RELATIVE PERFORMANCE EVALUATION WITH SYSTEMATIC RISK*

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Abstract

Most observed managerial compensation packages do not filter out systematic economy-wide fluctuations (e.g., market indices, oil prices, macroeconomic variables). However, given that such shocks do not appear informative on individual actions, the limited use of relative performance evaluation (RPE) is unexplained within the conventional model of incentives. Incorporating asset pricing considerations, this paper shows that the cost to a firm of a dollar of compensation is lower after a favorable market-wide shock than after an unfavorable shock. As a result, firms optimally choose a compensation structure in which pay depends on the systematic shock. Further, the systematic shock may be informative on the wealth of the manager, and thus useful to tailor the incentive scheme. Specifically, the optimal contract may imply higher pay-for-performance in good states, because such states feature more personal wealth and thus greater risk tolerance. Under constant relative risk-aversion, a contract with pure RPE is suboptimal even if the manager can privately trade the market. The framework suggests that apparent rejections of RPE may be driven by rational asset pricing effects, and offers an empirical methodology to filter out these effects.

Keywords: Agency, General Equilibrium, Relative Performance, Markets

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Relative performance evaluation is solidly grounded in conventional agency theory. In the standard model, risk-neutral shareholders should use information that helps infer which actions have been taken, but insure the manager against other forms of uncertainty that are statistically independent from privately-observed actions. In the context of executive compensation, the informativeness principle suggests that executives should be rewarded relative to their peers, and not for systematic economy-wide fluctuations.[1]

Yet, relative performance evaluation (hereafter RPE) represents a relatively small fraction of total managerial pay and most empirical studies reject the forms of RPE obtained from theory.[2] The empirical failure of RPE is problematic for several reasons. First, it questions the validity of the informativeness principle, a cornerstone of information economics in accounting research, and suggests that the benchmark model ignores essential contracting frictions. Second, the standard theory fails to provide useful guidelines for the observed non-trivial relationship between incentive contracts and statistically uninformative systematic shocks. Despite many attempts by observers to popularize relative performance, pure RPE has received very little support outside of the academic profession.[3]

This paper extends the standard agency model to a general equilibrium framework with systematic shock, and explicitly model the risk premia demanded by well-diversified investors (an aspect absent in the standard agency model). We show that RPE can be suboptimal in economies with systematic economy-wide shocks, in apparent contrast to the informativeness principle. In a competitive equilibrium, firms value a dollar of compensation in a good state less than a dollar of compensation in a bad state; this creates rational motives to offer contracts that are sensitive to systematic risk factors. We characterize the optimal contract and find that, in general, pay-for-performance coefficients increase after a favorable systematic shock. In practice, such a dependence can occur with standard options, since the delta of the option will be greater after an increase of the stock price due to a market-wide movement. This property rationalizes the empirical findings of Hall and Knox (2004) who report that pay-for-performance increases (decreases) after an increase (decrease) in the market, even when accounting for new option grants.

[1]Holmström (1982) provides a proof of this statement (Theorems 7, 8), and interprets this result in the context of an economy with systematic risk.

[2]Several studies document the relationship between executive pay and systematic risk factors such as market indices (Gibbons and Murphy (1990), Himmelberg and Hubbard (2000), Jin (2002)), oil prices (Bertrand and Mullainathan (2001)), industry risk (Antle and Smith (1986), Janakiraman, Lambert, and Larcker (1992)). Murphy (1999) and Bannister and Newman (2003) review the compensation plans of a panel of firms, and find that explicit RPE is relatively uncommon. Perhaps the strongest empirical support of RPE is among subsidiaries of a firm or institution (Bushman, Indjejikian, and Smith (1995), Matsumura and Shin (2006)).

[3]Abowd and Kaplan (1999) ask: "Why should CEOs be rewarded for doing nothing more than riding the wave of a strong bull market?" They propose to improve managerial compensation by linking the exercise price of executive options to stock indices. Following a similar argument, Bertrand and Mullainathan (2001) explain that in their sample appear to be paid for "luck" rather than skill.

Finally, we consider whether RPE may be optimal, if the manager can hedge the market portfolio on his own. In other words, can the firm offer a wage that is insensitive to market risk, but letting the agent recreate an efficient exposure to market risk via portfolio rebalancing decisions. Using a general equilibrium approach, we show that such a separation between the compensation and the portfolio decisions, always fails except with constant absolute risk-aversion and identify wealth effects as a necessary and sufficient condition for the failure of RPE. We further discuss implications of the model for accounting research and propose methodologies to test for the presence of RPE and, over standard parametric classes of preferences, test for violations of efficient contracting.

Related Literature

This paper contributes to the body of existing literature on RPE. Several authors relate the limited use of relative performance to financial market imperfections. Jin (2002) analyzes cases in which the manager cannot invest, via his own portfolio decisions, on financial markets. He shows that, in such situations, the optimal contract induces some exposure to market variations, an apparent failure of RPE. However, in his model, RPE can again become optimal if the manager can trade on his own. Extending this setting, Garvey and Milbourn (2003) assume that trading the market portfolio may be more costly for the manager than for the firm. In both of these papers, the primary focus is the relationship between risk characteristics of the firm (e.g., volatility, beta) and the sensitivity of wages to systematic shocks - not the primitive reasons why RPE should fail if the manager is able to hedge the market. Finally, focusing on market imperfections, Oyer (2004) argues that the outside option of the manager may vary with the business cycle (e.g., the CEO may quit). Given that a contract cannot commit the manager to remain with the firm, the firm must increase the wage after a positive productivity shock, thus violating an efficient insurance against the aggregate shock from an ex-ante perspective.

Our approach is complementary to this literature. In comparison this set of papers, we do not assume any friction that impairs efficient contracting, yet show that RPE may fail even in the standard incentive model. Our approach further leads to an extended informativeness principle: RPE may be violated for any variable that is informative on the agent's wealth, such as systematic economy-wide shocks. On the other hand, as in the standard agency model, our model cannot explain violations of RPE for diversifiable risk (that does not incorporate *any* economy-wide risk) or uninformative variables once the economy-wide risk has been filtered out.[4]

[4] In particular, this implies that the dependence of the compensation on market movements is consistent with the standard agency model. However, the compensation should not depend on industry movements, once the share of these movements explained by aggregate risk factors has been filtered out.

A different set of papers focuses on how agents in a firm may affect total profits or economic efficiency in the industry: a common shock can be informative on a manager's diligence and thus should be used as part of the compensation structure. Dye (1992) shows that RPE may induce manager to enter markets in which they are comparatively better than their competitors, although not necessarily efficient in absolute terms. Aggarwal and Samwick (1999) show that RPE generates excess competition on the product market. A common ground in these papers is that efficient production requires some coordination among managers from different firms - in contrast, RPE may give inappropriate incentives to perform better than peers. In comparison to these papers, we show that RPE can fail even when managers do not affect total industry profits, and thus there is no incentive alignment problem at the industry level. Consistently with this idea, we focus on economy-wide risk and model each firm as being small relative to the rest of the market.

In the context of the existing literature on agency theory, the paper provides three main contributions. First, we discuss the design of an optimal contract with systematic risk, proving that under reasonable conditions filtering out systematic risk is suboptimal. Our approach further provides a novel rationale for using options whose strike do not filter out movements in the market. In addition, we provide novel modeling techniques to incorporate agency (and possibly other informational frictions) in a competitive equilibrium framework. Second, we solve for the risk premia demanded on systematic risk in an economy featuring moral hazard. Our main result is that risk premia are, to a large extent, unaffected by the informational friction. Third, we argue that existing tests of RPE do not necessarily indicate violations of efficient contracting, and derive from the theory several alternative empirical procedures that are robust to systematic risk.

1. Preliminaries

To keep the model as transparent as possible, we present a simplified version of the model with identical managers and firms, and a single time period - these assumptions are unimportant for the main results and will be relaxed in Section 5. Assume that the economy consists of a continuum of identical firms and managers with mass normalized to one and indexed by $k \in [0,1]$. In the model, each firm and manager will be small as compared to the economy; this assumption is meant to capture the idea that individual outcomes have no effect on aggregate outcomes, and to distinguish our model from other papers in which the manager controls the systematic signal.

• **Managers**

Each manager receives a utility $u_k(c) = u(c_k) - \psi(a_k)$ where c_k is final consumption and $a_k \in [\underline{a}, \overline{a}]$ is a privately observed effort decision. Assume that $u(.)$ is twice-differentiable, strictly increasing and strictly concave, and satisfies standard Inada conditions. The cost of effort $\psi(.)$ is convex, increasing and differentiable with $\psi(0) = \psi'(\underline{a}) = 0$ and $\lim_{a \to \overline{a}} \psi'(a) = +\infty$.

• Production

Each firm requires the services of a single manager to operate and, without loss of generality, assume that firm k employs manager k.[5] A firm produces an output $\pi_k = \alpha y_k$, where $\alpha \geq 0$ is drawn from an absolutely continuous distribution with a differentiable density $h(.)$ and full support on $[\underline{\alpha}, +\infty]$ (where $\underline{\alpha} > 0$). It is convenient to interpret α as an economy-wide systematic shock that affects all managers, such as a market index change or a macroeconomic aggregate fluctuation.[6] Note however that, given that industry risk should also incorporate some economy-wide shocks, the model also has implications for RPE within comparable industries.[7]

The distribution of y_k depends on the effort of the employed manager, a_k. Conditional on a_k, the distribution of y_k is denoted $F(.|a_k)$ and has a density $f(.|a_k)$, twice-differentiable in a_k and with compact positive-valued support $[\underline{y}, \overline{y}]$. Since the objective of the analysis is to explain the dependence of pay on systematic risk, it is assumed here that $y_k|a_k$ does not depend on α. In other words, the common shock α is not statistically informative on effort and should not be part of the optimal contract in the standard agency model with a risk-neutral principal. To avoid other well-known situations in which relative performance pay is desirable, assume as well $(y_k)_{k' \neq k}$ is independent of y_k.[8] Assume that $f_a(y|a)/f(y|a)$ is strictly increasing in y (monotone likelihood ratio). This assumption implies in particular that for any $a > a'$, $y|a$ first-order stochastically dominates $y|a'$. To simplify notations, the mean of $y|a$ is normalized to a.

[5] Note that although in this version of the model all managers are identical, the generalized version of the model in Section 5 accommodates heterogeneity across agent preferences and production (e.g., CEO, plant managers, regular employees, etc.).

[6] We choose the multiplicative form αy_k is assumed here for presentation purposes and because it can then be defined as total factor productivity (TFP) as in Kydland and Prescott (1982) and the follow-up literature on aggregate fluctuations. However, this functional form does not drive the results and it is relaxed in Section 5.

[7] We are not aware of any paper that attempts to filter out economy-wide risk from industry movements when testing for RPE - stepping ahead, this paper shows that a direct test of the RPE hypothesis of Holmström (1982) should first involve filtering out systematic risk from the contracting variables.

[8] In other words, we summarize the covariation between firms by the parameter α. Given a richer correlation structure, RPE would fail for any movement in the industry that leads to effects on the total production in the economy. This can be modeled easily in this framework by rewriting α as a multi-dimensional variable and, as in Section 5, incorporating differences across firms. A consequence is that pure RPE should hold in our model only for the portion of industry risk that is diversifiable in the sense of the capital asset pricing model.

2. Market Economy

We present now the decentralized market economy. Firms are viewed as principals and can offer a state-contingent contract. Firm k's contract is denoted $L^k = (a_k, w(\alpha, y_k))$ and prescribes the effort to be chosen by the manager and the wage to be paid conditional on each state. Note that the wage may depend on the common shock α and the firm-specific shock y_k.[9]

• Firm Contract

A contract L^k is feasible and incentive-compatible if it meets the following two constraints. First, it must be individually rational for manager k to accept the contract. Let R denote the utility achieved by managers when contracting with other firms in the economy. Then, the wage offered by firm k must satisfy:

$$\int\int h(\alpha)u(w_k(\alpha, y_k))f(y_k|a_k)dy_k d\alpha - \psi(a_k) \geq R \tag{2.1}$$

Second, the choice of a_k is not observable by the firm. Thus, the recommended effort a_k in the contract must be incentive-compatible for the manager. Taking the first-order condition on the problem of the manager, the choice of a_k must satisfy the following Equation.

$$\int\int h(\alpha)f_a(y_k|a_k)u(w_k(\alpha, y_k))dy_k d\alpha = \psi'(a) \tag{2.2}$$

Remark 1: Typically, the components y_k and α are not directly observable. However, in this model, they can be perfectly inferred from actual profits as follows. Let A denote the average effort anticipated in equilibrium in the economy and define

$$\begin{aligned} \hat{\alpha} &= \int \pi_k dk/A \\ \hat{y_k} &= \pi_k/\hat{\alpha} \end{aligned} \tag{2.3}$$

Then $\hat{\alpha}$ represents a perfect estimate of the common shock in the economy and $\hat{y_k}$ represents a perfect estimate of the performance of firm k relative to its peers. Note that, by construction of $\hat{y_k}$, any compensation that depends only on y corresponds to RPE, in that it filters out the common component $\hat{\alpha}$ in π_k.

[9] As before and for obvious reasons, there would be no purpose in offering a wage that depends on the firm-specific shock of other firms.

• **State-Contingent Assets**

In this economy, an *aggregate state of the world* is defined as a realization of α. To solve for an optimal contract, it is first necessary to model the value to a well-diversified investor of a claim on the aggregate state of the world. We follow the asset pricing model of Debreu (1972) to price systematic risk. Assume that there is a continuum of state-contingent claims $(Q(\alpha))$ that are traded: an investor owning one unit of a claim $Q(\alpha)$ receives one when the state of the world is α is realized and zero else. Let $q(\alpha)$ be the price of a claim $Q(\alpha)$ (or state price) before α is known.[10] As is usual, the state prices are normalized to $\int q(\alpha)d\alpha = 1$. These claims are traded financial assets that are in zero net supply in the economy.

Remark 2: Following this normalization, the cost of buying an asset that pays one regardless of the state of the world α, is $\int q(\alpha)d\alpha = 1$. This means that the risk-free rate is normalized to one. In the finance literature, $q(\alpha)$ is also referred to as the risk-neutral probability measure, i.e. each asset can be valued as if by a risk-neutral manager but using the distribution $q(\alpha)$, instead of the objective distribution $h(\alpha)$.

For any possible realization of α and $(y_k)_{k\in[0,1]}$, $\int \alpha y_k dk$ exists and represents total output available in the economy. Conditional on efforts $(a_k)_{k\in[0,1]}$, the law of large numbers implies that $\int \alpha\mathbb{E}(y_k|a_k)dk = \alpha \int a_k dk$ is available in the economy. Thus, the state price $q(\alpha)$ can also be understood as an asset paying one unit of good when the aggregate endowment in the economy is $\alpha \int a_k dk$.[11]

• **Firm's Problem**

The objective of firms is to maximize the value of their production plan - here: their market value from the perspective of well-diversified investors. Formally, consider a firm with a production plan $\Pi(\alpha)$ in each state α. It can also be viewed as a portfolio of $\Pi(\alpha)$ units of each claim $Q(\alpha)$. By arbitrage, its value should be the same as the value of such a portfolio, that is: $\int q(\alpha)\Pi(\alpha)d\alpha$, that is the summation of the number of units produced $\Pi(\alpha)$ in each aggregate state weighted by the Arrow-Debreu price $q(\alpha)$ associated to this aggregate state.[12]

[10] For example, an investor buying two units asset $Q(.5)$ will pay $2q(.5)$. If the realization of α is .5, this investor will receive two units of consumption.

[11] Note that from the perspective of any single manager in the economy, $\alpha \int a_k dk$ is not controlled; and thus every agent is equally informed on the realization of the aggregate endowment.

[12] It is well-known that firms will behave as risk-neutral to any shock that does not affect the aggregate endowment (see Duffie (2001) for references).

In the context of this model, each firm produces a state-contingent expected number of units $\Pi_k = \int \alpha y_k - w_k(\alpha, y_k) f(y_k|a_k) dy_k$. Taking state prices $q(.)$ as given, the value to firm k of a feasible contract L^k is:

$$V(L^k) = \int \int (\alpha y_k - w_k(\alpha, y_k)) f(y_k|a_k) q(\alpha) dy_k d\alpha \tag{2.4}$$

The optimal contract L^k is then given by a solution to the following problem.

$$(P_k) \quad \max_{L^k} \quad V(L^k)$$

$$\text{s.t.} \qquad \text{(2.1) and (2.2)}$$

Remark 3: In $V(L^k)$, the firm is held by well-diversified investors. These investors may not value a payment when α is high in the same way as a payment when α is low, and thus calculating its profit with respect to the objective probability distribution $h(\alpha)$ is inappropriate: the value of the firm must be decomposed as a portfolio of state-contingent securities with value $q(\alpha)$.

• Competitive Equilibrium

Firms are competitive and make zero profit.[13] In each aggregate state, each firm will offer a wage $w_k(\alpha, y_k)$, for a total consumed $\int \int w_k(\alpha, y_k) f(y_k|a_k) dy_k dk$. This must be equal to the total endowment in the economy $\alpha \int a_k dk$. For all α,

$$\int \int w_k(\alpha, y_k) f(y_k|a_k) dy_k dk = \alpha \int a_k dk \tag{2.5}$$

The equilibrium is formally defined below.

Definition 2.1 *A (symmetric) competitive equilibrium is* $\Gamma = (a, w(.,.), q(.), R)$ *such that:*

1. $(a, w(.))$ *maximizes* (P_k) *for all* k.

2. *Equation* (2.5) *holds for all* α.

A first-best competitive equilibrium is defined in the same manner but omitting Equation (2.2), the incentive-compatibility, in the contract design problem (P_k). In the rest of the analysis, the index k will

[13]The assumption of zero profits can be justified by equilibrium reasoning. Suppose that firms may make positive profits and are equally owned by all agents. When a contract is signed, agents transfer their assets to the firm and are given the contractual consumption $w(\alpha, y)$ (the firm receiving the dividends from the agent's assets). In this case, equilibrium dictates that total consumption equal to total endowment in each state (Equation (2.5)). This in turn implies zero profit for firms. In the more general case discussed in Section 5, this property is less immediate and formally proved.

be omitted to simplify notations. The main focus of this paper is the analysis of the optimal contract, and thus a competitive equilibrium is assumed to exist and feature a contract $w(.,.)$ that is a smooth function of both parameters.

3. Analysis of the Contract

3.1. First-Best

As a benchmark to the analysis, the solution to the first-best economy is formally stated.

Proposition 3.1 *A first-best competitive equilibrium is Pareto-efficient and satisfies (up to sets with zero probability):*

$$\psi'(a) = \int \alpha h(\alpha) u'(\alpha a) d\alpha \tag{3.1}$$

$$w(\alpha, y) = \alpha a \tag{3.2}$$

$$q(\alpha) = \frac{h(\alpha)u'(\alpha a)}{\int h(x)u'(xa)dx} \tag{3.3}$$

$$R = \int\int h(\alpha) f(y|a) u(\alpha a) dy d\alpha - \psi(a) \tag{3.4}$$

The first-best solution to the model is such that the disutility of effort is equal to the gains from greater consumption in all states. It is a simple implication of the first welfare theorem in economies with production (Debreu 1972) (since firms are producing state-contingent units of good). Further, the competitive equilibrium is Pareto efficient and prescribes complete insurance against idiosyncratic risk. On the other hand, the systematic shock is not insured and each manager receives a wage αa.

3.2. Second-Best Contract

In second-best, the firm must be mindful of the need to provide incentives to the manager. The following preliminary result establishes the optimality of providing some incentives regardless of the state prices or cost of effort.

Lemma 3.1 *A competitive equilibrium must be such that* $a > \underline{a}$.

While proved here in the presence of systematic risk, this property is similar to Shavell (1979) (Proposition 2, p. 59). The firm can increase effort at almost no cost by shifting from perfect insurance to a pay that depends on both idiosyncratic (to induce effort) and systematic risk (to provide insurance).

Let λ (resp. μ) denote the multiplier associated to Equation (2.1) (resp. Equation (2.2)) in Problem (P). Taking the first-order condition with respect to $w(\alpha, y)$ yields the following characterization of the optimal contract:

$$u'(w(\alpha, y)) = \underbrace{\frac{q(\alpha)}{h(\alpha)}}_{G(\alpha)} \underbrace{\frac{1}{\lambda + \mu \frac{f_a(y|a)}{f(y|a)}}}_{H(y)} \tag{3.5}$$

Because $a > \underline{a}$ is elicited (Lemma 3.1), the wage cannot be decreasing for all y and thus the multiplier μ must be strictly positive. In Equation (3.5), the marginal utility of the manager can be decomposed into a systematic component $G(\alpha)$ and an idiosyncratic component $H(y)$. The systematic component captures risk-aversion to aggregate risk and is driven by the common labor productivity shock α. The idiosyncratic component captures the role of incentives and its interpretation is similar to a standard agency model with no systematic risk (Holmström 1979).

Proposition 3.2 *$w(\alpha, y)$ is strictly increasing in α and y.*

The manager is paid more in good states of the world. This is consistent with evidence that, when controlling for firm-specific performance, contracts depend on systematic risk factors (Gibbons and Murphy (1990), Himmelberg and Hubbard (2000)). To better understand the statement, it is helpful to reframe the result in intuitive terms starting from two perspectives. First, the Arrow-Borch conditions for perfect risk-sharing reveal that, in situations such that firm and agents are risk-averse to systematic shocks, the optimal contractual arrangement should *not* provide perfect insurance and wages should depend on common risk factors. Here, the risk-aversion of the firm is captured by $q(\alpha)/h(\alpha)$ decreasing. In other words, in this economy, even well-diversified investors demand a risk premium to be exposed to the systematic risk factor.[14] Second, from a general equilibrium perspective, the market clearing condition makes perfect insurance *at any price* impossible. The aggregate shock is effectively allocated to the agents in the economy as consumption risk.[15]

3.3. Performance Pay

Since the wage is affected by aggregate shocks, the optimal performance-pay coefficients may also depend on the realization of the aggregate shocks. The next Proposition establishes the relationship

[14] The risk premium demanded by investors for holding the market can be obtained as $RP = \int h(\alpha)\alpha d\alpha / \int q(\alpha)\alpha d\alpha - 1$ - that is the ratio of the expected value of the market by the ex-ante cost of buying an asset that pays the market. As in standard asset pricing theory, $q(\alpha)/h(\alpha)$ decreasing implies that $RP > 0$.

[15] Note that the market clearing condition implies only that the expected wage $\mathbb{E}(w(\alpha, y)|\alpha)$ should be increasing in α, which is weaker than the claim in Proposition 3.2.

between the sensitivity of the manager's pay to idiosyncratic risk and the state of the economy.

Proposition 3.3 *If absolute risk aversion is decreasing (resp. increasing), then the performance-pay coefficient (w_y) is increasing in α.*

Corollary 3.1 *In the HARA class of preferences (i.e., $u(x) = \frac{\gamma}{1-\gamma}(b_1 + b_2 x)^{1-\gamma}$), $\log w_\alpha(\alpha, y) = \kappa_1(\alpha) + \kappa_2(y)$ and $\log w_y(\alpha, y) = \kappa_3(\alpha) + \kappa_4(y)$ can be written additively in α and y.*

In the class of preferences with decreasing absolute risk aversion (DARA), performance pay should increase during expansions and decrease during recessions. Here, the optimal contract should prescribe more incentive pay in states with lower risk-aversion. This implies in particular more pay for performance in states such that managers are relatively wealthy which are, by market clearing, states with a high systematic shock. DARA preferences have appeal in the context of executive compensation since, unlike with constant absolute risk-aversion (CARA) utility, they do not presuppose the same portfolio of risky holdings regardless of personal wealth (e.g., under CARA, a wealthy manager would the same value invested in the market as a regular employee).[16] Further, they may be appropriate given the large variations in personal wealth experienced by a CEO.

This property of the optimal contract can help rationalize a long-standing puzzle regarding the use of option compensation. Many observers have noted that, if the market decreases, the pay for performance sensitivity of executives paid with options also decreases. This is because, when the underlying security decreases in value due to a market shock, its delta (i.e., the derivative of its price with respect to the stock price) decreases as well. Within the conventional agency model, the dependence of pay-for-performance coefficients on uninformative variables may seem surprising. In contrast, Proposition 3.3 shows that a performance sensitivity linked to systematic risk can be desirable because of its ex-ante risk-sharing properties. Indeed, consistently with the DARA class of preferences, the performance-pay of managers paid with options should decrease after a systematic decrease in the market.

The class of hyperbolic absolute risk-aversion (or HARA) preferences includes as a special case quadratic, constant absolute risk-aversion (CARA) and constant relative risk-aversion (CRRA) preferences. Corollary 3.1 offers a simple non-parametric procedure to test for efficient risk-sharing in the HARA class. The model suggests that the regression of the log sensitivity of the compensation to firm-specific or systematic shocks should be additively separable in both shocks.

[16]Note that the CAPM framework used in Jin (2002) and Garvey and Milbourn (2003) requires linear contracts and CARA preferences and thus, unlike in the framework presented here, cannot accommodate a state-contingent pay-for-performance w_y.

3.4. State Prices

The previous Propositions investigate how state prices (and systematic risk) can affect the contract design problem. Conversely, this Section explores the effect of informational frictions on risk premia. To begin with, note that the first-best state prices (as given by Equation (3.3)) correspond to the state prices in a standard representative agent framework. In second-best, however, managers will face idiosyncratic wage shocks and thus aggregation into a representative manager may potentially fail. The next Proposition explores this question in greater details.

Proposition 3.4 *Suppose that $u(.)$ is in the HARA class. Then, in a competitive equilibrium with moral hazard such that a is elicited, state prices must be given by:*

$$q(\alpha) = \frac{h(\alpha)u'(\alpha a)}{\int h(x)u'(xa)dx} \tag{3.6}$$

Corollary 3.2 *In the HARA class, conditional on the elicited effort, the state prices do not depend on the informational friction (i.e., same as in first-best).*

Proposition 3.4 establishes that, for a wide class of preferences, the informational friction does not affect the state prices and risk premia in the economy. Conditional on the chosen effort, the state prices are the same as in an economy with a single representative manager receiving an endowment αa (but not subject to moral hazard).[17] In other words, the informativeness of the information about the manager's actions may have an effect on expected output in the economy (via the effort decision) but, for a given effort, does not directly affect risk premia. This result contrasts with the standard intuition that aggregation should fail under incomplete markets (here, uninsurable labor income). One intuition for this result is that the optimal contract does not distort the manager's exposure to systematic shocks. Therefore, averaging over identical managers, the agency friction does not distort state prices.

Corollary 3.3 *An increase in effort a increases the aggregate risk premium RP if and only if preferences exhibit decreasing relative risk-aversion. In particular, under CRRA preferences, the aggregate risk premium RP does not depend on the informational friction (i.e., even after adjusting effort).*

[17] If effort is assumed to take only two values, there are therefore cases such that first-best and second-best state prices exactly coincide. For the more general case with continuous effort studied here, the second-best may affect state prices only via aggregate effort. Note that while all agents in the economy are ex-ante identical, they are not identical post realization of their firm cash flows y_k. In the general case, aggregation among agents with different endowments will occur with CRRA preferences (Huang and Litzenberger (1988), p. 147-148) - a more stringent condition that the HARA preferences required here.

While Corollary 3.2 is concerned with the pricing of each state-contingent asset, Corollary 3.3 focuses on the aggregate risk premium, i.e. how much return do agents require to hold one unit of the market portfolio. Suppose here that an increase in earnings informativeness leads to an increase in effort.[18] In this case, the effect of this greater effort on risk premia (i.e., the expected return of the market) depends on agents' preferences. When agents are less risk-averse per unit of wealth (decreasing relative risk-aversion), a proportional increase in the economy's cash flows leads to a decrease in risk premia while, vice-versa, greater risk-aversion per unit of wealth leads to an increase in risk premia.

Corollary 3.2 has implications for a recent literature on accounting information and cost of capital. Several authors discuss whether the presence of more information should affect cost of capital. In particular, Easley and O'Hara (2004) show that the aggregate risk premia in the economy incorporate the severity of informational frictions between investors. We revisit here these questions in the context of imperfect information about managerial actions, as well as general preferences. For a given level of effort, we show that more or less informative earnings should not lead to different risk premia in the economy, when conditioning on the economy's aggregate wealth a. However, the paper offers a simple reason why risk premia may be affected by informational frictions, namely via their effect on total output.[19]

Corollary 3.4 *In the HARA class, there exists $K > 0$, a constant that does not depend on α or y, such that:*

$$w_\alpha(\alpha, y) = KH(y)\frac{u''(\alpha a)}{u''(w(\alpha, y))} \tag{3.7}$$

In the HARA class of preferences, the coefficient of pay for systematic risk can be captured by the ratio of the curvature of the utility function evaluated at: (i) the consumption of the representative manager (receiving αa), (ii) the consumption of the manager (receiving $w(\alpha, y)$). Here, an agent who is more successful should be given more exposure to the systematic risk factor. In other terms, a high idiosyncratic performance magnifies the dependence of consumption on the systematic shock.

3.5. An Example

The example of constant relative risk-aversion (CRRA) preferences is developed in more details to illustrate how systematic risk affects the optimal contractual arrangement. Suppose here that $u(x) = x^{1-\gamma}/(1-\gamma)$ where $\gamma > 0$ (if $\gamma = 1$, $u(x)$ is assumed to be logarithmic). By Equation (3.5), the inverse

[18] Since our focus is on systematic risk, we leave aside here the analysis of more primitive conditions under which this statement holds. In a two-effort problem, for example, an extremely low informativeness should lead to a choice of $\underline{a}$, while a sufficiently high informativeness should lead to a choice of $\overline{a}$.

[19] We do not attempt to address here another existing puzzle, namely that informational asymmetries may have an effect on a firm's cost of capital - here, informational asymmetries only affect the aggregate risk premium.

of the marginal utility of the manager will be linear in the likelihood ratio and the ratio of the objective probabilities to the state prices.

$$w(\alpha, y)^{\gamma} = \frac{h(\alpha)}{q(\alpha)}(\lambda + \mu \frac{f_a(y|a)}{f(y|a)}) \tag{3.8}$$

Therefore:

$$w(\alpha, y) = (\frac{h(\alpha)}{q(\alpha)})^{1/\gamma}(\lambda + \mu \frac{f_a(y|a)}{f(y|a)})^{1/\gamma} \tag{3.9}$$

Pre-multiplying both sides of this Equation by $f(y|a)$, integrating with respect to y and using the market clearing conditions yields that:

$$\alpha a = \frac{h(\alpha)}{q(\alpha)}^{1/\gamma}(\int (\lambda + \mu \frac{f_a(y|a)}{f(y|a)})^{1/\gamma} dy) 1/\gamma \tag{3.10}$$

One can then substitute $\frac{h(\alpha)}{q(\alpha)}^{1/\gamma}$ from Equation (3.10) in Equation (3.9).

$$w(\alpha, y) = \alpha a \frac{(\lambda + \mu \frac{f_a(y|a)}{f(y|a)})^{1/\gamma}}{\int f(y'|a)(\lambda + \mu \frac{f_a(y'|a)}{f(y'|a)})^{1/\gamma} dy'} \tag{3.11}$$

The wage can thus be written as a share of the aggregate endowment αa; this share is greater for outcomes that are more informative about effort. In addition, a greater risk-aversion γ reduces the increase in this share for a greater idiosyncratic performance, as is intuitive.

With logarithmic utilities, the wage can be simplified as: $w(\alpha, y) = \alpha a(1 + \frac{\mu}{\lambda} \frac{f_a(y|a)}{f(y|a)})$, that is one unit of the market portfolio as well as a bonus that depends on performance. Then, when the likelihood ratio is zero, the manager receives exactly the aggregate endowment. In particular, if $f(y|a)$ is Normal with mean a and variance one, the likelihood ratio will be linear in y, and thus the optimal contract will take the form of a combination of standard company stock and a unit of the market.

In the logarithmic case, the manager's participation constraint and incentive-compatibility can be further simplified as follows:

$$\int \log(\alpha a) h(\alpha) d\alpha + \int \log(1 + \frac{\mu}{\lambda} \frac{f_a(y|a)}{f(y|a)}) f(y|a) dy - \psi(a) = R \tag{3.12}$$

$$\int f_a(y|a) \log(1 + \frac{\mu}{\lambda} \frac{f_a(y|a)}{f(y|a)}) dy = \psi'(a) \tag{3.13}$$

To state the objective of firm k, it is now important to distinguish the average effort in the economy, a, from the effort chosen by firm k, a_k. Denoting $\phi(y) = (1 + \frac{\mu}{\lambda}\frac{f_a(y|a)}{f(y|a)})$ (a function that only depends on y), the value of firm k can be written:

$$
\begin{aligned}
V(L^k) &= \int\int q(\alpha)(\alpha a_k - \alpha a\phi(y_k))f(y_k|a_k)dy_k d\alpha \\
&= \int q(\alpha)\alpha d\alpha \int (a_k - a\phi(y_k))f(y_k|a_k)dy_k
\end{aligned}
$$

Let $R' = R - \int \log(\alpha)h(a\alpha)d\alpha$ denote a normalized reserve utility. One can then rewrite a reduced contract design problem eliminating the dependence on systematic risk.

$$
(P_2) \qquad \max_{\phi(.),a_k} \int (a_k - a\phi(y_k))f(y_k|a_k)dy_k
$$

s.t.

$$
\int f(y_k|a)u(\phi(y_k))dy_k - \psi(a_k) = R' \tag{3.14}
$$

$$
\int f_a(y_k|a_k)u(\phi(y_k))dy_k = \psi'(a_k) \tag{3.15}
$$

For an aggregate effort a and a (net) cost of labor R', the firm does not need to know the distribution $h(.)$ of the systematic component or the state prices $q(.)$. Thus, with Logarithmic utility functions, the systematic shock has only a simple scaling effect on payments from the contract: the firm will inflate the wage by the total endowment.

4. Manager's Portfolio Choice

4.1. Private Hedging

In the optimal contract as defined here, the manager cannot trade the market outside of the contract. As a result, the model constrains the manager to trade on systematic risk *through* the labor contract. This assumption, while convenient as a solution technique, is not innocuous when interpreting the findings in a practical context. Proponents of filtering out systematic risk would generally argue that it is desirable only if the manager can privately buy and sell the market portfolio outside of the contract (or else, holding the market portfolio would generate diversification benefits). In other words, proving the dependence of consumption on aggregate risk may not be sufficient to make a case against pure RPE. In addition, it must be established that the optimal contract cannot be replicated using a pure relative performance payment but letting the manager freely trade on capital markets (although, for obvious reasons, not on the firm's idiosyncratic risk).

The notion of private trading is now formally defined. Let $\eta(\alpha)$ be the number of Arrow-Debreu assets purchased by the manager. To avoid unraveling of the firm's idiosyncratic risk, manager k may not trade on y_k. In the presence of trading, the participation of an agent incorporates the return from the Arrow-Debreu assets purchased by the manager.

$$\int\int h(\alpha)u(w_k(\alpha, y_k) + \eta(\alpha))f(y_k|a_k)dy_k d\alpha - \psi(a_k) \geq R \tag{4.1}$$

Since trading is private, the incentive-compatibility of the manager incorporates now effort and trading. That is, for a given contract, the manager will be facing the following problem.

$$(\Lambda_k) \qquad \max_{\tilde{\eta}(.),\tilde{a},A} \int\int h(\alpha)u(w(\alpha, y_k) + \tilde{\eta}(\alpha) - A)f(y_k|\tilde{a})dy_k d\alpha - \psi(\tilde{a})$$

$$\text{s.t.} \qquad \int q(\alpha)\eta(\alpha)d\alpha = A$$

The constraint on (Λ_k) is a budget constraint and states that the manager must pay the portfolio of assets at its current state prices. A value-maximizing contract is given by $(L^k, \eta_k(.))$ and maximizes $V(L^k)$ subject to participation and incentive compatibility.

$$(S_k) \qquad \max_{L^k,\eta_k(.)} \quad V(L^k)$$

$$\text{s.t.} \qquad \text{(4.1) and } (a_k, \eta_k) \text{ maximizes } (\Lambda_k)$$

Finally the market clearing must reflect private purchases by the manager, i.e. for all α,

$$\int (w_k(\alpha, y_k) + \eta_k(\alpha))dk = \alpha \int a_k dk \tag{4.2}$$

Definition 4.1 *A (symmetric) competitive equilibrium with trading is* $\Gamma^s = (a, w(.), \eta(.), q(.), R)$ *such that:*

1. $(a, w(.), \eta(.))$ *maximizes* (S_k) *for all* k.

2. *Equation* (4.2) *holds for all* α.

A first-best competitive equilibrium with trading is defined similarly by omitting a_k in the maximization (Λ_k). A relative performance payment is a wage $w(.,.)$ that depends only on y but not α. Note that, because the manager may now trade outside of the contract, a wage that does not depend on α is not necessarily incompatible with market clearing. In short-hand, a competitive equilibrium with trading will now refer to the equilibrium concept in which the manager can privately trade (Definition 4.1), while a competitive equilibrium will refer to the equilibrium concept in which the manager may not trade outside of the contract (Definition 2.1).

4.2. Incentive-Compatibility

In managerial contexts, the investment of a manager is his/her own stock is heavily controlled as the manager would prefer to unravel the idiosyncratic factor on financial markets (e.g., by short-selling company stock). A natural concern is whether the same caveats apply to the dependence of w on α: Should the firm contractually commit the manager not to offset the dependence of the wage on α? The next Proposition establishes that such constraints are unnecessary when firms offer an optimal contract.

Proposition 4.1 *Suppose* $\Gamma = (w(.,.), a, q(.), R)$ *is a competitive equilibrium (i.e., the manager cannot privately trade), then* $\Gamma^s = (w(.,.), \eta(.), a, q(.), R)$ *with* $\eta(\alpha) = 0$ *for all* α *is a competitive equilibrium with trading.*

The intuition at play in Proposition 4.1 is similar to that underlying Proposition 3.6. In an optimal contract where the manager cannot trade, the principal should not distort exposure to systematic risk. As a result, even if the manager were to be given the option to trade state-contingent assets, it would not be optimal to trade.[20]

To further illustrate the result, it is useful to revisit a widespread idea in the relative performance evaluation literature. Describing previous work in the area, Bebchuk and Fried (2004) explain: "compensation for sector or market increases exposes executives to market risk. Given that managers are risk-averse and therefore value risky compensation less, it would be cheaper (...) to give agents indexed options and cash equal to the value of the market component of their options" (p.157).[21] Johnson and Tian (2000) argue that offering indexed options would reduce total compensation cost by two thirds. In

[20] To some extent, this feature is caused by the assumption that the number of firms is large. If the number of firms was finite, the manager could unravel some of the firm's idiosyncratic risk by trading the market portfolio; this would create a motive to avoid private trading (however, the result would carry over provided the manager were constrained to trade only on other firms).

[21] This argument is, for example, present in Abowd and Kaplan (1999) as a recommendation to improve executive compensation. Note that Bebchuk and Fried do not support this explanation; their observation is that CEOs use this argument to demand higher expected wages which, as shown here, is incorrect.

fact, as Proposition 4.1 shows, doing so in the manner suggested would require the *exact* same monetary expense for the firm and, eventually, the manager would unravel the transaction and hold the same consumption.[22] Therefore, such a change to the contract would be irrelevant to the effort choice, consumption choice or total surplus created. In intuitive terms, firms and manager are “equally” risk-averse to market risk and thus reallocating market risk between the firm and the manager would not provide more efficient insurance.

4.3. Relative Performance

As a benchmark to the analysis, Proposition 4.2 analyzes whether relative performance can be optimal in the absence of informational frictions. A relative performance payment is defined as a wage that does not depend on the realization of α.

Proposition 4.2 *Let* $\Gamma^s = (w(.,.), a, q(.), R)$ *be a first-best competitive equilibrium. Then, there exists a first-best competitive equilibrium with private trading* $\Gamma = (\tilde{w}(.,.), \eta(.), a, q(.), R)$ *where* $\tilde{w}(.,.)$ *does not depend on* α *(pure relative performance).*

Proof: Set $w^{**}(\alpha, y) = \int \alpha q^*(\alpha) d\alpha a^*$ (constant). It is then immediately verified that $\eta^{**}(y) = \alpha a^*$ is a solution to Λ^*.□

It turns out that, in first-best, the optimal contract can be rewritten in terms of a compensation that does not depend on α but letting the manager privately trade. To summarize, while the first-best competitive equilibrium does not exclude pay as a function of systematic risk, it suggests that filtering out systematic shocks is also possible and may even be, presumably, more transparent to shareholders.

This question is now discussed when effort is unobservable. To begin with, consider the case of constant absolute risk aversion. By Proposition (3.3), the wage in an equilibrium with no trading can be written additively as $w(\alpha, y) = w_1(\alpha) + w_2(y)$. Then, integrating with respect to y and using the market-clearing constraint yields that: $w_1(\alpha) = \alpha a - \int f(y|a) w_2(y) dy$. The optimal contract with CARA preferences is thus the sum of a unit of the market portfolio and a relative performance payment. Suppose now that, instead of offering $w(.,.)$, the principal offers a wage $\hat{w}(\alpha, y) = w_b^*(y) + w_f^*$, composed of a relative performance payment $w_b^*(y) = w_2(y) - \int f(y|a) w_2(y) dy$ and a fixed salary $w_f^* = a \int q(\alpha) \alpha d\alpha$. Then, assume that the principal lets the manager trade on the Arrow-Debreu assets. Proposition 4.1

[22]For example, when offered the contract with market indexed options, the manager would achieve a utility level that is strictly less than under the previous contract and may then seek employment with another firm.

implies that the manager will use w_f^* to purchase one unit of the market portfolio, thus consuming exactly $w(\alpha, y)$. Collecting these observation yields the next result.

Proposition 4.3 *Suppose $u(.)$ is CARA. Suppose $\Gamma = (w(.,.), a, q(.), R)$ is a competitive equilibrium with moral hazard, then there exists a competitive equilibrium with moral hazard and private trading $\tilde{\Gamma} = (\tilde{w}(.,.), \eta(.), a, q(.), R)$ such that $\tilde{w}(.,.)$ does not depend on α and $w(\alpha, y) = \tilde{w}(\alpha, y) + \eta(\alpha)$.*

In the case of CARA preferences, the trading and contracting decisions can be separated. The manager can be given a contract that only depends on the systematic component and use this expected surplus to trade on capital markets. This will yield a consumption that is the same as that of the optimal contract with commitment not to trade ($w(\alpha, y)$). The result corresponds to relative performance-pay in the conventional sense.

RPE and private trading can be used to replicate the optimal contract only if the optimal contract is additive in the systematic and firm-specific shock. However, if absolute risk-aversion is not constant, the consumption of the manager cannot be written as the sum of a function of the aggregate shock and a function of the idiosyncratic shock (since then $w_{\alpha,y} \neq 0$). Therefore, there will be no hope of replicating a consumption profile $w(\alpha, y)$ with a pure relative performance payment and private trading on capital markets (since, typically, $w(\alpha, y) \neq \tilde{w}(y) + \eta(\alpha)$). As a result, in a competitive equilibrium with private trading, firms would always offer labor contracts such that the wage depends on systematic risk factors. This observation is summarized next.

Proposition 4.4 *Suppose that $u(.)$ has non-constant absolute risk-aversion (on any interval). Then, in any competitive equilibrium with trading, $w(\alpha, y)$ must depend on α.*

Unlike in first-best, the separation between the manager's wage and trading does not hold with non-constant absolute risk-aversion. One reason for this is that the proper amount of exposure to systematic risk should depend on the realization of the idiosyncratic shock. For example, with CRRA preferences, a manager who is more successful than his/her peers should bear more systematic risk. The problem with pure relative performance is that the manager cannot choose an exposure to systematic risk that is conditional on the realization of the idiosyncratic shock.

To make relative performance optimal, more freedom is required on the trading choices of the manager. For the sake of the argument, it may be possible to instead allow managers to trade on capital markets and condition the *number* of the Arrow-Debreu securities purchased on the realization of the idiosyncratic shock y_k. In this case, Proposition 4.1 shows that relative performance would then yield the same consumption (and surplus) as the optimal contract without trading. From a more practical perspective, however, such an arrangement would require an external monitor controlling whether the manager is only trading on Arrow-Debreu assets and not unraveling the firm's idiosyncratic risk. Since the final payment from trading would depend on y_k, such monitoring may be difficult to implement.

5. Extensions

5.1. General Production Technology and Heterogeneity

The model is extended to incorporate: non-identical managers, a general production technologies in α and y. Assume now that manager $k \in [0,1]$ can produce output with a technology $\phi_k(\alpha, y_k)$ (instead of αy_k), increasing in the systematic shock α and the idiosyncratic shock y_k. The distribution of y_k is denoted $f^k(y|a_k)$ and may depend on k as well as effort. Each manager has a utility $u_k(c_k) - \psi_k(a_k)$, such that risk-aversion and cost of effort may depend on k as well.[23] It is now convenient to formulate the problem of manager k seeking an employment contract from firm k.

$$(P_k) \quad \max_{w(.,.),a} \quad \int\int h(\alpha) f^k(y|a) u_k(w(\alpha, y)) d\alpha dy - \psi_k(a)$$

s.t.

$$\int\int q(\alpha) f^k(y|a)(\phi_k(\alpha, y) - w(\alpha, y)) dy d\alpha \geq R^f \qquad (\lambda) \qquad (5.1)$$

$$\int\int h(\alpha) f^k_a(y|a) u_k(w(\alpha, y)) dy d\alpha = \psi'_k(a) \qquad (\mu) \qquad (5.2)$$

Equation (5.1) is the participation of the firm, where R^f denotes the expected profit expected by a firm on an employment contract. Equation (5.2) is the manager's incentive-compatibility.

Definition 5.1 *A competitive equilibrium is* $\Gamma = ((a_k, w_k(.,.))_{k \in [0,1]}, q(.), R^f)$ *such that:*

1. *For all* k, $(a_k, w_k(.,.))$ *maximizes* (P_k).
2. *For all* α, $\int \phi_k(\alpha, y) f^k(y|a_k) dy dk = \int w_k(\alpha, y) f^k(y|a_k) dy dk$.

Lemma 5.1 verifies that firms make zero profit.

Lemma 5.1 *In a competitive equilibrium,* $R^f = 0$.

[23] The results are unchanged if firms are non-identical and owned by agents in the economy. With non-identical agents and firms, there may be employer-employee matching and bargaining, which would require a full-fledged model of job matching.

The first-order optimality condition for (P_k) is:

$$u_k'(w(\alpha,y))\underbrace{\{1+\mu_k\frac{f_a^k(y|a)}{f^k(y|a)}\}}_{G_k(y)}=\underbrace{\lambda_k\frac{q(\alpha)}{h(\alpha)}}_{T_k(\alpha)} \tag{5.3}$$

Lemma 5.2 *In a competitive equilibrium, λ_k and μ_k are strictly positive.*

Lemma 5.3 *$G_k(y)$ is is strictly positive and strictly increasing in y and $T_k(\alpha)$ is strictly decreasing in α.*

Proposition 5.1 *In a competitive equilibrium, $w_k(\alpha, y)$ is increasing in α and y. Further, if $u_k(.)$ has decreasing absolute risk-aversion, w_y is increasing in α.*

Proposition 5.1 confirms the intuition of Proposition 3.3 in the more general case of a general production technology and, possibly, heterogeneity across agents. In the DARA class of preferences, the consumption cannot be additively separated in α and y and thus a relative performance payment cannot replicate $w_{k,\alpha,y}$ even if the manager can trade the market portfolio.

5.2. Multiple Periods and Serial Correlation

The basic model is now extended to dynamic moral hazard and, possibly, serial correlation across periods. Time is discrete with $t = 0, \ldots, +\infty$. At each period, firm k produces $\alpha^t y_k^t$ where α^t is current productivity shock and y_k^t is current output. To model serial correlation, let $\mathbf{y_k^t} = (\mathbf{y_k^0}, \ldots, \mathbf{y_k^{t-1}})$ and $\alpha^{\mathbf{t}} = (\alpha^{\mathbf{0}}, \ldots, \alpha^{\mathbf{t-1}})$ denote previous histories. The distribution of y_k^t is denoted $f(.|\alpha^{\mathbf{t}}, \mathbf{y^t}, \mathbf{a_{kt}})$ where a_{kt} is period t effort for firm k. The distribution of α^t is denoted $h(.|\alpha^{\mathbf{t}})$, and may possibly depend on all past realizations. A wage at date t is denoted $w(\mathbf{y^t}, \alpha^{\mathbf{t}}, \alpha, \mathbf{y})$, and may depend on the firm's past history $\mathbf{y^t}$, past aggregate shocks $\alpha^{\mathbf{t}}$, the current aggregate shock α and the current firm shock y. The price of receiving one unit of consumption at date t conditional on $(\alpha^{\mathbf{t}}, \alpha)$ is denoted $q(\alpha^{\mathbf{t}}, \alpha)$. In short, these functions are denoted $w_{kt}(\alpha, y)$, $q_t(\alpha)$, $f^{kt}(y|a)$ and $h_t(\alpha)$. The dependence on firm k is omitted when there is no ambiguity.[24] Finally, let $\mathcal{L}_\alpha^t$ and $\mathcal{L}_y^t$ denote the probability measure associated to outcomes $\alpha^{\mathbf{t}}$ and $\mathbf{y^t}$, respectively.

The firm and the agent sign a contract with commitment at date 0, which may dynamically insure the agent against systematic and idiosyncratic shocks.

[24]The previous assumptions are maintained. Note that, as a result, $f_t(y|a)$ must have mean a which is with loss of generality. However, this restriction is mostly for convenience and can be easily relaxed.

$$\max_{w_t,a_t} \int\int \sum_{t=0}^{\infty} q_t(\alpha)(\alpha y - w_t(\alpha,y)) f^t(y|a_t) dy d\alpha d\updownarrow^t_\alpha d\updownarrow^t_y$$

$$\text{s.t.} \quad \forall \alpha^{\mathbf{t}}, \mathbf{y}^{\mathbf{t}}$$

$$\int\int \sum_{t=0}^{\infty} h_t(\alpha) u(w_t(\alpha,y)) f^t(y|a_t) dy d\alpha d\updownarrow^t_\alpha d\updownarrow^t_y - \psi(a_t) \geq R \tag{5.4}$$

(5.5)

$$\int\int \sum_{t=0}^{\infty} h_t(\alpha) u(w_t(\alpha,y)) f^t_a(y|a_t) dy d\alpha d\updownarrow^t_\alpha d\updownarrow^t_y - \psi'(a_t) = 0 \tag{5.6}$$

In the above program, both the wage w_t and the elicited effort can depend on $\alpha^{\mathbf{t}}$ and $\mathbf{y}^{\mathbf{t}}$. The definition of a competitive equilibrium is similar to the definitions given before and not repeated here; it requires contracts to solve the above program and market clearing for any history and any date. The first-order optimality condition for this dynamic contract design is given next.

$$\frac{q_t(\alpha)}{\beta^t h_{kt}(\alpha)} = u'(w_{kt}(\alpha,y))(\lambda_{kt} + \mu_{kt} \frac{f^{kt}_a(y|a_t)}{f^{kt}(y|a_t)}) \tag{5.7}$$

This optimality condition has the same form as Equation (3.5), except that the dependence on past history is explicitly modeled. It can then be easily verified with the same steps as in Proposition 5.1 that the wage will be increasing will be increasing in both systematic and idiosyncratic risk. In addition, with DARA preferences, the pay-for-performance coefficient will be greater when α is high.

5.3. Empirical Implications

Our model provides a number of guidelines to empirically test RPE, and match it to the predictions of agency theory. These are summarized next:

(i) Testing for pure RPE as recommended by the conventional model of incentives requires to filter out any economy-wide systematic risk. In other words, such tests cannot be done with the market index. When testing for RPE using industry-wide variables, the systematic economy-wide component in these variables should be filtered out first.

(ii) In the regression $w(\alpha, y) = k_0 + k_1\alpha + k_2 y + k_3 \alpha * y$ (*), the interaction term k_3 should be positive under decreasing absolute risk-aversion (Proposition 3.3).

(iii) Let $c(\alpha, y)$ denote the actual consumption of the manager, and consider a linear decomposition: $c(\alpha, y) = k'_0 + k'_1\alpha + k'_2 y + k'_3 \alpha * y$ (**). Then, the parameters k'_0 and k'_1 are not separately identified from k_0 and k_1 (derived from the wage regression (*)) given that the manager may rebalance a portfolio of assets (Proposition 4.1). More precisely, if k_0' and k_1' are the required coefficients in (**) to ensure an optimal incentive scheme, then this can be achieved with any wage of the form (*) satisfying $k_0 + k_1 \int \alpha q(\alpha) d\alpha = k_0' + k_1' \int \alpha q(\alpha) d\alpha$.

(iv) For certain classes of preferences, one can test for optimal risk-sharing with respect to systematic risk. In the class of HARA preferences, the regression of $\log w_\alpha$ (log market sensitivity) and $\log w_y$ (log performance-pay) on functions of α and y should feature no interaction terms between α and y (Corollary 3.1).

On the other hand, the model suggests that empirically testing RPE may be a more difficult than generally believed. First, testing RPE requires an empirical model of systematic risk factors, a difficult task given the low explaining power of commonly used risk factors, as well as the presence of unexplained asset pricing anomalies. Second, performance variables that do not have large systematic components tend to be informative on managerial actions - for example, the existing literature shows that there may be benefits to contract on the performance of peers if the number of firms in the industry is small.

6. Concluding Remarks

In this paper, we revisit the informativeness principle of agency theory. A common version of this principle states that a contract between an agent and well-diversified shareholders should condition pay only on variables that are informative on the agent's actions. We show that this principle does not apply to market-wide economic shocks or variables that incorporate some of this risk. Intuitively, although the realization of a non-diversifiable risk can be uninformative on effort, it can be informative on the ex-post wealth of agents in the economy (after the shock occurred) and thus the agent's state-contingent risk tolerance. For common preferences, the pay-for-performance coefficient increase after a favorable shock, a well-known property of standard options.

Since we focus here on a traditional agency in which the manager chooses effort, several questions are left unanswered within the current model and may offer interesting directions for future research. In practice, a manager may have superior information about the firm's exposure to systematic risk, or may have some control over it. Empirically, for example, most firms engage in hedging systematic risk, an operation that is value-irrelevant in an environment without capital market frictions. Another set of questions concerns the effect of limited liability constraints on the contract design problem - in particular, whether such constraints may increase the downward rigidity of the wage to shocks of the business cycle, and may induce a need to monitor the manager's personal portfolio choices.

Technical Appendix

First-Order Approach: To guarantee that the first-order approach is valid, it must be established that, when using the contract given by Equation (3.5), there exists a unique solution for a in the problem of the agent.

Writing the expected utility of the agent $U(a)$,

$$\begin{aligned} U(a) &= \int h(\alpha) \int u(w(\alpha, y)) f(y|a) dy d\alpha - \psi(a) \\ &= \int h(\alpha)([u(w(\alpha, y))F(y|a)]_{\underline{y}}^{\overline{y}} - \int u'(w(\alpha, y))F(y|a) w_y(\alpha, y) dy) d\alpha - \psi(a) \end{aligned} \quad \text{(A-1)}$$

By Proposition 3.2, $w_y(\alpha, y) > 0$. Then, it is readily verified that $U(a)$ is strictly concave whenever $F_{aa}(y|a) > 0$. This is the convexity of the distribution function condition (CDFC) given in Rogerson (1985).□

Proof of Proposition 3.1: Note first that constraint (2.1) is binding and denote λ^* its associated Lagrange multiplier. The first-order condition in $w(\alpha, y)$ can be written:

$$\lambda^* h(\alpha) u'(w(\alpha, y)) = q(\alpha) \quad \text{(A-2)}$$

Thus, w depends only on α.

The first-order condition in a yields that:

$$\int q(\alpha) f_a(y|a)(\alpha y - w(y, \alpha)) dy d\alpha - \lambda^* \psi'(a) + \lambda \int \int h(\alpha) u(w(\alpha, y)) f_a(y|a) dy d\alpha \quad \text{(A-3)}$$

Because the w does not depend on y, the above expression can be simplified as follows:

$$\int \int q(\alpha) \alpha d\alpha = \lambda^* \psi'(a) \quad \text{(A-4)}$$

Together, Equations (A-2), (A-4) and (2.1) characterize a solution to the contracting problem. Note first that $\lambda^* > 0$ or else all Arrow-Debreu prices would be equal to zero. It follows that the solution to Equation (A-4) is unique.

Consider next the restrictions imposed by the market clearing condition. From Equation (2.5), $w(\alpha, y) = \alpha a$ for all α, y. This wage must also be a solution to Equation (A-2), i.e.:

$$\lambda^* h(\alpha) u'(\alpha a) = q(\alpha) \quad \text{(A-5)}$$

Integrating both sides of this Equation with respect to α yields:

$$\lambda^* = \frac{1}{\int h(\alpha) u'(\alpha a) d\alpha} \quad \text{(A-6)}$$

Plugging this multiplier into Equation (A-5),

$$q(\alpha) = \frac{h(\alpha)u'(\alpha a)}{\int h(x)u'(xa)dx} \tag{A-7}$$

Rewriting Equation (A-4) with the state prices:

$$\int h(\alpha)u'(\alpha a)\alpha dy d\alpha = \psi'(a) \tag{A-8}$$

To prove efficiency, consider the following planning problem.

$$(P^{**}) \qquad \max_{w(.,.),a} \int\int h(\alpha)f(y|a)u(\phi(\alpha,y))dyd\alpha - \psi(a)$$

The solution to this planning problem corresponds to the choice of effort given by Equation (A-8).□

Proof of Lemma 3.1: Suppose $w(\alpha, y)$ does not depend on y. Then, by Equation (2.2), all firms choose to elicit $a = \underline{a}$ and therefore, in the competitive equilibrium, pay is equal to $w(\alpha, y) = \alpha\underline{a}$. For $v \geq 0$, consider next the following alternative wage schedule:

$$u(\hat{w}(\alpha, y)) = \frac{\psi'(v)}{\int \alpha h(\alpha)d\alpha}(\alpha y - \alpha v) + \alpha\underline{a} + \psi(v) \tag{A-9}$$

Writing the incentive-compatibility condition for this wage:

$$\psi'(a) = \int\int h(\alpha)f_a(y|a)(\frac{\psi'(v)}{\int \alpha' h(\alpha')d\alpha'}(\alpha y - \alpha v) + \alpha\underline{a} + \psi(v))dyd\alpha \tag{A-10}$$

$$= \psi'(v) \tag{A-11}$$

So that this wage induces an effort equal to v. Next, note that:

$$\int\int h(\alpha)u(\hat{w}(\alpha, y))f(y|a)dyd\alpha - \psi(a) = \int \alpha\underline{a}h(\alpha)d\alpha = R \tag{A-12}$$

So that conditional on choosing effort v, the agent achieves the reserve R. It follows from Equations (A-10) and (A-12) that the new contract verifies the incentive-compatibility condition and the participation constraint.

It must now be established that this new contract makes the principal better-off. To simplify notations, let $T[y] \equiv u^{-1}(\alpha\underline{a} + \psi(v) + \frac{\psi'(v)}{\int \alpha h(\alpha)d\alpha}(\alpha y - \alpha v))$. The firm optimizes its profit $M(v)$ over $v \geq 0$:

$$M(v) = v\int \alpha q(\alpha)d\alpha - \int\int q(\alpha)f(y|v)T[y]dyd\alpha$$

The first-order condition in v yields:

$$M'(v) = \int q(\alpha)\alpha d\alpha - \int\int q(\alpha)f_v(y|v)T[y]dyd\alpha - \int\int q(\alpha)f(y|v)\frac{\frac{\psi''(v)(\alpha y - \alpha v) - \alpha\psi'(v)}{\int \alpha h(\alpha)d\alpha} + \psi'(v)}{u'(T[y])}$$

Evaluating this expression at $v = 0$, $M'(0) = \int q(\alpha)\alpha d\alpha > 0$.□

Proof of Proposition 3.2: Differentiating the market clearing condition with respect to α:
$w_\alpha(\alpha, y)f_1(y)dy = A$. Thus, there exists values of y, say $\hat{y}$, such that $w_\alpha(\alpha, \hat{y}) > 0$. Differentiating both sides of Equation

(3.5) with respect to α:

$$w_\alpha(\alpha, y) = H(y)\frac{G'(\alpha)}{u''(w(\alpha, y))} \tag{A-13}$$

Therefore, for all y, $Sign(w_\alpha(\alpha, y)) = Sign(-G'(\alpha))$. This is true in particular at $y = \hat{y}$ and thus $q(\alpha)/h(\alpha)$ is strictly decreasing.□

Proof of Proposition 3.3: Differentiating Equation (3.5),

$$G(\alpha)H'(y) = u''(w(\alpha, y))w_y(\alpha, y) \tag{A-14}$$

$$G'(\alpha)H'(y) = u'''(w(\alpha, y))w_\alpha(\alpha, y)w_y(\alpha, y) + u''(w(\alpha, y))w_{\alpha,y}(\alpha, y) \tag{A-15}$$

Using Equations (A-13) and (A-14) to substitute w_y and w_α in Equation (A-15) yields that:

$$w_{\alpha,y}(\alpha, y) = \frac{G'(\alpha)H'(y)}{-u''(w(\alpha, y))}(\frac{u'(w(\alpha, y))u'''(w(\alpha, y))}{(u''(w(\alpha, y)))^2} - 1) \tag{A-16}$$

Note that the term on the right-hand side is the derivative of the inverse of the Arrow-Pratt absolute risk-aversion.□

Proof of Corollary 3.1: Equation (A-16) in the HARA case with $u(x) = \frac{\gamma}{1-\gamma}(b_1 + b_2 x)^{1-\gamma}$ can be written:

$$w_{\alpha,y}(\alpha, y) = \frac{G'(\alpha)H'(y)}{-u''(w(\alpha, y))}/\gamma \tag{A-17}$$

From Equation (A-13), this implies that:

$$w_{\alpha,y}(\alpha, y) = w_\alpha(\alpha, y)\frac{-H'(y)}{H(y)}/\gamma \tag{A-18}$$

Rearranging terms:

$$\frac{\partial \log w_\alpha(\alpha, y)}{\partial y} = \frac{\partial \log H(y)}{\partial y}/\gamma \tag{A-19}$$

Therefore:

$$\log w_\alpha(\alpha, y) = \log H(y) + \kappa_1(\alpha) \tag{A-20}$$

One can also use Equation (A-14) to write Equation (A-17) as follows:

$$w_{\alpha,y}(\alpha, y) = w_\alpha(\alpha, y)\frac{-G'(\alpha)}{G(\alpha)}/\gamma \tag{A-21}$$

Thus leading to the following expression:

$$\log w_y(\alpha, y) = \log(G(\alpha))/\gamma + \kappa_4(y) \tag{A-22}$$

Equations (A-20) and (A-22) imply that $\log w_\alpha$ and $\log w_y$ can be decomposed additively as the sum of a function of α and a function of y.□

Proof of Proposition 3.4: Let $u(x) = \frac{\gamma}{1-\gamma}(b_1 + b_2 x)^{1-\gamma}$. By Equation (3.5),

$$b_2\gamma(b_1 + b_2 w(\alpha, y))^{-\gamma} = \frac{q(\alpha)}{h(\alpha)}\frac{1}{\lambda + \mu\frac{f_a(y|a)}{f(y|a)}} \tag{A-23}$$

Therefore:

$$(b_2\gamma)^{-1/\gamma}(b_1+b_2w(\alpha,y)) = (\frac{q(\alpha)}{h(\alpha)})^{-1/\gamma}(\frac{1}{\lambda+\mu\frac{f_a(y|a)}{f(y|a)}}))^{-1/\gamma} \tag{A-24}$$

Pre-multiplying both sides of this Equation by $f(y|a)$, integrating with respect to y and using the market clearing conditions yields that:

$$(b_2\gamma)^{-1/\gamma}(b_1+b_2\alpha a) = (\frac{q(\alpha)}{h(\alpha)})^{-1/\gamma}\underbrace{\int f(y|a)(\frac{1}{\lambda+\mu\frac{f_a(y|a)}{f(y|a)}})^{-1/\gamma}dy}_{K} \tag{A-25}$$

Solving for q,

$$q(\alpha) = h(\alpha)(\frac{(b_1+b_2\alpha a)(b_2\gamma)^{-1/\gamma}}{K})^{-\gamma} \tag{A-26}$$

Integrating both sides with respect to α and solving for K,

$$K = (\int h(\alpha)(b_1+b_2\alpha a)^{-\gamma}(b_2\gamma)d\alpha)^{-1/\gamma} \tag{A-27}$$

Reinjecting K in Equation (A-26), implies Equation (3.3) in the case of a HARA utility function.□

Proof of Corollary 3.3: The risk premium is defined as:

$$\begin{aligned} RP &= \frac{\int \alpha a h(\alpha)d\alpha}{\int q(\alpha)\alpha d\alpha} && \text{(A-28)} \\ &= \int h(\alpha)\alpha d\alpha \underbrace{\frac{\int h(\alpha)u'(\alpha a)d\alpha}{\int h(\alpha)\alpha u'(\alpha a)d\alpha}}_{B(a)} && \text{(A-29)} \end{aligned}$$

Differentiating $B(a)$ with respect to a,

$$\begin{aligned} B(a) &= \frac{\int h(\alpha)\alpha u''(\alpha a)d\alpha \int \alpha u'(\alpha a)h(\alpha)d\alpha - \int h(\alpha)u'(\alpha a)d\alpha \int \alpha^2 h(\alpha)u''(\alpha a)d\alpha}{(\int h(\alpha)\alpha u'(\alpha a)d\alpha)^2} && \text{(A-30)} \\ &= \frac{\int h(\alpha)u'(\alpha a)d\alpha \int \alpha h(\alpha)\theta(\alpha a)u'(\alpha a)d\alpha - \int h(\alpha)u'(\alpha a)\theta(\alpha a)d\alpha \int \alpha h(\alpha)u'(\alpha a)d\alpha}{a(\int h(\alpha)\alpha u'(\alpha a)d\alpha)^2} && \text{(A-31)} \end{aligned}$$

where in the above Equation $\theta(x) = -xu''(x)/u'(x)$ is the relative risk-aversion coefficient. It follows from the Rearrangement theorem that the above expression is positive when $\theta(.)$ is increasing, and negative otherwise. To conclude, note that the above expression is zero in the CRRA case. In this case, given that RP does not depend on a directly, it does not depend on the informativeness of the distribution $f(.|a)$.□

Proof of Corollary 3.4: In the HARA class,

$$q'(\alpha) = \frac{h'(\alpha)u'(\alpha a) + ah(\alpha)u''(\alpha a)}{\int h(x)u'(xa)dx} \tag{A-32}$$

Then:

$$\begin{aligned} G'(\alpha) &= \frac{q'(\alpha)h(\alpha) - h'(\alpha)q(\alpha)}{h(\alpha)^2} \\ &= \frac{h(\alpha)h'(\alpha)u'(\alpha a) + ah(\alpha)^2u''(\alpha a) - h'(\alpha)h(\alpha)u'(\alpha a)}{h(\alpha^2)\int h(x)u'(xa)dx} \\ &= \frac{au''(\alpha a)}{\int h(x)u'(xa)dx} \end{aligned}$$

Reinjecting $G'(\alpha)$ in Equation (A-13),

$$w_\alpha(\alpha, y) = KH(y)\frac{u''(\alpha a)}{u''(w(\alpha, y))} \tag{A-33}$$

where $K \equiv a/(\int h(x)u'(xa)dx)$.□

Proof of Proposition 4.1: Suppose the wage is given by Equation (3.5),

$$\max_{\eta(.),a,A} \int h(\alpha) \int u(w(\alpha, y) + \eta(\alpha) - A)f(y|a)dyd\alpha - \psi(a)$$

s.t. $\int q(\alpha)\eta(\alpha)d\alpha = A(\rho)$

Holding a fixed, consider first the trading choice $\eta(.)$. Note that the problem is concave in each $\eta(.)$ and thus it has a unique solution. The first-order condition in $\eta(.)$ implies:

$$\int h(\alpha)u'(w(\alpha, y) + \eta(\alpha))f(y|a)dy = \rho q(\alpha) \tag{A-34}$$

At $\eta(\alpha) = 0$, this condition can be written:

$$\begin{aligned} \rho q(\alpha) &= \int h(\alpha)u'(w(\alpha, y))f(y|a)dy \\ &= q(\alpha)\int \frac{f(y|a)}{\lambda + \mu\frac{f_a(y|a)}{f(y|a)}}dy \end{aligned} \tag{A-35}$$

And thus the solution is obtained with $\rho = \int \frac{f(y|a)}{\lambda+\mu\frac{f_a(y|a)}{f(y|a)}}dy$. Thus $\eta(\alpha) = 0$ is an optimum to the problem regardless of a.□

Proof of Lemma 5.1: The following holds:

$$0 = \int\int (w_k(\alpha, y) - \phi_k(\alpha, y))f^k(y|a_k)dydk \tag{A-36}$$

$$= \int q(\alpha)\int\int (w_k(\alpha, y) - \phi_k(\alpha, y))f^k(y|a_k)dydkd\alpha \tag{A-37}$$

$$= \int\int\int q(\alpha)(w_k(\alpha, y) - \phi_k(\alpha, y))f^k(y|a_k)dyd\alpha dk \tag{A-38}$$

$$= \int Rdk = R \tag{A-39}$$

The first Equation is true for all α by market clearing. The second Equation follows by pre-multiplying with respect to $q(\alpha)$ and integrating with respect to α. The third Equation is true by Fubini's theorem. The fourth Equation follows from Equation (5.1).□

Proof of Lemma 5.2: Since the argument given in Lemma 3.1 is unchanged, $a > \underline{a}$ is still elicited. Suppose that $\lambda_k = 0$. Then, $G_k(y) = 0$ for all y, a contradiction. Suppose that $\mu_k \le 0$, then $w_k(\alpha, y)$ is decreasing in y. Define y_0 such that $f_a^k(y_0|a) = 0$.

$$\int\{\int_{-\infty}^{y_0} f_a^k(y|a)u_k(w_k(\alpha, y))dy + \int_{y_0}^{+\infty} f_a^k(y|a)u_k(w_k(\alpha, y))dy\}d\alpha \tag{A-40}$$

$$\le \int h(\alpha)\{\int_{-\infty}^{y_0} f_a^k(y|a)u_k(w_k(\alpha, y_0))dy + \int_{y_0}^{+\infty} f_a^k(y|a)u_k(w_k(\alpha, y_0))dy\}d\alpha = 0 \tag{A-41}$$

This contradicts $a > \underline{a}$ and Equation (5.2).□

Proof of Lemma 5.3: The properties of $G_k(y)$ follow immediately from Lemma 5.2. Suppose next that $T_k(\alpha)$ is not strictly decreasing in α for some k. Then, there exists $\alpha' < \alpha$ such that $T_k(\alpha') \leq T_k(\alpha)$.

For all k',

$$\begin{aligned} T_{k'}(\alpha') &= \lambda_{k'} \frac{q(\alpha)}{h(\alpha)} && \text{(A-42)} \\ &= \frac{\lambda_{k'}}{\lambda_k} T_k(\alpha') && \text{(A-43)} \\ &\leq \frac{\lambda_{k'}}{\lambda_k} T_k(\alpha) && \text{(A-44)} \\ &\leq T_{k'}(\alpha) && \text{(A-45)} \end{aligned}$$

Therefore $T_k(\alpha') \leq T_k(\alpha)$ for all k.

Next, $T_k(\alpha') \leq T_k(\alpha)$ implies that for all y, $u'_k(w_k(\alpha', y))G_k(y) \leq u'_k(w_k(\alpha, y))G_k(y)$ and therefore by Lemma 5.2, $w_k(\alpha', y) \geq w_k(\alpha, y)$ for all y.

Collecting these two observations, $w_k(\alpha', y) \geq w_k(\alpha, y)$ for all y and k. But then:

$$\int\int f_k(y|a_k) w_k(\alpha', y) dy dk \geq \int\int f_k(y|a_k) w_k(\alpha, y) dy dk \qquad \text{(A-46)}$$

Therefore:

$$\int\int f_k(y|a_k)(\phi_k(\alpha', y) - \phi_k(\alpha, y)) dy dk \geq 0 \qquad \text{(A-47)}$$

But $(\phi_k(\alpha', y) - \phi_k(\alpha, y))$ is strictly negative, a contradiction.□

Proof of Proposition 5.1: Define $M_k(y) = 1/G_k(y)$. The following holds:

$$\begin{aligned} T'_k(\alpha)M_k(y) &= w_{k\alpha}(\alpha, y)u''_k(w_k(\alpha, y)) && \text{(A-48)} \\ T_k(\alpha)M'_k(y) &= w_{ky}(\alpha, y)u''_k(w_k(\alpha, y)) && \text{(A-49)} \\ T'_k(\alpha)M'_k(y) &= w_{k\alpha y}(\alpha, y)u''_k(w_k(\alpha, y)) + T'_k(\alpha)T_k(\alpha)M'_k(y)M_k(y)\frac{u'''_k(w_k(\alpha, y))}{(u''_k(w_k(\alpha, y)))^2} && \text{(A-50)} \end{aligned}$$

Finally note that:

$$\begin{aligned} \frac{w_{k,\alpha,y}(\alpha, y)u''_k(w_k(\alpha, y))}{T'_k(\alpha)M'_k(y)} &= 1 - M_k(y)T_k(\alpha)\frac{u'''_k(w_k(\alpha, y))}{(u''_k(w_k(\alpha, y)))^2} && \text{(A-51)} \\ &= 1 - u'_k(w_k(\alpha, y))\frac{u'''_k(w_k(\alpha, y))}{(u''_k(w_k(\alpha, y)))^2} && \text{(A-52)} \\ &= \frac{\partial}{\partial y}(\frac{u'_k}{u''_k})(w_k(\alpha, y)) && \text{(A-53)} \end{aligned}$$

In the DARA case, $w_{k,\alpha,y} > 0$.□

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