Proceedings of the ASME 2011 Dynamic Systems and Control Conference
DSCC2011
October 31 - November 2, 2011, Arlington, VA, USA

DSCC2011-6078

MODEL-BASED HIERARCHICAL OPTIMAL CONTROL DESIGN FOR HVAC SYSTEMS

Mehdi Maasoumy, Alessandro Pinto and Alberto Sangiovanni-Vincentelli*†

ABSTRACT

A hierarchical control architecture for balancing comfort and energy consumption in buildings is presented. The control design is based on a simplified, yet accurate model of the temperature within each room of the building. The model is validated against real measurements. The control architecture comprises a first level that regulates low level quantities such as air flow, and a second level that balances comfort (i.e. distance between the desired and actual temperature) and energy consumption (i.e. total energy consumed for the required level of comfort). We show the effectiveness of our approach by simulation using validated models.

1 Introduction

Advanced control algorithms are considered critical enablers to achieve low energy consumption in commercial buildings. Entire sections of the ASHRAE 90.1 standard [15] are dedicated to the specification of control requirements. Although the optimal control of an HVAC system is a complex multi-variable problem, it is standard practice to rely on simple control strategies that include bang-bang controllers with hysteresis, and PID controllers. In most cases, standard sequences of operations for *typical* installations are used by control contractors. Each sequence controls the HVAC equipment during an operation phase such as optimal start, safety shutdown and normal operation. After installation and tuning, the building is inspected by a commissioning agent that mainly verifies that the building satisfies the owner's expectations. The commissioning agent does not only verifies the expected performance right after installation, but also after the building has started his operations.

This short snapshot of design and validation practices in the building industry shows the importance of a model-based design flow for building controls. To attain energy efficiency, control algorithms need to be tailored to the physical properties of the building at hand rather than being an adaptation of a standard sequence designed for a typical building. Thus, a thermal model of the building is needed that is also suitable for optimal control design. Once such model is made available, it can be used to design an optimal controller that balances comfort and energy usage. To achieve building-level energy-optimality, the model should be able to capture the interaction between physically connected spaces in the building, occupancy schedules, and state and input constraints.

A variety of related work can be found in the literature. The authors of [11] have proposed a nonlinear model of the overall cooling system including the chillers, the cooling towers and the thermal storage tank, and have developed an MPC scheme for minimizing the energy consumption. In [14] and [7], the authors use a model of the building which is bilinear between inputs, states and weather parameters. The approach that they take is a form of Sequential Quadratic Programming (SQP) for solving non-linear problems in which they iteratively linearize the non-convex constraints around the current solution, solve the optimization problem and repeat until a convergence condition is met. In [10] authors apply common available methods for wall thermal analysis. In [3] the authors presented a simplified building zone model based on an RC electrical network which is directly coupled to a complete HVAC system. In [4], the authors proposed a building thermal model based on an RC-

*M. Maasoumy and A. Sangiovanni-Vincentelli are with the Departments of Mechanical Engineering and Electrical Engineering at University of California, Berkeley, respectively. CA 94720 USA mehdi@me.berkeley.edu alberto@eecs.berkeley.edu

†A. Pinto is with the Systems Department, United Technologies Research Center Inc., Berkeley, CA 94705, USA pintoa@utrc.utc.com

network, with a large number of coupled linear differential equations and reduce the order of the model via aggregation of states. In [8], the authors investigated the potential of building thermal storage inventory, in particular the combined utilization of active and passive inventory, for the reduction of electrical utility cost using common time-of-use rate differentials. In [6], the authors proposed a simple model where the model inputs are divided into manipulated variables and disturbance inputs. In [13], the authors elaborated a dynamic multinodal lumped-capacitance non-linear model to describe a building, considering conduction heat fluxes, envelope thermal capacity, lighting and people loads, infiltration, fenestration and thermal inertia of heating systems. In [2], the authors presented a nonlinear disturbance rejection state feedback controller for an HVAC system.

The contribution of this paper is twofold. First, we propose a model-based approach to the design of control algorithms for HVAC systems. The model-based approach enables design methods that are rigorous and verifiable. Further, the same models can be used during building operation for diagnostic purposes. For this purpose we have developed a nonlinear model that takes into account heat transmission and storage in building elements such as walls and room air, and external and internal heat gains from different sources such as the sun and building occupants. The model has then been calibrated against real data of an existing building. Simulation results are provided to confirm the validity of the model. Second, we propose a hierarchical approach to the control problem whose complexity is manageable and that can be automatically solved. The proposed architecture is also effective in managing the trade-off between comfort and energy consumption.

The paper is organized as follows. Section 2 presents the high level thermal model for building that we developed. We also present the validation methodology and the validation results in the same section. Section 3 shows the control architecture and presents algorithms for the automatic synthesis of the control algorithm parameters. Finally, Section 4 presents results obtained by simulation.

2 Modeling and validation

Heat storage capacity and *heat transmissibility* are the essential thermal properties of building elements. Walls, ceilings, floors and the air inside enclosed spaces are building components that can store energy. The capacity of these elements in storing energy is a function of their mass and their specific heat capacity. Heat is not only stored, but it can be transmitted through building elements in different ways. A useful representation of a thermal network is by means of a circuit analog where heat storage is represented by capacitors and heat transmission by resistors. We develop a model for building thermal elements such as rooms and walls using the *lumped-capacitance method*. To be useful for control design, the model has to be simple yet accurate enough so that relevant dynamic behaviors of thermal elements are retained.

2.1 From thermal network to circuit network

For an object with mass m and specific heat capacity c_p, a rate of change of temperature $\dot{T}$ corresponds to the heat flow $Q = mc_p\dot{T}$. Heat transfer takes place via *conduction*, *convection*, and *radiation*.

To describe the heat transmission process, we refer to the one-dimensional plane wall having a temperature distribution $T(x)$. The heat transfer process is expressed by the *Fourier's law* $Q_x = -kAdT/dx$, where Q_x is the heat transfer rate in the x direction, k is the *thermal conductivity* (a characteristic of the wall material), and A is the area of the wall. Under the *steady state* condition the temperature distribution is *linear*, and the temperature gradient may be expressed as $dT/dx = \frac{T_2-T_1}{L}$ where L is the thickness of the wall, and T_1 and T_2 are the temperatures of the wall surfaces. Hence, the heat flow can be written as $Q = kA(T_1 - T_2)/L$.

In a convective heat transfer process the rate of heat being transferred is $Q = hA(T_s - T_\infty)$, where the convective heat transfer is proportional to the difference between the surface and the air temperatures, T_s and T_∞, respectively. The proportionality constant h is referred to as the *convection heat transfer coefficient*.

The heat flux emitted by a surface at temperature T, is given by $E = \varepsilon\sigma T_s^4$ where ε is a relative property of the surface called the *emissivity*. The radiation may originate from a special source, such as the sun, or from other surfaces to which the surface of interest is exposed. Irrespective of the source(s), we designate the rate at which the radiation is incident on a unit area of the surface as the *irradiation* G. A portion or all the irradiation may be *absorbed* by the surface, thereby increasing the thermal energy of the material. The rate at which radiant energy is absorbed per unit surface area may be evaluated from the knowledge of surface radiative property called *absorptivity* α. That is $G_{abs} = \alpha G$. In the context of building thermal analysis, we will ignore the radiation heat transfer among the internal walls in the building due to relative low range of temperatures inside the building (details in [12]), but we will consider the irradiation from the sun on the external sides of the walls in deriving the differential equations of the temperature distribution in the walls and rooms.

We now leverage the analogy between the diffusion of *heat* and *electrical charge*. Temperature, or *thermal potential*, plays the role of voltage in electrical circuits. The temperature of a point is fixed in steady state heat transfer, while it varies with time in transient heat transfer or heat storage. Just as an electrical resistance is associated with the conduction of electricity, a thermal resistance may be associated with the conduction of heat [9]. Defining resistance as the ratio of a driving potential to the corresponding transfer rate, the *thermal resistance for con-*

duction in a plane wall is $R'_{cond} = \frac{T_{s,1}-T_{s,2}}{Q} = \frac{L}{kA}$ Similarly, the *thermal resistance for convection* is then $R'_{conv} = \frac{T_s - T_\infty}{Q} = \frac{1}{hA}$. We denote the internal convective heat transfer coefficient, by h_i and the external convective heat transfer coefficient, by h_o. Accordingly the thermal resistance for convection on the internal and external sides of the peripheral walls, denoted by R_i and R_o, respectively, can be defined as follows $R'_i = \frac{1}{h_i A}$ and $R'_o = \frac{1}{h_o A}$ Notice that the R-value of materials which can be found in tables can be used to calculate R' for each wall. The relation of R and R' is $R' = \frac{R}{A}$ where A is the area of the wall.

To analyze the transient thermal behavior of the building model, we introduce the concept of *thermal capacitance*. Thermal capacitance or heat capacity is the capacity of a body to store heat.

In the context of building design, thermal mass provides "inertia" against temperature fluctuations, sometimes known as the thermal flywheel effect [16]. For example, when outside temperatures are fluctuating throughout the day, a large thermal mass within the insulated portion of a house can serve to "flatten out" the daily temperature fluctuations, since the thermal mass will absorb heat when the surroundings are hotter than the mass, and give heat back when the surroundings are cooler. This is distinct from a material's insulating value, which reduces a building's thermal conductivity, allowing it to be heated or cooled relatively separate from the outside, or even just retain the occupants' body heat longer.

Time varying conduction heat transfer investigations of walls are very important for the prediction of heating and cooling loads in air conditioning practice. The walls store heat, absorb and dissipate a fraction of it and transmit the rest into the conditioned space at a later time, which depends on the wall thermal inertia. The author of [16] concludes that the accurate prediction of time varying conduction heat transfer which strongly influences the air conditioning load, is vital since it may considerably lag the direct heat gain load components which occur in phase with ambient temperature and solar radiation incident at the outer building envelope.

To capture the variations with time of temperatures of walls and rooms, we assign a node to each wall or room, and a capacitance with capacity $C = mc_p$ to each node in the thermal circuit. Notice that bodies of distributed mass like walls and air are considered as *nodes* in our modeling. This approximation is done based on some assumptions that will be presented in Section 2.2.

Example 2.1.1 Peripheral Wall

In this example we consider a peripheral wall (i.e. one side of the wall is exposed to the outside air and to the sun, and the other side is exposed to the inside air) as shown in Figure (1). The wall has a window with area A_w, thickness t and conductive heat transfer coefficient k_w. The total area of the wall, not including the window, is A. The circuit model for this example has three nodes with potentials T_1, T_2 and T_3 corresponding to the outside air, the

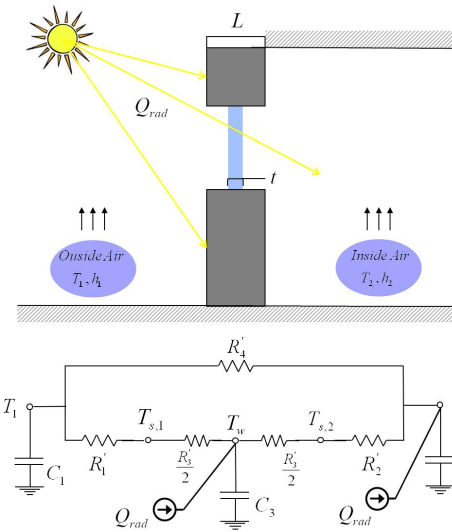


Figure 1. Thermal and circuit network for a peripheral wall with window

inside air and the wall, respectively. The nodes are connected to ground via capacitors with capacitance value $C_1 = m_{a_1} c_a$ for the outside air, $C_2 = m_{a_2} c_a$ for air in in the room, and $C_3 = m_w c_w$ for wall material where m_{a_1}, m_{a_2}, and m_w are the masses fo each element, and c_a and c_w are specific heat capacitance of air and wall material respectively.

Heat propagates from the outside to the inside through two parallel paths: the walls and the window. The path through the walls is modeled by convective thermal resistances $R'_1 = \frac{1}{h_1 A}$ between the outside air and the wall, $R'_2 = \frac{1}{h_2 A}$ between the inside air and the wall, and a conductive thermal resistance $R'_3 = \frac{L}{kA}$ of the wall. The path through the window is modeled by a resistance $R'_4 = \frac{1}{h_1 A_w} + \frac{t}{k_w A_w} + \frac{1}{h_2 A_w}$. The radiative heat gain from the sun Q_{rad} is also shown on the circuit network by a variable *current source*. The output of the current source that calculates the corresponding sun radiation is a function of the *altitude* and *azimuth* angle of the location of the building on the Earth, orientation of the considered wall or window, day of the year, time of the day, outside weather and sky condition, and etc.

Remark. One advantage of the proposed model to the other models that can be found in the literature is that the parameters of the proposed model have physical meanings and interpretations. This feature makes it possible to modify the model very easily when the parameters of the real building are changed from one building to another. For example if the glazing of the windows are changed this change can be applied to the model by only correcting the transmissivity of the windows in the model and keeping other parameters intact. Also when analyzing two buildings with the same wall materials and different wall thicknesses, we know that the $Rval_w$ of the two buildings are the same, therefore R_{ij} between any two neighboring nodes for the new building can be obtained by using the $Rval_w$ and $Rval_{in}$ of the old building and the dimensions of the new building.

2.2 Plant modeling

We derive a mathematical model of the thermal behavior of buildings that can be effectively used in control design. The model is for rooms on the same floor and for the sake of simplicity are considered isolated from rooms on adjacent floors. We use a lumped model [5] where the air in a room has one temperature across its volume, and the temperature of a wall across its volume is assumed to be equal to its centerline temperature. We assume that all rooms are at the same pressure which is equal to the pressure used in the heating and cooling ducts. Air exchange between a room and a vent is then *isobaric*, so the air mass in the room will not change in the process. We denote the air mass in the room by m, the rate of air mass entering the room (and also leaving the room) by $\dot{m}$, and the temperature of the conditioned air entering room i by T_{s_i}. The temperature of the air leaving the room equals the current temperature of the room. We ignore the capacitance of windows since their mass is negligible compared to the mass of walls (i.e. windows are modeled as pure resistances in the thermal circuit). The radiative heating for each building face is regarded as a disturbance to the plant model. Finally, the specific heat of air, c_p, is considered constant at 1.007. In reality, c_p is 1.006 at *250 K* and 1.007 at *300 K*, so our assumption is accurate within 0.1% error over the range of temperatures that would occur in a building during normal operation.

Our network consists of two types of nodes: walls and rooms. There are in total n nodes, m of which represent rooms and the remaining $n-m$ nodes represent walls. The temperature of the i-th wall is governed by the following equation:

$$\frac{dT_{w_i}}{dt} = \frac{1}{C_{w_i}}\left[\sum_{j\in\mathcal{N}_{w_i}} \frac{T_j - T_{w_i}}{R'_{ij}} + r_i \alpha_i A_i q''_{rad_i}\right]$$

$\mathcal{N}_{w_i}$ is the set of all of neighboring nodes to node w_i and, r_i is equal to 0 for internal walls, and to 1 for peripheral walls. The temperature of the i-th room is governed by the following equation:

$$\frac{dT_{r_i}}{dt} = \frac{1}{C_{r_i}}\left[\sum_{j\in\mathcal{N}_{r_i}} \frac{T_j - T_{r_i}}{R'_{ij}} + \dot{m}_{r_i} c_p (T_{s_i} - T_{r_i}) + \right.$$
$$\left. + \; w_i \tau_{win_i} A_{win_i} q''_{rad_i} + \dot{q}_{int}\right]$$

Where $\mathcal{N}_{r_i}$ is the set of all of the neighboring nodes to room i and, w_i is equal to 0 if none of the walls surrounding room i has window, and is equal to 1 if at least one of them has.

As an example we consider a building with three thermal zones. A schematic of the building with the thermal network and circuit network for one of the thermal zones is shown in Figure (2).

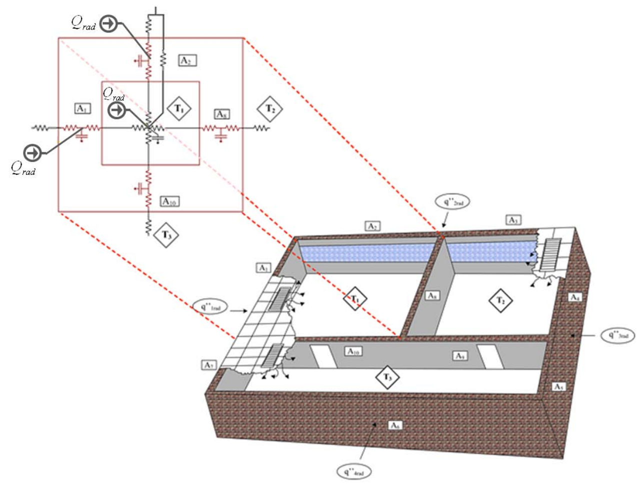


Figure 2. Three-room building with thermal and circuit network shown for one room and surrounding walls

The variations of temperature in wall 1 and room 1 are given by following equations:

$$\frac{dT_{w1}}{dt} = \frac{1}{C_{w1}}\left[\frac{T_{out} - T_{w_1}}{R'_{out,w_1}} + \frac{T_{r_1} - T_{w_1}}{R'_{r_1,w_1}} + \alpha A_1 q''_{rad_1}\right]$$

$$\frac{dT_{r_1}}{dt} = \frac{1}{C_{r_1}}\left[\frac{T_{w_1} - T_{r_1}}{R'_{w1,r1}} + \frac{T_{w_2} - T_{r_1}}{R'_{w2,r1}} + \frac{T_{w_8} - T_{r_1}}{R'_{w8,r1}} \right.$$
$$\left. + \frac{T_{w_{10}} - T_{r_1}}{R'_{w10,r1}} + \dot{m}_1 c_p (T_{s_1} - T_{r_1}) + \tau_{win_1} A_{win_1} q''_{rad_1} + \dot{q}_{int_1}\right]$$

If we write the heat transfer equation for every wall and room in the building and represent the equations in a state space form we get $\dot{x}(t) = Ax(t) + f(x(t), u(t))$ and $y(t) = Cx(t)$. Where $x \in \mathbb{R}^{n\times 1}$ is the state vector representing the temperature of the nodes in the thermal circuit, $u \in \mathbb{R}^{m\times 1}$ is the input vector which in our case is the mass flow rate of conditioned air into each thermal zone, and $y \in \mathbb{R}^{m\times 1}$ is the output vector of the system which represents the temperature of the thermal zones. A is a square $n \times n$ matrix and $C \in \mathbb{R}^{m\times n}$ determines the outputs. Matrix $C \in \mathbb{R}^{m\times n}$ determines which states are used as the output of the system. Vector $f(x(t), u(t)) \in \mathbb{R}^{n\times 1}$ is composed of both the input and the disturbance to the system. We have linearized the system dynamics around the equilibrium points of the system (details in [12]). Note that since the range of thermal zone temperature that the system experience in the course of a day is not so wide (usually 18 - 22 ° C), linearizing about the equilibrium point does not introduce significant error. On the other hand dealing with a linear system dramatically decreases the computational efforts and makes it possible to use linear optimal control techniques.

Table 1. Parameter identification results

Parameter	Value (kJ/K)	Parameter	Value ($m.\mathrm{K}/W$)
C_{r_1}	1.673×10^3	$Rval_w$	1.659
C_{w_1}	2.707×10^4	$Rval_{gl}$	0.124
C_{w_2}	2.730×10^4	$Rval_{in}$	0.062
C_{w_3}	1.895×10^4	$Rval_{out}$	2.149
C_{w_4}	3.898×10^4		

2.3 Validation

For validating the model that was developed in section 2 we have used the data of zone temperature of a specific zone at Bancroft library of UC Berkeley campus along with airflow, discharge air temperature (DAT) and outside air temperature (OAT) data to simulate the thermal behavior of that specific zone and then compare the simulation results with the measured temperature of the zone. We have used the WebCTRL of Automated Logic Corporation (ALC) to download the temperature data.

2.3.1 Parameter Identification In order to estimate the parameters of the model such as the overall thermal resistance of each wall we have used some typical R-values for walls from ASHRAE handbook [15] as the initial guesses and then we have used the *fmincon* function in MATLAB to solve for the optimal parameters by minimizing the error between the measured temperature and the simulated temperature of the zone with respect to constraints on the parameters. The results of model validation is shown in Figure 3. In this optimization problem, the optimization parameters include the thermal resistances of the walls and the masses in the energy balance equation. The parameters of the model are reported in Table 1.

Note that since the presence of people in the room is very random and has not been considered in the modeling we have used the data of a weekend in order to minimize the disturbance effect of internal heat gains by the people in the system.

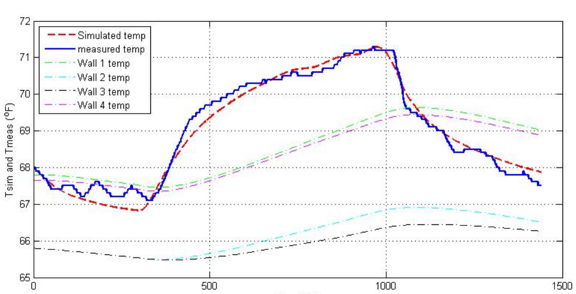


Figure 3. Simulated temp. vs. measured temp. of zone 8 (Oct 17, 2010)

2.3.2 Parameter Validation In order to validate the parameters listed in Table 1 we have simulated the temperature of the same thermal zone using the data of next weekend (Oct 24). The results of the simulation is presented in Fig. 4.

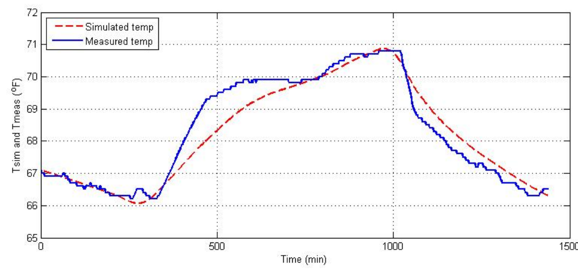


Figure 4. Simulated temp. vs. measured temp. of zone 8 (Oct 24, 2010)

Note that zone 8 is located in the north east corner of second floor of DOE library building. The walls on North and East side of the building face outside and have windows. There are two points in dealing with walls with windows. First, we have to take into account the R-value of windows which is in parallel with the R-value of the wall, and also we have to take into account the effect of radiation gain from the sun during the day and the radiation loss due to heat transfer between the room and the sky at night. We have assumed an additive sinusoidal disturbance to the model to represent the effect of solar radiation gain and heat loss to the sky at night. The amplitude of the sinusoidal wave and the transmissivity of the windows are the optimization variables that are obtained in the identification process.

3 Controller design

We propose the hierarchical control algorithm shown in Figure (5). At the lower level, each thermal zone is controlled by a PID controller while a model-based optimal control (e.g. LQR) is used at the higher level for a group of thermal zones. [1]

At each time step, the current desired temperature for each thermal zone (set by the building occupant) and the current temperature of each thermal zone are passed as inputs to the high level optimal controller. The high level controller solves an optimization problem to compute the new set-points for the lower level PID controllers. The objective of the optimization problem is to find the optimal trade-off between comfort and energy consumption. This methodology makes it possible to operate the HVAC system in an optimal, energy efficient fashion by incorporating detailed occupancy schedules for each thermal zone.

[1] In principle, the hierarchy could extend upwards to the whole building, and further to entire communities.

Table 2. Nomenclature

Parameter	Definition
A_i	Area of wall i
A_{win_i}	Total area of window on walls surrounding room i
α_i	Absorption coefficient of surface of wall i
τ_{win_i}	Transmissivity of glass of window i
q''_{rad_i}	Radiative heat flux density radiated to node i
$\dot{q}_{int_1}$	Internal heat generation in thermal zone i
c_a	Specific heat capacity of air
c_w	Specific heat capacity of wall material
C_{r_i}	Capacitance of thermal zone i
C_{w_i}	Capacitance of wall i
h	Convection heat transfer coefficient
k	Conduction heat transfer coefficient
$\dot{m}_{r_i}$	Mass flow rate of conditioned air entering thermal zone i
T_{r_i}	Temperature of room i
T_{s_i}	Supply air temperature into thermal zone i
T_{w_i}	Temperature of wall i
R'_{ij}	Thermal resistance between node i and node j
$Rval_w$	R-value of wall
$Rval_{gl}$	R-value of glass window
$Rval_{in}$	R-value of inside air film
$Rval_{out}$	R-value of outside air film

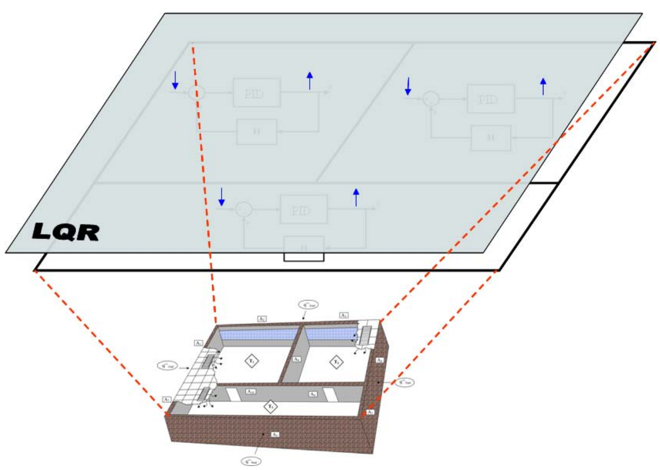


Figure 5. Hierarchical Control (low level PIDs and high level LQR)

The higher level controller can be in principle implemented using any optimal control algorithm including model-predictive control. In this paper, we have selected a *Linear Quadratic Regulator*. The main reason for this choice is computational complexity, which is low for LQR compared to other nonlinear model predictive control techniques. Because the size of the problems tend to be very large (as the number of nodes in the thermal network is typically large), computational complexity is of primary concern. To use an LQR, we linearized the system dynamics around the equilibrium points of the system. Note that since the range of thermal zone temperature that the system experience in the course of a day is not so wide (usually 18 - 22 $^\circ$ C), linearizing about the equilibrium point does not introduce significant error.

The type of LQR that we require is a *tracking LQR*, meaning that the controller output needs to track the desired temperature trajectories set by the occupants. The LQ tracking problem is formulated as follows:

$$\begin{aligned}&\min_{U_0} \frac{1}{2}\,[y_d(N)-y(N)]^T\,S\,[y_d(N)-y(N)]\\&\quad+\frac{1}{2}\sum_{k=0}^{N-1}\Big([y_d(k)-y(k)]^T Q[y_d(k)-y(k)]+u(k)^T Ru(k)\Big)\end{aligned} \tag{1}$$

Subject to $x(k+1)=Ax(k)+Bu(k)$, $y(k)=Cx(k)$ and $x(0)=x_0$. Where the state vector $x(k)\in\mathbb{R}^n$ at time k contains the temperatures of all the nodes in the thermal network, $x(k)=[T_{w_1}(k)\ \cdots\ T_{w_p}(k)\ T_{r_1}(k)\ \cdots\ T_{r_m}(k)]'$ and p is the number of walls, m is the number of rooms, and $p+m=n$. The vector $u(k)\in\mathbb{R}^m$ is the input at $t=k$, $u(k)=[\dot{m}_1(k)\ \cdots\ \dot{m}_m(k)]'$. Vector $y_d(k)$ is the desired output trajectory, specified $\forall\ k=1,2,\cdots,N$ and $U_k:=[u(k)\ u(k+1)\ \cdots\ u(N-1)]'$.

The LQR finds the optimal control input of a linear system according to a quadratic cost function of the states and the inputs. The states and the inputs are assigned weight matrices called Q and R, respectively. The controller can be tuned by varying the weight matrices.

Now, define:

$$\begin{aligned}J_k&=\frac{1}{2}\,[y_d(N)-y(N)]^T\,S\,[y_d(N)-y(N)]\\&\quad+\frac{1}{2}\sum_{i=k}^{N-1}\{[y_d(i)-y(i)]^T Q[y_d(i)-y(i)]+u(i)^T Ru(i)\}\end{aligned}$$

Using *Bellman's principle of optimality*, a recursive relation can be obtained between $J_k^o[x(k)]$, i.e. the optimal cost to go from $x(k)$ to $x(N)$, and $J_{k+1}^o[x(k+1)]$. This optimization problem suggests a recursive algorithm backwards in time to determine the optimal control law. This algorithm is referred to as *dynamic programming*. It is shown in [12] that the optimal control law is given by the following equation:

$$\begin{aligned}u^o(k)&=F(k)b(k+1)-K(k)x(k)\\K(k)&=[R+B^T P(k+1)B]^{-1}B^T P(k+1)A\\F(k)&=-[R+B^T P(k+1)B]^{-1}B^T\end{aligned}$$

Where P and b can be calculated backwards in time using

$$\begin{aligned}P(k-1)\ &=\ C^T QC+A^T P(k)A-\\&\qquad A^T P(k)B[R+B^T P(k)B]^{-1}B^T P(k)A\\b(k-1)\ &=\ A^T b(k)-C^T Qy_d(k-1)-\\&\qquad A^T P(k)B[R+B^T P(k)B]^{-1}B^T b(k)\end{aligned}$$

with the final values $P(N) = C^T SC$ and $b(N) = -C^T Sy^d(N)$. Note that $K(k)$ can be regarded as the feedback gain and $F(k)$ is the feed forward gain [1].

The existence of a closed-form solution for the tracking LQR problem eliminates the need to solve a large constrained optimization problem at each time step. In fact, there is no need to recalculate the matrices $P(k)$ and $b(k)$ as long as the regulating matrices Q and R remain unchanged.

3.1 Tuning the LQR parameters.

We define the concept of *comfort level* to be inversely proportional to the difference between the actual temperature of a room to the temperature requested by the occupants. Together with comfort level, we consider *energy usage*. For a given level of comfort the objective is to utilize the least amount of energy possible. It is obvious that one can rise the level of comfort using a higher actuation authority and therefore higher energy consumption.

To trade-off comfort and energy usage, we rely on the two matrices Q and R. Matrix Q multiplies the difference between the desired and actual temperature which is directly related to comfort, while matrix R multiplies the flow rates which are directly related to energy usage. For example, the entry in R corresponding to a room where an important and widely attended meeting is known to take places at a certain time can be decreased since we do not wish to save energy in that area of the building. On the other hand, the entries of Q corresponding to unoccupied zones can be decreased as to say that comfort is not important in those zones.

3.2 Control Algorithm Implementation

We model the heat transfer system in Simulink based on the equations derived in Section 2, and the implementation of the control algorithm introduced in Section 3. A library was also developed for future use which has some basic elements like the model of a wall and a room, which can be combined to make an arbitrary building. In Figure (6) we show the interconnection of two layers of controllers which was described above.

As we show in Figure (6) the system dynamics is solved in the left box labeled as “Three Room Plant Model” with the inputs of the block being the mass air flow inputs from the dampers. This block simulates the dynamic behavior of the model and solves for the temperatures of the rooms. These temperatures are fed to the block in the middle labeled “LQR”. In this block the optimal tracking problem is solved using a *Dynamic Programming* approach. The solution of the optimal tracking problem is the optimal input which is fed to the lower level PID controllers. The dynamics of the damper is considered in the block between the PID controllers and the Plant. The optimal input is fed to the PID controllers which track this reference signal. The output of the PID is the controlling signal which is given to the dampers to let the required amount of air mass flow into the rooms. So, the loop is closed by feeding the input to the plant model. A detailed view of what takes place in the Plant block and the LQR block is shown in Figure (6).

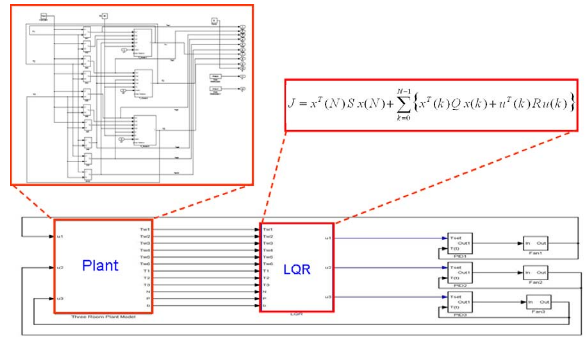


Figure 6. A detailed view of the inside of Plant and LQR blocks

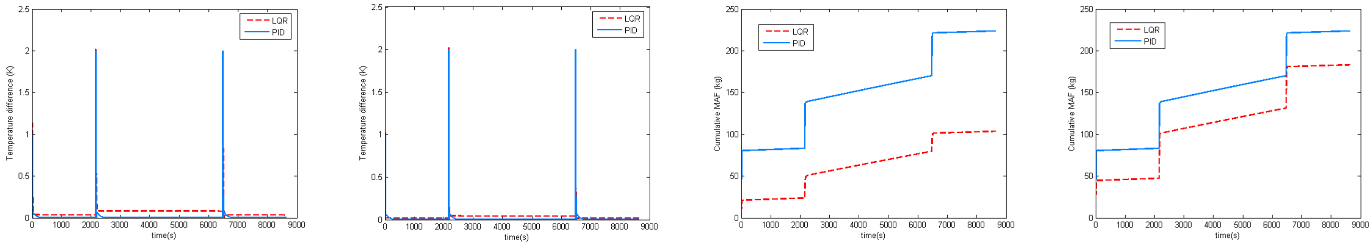
4 Experimental results

We compare two control architectures. In the standard architecture, we only implemented the local PID controllers. The temperature of a zone is fed back to the PID controller which tries to track the given set point without having any information about the temperature trajectory in the future. In our architecture, we applied both the PID controller and the LQR controller to optimally track the set point temperatures of the zones.

For the following simulation results we assumed the set-point trajectory to be $T = 18(^oC)$ from 12 am to 6 am, $T = 20(^oC)$ from 6 am to 6 pm and $T = 18(^oC)$ from 6 pm to 12 am. The initial temperature of the walls, of the air in the rooms, and of the outside temperature are assumed to be $T = 16(^oC)$.

Case 1. In this case, we set $R = eye(3)$ and $Q = eye(3)$. This choice of Q and R matrices imply that there is no preference to either put more weight on the output or on the input in the cost function. The results of the simulation for the comfort and the energy usage comparing two different cases, one with only PID controller and the other with both PID and LQR is shown in Figure (7(a)) and (7(c)), respectively.

Case 2. In this case, we set $R = 10^{-2} \times eye(3)$ and $Q = 10^3 \times eye(3)$. Here, more weight was placed on the output in order to have an output which is closer to the desired profile. Further, the weight on the input has been lowered, meaning that more control effort has been allowed. The simulation results are shown in Figure (7(b)) and (7(d)), respectively. Note that in the energy plots, the cumulative air flow is shown which is proportional to the summation of heating and cooling energy consumption based on the assumption of constant supply air temperatures. As observed



(a) Temperature tracking plot for case 1 (b) Temperature tracking plot for case 2 (c) Energy plot for case 1 (d) Energy plot for case 2

from the figures the response time of the system is about 200 (s) which is a typical response time for a room of this size in this building which confirms the validation of the proposed model.

5 Conclusion

We presented a model based hierarchical control strategy that balances comfort and energy consumption. The building is modeled by a thermal network that captures the relevant dynamics of the temperature for each room taking into account the interactions between rooms, external loads, and building occupancy. We linearize the model around its operating point and we use an LQR supervisory controller that selects the optimal set-points for the lower level PID controllers. As future work we plan to extend the hierarchy to the building and campus level and to provide bounds on the optimality of our control strategy.

REFERENCES

[1] B.D.O. Anderson and J.B. Moore. *Linear optimal control*. Prentice-Hall Englewood Cliffs, NJ, 1971.

[2] B. Arguello-Serrano and M. Velez-Reyes. Nonlinear control of a heating, ventilating, and air conditioning system with thermal load estimation. *Control Systems Technology, IEEE Transactions on*, 7(1):56–63, 2002.

[3] S. Bertagnolio, G. Masy, J. Lebrun, P. André, and H.R. Sualem. Building and HVAC System simulation with the help of an engineering equation solver. In *Proceedings of the Simbuild 2008 Conference, Berkeley, USA*, 2008.

[4] K. Deng, P. Barooah, P.G. Mehta, and S.P. Meyn. Building thermal model reduction via aggregation of states. In *American Control Conference (ACC), 2010*, pages 5118–5123. IEEE, 2010.

[5] F. Felgner, R. Merz, and L. Litz. Modular modelling of thermal building behaviour using Modelica. *Mathematical and Computer Modelling of Dynamical Systems*, 12(1):35–49, 2006.

[6] M. Gwerder and J. Toedtli. Predictive control for integrated room automation. In *8th REHVA World Congress Clima*, 2005.

[7] D. Gyalistras and M. Gwerder. Use of weather and occupancy forecasts for optimal building climate control (OptiControl): Two years progress report. *Terrestrial Systems Ecology ETH Zurich, Switzerland and Building Technologies Division, Siemens Switzerland Ltd., Zug, Switzerland*, 2010.

[8] G.P. Henze, C. Felsmann, and G. Knabe. Evaluation of optimal control for active and passive building thermal storage. *International Journal of Thermal Sciences*, 43(2):173–183, 2004.

[9] F.P. Incropera and D.P. DeWitt. *Introduction to heat transfer*. John Wiley & Sons New York, 1996.

[10] S.F. Larsen, C. Filippín, and G. Lesino. Thermal behavior of building walls in summer: Comparison of available analytical methods and experimental results for a case study. In *Building Simulation*, volume 2, pages 3–18. Springer, 2009.

[11] Y. Ma, F. Borrelli, B. Hencey, B. Coffey, S. Bengea, and P. Haves. Model predictive control for the operation of building cooling systems. In *American Control Conference (ACC), 2010*, pages 5106–5111. IEEE, 2010.

[12] Mehdi Maasoumy Haghighi. Modeling and optimal control algorithm design for hvac systems in energy efficient buildings. Master's thesis, EECS Department, University of California, Berkeley, Feb 2011.

[13] N. Mendes, G.H.C. Oliveira, and H.X. de Araújo. Building thermal performance analysis by using matlab/simulink. In *Seventh International IBPSA Conference, Rio de Janeiro, Brazil*, 2001.

[14] F. Oldewurtel, A. Parisio, C.N. Jones, M. Morari, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and K. Wirth. Energy efficient building climate control using stochastic model predictive control and weather predictions. In *American Control Conference (ACC), 2010*, pages 5100–5105. IEEE, 2010.

[15] A. Standard. 90.1-1989. *American Society of Heating, Refrigerating, and Air-Conditioning Engineers*, 1989.

[16] PT Tsilingiris. Thermal flywheel effects on the time varying conduction heat transfer through structural walls. *Energy and Buildings*, 35(10):1037–1047, 2003.