Tuning of PID Controller of Inverted Pendulum Using Genetic Algorithm

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Abstract
This paper presents different types of mathematical modelling of Inverted Pendulum and also a Proportional-Intregal-Derative (PID) controller is designed for its stabilization. After desiging of PID controller some reference stable system has been selected and then different types of error has been optimized (minimized) by using Genetic algorithms. The proposed system extends classical inverted pendulum by incorporating two moving masses. Also a tuning mechanism is done by genetic algorithm for optimizing different gains of controller parameter. Alsohere different performance indeces are calculated in MATLAB environment. This paper addresses to demonstrate the capability of genetic algorithm's to solve complex and constraint optimization problems via utilizing GA's as ageneral purpose optimizing tool to solve different control system design problems.

Keyword
Inverted Pendulum,Mathematical Modelling,Swing Up Control, PID Controller, Tuning, Genetic Algorithm, Performance Indeces, Error Minimizatio.

I. Introduction

The inverted pendulum is a classical problem in dynamics and control theory [1] and is widely used as a benchmarkfor testing control algorithms (PID controllers, Linear Quadratic Regulator (LQR), neural networks, fuzzy logic control, genetic algorithms, etc).The inverted pendulum is unstable in the sense that it may fall over any time in any direction unless a suitable control force is applied. The control objective of the inverted pendulum is to swing up [4] the pendulum hinged on the moving cart by a linear motor from stable position (vertically down state) to the zero state(vertically upward state) and to keep the pendulum in vertically upward statein spite of the disturbance [5].

In the field of engineering and technology the importance of benchmark needs no explanation. They make it easy to check whether a particular algorithm [6] is giving the requisite results. A lot of work has been carried out on the inverted pendulum in terms of its stabilization. Many attempts have been made to control it using classical control [3]. In this paper ,it is proposed that the controller be tuned using Genetic Algorithm (GA) technique. Genetic Algorithm [2] have been shown to be capable of locating high performance areas in complex domains without experiencing the difficulties, also associated with high dimensionality or false optima as many occur with some other optimization method.

II. Mechanical Construction

The system comprises of a horizontal plate that is connected to two wheels through a connecting rod. The wheels are independent of each other and are placed in the centre of the rail. Thus the platform can move on a horizontal surface and is able to rotate about the axis of wheels. There are two masses on top of the system that can slide along the horizontal rail, the masses being on both sides of the rail. The system is shown in fig. 1.

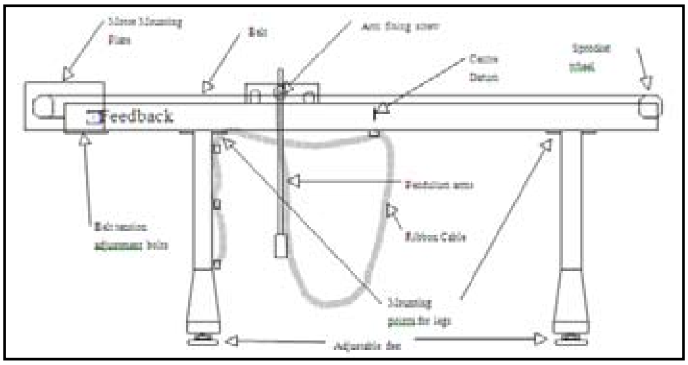

Fig. 1: Pendulum System

III. Mathematical Model of the Plant

Defining the angle of the rod from the vertical (reference) line as θ and displacement of the cart as x. Also assume the force applied to the system is F, centre of gravity of the pendulum rod is at its geometric centre and l be the half length of the pendulum rod. The physical model of the system is shown in fig (2).

The Lagrangian of the entire system is given as,

$$L=\frac{1}{2}(m\dot{x}^2+2ml\dot{x}\dot{\theta}\cos\theta+ml^2\dot{\theta}^2+M\dot{x}^2)+\frac{1}{2}I\dot{\theta}^2]-mgl\cos\theta$$

The Euler-Lagrange's equation for the system is given as

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}}\right)-\frac{\delta L}{\delta x}+b\dot{x}=F$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\theta}}\right)-\frac{\delta L}{\delta \theta}+d\dot{\theta}=0$$

Using these two above equations we get the dynamics of the entire system

$$(I+ml^2)\ddot{\theta}+ml\cos\theta\,\ddot{x}-mgl\sin\theta+d\dot{\theta}=0 \quad (1)$$

$$(M+m)\ddot{x}+ml\cos\theta\,\ddot{\theta}-ml\sin\theta\,\dot{\theta}^2+b\dot{x}=F \quad (2)$$

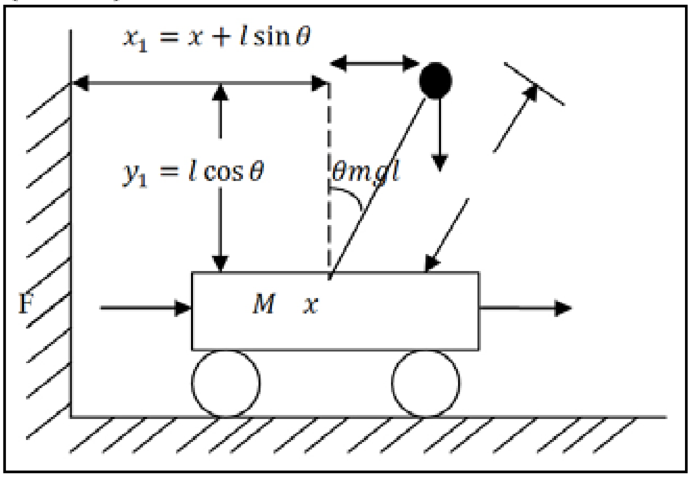

Fig. 2: The Inverted Pendulum System

The above equation (1&2) shows the dynamics of the entire system.

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In order to derive the linear differential equation modelling, we need to linearaize the non linear differtional equationobtained as above so far. For small angle deviation around the upright equilibrium (fig. 2) point assume

$$\sin\theta = \theta, \cos\theta = 1, \qquad \dot{\theta}^2 = 0$$

Using above relation we can write as,

$$r\ddot{\theta}+q\ddot{x}-k\theta+d\dot{\theta}=0 \quad (3)$$

$$p\ddot{x} + q\ddot{\theta} + b\dot{x} = F \quad (4)$$

Where, (M+ m)= p, mgl=k, ml=q, I+ml²=r

Eq (3 & 4) is the linear differential eqution modelling of the entire system. In order to find the transfer function of the system , Laplace transform of Eq(3&4) has been taken out and substituting the parameter value (table 1), we got

$$\frac{\theta(s)}{F(s)} = \frac{-qs^2}{rs^2 - k + ds}$$

$$\frac{\theta(s)}{F(s)} = \frac{-0.04283097s^2}{0.1539s^4 + 0.01265\, s^3 - 0.6167s^2 - 0.02099\, s}$$

$$\frac{\theta(s)}{F(s)} = \frac{-0.2783\, S^2}{s\,(s+2.026)\,(s-1.978)\,(s+0.03402)}$$

and

$$\frac{X(s)}{F(s)} = \frac{rs^2 - k + ds}{(pr - q^2)s^4 + (pd + br)s^3 + (bd - pk)s^2 - kbs}$$

$$\frac{X(s)}{F(s)} = \frac{0.106\, S^2 + 0.005\, S - 0.4197}{0.1539\, S^4 + 0.01265\, S^3 - 0.6167\, S^2 - 0.02099\, S}$$

$$\frac{X(s)}{F(s)} = \frac{0.68843\,(s+2.014)\,(s-1.967)}{s\,(s+2.026)\,(s-1.978)\,(s+0.03402)}$$

The system poles lies on R.H plane, hence system is unstable.

Table 1: Parameters of the System from Feedback Instrument U.K.

Parameter	Value	unit
Cart mass()	1.206	Kilo gram
Mass of the pendulum()	0.2693	Kilo gram
Half Length of pendulum()	0.1623	meter
Coefficient of frictional force()	0.05	Ns/m
Pendulum damping coefficient(q)	0.005	/rad
Moment of inertia of pendulum()	0.099	/
Gravitation force()	9.8	/

IV. Performance Indeces

The design of a control system is an attempt to meet a set of specifications which define the overall performance of the system in terms of certain measurable quantities. In the normal way design of control system, some specific parametric values of the system are assumed and the control system is designed accordingly to meet desired performance of the system. Here we used four most commonly mathematical functions as a performance index associated with error of a closed loop system. A performance index is a number which indicates goodness of system performance. The objective is to design an optimal system by proper choice of its parameters such that the specified performance index is extremum- either minimum or maximum. A performance index must be a single positive number or zero, the latter being obtained if and only if the measure of the deviation becomes identically zero.

The commonly used performance indeces (PI) are:

Integral of squared error (ISE),

$$J = \int_0^\infty e^2(t)dt$$

Integral of time multiplied squared error (ITSE),

$$J = \int_0^\infty te^2(t)dt$$

Integral of absolute error (IAE),

$$J = \int_0^\infty |e(t)|\, dt$$

Integral of time multiplied absolute error (ITAE),

$$J = \int_0^\infty t|e(t)|\, dt$$

Here the error is define as $e(t) = x(t) - y(t)$ We have taken the stable reference model for angle whose transfer function is

$$\frac{\theta_1(s)}{F_1(s)} = \frac{s+2}{s^3 + 3.125s^2 + 17.94s + 35}$$

And the transfer function of our system when angle be the output is

$$\frac{\theta(s)}{F(s)} = \frac{-0.04283097s^2}{0.1539s^4 + 0.01265\, s^3 - 0.6167s^2 - 0.02099\, s}$$

Table 2: The Different Performance Indeces When the Angle of the Pendulum is Output

Performance Indexes	PID
ISE	$\frac{-50.87s^2 - 58.99s - 149.5}{s}$
ITSE	$\frac{-50.72s^2 - 58.95s - 150.8}{s}$
IAE	$\frac{-50.66s^2 - 58.87s - 149.1}{s}$
ITAE	$\frac{-50.05s^2 - 58.41s - 151.4}{s}$

Table 3: The Different Performance Indeces When Position of the Cart is Output

Performance Indexes	PID
ISE	$\frac{45.66s^2 - 58.99s - 149.5}{s}$
ITSE	$\frac{42.78s^2 + 174.6s + 22.42}{s}$
IAE	$\frac{44.56s^2 + 187.9s + 22.4}{s}$
ITAE	$\frac{43.09s^2 - 175.7s - 38.28}{s}$

Also we have taken the stable reference model for position of the cart whose transfer function is
Reference Model

$$\frac{X_1(s)}{F_1(s)} = \frac{4.77s^4 + 21.43\,s^3 - 15.32s^2 - 83.82\,s - 10.16}{0.1539s^5 + 4.783\,s^4 + 20.81s^3 - 15.34s^2 - 83.82\,s - 10.16}$$

And the transfer function of our system when position be the output is

$$\frac{X(s)}{F(s)} = \frac{0.106s^2 + 0.005s - 0.4197}{0.1539s^4 + 0.01265\,s^3 - 0.6167s^2 - 0.02099\,s}$$

V. Genetic Algorithm

Genetic Algorithms (GA) are search procedures inspired by the laws of natural selection and genetics. They can be viewed as a general-purpose optimization method and have been successfully applied to search, optimization and machine learning tasks. GA has the ability to solve difficult, multi dimensional problems with little problem-specific information and hence has been chosen as the optimization technique to solve various problems in control systems.
It has been shown that compared with other traditional heuristic optimization method, Genetic Algorithm is likely to be more computationally efficient. The controller parameters are usually determined by trial-and-error through simulation. In such case, the paradigm of GA appear to offer an effective way for automatically and efficient searching for a set of control performance.

VI. Simulation & Results

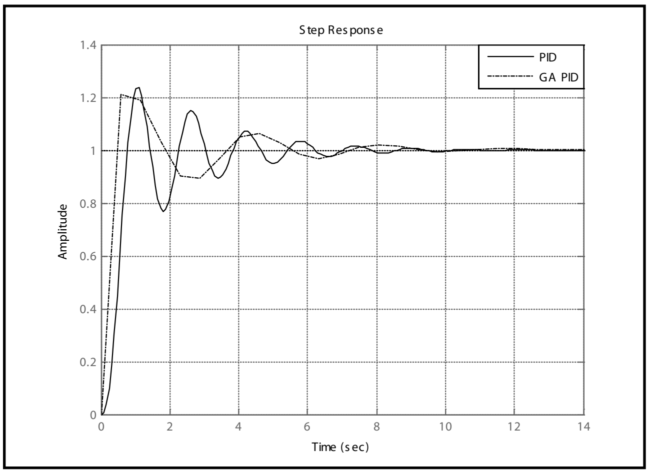

Fig. 3: ISE GA PID Controller of Angle

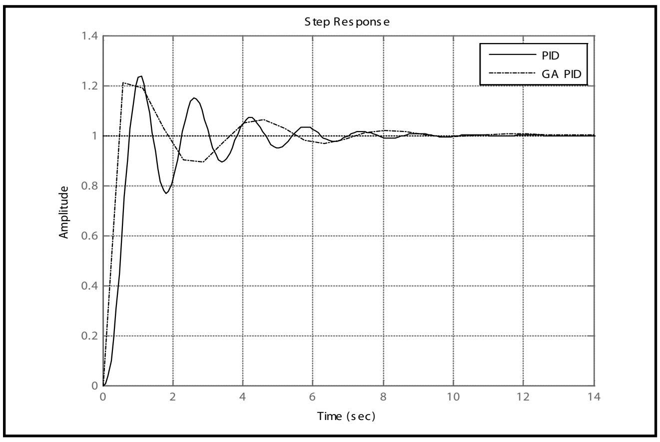

Fig. 4: ITSE GA PID Controller of Angle

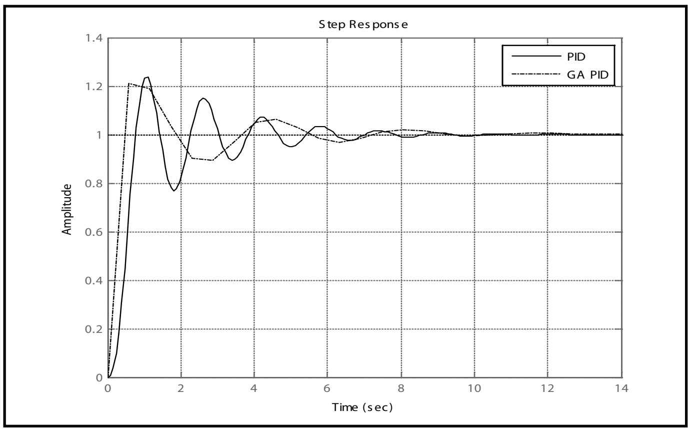

Fig. 5: IAE GA PID Controller of Angle

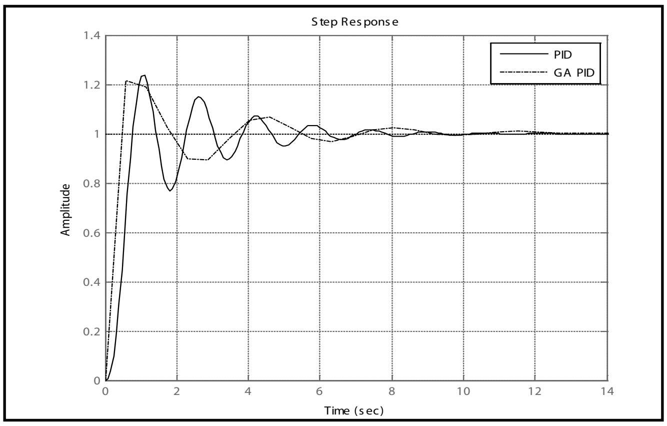

Fig. 6: ITAE GA PID Controller of Angle

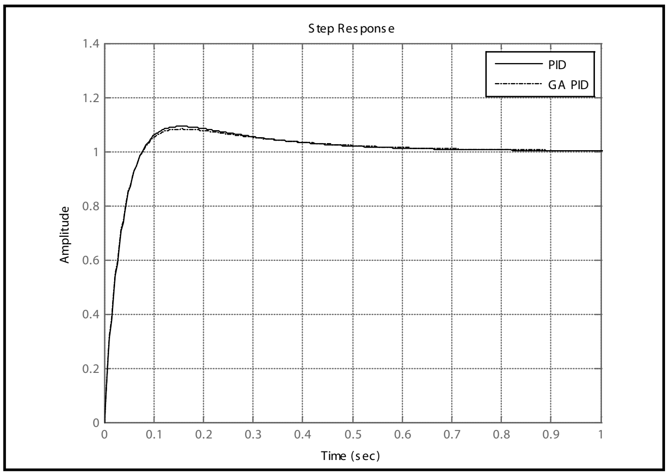

Fig. 7: ISE GA PID Controller of Cart

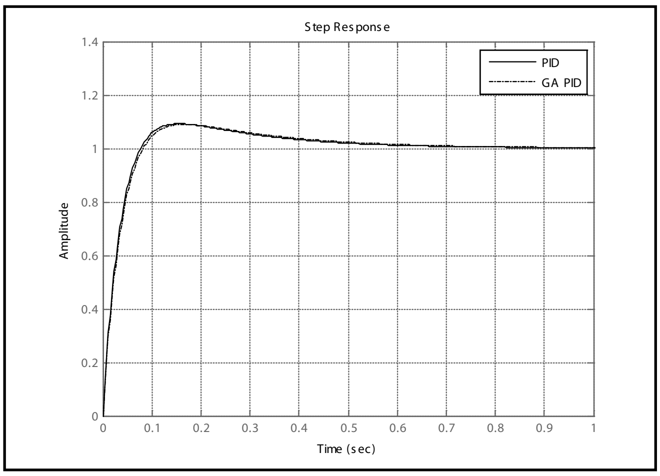

Fig. 8: ITSE GA PID Controller of Cart

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VII. Conclusion

Modelling of inverted pendulum shows that system is unstable with non-minimum phase zero. Results ofapplying PID controllers show that the systemcan be stabilized. while PIDcontroller method is cumbersome because ofselection ofconstants ofcontroller, Constant of the controllers can be tuned by some Genetic Algorithm technique for better result if same order of reference model like our system is chosen then the result will be comparatively better. If Walsh function can be used in equation (1&2) then we will directly find out the solution of non-linear differential equations and non-linear controller can be designed.

VIII. Acknowledgement

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