communication between t_1 and r_1 will remain in admissible quality only if the following conditions are satisfied:

1) Each interferer contributes up to $\frac{IT_{1,c}}{3}$;
2) Up to two of the interferers contributes up to $\frac{IT_{1,c}}{2}$ each;
3) At most one interferer contributes more than $\frac{IT_{1,c}}{2}$.

We observe that there exist scenarios satisfying conditions (1)-(3) where the communication quality for $(t_1,\ r_1)$ remains acceptable in the presence of co-channel interferers. In other words, transmitter-centric constraints that prohibit any co-channel interferer from transmitting may result

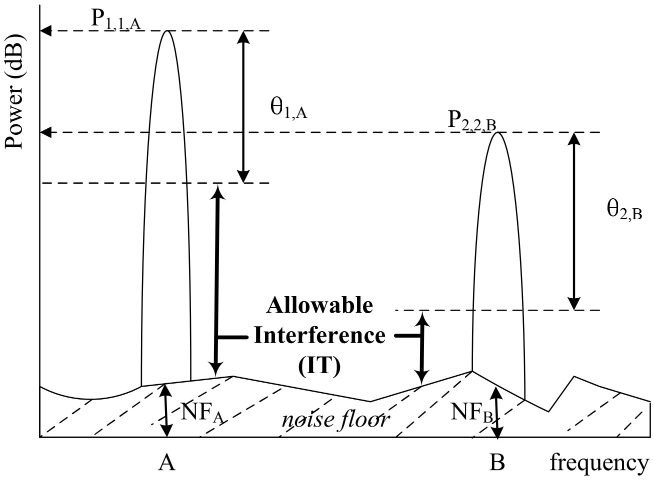
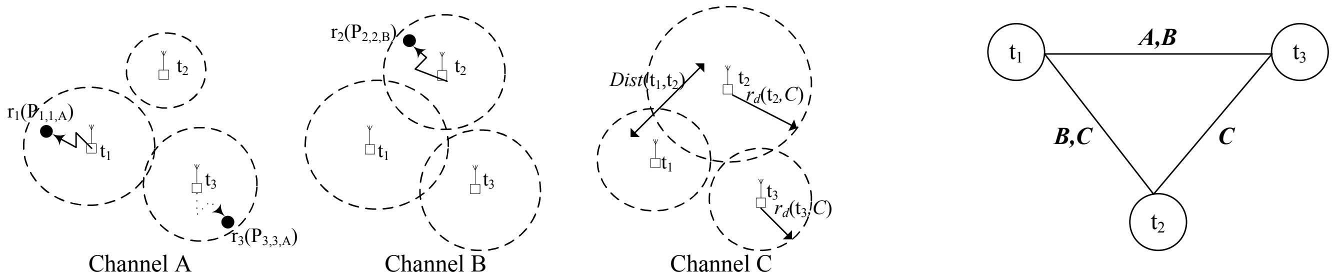


Fig. 1. An interference temperature metric to determine the additional allowable co-channel interference to enable additional communication without degrading ongoing communication. This is given by $P_{i,i,c}$ - C-$I_{th,i,c}$ - NF_i, where $P_{i,i,c}$ is the received power of the ongoing transmission from user *i* in channel *c*, NF_c is the noise floor and C-$I_{th,i,c}$ is the carrier-to-interference ratio threshold for admissible call quality.



(a) Transmitter-centric interference constraints

(b) Graph Colouring (GC) Model

Fig. 2. (a) An illustration of binary and transmitter-centric interference constraints and (b) the corresponding graph colouring model for allocating 3 channels, $\{A, B, C\}$ amongst 3 transmitting users, $\{t_1, t_2, t_3\}$ (represented by vertices). Each dotted circle represents the detection range of a node and the label on edge i–j indicates spectrum unusable by nodes *i* and *j* simultaneously.

in call denial. On the other hand, there exist scenarios that satisfy Eq. (4) while violating conditions (1)-(3) e.g., $P_{j,1,c}$ = $\frac{IT_{1,c}}{2}$, j = $\{2,3,4\}$, i.e., binary receiver-centric constraints may permit channel assignments that result in inadequate C-I for communication.

In general, we have the following theorem:

Theorem 1: GC models used for solving binary interference constraints cannot adequately capture the interference temperature metric. In other words, the resulting channel assignments may (a) permit co-channel communication that incurs excessive interference and/or (b) unnecessarily deny co-channel communication that would have otherwise maintained acceptable call quality.

Proof: Consider a communication pair (t_i, r_i) in channel c with C-I threshold, θ. Let $disk_i$ denote the area covered by the circle of radius $r_d[t_i,c]$ centered at t_i. Let υ_i represent a upper limit on $P_{j,i,c}$ for any transmitter t_j and λ_i represent a lower limit on $\frac{P_{i,i,c}}{P_{j,i,c}}$.

A transmitter t_j is a *potential interferer* at r_i under the following assumptions:

1) (transmitter-centric ratio-based) if $\frac{P_{i,i,c}}{P_{j,i,c}} < \lambda_i$ anywhere on $disk_i$;
2) (transmitter-centric absolute-based) if $P_{j,i,c} > \upsilon_i$ anywhere on $disk_i$;
3) (receiver-centric ratio-based) if $\frac{P_{i,i,c}}{P_{j,i,c}} < \lambda_i$ at r_i;
4) (receiver-centric absolute-based) if $P_{j,i,c} > \upsilon_i$ at r_i.

For a *transmitter-centric ratio-based GC model*, place an edge between the two transmitters t_i and t_j if either t_j is a transmitter-centric ratio-based potential interferer for t_i, or vice versa. The edge indicates that the two transmitters cannot share a channel. Transmitter-centric absolute-based, receiver-centric ratio-based and receiver-centric absolute-based GC models are defined similarly. A channel assignment satisfies a GC model if each pair of transmitters connected by an edge in the model has no common channel. That is, for each pair (t_i, t_j) linked in the graph, $c_i \cap c_j = \phi$.

We will show that, for any choice of λ (the pairwise ratio threshold) or υ (the absolute upper bound) in the corresponding model, either (i) we can find an assignment of channels which are allowed by the GC model, but which fails the IT metric, or (ii) we can find an assignment of channels which satisfies the IT metric, but which is rejected by the GC model. Without loss of generality, we consider the case where there is only one channel available (so we can drop the channel subscript c).

Part A: Transmitter-centric ratio-based model

(i) Suppose $\lambda \leq 2\theta$ and we have 4 transmitters. Position them such that $P_{2,1} = P_{3,1} = P_{4,1} = \frac{P_{1,1}}{\lambda+\epsilon(2\theta-\lambda)}$ for some ϵ s.t. $0 < \epsilon < 1$.

For the GC model,
$$\begin{aligned}\frac{P_{1,1}}{P_{2,1}} &= \frac{P_{1,1}}{\frac{P_{1,1}}{\lambda+\epsilon(2\theta-\lambda)}}\\ &= \lambda + \epsilon(2\theta - \lambda)\\ &\geq \lambda \text{ (since } \epsilon > 0 \text{ and } 2\theta \geq \lambda),\end{aligned}$$

and similarly for $\frac{P_{1,1}}{P_{3,1}}$ and $\frac{P_{1,1}}{P_{4,1}}$. So the GC model does not impose any constraint, and allows t_1,t_2,t_3 and t_4 to be all co-channel.

But
$$\begin{aligned}\frac{P_{1,1}}{P_{2,1}+P_{3,1}+P_{4,1}} &= \frac{P_{1,1}}{\frac{3P_{1,1}}{\lambda+\epsilon(2\theta-\lambda)}}\\ &= (\lambda + \epsilon(2\theta - \lambda))/3\\ &= \lambda/3 - \epsilon\lambda/3 + \tfrac{2}{3}\epsilon\theta\\ &= (1-\epsilon)\lambda/3 + \tfrac{2}{3}\epsilon\theta\\ &\leq \tfrac{2}{3}(1-\epsilon)\theta/ + \tfrac{2}{3}\epsilon\theta\\ &= \tfrac{2}{3}\theta\\ &< \theta,\end{aligned}$$

and thus according to the IT metric, t_1, t_2, t_3 and t_4 cannot all be co-channel.

(ii) Suppose now that $\lambda > 2\theta$ (and hence $\theta < \lambda/2$) and we now have two transmitters, located such that $P_{2,1} = \frac{P_{1,1}}{\theta+\lambda/2}$ and $P_{1,2} = \frac{P_{2,2}}{\theta+\lambda/2}$.

Then, for the GC model,
$$\begin{aligned}\frac{P_{1,1}}{P_{2,1}} &= \frac{P_{1,1}}{P_{1,1}/(\theta+\lambda/2)}\\ &= \theta + \lambda/2\\ &< \lambda/2 + \lambda/2\\ &= \lambda,\end{aligned}$$

and so there is an edge in the graph between t_1 and t_2. Since there is only one channel, the GC model is unable to find a satisfying assignment of channels.

But
$$\begin{aligned}\frac{P_{1,1}}{P_{2,1}} &= \theta + \lambda/2 \text{ (as before)}\\ &> \theta + \theta\\ &> \theta,\end{aligned}$$

and similarly for $\frac{P_{2,2}}{P_{1,2}}$. So, by the IT metric, the two transmitters can share the same channel.

Part B: Receiver-centric ratio-based model

The proof is the same as for Part A, as long as we can make sure that the transmitters and the receivers are placed appropriately.

Part C: Transmitter-centric absolute-based model

(i) Suppose $\upsilon \geq P_{1,1}/\theta$ and we have 2 transmitters. Position them such that $P_{2,1} = P_{3,1} = \frac{2}{3}P_{1,1}\theta < P_{1,1}/\theta \leq \upsilon$ and so both t_2 and t_3 are within the acceptable pairwise interference limits at r_1, and so there are no edges in the graph between t_1 and either t_2 or t_3. Hence the GC model says that t_1, t_2 and t_3 can all be co-channel simultaneously.

$$\begin{aligned} \text{But } \frac{P_{1,1}}{P_{2,1}+P_{3,1}} &= \frac{P_{1,1}}{\frac{2}{3}*P_{1,1}*\theta+\frac{2}{3}*P_{1,1}*\theta} \\ &= \frac{P_{1,1}}{\frac{4}{3}*P_{1,1}*\theta} \\ &= \frac{3}{4}\theta \\ &< \theta, \end{aligned}$$

and thus by the IT metric, t_1, t_2 and t_3 cannot all be co-channel.

(ii) Suppose now that $\upsilon < P_{1,1}/\theta$. So we can say $\upsilon = \epsilon P_{1,1}/\theta$ for some $0 < \epsilon < 1$.

Suppose we now have two transmitters, located such that $P_{2,1} = \sqrt{\epsilon}P_{1,1}/\theta > \upsilon$ and $P_{1,2} = \sqrt{\epsilon}P_{1,1}/\theta > \upsilon$ and so the GC model forbids t_1 and t_2 from being co-channel.

$$\begin{aligned} \text{But } \frac{P_{1,1}}{P_{2,1}} &= \frac{P_{1,1}}{\sqrt{\epsilon}P_{1,1}/\theta} \\ &= \theta/(\sqrt{\epsilon}) \\ &> \theta, \end{aligned}$$

and similarly for $\frac{P_{2,2}}{P_{1,2}}$. So, by the IT metric, the two transmitters can share the same channel.

Part D: Receiver-centric absolute-based model

The proof is the same as for Part C, as long as we can place the receivers correctly. ■

E. Non-binary Receiver-centric Constraints

In Section II-D, we demonstrated the inadequacy of using binary interference constraints to construct channel assignments. This inadequacy stems from the fact that the scope of the constraint is limited to a single variable. To overcome this, we can instead surround several transmitters with a *hyperedge*, and form a non-binary receiver-centric constraint (refer to Fig. 3(c)).

While such constraints have been considered previously for cellular network problems [7], we refine them for use in CR networks and describe the algorithm to generate them.

1) Algorithm for Constraint Generation: For the communication pair $(t_i,\ r_i)$ transmitting in channel c, the *scope*, S, is an array of n_i potential *interferers*, $\{s_{i,0},\ s_{i,1},\ \cdots,\ s_{i,n_i-1}\}$, where $P_{s_{i,0},i,c} \geq P_{s_{i,1},i,c} \geq \cdots P_{s_{i,n_i-2},i,c} \geq P_{s_{i,n_i-1},i,c}$. The *relation*, R, to be formed is a set of tuples, T, each reflecting a *maximally interfering*, yet within the C-I threshold, *relative* assignment, $A = \{a_{s_{i,0}}, a_{s_{i,1}}, \cdots, a_{s_{i,n_i-1}}\}$, on the scope. Here, a_j = 0 (1) means that transmitter j in the scope is given a co-channel (off-channel or non-interfering) assignment relative to t_i.

The algorithm (shown in the Java-like pseudo-code of Algorithm 1) is essentially a potentially exhaustive depth-first-search of the search space, and it loops between two mechanisms (a) *step-forward* and (b) *back-track*. The step-forward mechanism performs a *greedy* assignment that ensures that no interferer would contribute excessive interference (i.e., C-I threshold exceeded at r_i). Since this (maximal) assignment may be conservative, the back-track mechanism performs *reassignment* to ensure that the relation stores only those tuples which impart the maximum amount of tolerable interference, i.e., *minimal* assignments. This is because any assignment, strictly greater lexicographically, which offers less interference than existing tuples is redundant. Thus, minimal assignments are incomparable, i.e. for two currently stored (minimal) tuples representing assignments A and A', if $\exists a_{i,j} \geq a'_{i,j}$ then there MUST $\exists a_{i,k} \leq a'_{i,k}$. A brief description of the main functions follows.

For the step-forward mechanism, the *greedilyAssign* function takes *index* as an argument, and proceeds to assign values to $\{s_{i,index},\ s_{i,index+1},\cdots,\ s_{i,n_i-1}\}$ while ensuring that the corresponding interference contribution satisfies the C-I threshold at r_i. Next, the function *addMinimalTuple* simply checks the current assignment against existing tuples in the relation (if any) and adds it if it is minimal and discards it otherwise.

For the back-track mechanism, we term the reassignment operation 0 $\leftarrow$ 1 (1 $\leftarrow$ 0) as *loosening* (*tightening*) to possibly accommodate more (reduce) interference at r_i. The *jumpBack* function finds the first "profitable" index, y, to reassign in the tuple. It does this by first skipping over the parts of the search space (in descending order of indices) where an alternative assignment provides even *less* interference (0 $\rightarrow$ 1), i.e., 0's are skipped. Next, it skips over any 1's (since we need to find an index whose value can be loosened with potential for subsequent tightening). For example, assume that n_i = 8 and the assignment $\{a_{s_{i,0}},\ a_{s_{i,1}},\ \cdots,\ a_{s_{i,7}}\}$ = {0,1,1,0,1,1,0,0} is found. Clearly, stepping back chronologically and assigning $a_{s_{i,7}} \leftarrow 1$ is wasteful, similarly for $a_{s_{i,6}}$. Since $a_{s_{i,5}}$ and $a_{s_{i,4}}$ cannot be loosened, simply step back over them until $a_{s_{i,3}}$. However, by loosening the assignment at $a_{s_{i,3}}$, it is possible (though not necessarily the case) that a tighter assignment to $a_{s_{i,4}}$ and/or $a_{s_{i,5}}$ could be found to be minimal.

Our constraint generation algorithm satisfies the following:

Lemma 2: Since the *jumpBack* function only steps over redundant tuples, and does not miss potentially minimal tuples, it is correct.

Theorem 3: The generateNbctr() algorithm is correct.

Proof: The *generateNbctr()* algorithm is essentially a depth-first backtracking style search augmented with an enhanced back-jumping scheme. The correctness of this back-jumping step (c.f. Lemma 2) and the correctness of backtracking implies the algorithm *generateNbctr()* is correct. This also applies to the improved algorithm with bounded step-forward search space. ■

Algorithm 1 generateNbctr()

S ←sort(scope)
index ← 0 $\{0 \leq \text{index} \leq n_i$ -1$\}$
$\{a_{s_{i,index}}, a_{s_{i,index+1}}, \cdots, a_{s_{i,n_i-1}}\}$ ← greedilyAssign(index) {first tuple is found greedily}
while index > -1 **do**
  addMinimalTuple(assignment) {jump back to index to "loosen"}
  index ← jumpBack()
  if index ≥ 0 **then**
    loosen assignment(index)
  end if
  $\{a_{s_{i,index+1}}, a_{s_{i,index+1}}, \cdots, a_{s_{i,n_i-1}}\}$ ←greedilyAssign(index+1)
end while

2) Illustration of Non-binary, Receiver-centric Constraints: We refer to the multiple-interferer scenario in Fig. 3. Here the potential interferers t_2, t_3 and t_4 are permitted to be co-channel (have a channel separation of 0 channels), or non-interfering (have a channel separation of at least 1 channel) with t_1. Using Algorithm 1, non-binary, receiver-centric interference constraints can be generated, and an example constraint with a relation that comprises three tuples is shown in Fig. 3(c).

Let us consider the first tuple (t_2:0, t_3:0, t_4:1). This implies that co-channel assignment is permitted in t_2 and t_3 *if and only if* t_4 has a separation of ≥ 1 (i.e. is non-interfering) with t_1. By observing the remaining tuples, we note that this simple example is symmetrical, i.e., up to two of the three devices may interfere *provided* that the remaining one does not. Should any of the transmitters have larger separations in channel space (especially those permitted to be co-channel) then they contribute less interference, giving rise to an increase in C-$I_{1,c}$.

3) Improved Constraint Generation Algorithm by Bounding Future Search Space: While the back-track mechanism in Section II-E.1 only prevents the search from examining redundant parts of the current sub-tree, the step-forward routine can be improved to similarly avoid redundant search, by bounding the extent of the space in which useful tuples may be found.

Let T be the first acceptable tuple (assignment) on the scope in the constraint relation, R, $T \in R$. Since the search amounts to a lexicographical traversal of the search space, we can define a sub-tuple $\underline{T}$ such that $\underline{T}_{j,n_i}$ is the corresponding assignment indexed from j to n_i.

We can use the lexicographically largest $\underline{T}_{j,n_i}$ to bound the search space following a back-track step. Since the back-track step loosens the assignment on j, any subsequent assignment to $\{s_{i,j}, \cdots, s_{i,n_i-1}\}$ which is greater or equal to the bound is necessarily redundant. We can now back-track again from index j without losing solutions (we have searched the entirety of the space in which a minimal tuple could be found). If we *do* find a new non-redundant solution within the bound given by $\underline{T}_{j,n_i}$, we simply add the tuple T_{0,n_i} to the relation and continue the search.

III. Comparison of Constraint Models

In this section, we compare the various constraint models intuitively as well as through numerical experiments. We term the binary transmitter-centric constraints as *"CauTious"* (or CT), since they eliminate the possibility of interference from other transmitters. We term the non-binary receiver-centric constraints as *"Interference Temperature"* (or IT), since they accurately map the possible interference contributions at a link level to ensure that the interference temperature for that link is not met or exceeded. We will ignore the simple binary receiver-centric constraints of Section II-C.2. Though they may work in practice if there is a further requirement that channel use is well spread out throughout the available spectrum band, solutions to these constraints are not necessarily suitable assignments in a network.

Since our objective is to minimize the number of channels needed to maintain acceptable communication quality for p communication pairs in a network with N CR-enabled secondary users, we denote by $NumCh_p^T$ and $\mathbf{C}_p^T$ the number of channels needed and set of channel assignments using constraints T, $T \in \{CT, IT\}$. We assume a *homogeneous* CR network, where each communication pair has a C-I threshold of θ and each transmitter has a detection range r_d. We consider the following questions:

1) *Does any spectrum assignment* $c_p^{CT} \in \mathbf{C}_p^{CT} \implies c_p^{CT} \in \mathbf{C}_p^{IT}$*?*

   The answer is clearly yes. CT does not permit interference from any other user at all, whereas IT does, provided the total interference is below the interference temperature. In the worst case when IT is as restrictive as CT, the set of potential interferers = { } and the corresponding C-I = $P_{i,i,c} - NF_c$ which will be greater than any likely C-I threshold.

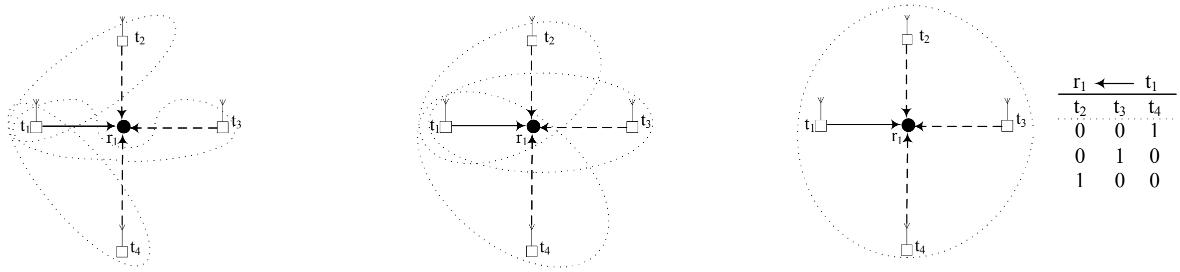
2) *Does* $\exists\ c_p^{IT} \in \mathbf{C}_p^{IT}$ *such that* $c_p^{IT} \notin \mathbf{C}_p^{CT}$*?*

   Again the answer is clearly yes. For a receiver r_i with C-I threshold = θ, we have a spectrum assignment $c_p^{IT} \notin C_p^{CT}$ as long as $\sum_j P_{j,i,c} + NF_c < P_{i,i,c} - \theta$.

We can infer from the above discussion that IT constraints results in *better* (or at worst, equal) spectrum utilisation than (to) that obtained with CT constraints. However, since the receivers are involved in generating receiver-centric constraints, we expect higher levels of overhead with the IT approach due to exchange of signalling messages over the CSCC.

Next, we present numerical results that compares $NumCh_p^{CT}$ and $NumCh_p^{IT}$ for p= {10, 20, $\cdots$, 50} for N=200, where the network becomes more *congested* as p increases. For each scenario (fixed p), we select p communication pairs randomly (while the remaining users are assumed to be idle), generate the interference constraints and construct the channel assignment for a C-I threshold of θ dB. We repeat the simulations for each scenario 1000 times and compute the average $NumCh_p^{CT}$ and $NumCh_p^{IT}$. The results are plotted in Fig. 4 for θ = 9 dB.

We observe that, as the network becomes more congested, the spectrum requirement increases using both types of constraints. This is expected since the likelihood of mutual interference increases as more users are communicating. Also, IT has a lower spectrum requirement than CT since it is tolerable to some interference (subject to the C-I threshold) while IT does not permit any co-channel interference at all. For the same reason, the increase in spectrum requirement is more gradual with IT and the gain achieved with IT compared with CT increases as the network becomes more congested.



(a) binary, transmitter-centric (b) binary, receiver-centric (c) non-binary, receiver-centric

Fig. 3. Comparison of interference constraints for a multiple-interferer scenario: (a) Scope of binary transmitter-centric constraints (b) Scope of binary receiver-centric constraints and (c) Scope of non-binary receiver-centric constraints and an example of a relation comprising three tuples (right).

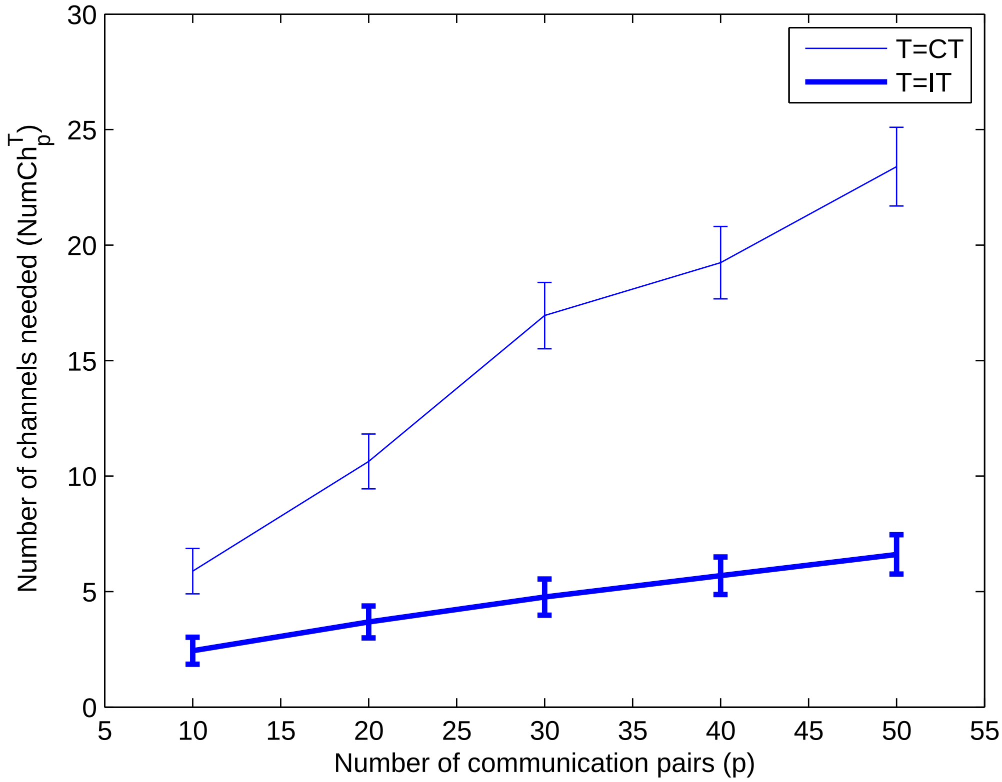


Fig. 4. Comparison of the number of channels needed ($NumCh_p^T$) to ensure acceptable communication (C-I threshold = 9 dB) amongst p communication pairs using CauTious (T=CT) and Interference Temperature (T=IT) constraints for channel assignment in a CR network with N=200 users.

IV. Conclusions

With the advent of cognitive radio technology, new paradigms for spectrum access can achieve near-optimal spectrum utilisation by letting each user sense and utilise available spectrum opportunistically while regulating the interference it imposes on other users through interference constraints. However, the most common forms of binary and transmitter-centric constraints are often inefficient since they only consider pair-wise sets of transmitters and prohibit any interference at receivers.

In line with the recently proposed interference temperature metric, we propose a non-binary receiver-centric constraints model that constraints whole subsets of transmitters and is easy to generate and test. By permitting interfering signals to be introduced, additional communication can be supported, leading to improved spectrum utilisation in cognitive radio networks compared with using transmitter-centric constraints. This is verified intuitively as well as through numerical experiments in the paper.

Our proposed constraints are currently used to devise a dynamic, negotiated etiquette for a CR network offering heterogeneous services in a wireless office scenario. While this extends and builds on the work in [17], the following differences are noted:

1) As opposed to binary transmitter-centric constraints, we apply non-binary receiver-centric constraints that more accurately models the interference, resulting in better spectrum utilisation;
2) Instead of assuming continuously backlogged homogeneous users, we consider a highly dynamic traffic model for users that operate over a number of dissimilar services (each requiring different minimum C-I levels and having different effective bandwidths).

References

[1] A. Petrin and P. G. Steffes, "Analysis and comparison of spectrum measurements performed in urban and rural areas to determine the total amount of spectrum usage," *Proc. of the ISART*, March 2005.

[2] W. Lehr and J. Crowcroft, "Managing shared access to a spectrum commons," *Proc. of the IEEE DySPAN*, pp. 420–444, November 2005.

[3] J. M. III, "Cognitive Radio: An Integrated Agent Architecture for Software Defined Radio," Ph.D. dissertation, KTH Royal Institute of Technology, 2000.

[4] S. Haykin, "Cognitive Radio: Brain-Empowered Wireless Communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, February 2005.

[5] L. Berry, "Spectrum metrics and spectrum efficiency," *IEEE Trans. Electromagnetic Compatability*, vol. EMC19, no. 3, pp. 254–260, 1977.

[6] W. Hale, "Frequency assignment: Theory and applications," *Proc. of the IEEE*, vol. 68, pp. 1497–1514, 1980.

[7] J. Bater, "Non-binary (higher-order) modeling and solution techniques for frequency assignment in mobile communication networks," Ph.D. dissertation, University of London, 2000.

[8] J. Neel, J. Reed, and A. MacKenzie, "Cognitive radio network performance analysis," *Cognitive Radio Technology*, August 2006.

[9] C. Peng, H. Zheng, and B. Zhao, "Utilization and fairness in spectrum assignment for opportunistic spectrum access," *Mobile Networks and Applications*, vol. 11, no. 4, pp. 555–576, August 2006.

[10] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *Proc. of the IEEE DySPAN*, pp. 251–258, November 2005.

[11] H. Zheng and C. Peng, "Collaboration and fairness in opportunistic spectrum access," *Proc. of the IEEE ICC*, vol. 5, pp. 3132–3136, May 2005.

[12] N. Nie and C. Comaniciu, "Adaptive channel allocation spectrum etiquette for cognitive radio networks," *Proc. of the IEEE DySPAN*, pp. 269–278, November 2005.

[13] E. P. K. Tsang, *Foundations of Constraint Satisfaction*. Academic Press, 1993.

[14] K. Marriot and P. J. Stuckey, *Programming with Constraints*. MIT Press, 1999.

[15] J. C. Regin, *Constraints and Integer Programming Combined*. Kluwer, 2003, ch. Global Constraints and Filtering Algorithms.

[16] P. J. Kolodzy, "Interference temperature: a metric for dynamic spectrum utilization," *International Journal of Network Management*, vol. 16, no. 2, pp. 103–113, March 2006.

[17] L. Cao and H. Zheng, "Distributed spectrum allocation via local bargaining," *Proc. of the IEEE SECON*, pp. 475–486, September 2005.