The Fate of the Accelerating Universe

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Abstract

Recently, the indication from supernova distance measurements that the expansion of the universe is accelerating has been extensively discussed, including the potential incompatibility of superstring theory with an event horizon in an eternally accelerating universe. In this paper, we study how quintessence and a cosmological constant may affect the fate of the universe. We wish to point out that the ultimate fate of our universe is much more sensitive to the presence of the cosmological constant than the quintessence or other matter content. In particular, the universe with a negative cosmological constant will always collapse eventually, even though the cosmological constant may be nearly zero and undetectable at all at the present time. This suggests a scenario in which the universe is accelerating now but may collapse eventually, thereby avoiding the cumbersome event horizon problem.

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1 Introduction

Recently the possible accelerating expansion of the present universe, as suggested by the type Ia supernova (SNIa) distance measurements [1, 2], is seriously considered. Combining SNIa results and the data from the measurements of the cosmic microwave background (CMB), it may be concluded that the universe has the critical density, consisting of 1/3 of ordinary matter and 2/3 of dark energy with a negative pressure (such that $p < -\frac{1}{3}\rho$) [3]. At the moment, the most promising candidate for the dark energy includes (i) a positive cosmological constant [4] and (ii) a slowly evolving scalar field called "quintessence" [5]. The existence of a positive cosmological constant is the simplest candidate for the dark energy. Unfortunately, some fine tuning problems need to be further explained if the acceleration is referred to such a constant energy source. On the contrary, quintessence provides dynamic negative pressure and may avoid the fine tuning problem, for example, the"tracker fields" proposed by Zlatev *et al.* entail solutions which will join a common cosmic evolutionary path, regardless of a wide range of initial conditions [6]. We note that the cosmological constant and the quintessence can be formulated in a single framework effectively. The cosmological constant can be incorporated into the quintessence potential as a constant which shifts the potential value, especially, the value of the minimum of the quintessence potential, where the quintessence field rolls towards. Conversely, the height of the minimum of the quintessence potential can also be regarded as a part of the cosmological constant. Usually, for separating them, the possible nonzero height of the minimum of the quintessence potential is incorporated into the cosmological constant and then set to be zero.

The cosmological constant can be provided by various kinds of matter, such as the vacuum energy of quantum fields and the potential energy of classical fields, and may also be originated in the intrinsic geometry. So far there is no sufficient reason or evidence to set the cosmological constant (or the height of the minimum of the quintessence potential) to be zero. In particular, some mechanisms to generate a negative cosmological constant have been pointed out by Shapiro and Solà [7], and recently by Gu and Hwang [8]. In Ref. [7], Shapiro and Solà investigated the running of the Higgs mass, couplings, and the vacuum term in the Standard Model from the Renormalization Group method. By the assumption that the cosmological constant is precisely cancelled at some point in the very far infrared (low energy) cosmic scale, it is shown that the consequent cosmological constant at present will be negative and situated at an energy scale comparable to the typical electron neutrino mass. In Ref. [8], we show that a negative cosmological constant, whose value is mainly governed by the mass of the scalar field rather than the original cosmological constant, is induced after the phase transition associated with the spontaneous symmetry breaking under the non-minimal interaction with a dynamic gravitational field.

For the accelerating expansion of the present universe, it is interesting to ask whether the expansion will keep accelerating forever or it will decelerate again after some time, which is related to another fascinating question: What dominates the fate of the universe? Several possibilities for an eternally accelerating universe have been studied [9, 10, 11]. If the expansion keeps accelerating perpetually, the universe will exhibit an event horizon, which raises an issue about the viability of string theory (or, more generally, quantum gravity). The existence of the event horizon brings challenges to string theory, such as the construction of suitable observables to displace the problematic conventional S-matrix in the universe with an event horizon [9, 10].

In this paper, we study how quintessence and a cosmological constant (which can be positive or negative) may affect the fate of the universe. We wish to point out the crucial role played by the cosmological constant in determining the fate of the universe. In particular, we find that a negative cosmological constant, even if nearly zero and undetectable at present, can make the universe collapse eventually. This suggests a scenario in which the universe is accelerating now caused by the quintessence but will decelerate and collapse eventually due to a negative cosmological constant (or the negative minimum of the quintessence potential), thereby avoiding the cumbersome event horizon.

2 The Basics

In the standard cosmological model, the universe at large scales is described by the Friedmann-Robertson-Walker (FRW) metric,

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right), \tag{1}$$

which follows from the cosmological principle, i.e., homogeneity and isotropy. In Eq. (1), $a(t)$ is the (cosmic) scale factor, and k can be chosen to be $+1$, -1, or 0 for spaces of positive, negative, or zero spatial curvature, corresponding to a closed, open, or flat universe, respectively. Provided that the matter content of the universe is taken to be a perfect fluid, the Einstein equation yields

$$\left(\frac{\dot a}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\tilde\rho = \frac{\Lambda}{3} + \frac{8\pi G}{3}\rho, \tag{2}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\tilde\rho + 3\tilde p) = \frac{\Lambda}{3} - \frac{4\pi G}{3}(\rho + 3p), \tag{3}$$

and also implies the energy-momentum conservation,

$$d(\tilde\rho a^3) = -\tilde p d(a^3), \tag{4}$$

where $\tilde\rho$ and $\tilde p$ are the effective energy density and pressure of the perfect fluid including the contributions from a cosmological constant Λ and other matter content, and G is the Newton's gravitational constant. From Eq. (3), we can see that the expansion of the universe is accelerating for a negative pressure p such that

$$\tilde p < -\frac{1}{3}\tilde\rho < 0\,, \quad \text{provided } \tilde\rho > 0. \tag{5}$$

The universe is usually considered to be composed of various kinds of perfect fluids with different types of equations of states:

$$p_i = \omega_i \rho_i\,, \tag{6}$$

where ρ_i and p_i are the energy density and pressure of the i-th component, and ω_i, to be called "state parameter" in this paper, may depend on the energy density ρ_i and time in general. Assuming that different species evolves independently and each ω_i is constant, we can obtain, from Eq. (4), a relation between the energy density ρ_i and the scale factor $a(t)$:

$$\rho_i \propto a^{-3(1+\omega_i)}, \tag{7}$$

which implies that the energy density of the component with a smaller state parameter ω will drop more slowly along with the expansion of the universe. As a result, eventually the universe will be dominated by the component with the smallest state parameter ω as long as the universe keeps expanding. This is a crucial point for the latter discussions about the ultimate fate of the accelerating universe.

3 The Fate of the Accelerating Universe

As one prominent candidate of the dark energy, quintessence may play a crucial role in swaying the fate of the universe. In addition, the cosmological constant, which entails the smallest state parameter $\omega = -1$, may take over to dominate the universe eventually. Therefore, in this section we will explore how these two kinds of matter will affect the ultimate fate of the presently accelerating universe.

Quintessence

We consider a Lagrangian density of a scalar field for the quintessence as follows:

$$\mathcal{L} = \sqrt{|g|}\left[\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + V(\phi)\right]. \tag{8}$$

Using the flat ($k = 1$) FRW metric in Eq. (1), the field equation of the scalar field is given by

$$\frac{\partial^2\phi}{\partial t^2} + 3H\frac{\partial\phi}{\partial t} - \frac{1}{a^2}\nabla^2\phi + V'(\phi) = 0, \tag{9}$$

and the energy density and pressure provided by the scalar field are given by

$$\rho_q = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{1}{2a^2}(\nabla\phi)^2 + V(\phi) \tag{10}$$

$$p_q = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{6a^2}(\nabla\phi)^2 - V(\phi). \tag{11}$$

In Eq. (9), the Hubble parameter H is defined by $H \equiv \dot{a}/a$, and $3H(\partial\phi/\partial t)$ is a damping term appearing due to the expansion of the universe. From Eqs. (10) and (11), we can see that the state parameter ($\omega_q \equiv p_q/\rho_q$) of the quintessence ranges from -1 to 1.

As summarized in [9, 10], there are various kinds of quintessence models leading to a perpetually accelerating universe and accordingly exhibiting an event horizon. For example, the potential proposed by Ratra and Peebles,

$$V(\phi) \sim \exp\left(-\sqrt{\frac{3}{2}(\kappa+1)}\phi\right), \tag{12}$$

with $\kappa < -1/3$ entails solutions which will approach to the equation of state $p = \kappa\rho$ eventually [12]. Consequently this class of potentials with $\kappa < -1/3$ will generate an perpetually accelerating universe. For another example, the potential (originally studied in [12])

$$V(\phi) = \frac{M^{4+n}}{\phi^n} \tag{13}$$

entails "track solutions" which approach to an equation of state $p = -\rho$ asymptotically, insuring the eternal acceleration [6]. Moreover, the potential in Eq. (13) can be classified to a wider class of potentials referred to as 'runaway scalar fields', in which V, V', V'', V'/V, and V''/V all approach to 0 as $\phi \to 0$ [13]. Steinhardt claims that the runaway scalar field will guarantee the eventual acceleration of the universe. Nevertheless, there are other possible quintessence models providing alternative results (also summarized in [10]). For example, for a potential which drops to a minimum at ϕ_0 and then becomes flat for $\phi > \phi_0$, the universe will be dominated by the kinetic energy after the scalar field passes the minimum, and then become decelerating with an equation of state $p = \rho$.

There are two more points which should be added before discussing the case of the cosmological constant. Firstly, as mentioned in Sec. 1, the height of the minimum of the quintessence potential, if nonzero, can make a contribution to the cosmological constant. We will regard the possible nonzero height of the potential's minimum as a part of the cosmological constant, and then set it to be zero. Secondly, we note that the kinetic energy and potential energy of the quintessence can be transferred to each other along with the evolution of the quintessence field: As the field rolls down, the potential energy is transferred to the kinetic energy, and the process reverses as the field climbs up. In addition, the kinetic energy will be dissipated, accompanying the expansion of the universe, due to the damping term $3H(\partial\phi/\partial t)$ in Eq. (9). As a result, the energy density of the quintessence will always be dissipated as the universe expands, and the state parameter ω_q can only skim over or approach -1 (rather than halt at that point) and hence should be always larger than the one of the cosmological constant.

Cosmological constant

The cosmological constant entails equation of state $p = -\rho$ with the smallest state parameter $\omega = -1$ (even smaller than the one of the quintessence as mentioned above). So the cosmological constant, if existing, will catch up the quintessence and other matter content, start to be predominant, and dominate the universe eventually, as long as the universe keeps expanding. In the following, using Eq. (2) and (3), we will see how a positive and a negative cosmological constants will affect the fate of the universe profoundly in extremely different ways.

(i) $\Lambda > 0$

In open ($k = -1$) and flat ($k = 0$) cases, the universe will expand forever, and the cosmological constant will take over eventually such that the expansion will eternally accelerate thereafter, consequently exhibiting an event horizon. For a closed ($k = 1$) universe, the situation is more complicated. It involves the competition between the three terms in the Friedmann equation (2): curvature term k/a^2, cosmological constant term $\Lambda/3$, and energy density term $\frac{8\pi G}{3}\rho$ from other matter contents. Roughly speaking, if the cosmological constant has been predominant when the universe expands to the extent that the curvature term is comparable to the matter density term, the universe will expand forever in an accelerating manner eventually and exhibit an event horizon. Conversely, if the cosmological constant is still comparatively small at that moment, the universe will collapse eventually.

(ii) $\Lambda < 0$

The situation for a negative cosmological constant is quite interesting: The universe will always collapse eventually! We can read off, from Eq. (2), that the universe will start to collapse when the expansion reaches the extent that

$$\frac{k}{a^2} = \frac{\Lambda}{3} + \frac{8\pi G}{3}\rho, \tag{14}$$

no matter the universe is open, flat, or closed and how close to zero the negative cosmological constant might be.

For a more concrete illustration, we analyze numerically the evolution of the universe which is considered to be composed of a negative cosmological constant (with the energy density ρ_Λ) and the quintessence with the potential

$$V(\phi) \propto \frac{1}{\phi^2}, \tag{15}$$

which belongs to the class of potentials in Eq. (13) (and also 'runaway scalar fields'). We note that this setup is equivalent to a quintessence field with the potential

$$V(\phi) = \frac{M^6}{\phi^2} + C, \tag{16}$$

where C is an arbitrary negative constant corresponding to the negative cosmological constant, that is, we set the minimum of the potential, where the quintessence field rolls towards, to be negative rather than zero. The results are presented in Fig. 1 and Fig. 2. These two figures sketch the evolution of the scale factor $a(t)$ and the Hubble parameter $H(t) \equiv \dot{a}/a$ of two universes: The solid line is for the case of $\rho_\Lambda = -\frac{1}{50}\rho_q(0)$, where $\rho_q(0)$ is the initial energy density of the quintessence, while the dash line sketches the accelerating and nearly exponential expansion of the universe in the case of $\rho_\Lambda = 0$ as a reference. As shown in these two figures, the universe with a negative cosmological constant will collapse (i.e. $\ln[a(t)/a_0]$ drops and $H(t)$ becomes negative) after some time, in contrast to the case of $\rho_\Lambda = 0$, even though the energy density of the cosmological constant in magnitude only accounts for 2 percent of the one of the quintessence initially. We note that the collapse takes place and proceeds in a violent manner since the original "damping" term $3H(\partial\phi/\partial t)$ in Eq. (9) turns to an "amplifying" term for $H < 0$ in the collapsing epoch.

The above discussions reveal an interesting fact: The ultimate fate of the universe is determined by the cosmological constant (if nonzero), even though the cosmological constant may be so tiny that there is no way to detect it at the present time!

4 Summary and Discussion

Recent observations indicate that the dark energy may accounts for a significant part of the energy density in the universe. Thus the quintessence, if accounting for the dark energy, will eventually become more predominant (due to its negative state parameter ω_q) than other ordinary matter, and will control the fate of the universe in a significant way. As summarized in [10], various quintessence potentials lead to an eternal acceleration and accordingly the existence of an event horizon, while there still exist other possibilities in the quintessence models to generate an accelerating universe which will decelerate again in the future. In particular, Steinhardt [13] pointed out that an eventual acceleration is ensured by a "runaway scalar field", a considerable class of quintessence models.

Nevertheless, as pointed out in Sec. 2, the specific content of the universe that entails the smallest state parameter ω will take over and eventually dominate the universe as long as the universe continues to expand. The cosmological constant is the one which has the smallest state parameter. A positive cosmological constant will lead to an eternally accelerating universe and an event horizon accordingly, unless the universe is a closed one and will collapse before the cosmological constant becomes predominant. Conversely and even more interestingly, the universe with a negative cosmological constant will always collapse eventually, no matter the universe is open, flat, or closed. Thus, an arbitrarily tiny negative cosmological constant can make the universe to decelerate in the future, even though it is accelerating now! This gives us an scenario in which the matter contents of the universe include the quintessence and a negative cosmological constant (which can be nearly zero and undetectable at present), or, equivalently, only one quintessence field which rolls towards a negative minimum of its potential, such that the universe is accelerating now caused by the quintessence evolving beyond zero of the quintessence potential, but will decelerate and collapse eventually due to the negative cosmological constant or the negative height of the minimum of the quintessence potential, thereby avoiding the cumbersome event horizon.

In conclusion, firstly, the fate of the universe is much more sensitive to the presence of the cosmological constant (or the nonzero height of the minimum of the quintessence potential) than other matter content, even though the cosmological constant may be extremely tiny and undetectable at all at the present time. Thus, before we pin down the magnitude and the sign of the cosmological constant from observations, it is hard to tell what the ultimate fate of the presently accelerating universe will be. Secondly, as suggested by the above discussions, the potential incompatibility of superstring theory with an event horizon in an eternally accelerating universe can be avoided in a self-consistent way if superstring theory can provide a negative cosmological constant in itself. We note that various background solutions in supergravity, which lead to an anti-de Sitter space (corresponding to a negative cosmological constant), have been extensively studied (for a review, see [14]). In addition, some mechanisms to generate a negative cosmological constant are also pointed out recently by Shapiro and Solà in [7], and by Gu and Hwang in [8].

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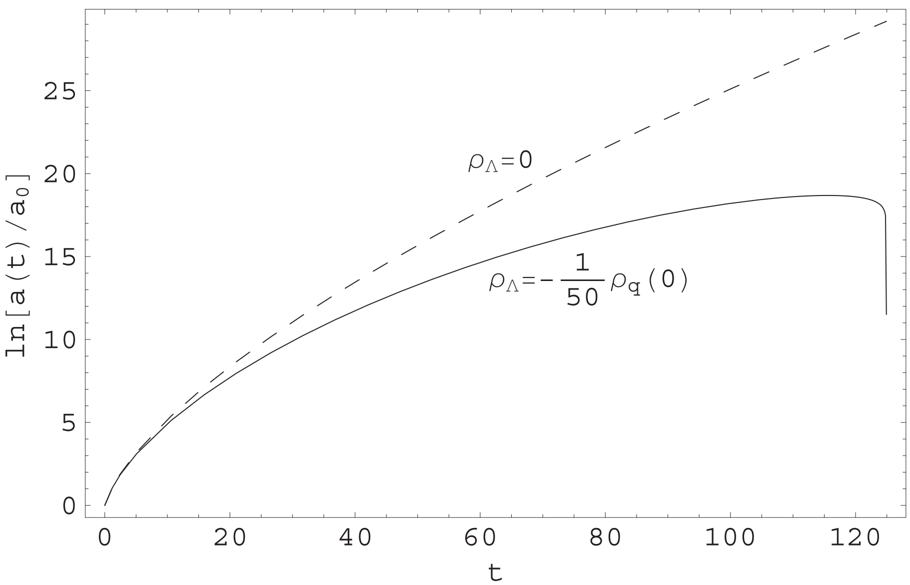

Figure 1: The evolution of the universes with $\rho_\Lambda = -\frac{1}{50}\rho_\phi(0)$ and $\rho_\Lambda = 0$ respectively — plot of $\ln[a(t)/a_0]$

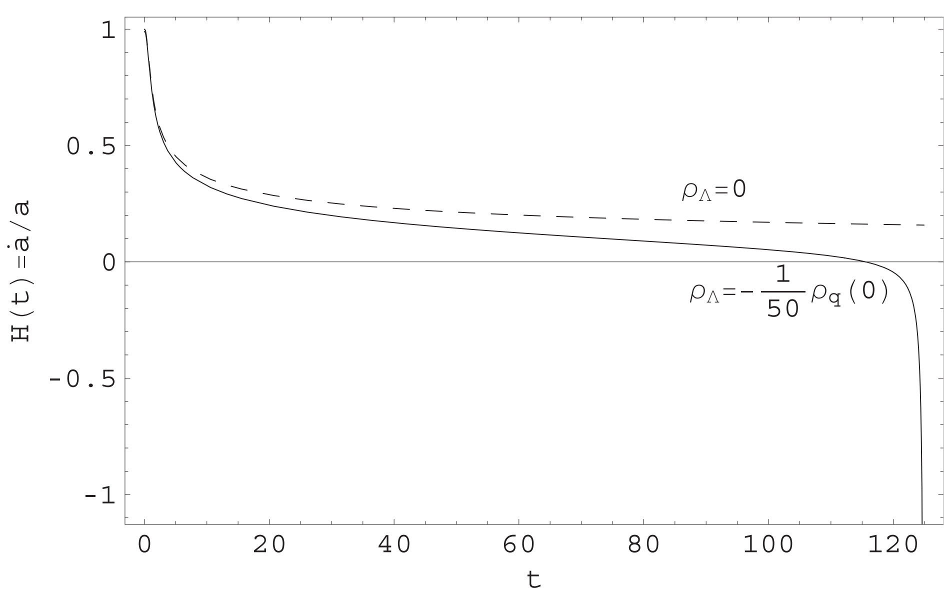

Figure 2: The evolution of the universes with $\rho_\Lambda = -\frac{1}{50}\rho_\phi(0)$ and $\rho_\Lambda = 0$ respectively — plot of $H(t)$