

## Modified Generalized Laguerre Functions for a Numerical Investigation of Flow and Diffusion of Chemically Reactive Species over a Nonlinearly Stretching Sheet

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**Abstract:** In this paper we provide a collocation method for the problem of flow and diffusion of chemically reactive species over a nonlinearly stretching sheet. This approach is based on a modified generalized Laguerre that it is an orthogonal function. Collocation method reduces the solution of these problems to the solution of systems of algebraic equations. We also compare this work with Homotopy Analysis Method (HAM). Moreover, in the graph of the  $\|Res\|^2$ , we show that the present solution is more accurate and faster convergence in this problem.

**AMS subject classifications:** 34B15 . 34B40 . 80A32

**Key words:** Chemical reaction . non-linear stretching . stretching sheet . modified generalized laguerre . collocation method . semi-infinite

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### INTRODUCTION

Mass and momentum transport by different power-law variations with constant, linear or nonlinear stretching velocity over continuously stretching surfaces has been modeled. Sakiadi [1, 2] worked about this problem with constant velocity and Crane [3] worked with stretching sheet of linear velocity [4].

In addition to the stretching surfaces, the heat and mass flow could make an effect on them by thermal diffusion, concentration differences, applied external forces, chemical reaction and the medium involved in many transport processes that exist in nature and industrial applications. The transfer of a chemically reactive species over a linearly stretching sheet for homogeneous first and higher order reactions has been investigated by Andersson *et al.* [5]. Thakar *et al.* [6] worked about this problem with non-zero velocity at the wall and applied magnetic field. Raptis and Perdakis [7] pursue their researches in this field by considering this problem under the influence of magnetic field over a nonlinearly (quadratic) stretching sheet. Also, Rajagopal *et al.* [8] had some researches in this field. The quality of the final product with desired characteristics could be achieved in some metallurgical and polymer processing applications by drawing the continuous strips or filaments throughout quiescent fluid or porous medium. Additionally there are some kinds of phenomena such as mass diffusion and mass convection that could be observed in fluid mixture that saturates a porous solid matrix for the migration of

moisture through fiberglass wool by Bejan [9]. In addition, several researchers expanded the problem of mass and momentum transport of chemically reactive species on linearly stretching sheet to porous medium and non-Newtonian fluids [4, 10-14].

Moreover, spectral methods have been successfully applied in the approximation of differential boundary value problems defined in unbounded domains. For problems whose solutions are sufficiently smooth, they exhibit exponential rates of convergence/spectral accuracy. The first approach is using Laguerre polynomials [15-17]. The Burgers equation and Benjamin-Bona-Mahony (BBM) equation on a semi-infinite interval are two equation that Guo [15] worked about them and suggested a Laguerre-Galerkin method for them. It is shown that the Laguerre-Galerkin approximations are convergent on a semi-infinite interval with spectral accuracy. In [16] proposed spectral methods using Laguerre functions and analyzed for model elliptic equations on regular unbounded domains. It is shown that spectral-Galerkin approximations based on Laguerre functions are stable and convergent with spectral accuracy in the Sobolev spaces. Siyyam [17] applied two numerical methods for solving initial value problem differential equations using the Laguerre Tau method. He generated linear systems and solved them. Maday *et al.* [18] proposed a Laguerre type spectral method for solving partial differential equations. They introduced a general presentation of the method and a description of the derivation discretization matrices and then determined

the optimum estimations in the adapted Hilbert norms. Recently, in [19] it used the modified generalized Laguerre. However, modified generalized Laguerre was used former. The second approach is reformulating original problem in semi-infinite domain to singular problem in bounded domain by variable transformation and then using the Jacobi polynomials to approximate the resulting singular problem [20]. The third approach is replacing semi-infinite domain with [0,L] interval by choosing L, sufficiently large. This method is named domain truncation [21]. The fourth approach of spectral method is based on rational orthogonal functions [22]. Boyd [23] defined a new spectral basis, named rational Chebyshev functions on the semi-infinite interval, by mapping to the Chebyshev polynomials. Guo *et al.* [24] introduced a new set of rational Legendre functions which is mutually orthogonal in  $L^2(0,+\infty)$ . They applied a spectral scheme using the rational Legendre functions for solving the Korteweg-de Vries equation on the half line. Among these, an approach consists in using the Pseudospectral method based on the nodes of Gauss formulas related to unbounded intervals [25]. Collocation method has become increasingly popular for solving differential equations also this is very useful in providing highly accurate solutions to differential equations. Recently, it used [25, 26].

In this paper, we aim to employ the collocation method to solving this problem. This paper is arranged as follows: In Section 2, we describe the formulation of flow and diffusion of chemically reactive species over a nonlinearly stretching sheet. In Section 3, we describe the formulation of modified generalized Laguerre functions. Section 4 summarizes the application of this method for solving flow and diffusion of chemically reactive species over a nonlinearly stretching sheet. In Section 5, we show results and in this section a comparison is made with homotopy analysis method. The conclusions are described in the final section.

### PROBLEM FORMULATION

Consider the steady two-dimensional incompressible fluid flow over a nonlinearly semi-infinite stretching sheet in a porous medium with the presence of a homogeneous first order chemical reaction. With the usual boundary-layer approximations, the governing equations for the momentum and concentration fields [4, 5, 7, 27, 28],

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \left( \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

Table 1: Nomenclature and greek symbols

Nomenclature	
MGLFM	Modified generalized Laguerre formulation method
HAM	Homotopy analysis method
C	Species concentration in the fluid
$C_w$	Concentration of the reactant at the wall
D	Mass diffusion coefficient
$B_0$	Strength of the magnetic field
F	Functions related to the velocity field
G	Functions related to the velocity field
H	Species concentrations in the fluid
S	Species concentrations in the fluid
K	Darcy permeability
Sc	Schmidt number
U	Local macroscopic velocity
V	Velocity component in y-direction
$k_1$	Chemical reaction parameter
Greek symbols	
$\nu$	Kinetic viscosity
$\beta$	Reaction rate parameter
$\rho$	Density of the fluid
$\eta$	Dimensionless variable
$\sigma$	Electrical conductivity

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 C \tag{3}$$

where x and y are the coordinates along and perpendicular to the sheet, u and v are the components of the velocity in the x and y directions, respectively.  $\nu$  is the kinematic viscosity,  $\rho$  is the fluid density,  $\sigma$  is the electrical conductivity,  $B_0$  is the strength of the magnetic field, C is the species concentration in the fluid, D is the mass diffusion coefficient and  $k_1$  is the chemical reaction parameter [4, 7, 27].

$$u(x,0) = ax + cx^2, \quad v(x,0) = 0, \quad C(x,0) = C_w \tag{4}$$

$$u \rightarrow 0, \quad C \rightarrow 0 \quad \text{as } y \rightarrow \infty \tag{5}$$

where a and c are constants and the subscript w denotes condition at the wall. Following Raptis and Perdakis [7], we introduce similarity transformations [4, 5, 7, 27, 28],

$$\eta = \sqrt{\frac{a}{\nu}} y, \quad u = ax f(\eta) + cx^2 g'(\eta) \tag{6}$$

$$v = -\sqrt{a\nu} f(\eta) - \frac{2cx}{\sqrt{\frac{a}{\nu}}} g(\eta) \tag{7}$$

$$C = C_w [H(\eta) + \frac{2cx}{a} S(\eta)] \tag{8}$$

$$\beta = \frac{k_1 sc}{a}, \quad k = \frac{\sigma B_0^2}{ap}, \quad sc = \frac{v}{D} \tag{9}$$

Applying the transformation of variables in Eqs. (6)-(9), the governing Eqs. (1)-(5) are transformed to a system of dimensionless nonlinear ordinary differential equations [4, 7, 27]

$$f''' + ff'' - (f')^2 - kf' = 0 \tag{10}$$

$$g''' + fg'' - 3fg' + 2f''g - kg' = 0 \tag{11}$$

$$H'' + scfH' - \beta H = 0 \tag{12}$$

$$S'' - scfS + scgH' + scfS' - \beta S = 0 \tag{13}$$

Subject to boundary conditions,

$$f(0)=0, \quad f'(0)=1, \quad f(\infty) = 0 \tag{14}$$

$$g(0)=0, \quad g'(0)=1, \quad g(\infty) = 0 \tag{15}$$

$$H(0)=1, \quad H(\infty) = 0 \tag{16}$$

$$S(0)=0, \quad S(\infty) = 0 \tag{17}$$

where  $f$  and  $g$  are functions related to the velocity field,  $H$  and  $S$  are the species concentrations in the fluid,  $k$  is the permeability parameter,  $sc$  is the Schmidt number and  $\beta$  is the reaction rate parameter. The primes denote differentiation with respect to  $\eta$  [4].

Different techniques have been used to obtain analytical and numerical solutions for this problem. Raptis and Perdakis [7] used the shooting method for this problem. Kechil and Hashim [27] obtained approximate analytical solution via Adomian decomposition method. Recently, in [4, 29] the homotopy analysis method was also applied for solving the above equation [4].

### MODIFIED GENERALIZED LAGUERRE FUNCTIONS

This section is devoted to the introduction of the basic notions and working tools concerning orthogonal modified generalized Laguerre. It has been widely used for numerical solutions of differential equations on infinite intervals.  $L_n^\alpha(x)$  (generalized Laguerre polynomial) is the  $n$ th eigenfunction of the Sturm-Liouville problem [25, 30, 31]:

$$x \frac{d^2}{dx^2} L_n^\alpha(x) + (\alpha + 1 - x) \frac{d}{dx} L_n^\alpha(x) + n L_n^\alpha(x) = 0 \tag{18}$$

$$x \in I = [0, \infty), n = 0, 1, 2, \dots$$

The generalized Laguerre in polynomial manner are defined with the following recurrence formula:

$$L_0^\alpha(x) = 1 \tag{19}$$

$$L_1^\alpha(x) = 1 + \alpha - x$$

$$n L_n^\alpha(x) = (2n - 1 + \alpha - x) L_{n-1}^\alpha(x) - (n + \alpha - 1) L_{n-2}^\alpha(x)$$

These are orthogonal polynomials for the weight function  $w_\alpha = x^\alpha e^{-x}$ . We define Modified generalized Laguerre functions (which we denote MGLF)  $\phi_j$  as follows [25]:

$$\phi_j(x) = \exp\left(\frac{-x}{2L}\right) L_j^1\left(\frac{x}{L}\right), L > 0 \tag{20}$$

This system is an orthogonal basis [32, 33] with weight function  $w(x) = \frac{x}{L}$  and orthogonality property [25]:

$$\langle \phi_m, \phi_n \rangle_{w_L} = \left(\frac{\Gamma(n+2)}{L^{2n} n!}\right) \delta_{nm} \tag{21}$$

where  $\delta_{nm}$  is the Kronecker function.

**Function approximation with Laguerre functions:** A function  $f(x)$  defined over the interval  $I = [0, \infty)$  can be expanded as

$$f(x) = \sum_{i=0}^{+\infty} a_i \phi_i(x) \tag{22}$$

where

$$a_i = \frac{\langle f, \phi_i \rangle_w}{\langle \phi_i, \phi_i \rangle_w} \tag{23}$$

If the infinite series in Eq. (22) is truncated with  $N$  terms, then it can be written as [25]

$$f(x) \cong \sum_{i=0}^{N-1} a_i \phi_i(x) = A^T \phi(x) \tag{24}$$

with

$$A = [a_0, a_1, a_2, \dots, a_{N-1}]^T \tag{25}$$

$$\phi(x) = [\phi_0(x), \phi_1(x), \phi_2(x), \dots, \phi_{N-1}(x)]^T \tag{26}$$

**Modified generalized Laguerre functions collocation method:** Laguerre-Gauss-Radau points and generalized Laguerre-Gauss-type interpolation were introduced by [25, 34-36].

Let

$$\mathfrak{R}_N = \text{span}\{1, x, \dots, x^{2N-1}\} \quad (27)$$

we choose the collocation points relative to the zeroes of the functions [25]

$$p_j(x) = \phi_j(x) - \left(\frac{j+1}{j}\right)\phi_{j-1}(x) \quad (28)$$

Let  $w(x) = \frac{x}{L}$  and  $x_j, j = 0, 1, \dots, N-1$ , be the N MGLF-Radau points. The relation between MGLF orthogonal systems and MGLF integrations is as follows [25, 37]:

$$\int_0^{+\infty} f(x)w(x)dx = \sum_{j=0}^{N-1} f(x_j)w_j + \left(\frac{\Gamma(N+2)}{(N)!(2N)!}\right) f^{2N}(\xi)e^{\xi} \quad (29)$$

Where  $0 < \xi < \infty$  and

$$w_j = x_j \frac{\Gamma(N+2)}{(L(N+1)!(N+1)\phi_{N+1}(x_j))^2}$$

$j = 0, 1, 2, \dots, N-1$ . In particular, the second term on the right-hand side vanishes when  $f(x)$  is a polynomial of degree at most  $2N-1$  [25]. We define

$$I_N u(x) = \sum_{j=0}^{N-1} a_j \phi_j(x) \quad (30)$$

it as:  $I_N u(x_j) = u(x_j), j = 0, 1, 2, \dots, N-1$ .  $I_N u$  is the orthogonal projection of  $u$  upon  $\mathfrak{R}_N$  with respect to the discrete inner product and discrete norm as [25]:

$$\sum_{j=0}^{N-1} a_j \phi_j''(\eta) + \sum_{j=0}^{N-1} a_j \phi_j(\eta) \sum_{j=0}^{N-1} a_j \phi_j''(\eta) - \left(\sum_{j=0}^{N-1} a_j \phi_j'(\eta)\right)^2 - k \sum_{j=0}^{N-1} a_j \phi_j'(\eta) = 0 \quad (38)$$

$$\sum_{j=0}^{N-1} b_j \phi_j''(\eta) + \sum_{j=0}^{N-1} a_j \phi_j(\eta) \sum_{j=0}^{N-1} b_j \phi_j''(\eta) - 3 \sum_{j=0}^{N-1} a_j \phi_j'(\eta) \sum_{j=0}^{N-1} b_j \phi_j'(\eta) + 2 \sum_{j=0}^{N-1} a_j \phi_j''(\eta) \sum_{j=0}^{N-1} b_j \phi_j(\eta) - k \sum_{j=0}^{N-1} b_j \phi_j'(\eta) = 0 \quad (39)$$

$$\sum_{j=0}^{N-1} c_j \phi_j''(\eta) + sc \sum_{j=0}^{N-1} a_j \phi_j(\eta) \sum_{j=0}^{N-1} c_j \phi_j'(\eta) - \beta \sum_{j=0}^{N-1} c_j \phi_j(\eta) = 0 \quad (40)$$

$$\sum_{j=0}^{N-1} d_j \phi_j''(\eta) - sc \sum_{j=0}^{N-1} a_j \phi_j'(\eta) \sum_{j=0}^{N-1} d_j \phi_j(\eta) + sc \sum_{j=0}^{N-1} b_j \phi_j(\eta) \sum_{j=0}^{N-1} c_j \phi_j'(\eta) + sc \sum_{j=0}^{N-1} a_j \phi_j(\eta) \sum_{j=0}^{N-1} d_j \phi_j'(\eta) - \beta \sum_{j=0}^{N-1} d_j \phi_j(\eta) = 0 \quad (41)$$

$$\langle u, v \rangle_{w,N} = \sum_{j=0}^{N-1} u(x_j)v(x_j)w_j \quad (31)$$

$$\|u\|_{w,N} = \langle u, v \rangle_{w,N}^{\frac{1}{2}} \quad (32)$$

thus for the MGLF Gauss-Radau interpolation we have

$$\langle I_N u, v \rangle_{w,N} = \langle u, v \rangle_{w,N}, \forall u, v \in \mathfrak{R}_N \quad (33)$$

### SOLVING THE PROBLEM WITH MODIFIED GENERALIZED LAGUERRE FUNCTIONS

To apply modified generalized Laguerre collocation method to Eqs. (10)-(13) with boundary conditions Eqs. (14)-(17), at first we expand  $f(\eta), g(\eta), H(\eta)$  and  $S(\eta)$  as follows:

$$I_N f(\eta) = \sum_{j=0}^{N-1} a_j \phi_j(\eta) \quad (34)$$

$$I_N g(\eta) = \sum_{j=0}^{N-1} b_j \phi_j(\eta) \quad (35)$$

$$I_N H(\eta) = \sum_{j=0}^{N-1} c_j \phi_j(\eta) \quad (36)$$

$$I_N S(\eta) = \sum_{j=0}^{N-1} d_j \phi_j(\eta) \quad (37)$$

To find the unknown coefficients  $a_j$ 's,  $b_j$ 's,  $c_j$ 's and  $d_j$ 's we substitute the truncated series  $f(\eta), g(\eta), H(\eta)$  and  $S(\eta)$  into Eqs. (10)-(13) and boundary conditions in Eqs. (14)-(17). Also, we can construct the residual functions  $\text{Res}_1(\eta), \text{Res}_2(\eta), \text{Res}_3(\eta)$  and  $\text{Res}_4(\eta)$  for the four equations in the governing system, Eqs. (10)-(13) as:

$$\sum_{j=0}^{N-1} a_j \phi_j(0) = 0, \sum_{j=0}^{N-1} a_j \phi'_j(0) = 1 \tag{42}$$

$$\sum_{j=0}^{N-1} b_j \phi_j(0) = 0, \sum_{j=0}^{N-1} b_j \phi'_j(0) = 1 \tag{43}$$

$$\sum_{j=0}^{N-1} c_j \phi_j(0) = 1 \tag{44}$$

$$\sum_{j=0}^{N-1} d_j \phi_j(0) = 0 \tag{45}$$

$$\sum_{j=0}^{N-1} a_j \phi'_j(\infty) = 0, \sum_{j=0}^{N-1} b_j \phi'_j(\infty) = 0, \tag{46}$$

$$\sum_{j=0}^{N-1} c_j \phi_j(\infty) = 0, \sum_{j=0}^{N-1} d_j \phi_j(\infty) = 0$$

By applying  $\eta$  in Eqs. (38)-(41) with the  $N$  collocation points which are roots of functions  $L_{\alpha}^N$ , we have  $4N$  equations; also, we have six boundary equations Eqs. (42)-(45). Now, all of these equations can be solved by Newton method for the unknown coefficients. We should mention Eq. (46) is always true in these generated equations; therefore, we do not need to apply these boundary conditions.

### RESULT AND DISCUSSION

In this paper, we present the results of our research by  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$  in modified generalized Laguerre for solving this problem. Table 2-4 show the comparison between numerical method that was mentioned in [4], HAM and MGLFM solutions of velocity profile  $-f''(0)$ ,  $-H'(0)$  and  $-S'(0)$  for various  $k$  when  $\beta = 0.2$  and  $sc = 0.24$ . Table 5 and 6 show the comparison between numerical method that was mentioned in [4], HAM and MGLFM solutions of  $-H'(0)$  and  $-S'(0)$  for various  $sc$  when  $\beta = 0.2$  and  $k = 0.8$ . Table 7 and 8 show the comparison between numerical method that it was mentioned in [4], HAM and MGLFM solutions of  $-H'(0)$  and  $-S'(0)$  for various  $\beta$  when  $sc = 0.24$  and  $k = 0.8$ .

Figure 1 and 2 illustrate that the destructive chemical reaction rate parameter  $\beta (\beta > 0)$  reduces the concentration of  $H(\eta)$  and contrarily increases  $S(\eta)$ . This shows that the diffusion rate can be significantly altered by chemical reaction rate. Figure 3 and 4 show the same effect where both  $H(\eta)$  and  $S(\eta)$  decrease as

Table 2: Comparison between MGLFM and other solutions of velocity profile  $-f''(0)$  for various  $k$  at  $\beta = 0.2$  and  $sc = 0.24$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

k	HAM [4]	Present method	Numerical solution [4]
	$-f''(0)$	$-f''(0)$	$-f''(0)$
0.6	1.264934	1.265299	1.264911
0.8	1.341624	1.342081	1.341640
1.0	1.414017	1.414686	1.414213
1.5	1.5802313	1.5816435	1.581139

Table 3: Comparison between MGLFM and other solutions of velocity profile  $-H'(0)$  for various  $k$  at  $\beta = 0.2$  and  $sc = 0.24$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

k	HAM [4]	Present method	Numerical solution [4]
	$-H'(0)$	$-H'(0)$	$-H'(0)$
0.6	0.505910	0.505903	0.50590
0.8	0.503601	0.503811	0.50380
1.0	0.501943	0.501962	0.50191
1.5	0.498112	0.498129	0.49813

Table 4: Comparison between MGLFM and other solutions of velocity profile  $-S'(0)$  for various  $k$  at  $\beta = 0.2$  and  $sc = 0.24$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

k	HAM [4]	Present method	Numerical solution [4]
	$-S'(0)$	$-S'(0)$	$-S'(0)$
0.6	0.040063	0.040357	0.04035
0.8	0.040320	0.040262	0.04022
1.0	0.040070	0.040070	0.04004
1.5	0.039380	0.039351	0.03937

Table 5: Comparison between MGLFM and other solutions of velocity profile  $-H'(0)$  for various  $sc$  at  $\beta = 0.2$  and  $k = 0.8$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

sc	HAM [4]	Present method	Numerical solution [4]
	$-H'(0)$	$-H'(0)$	$-H'(0)$
0.24	0.503601	0.503811	0.50380
0.4	0.544218	0.544194	0.54429
0.6	0.596620	0.596549	0.59659
0.8	0.650062	0.649917	0.65001

Table 6: Comparison between MGLFM and other solutions of velocity profile  $-S'(0)$  for various  $sc$  at  $\beta = 0.2$  and  $k = 0.8$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

sc	HAM [4]	Present method	Numerical solution [4]
	$-S'(0)$	$-S'(0)$	$-S'(0)$
0.24	0.0403201	0.0402621	0.04022
0.4	0.0673822	0.0680449	0.06795
0.6	0.1014366	0.1022135	0.10221
0.8	0.1338458	0.1346625	0.13469

Table 7: Comparison between MGLFM and other solutions of velocity profile  $-H'(0)$  for various  $\beta$  at  $\beta = 0.8$  and  $sc = 0.24$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

$\beta$	HAM [4] $-H'(0)$	Present method $-H'(0)$	Numerical solution [4] $-H'(0)$
0.2	0.503601	0.503811	0.50380
0.8	0.933541	0.933529	0.93358
1.0	1.036527	1.036521	1.03652
1.2	1.129936	1.129923	1.12992

Table 8: Comparison between MGLFM and other solutions of velocity profile  $-S(0)$  for various  $\beta$  at  $\beta = 0.8$  and  $sc = 0.24$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

$\beta$	HAM [4] $-S(0)$	Present method $-S(0)$	Numerical solution [4] $-S(0)$
0.2	0.040320	0.040262	0.040220
0.8	0.030445	0.030681	0.030667
1.0	0.028971	0.029130	0.029101
1.2	0.027672	0.027872	0.027822

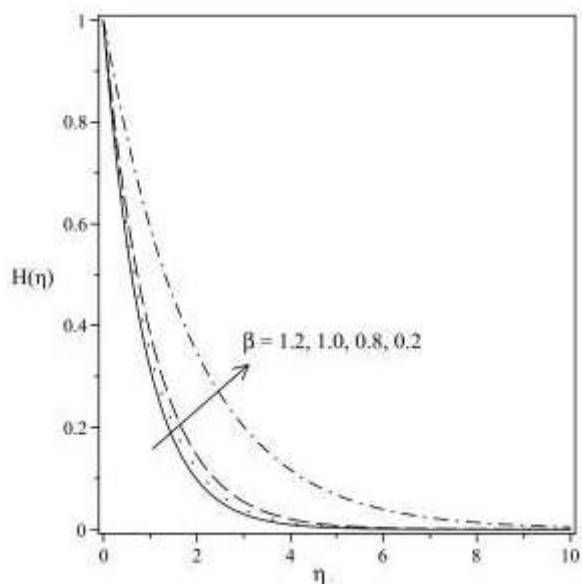


Fig. 1: Concentration profile  $H(\eta)$  for various  $\beta$  when  $k = 0.8$  and  $sc = 0.24$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

the Schmidt number  $sc$  increases. This is due to the contribution of mass diffusion which becomes less significant as the Schmidt number that represents the ratio of momentum and mass diffusivity increases.

Also, as shown in Figure 5 and 6 for the effects of magnetic parameter, the consumption of the low concentration  $H(\eta)$  and  $S(\eta)$  continues and reaches its peak minimum almost at the same distance of  $\eta$  from the wall before it oscillates back to zero. Figure 7

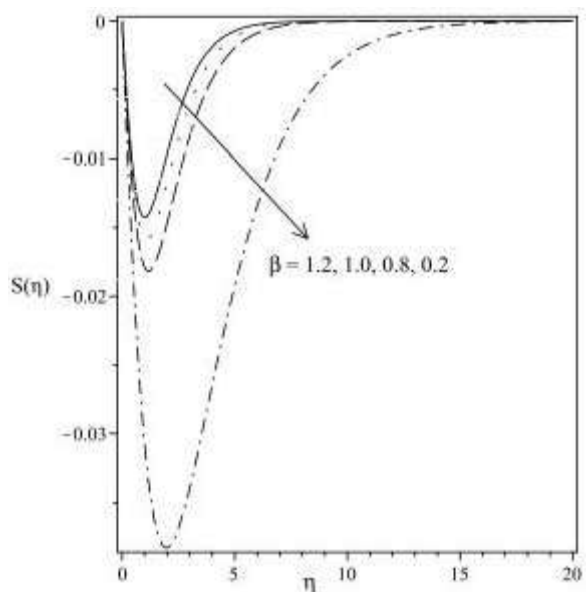


Fig. 2: Concentration profile  $S(\eta)$  for various  $\beta$  when  $k = 0.8$  and  $sc = 0.24$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

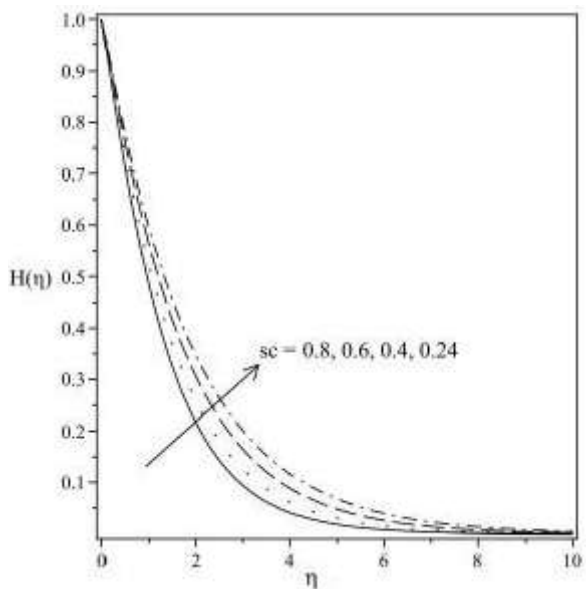


Fig. 3: Concentration profile  $H(\eta)$  for various  $sc$  when  $k = 0.8$  and  $\beta = 0.2$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

illustrates the influence of the magnetic parameter  $k$  on the velocity profile  $g'(\eta)$ . It shows that the parameter  $k$  have the few effect on the velocity profile.

Figure 8 illustrates the influence of the magnetic parameter  $k$  on the velocity profile  $f'(\eta)$ . In Fig. 8, we can observe an almost precise alignment between MGLFM and the exact solutions. Furthermore, Fig. 8

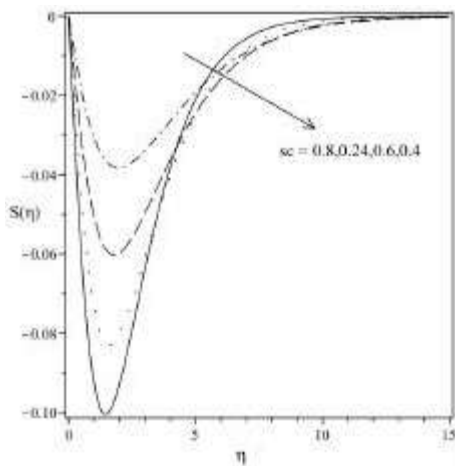


Fig. 4: Concentration profile  $S(\eta)$  for various  $sc$  when  $k = 0.8$  and  $sc = 0.2$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

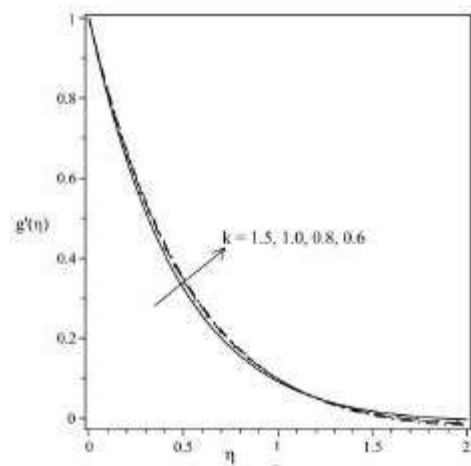


Fig. 7: Velocity profile  $g'(\eta)$  for various  $k$  when  $sc = 0.24$  and  $\beta = 0.2$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

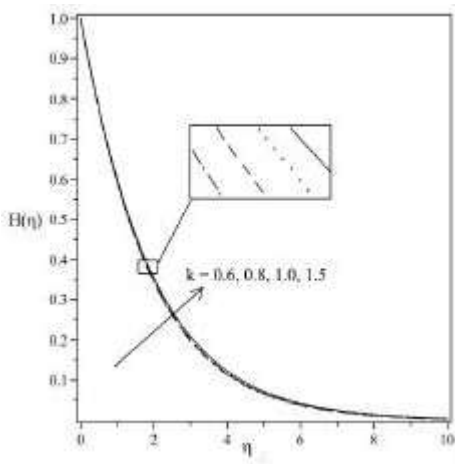


Fig. 5: Concentration profile  $H(\eta)$  for various  $k$  when  $sc = 0.24$  and  $sc = 0.2$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

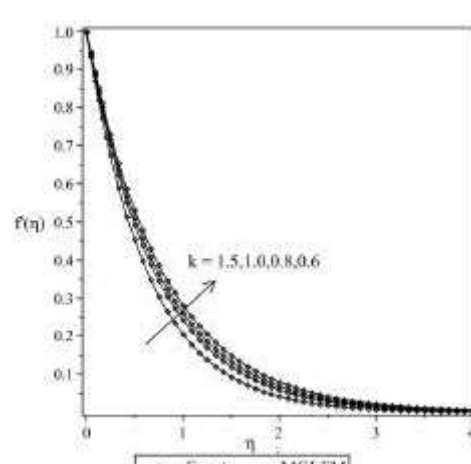


Fig. 8: Velocity profile  $f'(\eta)$  for various  $k$  and comparison with exact solutions when  $sc = 0.24$  and  $\beta = 0.2$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

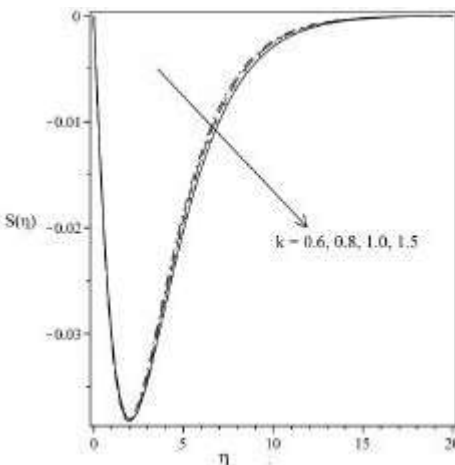


Fig. 6: Concentration profile  $S(\eta)$  for various  $k$  when  $sc = 0.24$  and  $\beta = 0.2$  with  $N = 20$ ,  $\alpha = 1$  and  $L = 0.99$

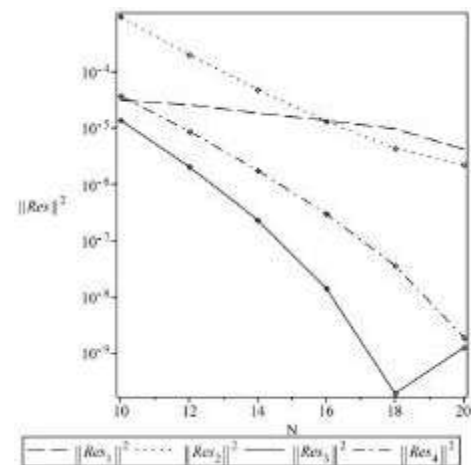


Fig. 9: Graph of  $\|Res\|^2$  by MGLFMs solution for  $\beta = 0.8$ ,  $k = 0.8$  and  $sc = 0.24$

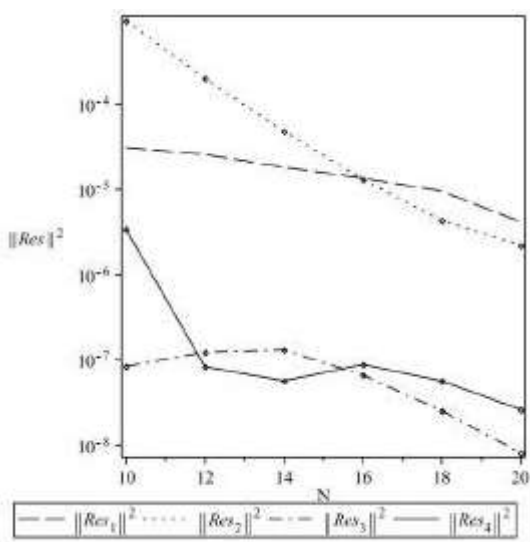


Fig. 10: Graph of  $\|Res\|^2$  by MGLFMs solution for  $\beta = 0.2, k = 0.8$  and  $sc = 0.4$

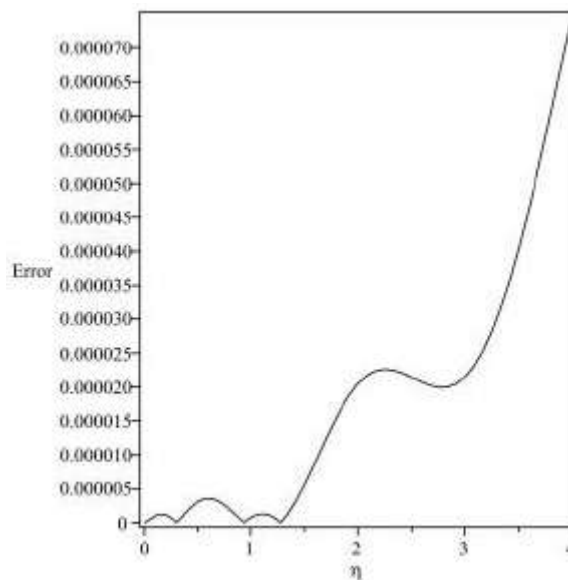


Fig. 12: Graph of Error by MGLFMs solution for  $\beta = 0.2, k = 0.6$  and  $sc = 0.24$

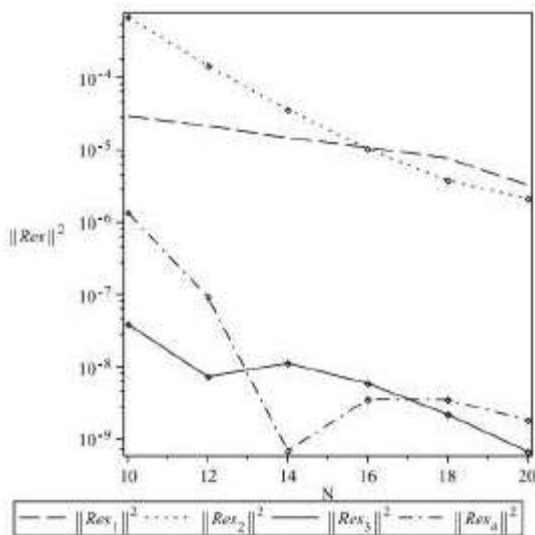


Fig. 11: Graph of  $\|Res\|^2$  by MGLFMs solution for  $\beta = 0.2, k = 0.6$  and  $sc = 0.24$

shows that the increment in the magnetic parameter reduces the velocity profile  $f'(\eta)$ . Here we note that the Eq.(10) subject to boundary conditions Eq.(14) has an exact solution [10] as

$$f(\eta) = \frac{1}{\sqrt{1+k}} (1 - e^{-\sqrt{1+k}\eta}) \quad (47)$$

while in the absence of the magnetic field where  $k = 0$ , the exact solution first obtained by Crane [3] is

$$f(\eta) = 1 - e^{-\eta} \quad (48)$$

The logarithmic graphs of the  $\|Res\|^2$  for MGLFM at  $(\beta = 0.8, k = 0.8$  and  $sc = 0.24)$ ,  $(\beta = 0.2, k = 0.8$  and  $sc = 0.4)$  and  $(\beta = 0.2, k = 0.6$  and  $sc = 0.24)$  are shown in Fig. 9-11, respectively. These graphs illustrate the convergence rate of the method.

The absolute error between MGLFMs solution and exact solution of the velocity profile  $f(\eta)$  for  $\beta = 0.2, k = 0.8$  and  $sc = 0.24$  is shown in Fig. 12.

## CONCLUSION

In this study, flow and diffusion of chemically reactive species over a nonlinearly stretching sheet was investigated numerically using MGLFM. Modified generalized Laguerre functions are orthogonal functions that solved the system of non-linear differential equations governing the problem on the semi-infinite domain without truncating it to a finite domain, imposing the asymptotic condition transforming and transforming the domain of the problem. Modified generalized Laguerre functions were proposed to provide a simple way to improve the convergence of the solution by collocation method. The graphs of the  $\|Res\|^2$  demonstrated the fact that the present solution using MGLFM was highly accurate.

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