CALT-68-2657

THE EARLY YEARS OF STRING THEORY: A PERSONAL PERSPECTIVE

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Abstract

This article surveys some of the highlights in the development of string theory through the first superstring revolution in 1984. The emphasis is on topics in which the author was involved.

Based on a lecture presented on June 20, 2007 at the Galileo Galilei Institute

1 Introduction

I am happy to have this opportunity to reminisce about the origins and development of string theory from 1962 (when I entered graduate school) through the first superstring revolution in 1984. Some of the topics were discussed previously in three papers that were written for various special events in 2000 [1, 2, 3]. Also, some of this material was reviewed in the 1985 reprint volumes [4], as well as the string theory textbooks [5, 6]. In presenting my experiences and impressions of this period, it is inevitable that my own contributions are emphasized.

Some of the other early contributors to string theory have presented their recollections at the Galileo Galilei Institute meeting on "The Birth of String Theory" in May 2007. Since I was unable to attend that meeting, my talk was given at the GGI one month later. Taken together, the papers in this collection should convey a fairly accurate account of the origins of this remarkable subject.[1]

The remainder of this paper is divided into the following sections:

- 1960 – 68: The analytic S matrix (Ademollo, Veneziano)
- 1968 – 70: The dual resonance model (Veneziano, Di Vecchia, Fairlie, Neveu)
- 1971 – 73: The RNS model (Ramond, Neveu)
- 1974 – 75: Gravity and unification
- 1975 – 79: Supersymmetry and supergravity (Gliozzi)
- 1979 – 84: Superstrings and anomalies (Green)

For each section, the relevant speakers at the May meeting are listed above. Since their talks were more focussed than mine, they were able to provide more detail. In one section (gravity and unification) my presentation provided more detail than the others.

2 1960 – 68: The analytic S matrix

In the early 1960s there existed a successful quantum theory of the electromagnetic force (QED), which was completed in the late 1940s, but the theories of the weak and strong nuclear forces were not yet known. In UC Berkeley, where I was a graduate student during the period 1962 – 66, the emphasis was on developing a theory of the strong nuclear force.

[1] Since the history of science community has shown little interest in string theory, it is important to get this material on the record. There have been popular books about string theory and related topics, which serve a useful purpose, but there remains a need for a more scholarly study of the origins and history of string theory.

I felt that UC Berkeley was the center of the Universe for high energy theory at the time. Geoffrey Chew (my thesis advisor) and Stanley Mandelstam were highly influential leaders. Also, Steve Weinberg and Shelly Glashow were impressive younger faculty members. David Gross was a contemporaneous Chew student with whom I shared an office.[2]

Geoffrey Chew's approach to understanding the strong interactions was based on several general principles [8, 9]. He was very persuasive in advocating them, and I was strongly influenced by him. The first principle was that quantum field theory, which was so successful in describing QED, was inappropriate for describing a strongly interacting theory, where a weak-coupling perturbation expansion would not be useful. A compelling reason for holding this view was that none of the hadrons (particles that have strong interactions) seemed to be more fundamental than any of the others. Therefore a field theory that singled out some subset of the hadrons did not seem sensible. Also, it was clearly not possible to formulate a quantum field theory with a fundamental field for every hadron. One spoke of *nuclear democracy* to describe this situation.[3]

For these reasons, Chew argued that field theory was inappropriate for describing strong nuclear forces. Instead, he advocated focussing attention on physical quantities, especially the S Matrix, which describes on-mass-shell scattering amplitudes. The goal was therefore to develop a theory that would determine the S matrix. Some of the ingredients that went into this were properties deduced from quantum field theory, such as unitarity and maximal analyticity of the S matrix. These basically encode the requirements of causality and nonnegative probabilities.

Another important proposal, due to Chew and Frautschi, whose necessity was less obvious, was maximal analyticity in angular momentum [10, 11]. The idea is that partial wave amplitudes $a_l(s)$, which are defined in the first instance for angular momenta $l = 0, 1, \ldots$, can be uniquely extended to an analytic function of l, $a(l, s)$, with isolated poles called Regge poles. The Mandelstam invariant s is the square of the invariant energy of the scattering reaction. The position of a Regge pole is given by a Regge trajectory $l = \alpha(s)$. The values of s for which l takes a physical value, correspond to physical hadron states. The necessity of branch points in the l plane, with associated Regge cuts, was established by Mandelstam.

[2] It was a particularly nice office, which was being reserved for Murray Gell-Mann, whom Berkeley was trying to hire. It was felt that students would be easier to dislodge than a faculty member. Gross and I wrote one joint paper in 1965 [7], which I felt was rather clever.

[3] The quark concept arose during this period, but the prevailing opinion was that quarks are just mathematical constructs. The SLAC deep inelastic scattering experiments in the late 1960s made it clear that quarks and gluons are physical (confined) particles. It was then natural to try to base a quantum field theory on them, and QCD was developed a few years later with the discovery of asymptotic freedom.

Their role in phenomenology was less clear.

The theoretical work in this period was strongly influenced by experimental results. Many new hadrons were discovered in experiments at the Bevatron in Berkeley, the AGS in Brookhaven, and the PS at CERN. Plotting masses squared versus angular momentum (for fixed values of other quantum numbers), it was noticed that the Regge trajectories are approximately linear with a common slope

$$\alpha(s) = \alpha(0) + \alpha' s \qquad\qquad \alpha' \sim 1.0\,(\mathrm{GeV})^{-2}\,.$$

Using the crossing-symmetry properties of analytically continued scattering amplitudes, one argued that exchange of Regge poles (in the t channel) controlled the high-energy, fixed momentum transfer, asymptotic behavior of physical amplitudes:

$$A(s,t) \sim \beta(t)(s/s_0)^{\alpha(t)} \qquad s \to \infty,\, t < 0.$$

In this way one deduced from data that the intercept of the ρ trajectory, for example, was $\alpha_\rho(0) \sim .5$. This is consistent with the measured mass $m_\rho = .76\,\mathrm{GeV}$ and the Regge slope $\alpha' \sim 1.0\,(\mathrm{GeV})^{-2}$.

The ingredients discussed above are not sufficient to determine the S matrix, so one needed more. Therefore, Chew advocated another principle called the *bootstrap*. The idea was that the exchange of hadrons in crossed channels provide forces that are responsible for causing hadrons to form bound states. Thus, one has a self-consistent structure in which the entire collection of hadrons provides the forces that makes their own existence possible. It was unclear for some time how to formulate this intriguing property in a mathematically precise way. As an outgrowth of studies of *finite-energy sum rules* in 1967 [12, 13, 14, 15, 16] this was achieved in a certain limit in 1968 [17, 18, 19]. The limit, called the *narrow resonance approximation* was one in which resonance lifetimes are negligible compared to their masses. The observed linearity of Regge trajectories suggested this approximation, since otherwise pole positions would have significant imaginary parts. In this approximation branch cuts in scattering amplitudes, whose branch points correspond to multiparticle thresholds, are approximated by a sequence of resonance poles.

The bootstrap idea had a precise formulation in the narrow resonance approximation, which was called *duality*. This is the statement that a scattering amplitude can be expanded in an an infinite series of s-channel poles, and this gives the same result as its expansion in an infinite series of t-channel poles.[4] To include both sets of poles, as usual Feynman diagram techniques might suggest, would amount to double counting.

[4]One defines divergent series by analytic continuation.

3 1968 – 70: The dual resonance model

I began my first postdoctoral position (entitled *instructor*) at Princeton University in 1966. For my first two and a half years there, I continued to do work along the lines described in the previous section (Regge pole theory, duality, etc.). Then Veneziano dropped a bombshell – an exact analytic formula that exhibited duality with linear Regge trajectories [20]. Veneziano's formula was designed to give a good phenomenological description of the reaction $\pi + \pi \to \pi + \omega$ or the decay $\omega \to \pi^+ + \pi^0 + \pi^-$. Its structure was the sum of three Euler beta functions:

$$T = A(s,t) + A(s,u) + A(t,u)$$

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))},$$

where α is a linear Regge trajectory

$$\alpha(s) = \alpha(0) + \alpha' s.$$

An analogous formula appropriate to the reaction $\pi + \pi \to \pi + \pi$ was quickly proposed by Lovelace and Shapiro [21, 22]. A rule for building in adjoint $SU(N)$ quantum numbers was formulated by Chan and Paton [23]. This symmetry was initially envisaged to be a global (flavor) symmetry, but it later turned out to be a local gauge symmetry.

The Veneziano formula gives an explicit realization of duality and Regge behavior in the *narrow resonance approximation*. The function $A(s,t)$ can be expanded in terms of the s-channel poles or the t-channel poles. The motivation for writing down this formula was mostly phenomenological, but it turned out that formulas of this type describe tree amplitudes in a perturbatively consistent quantum theory!

Very soon after the appearance of the Veneziano amplitude, Virasoro proposed an alternative formula [24]

$$T = \frac{\Gamma(-\frac{1}{2}\alpha(s))\Gamma(-\frac{1}{2}\alpha(t))\Gamma(-\frac{1}{2}\alpha(u))}{\Gamma(-\frac{1}{2}\alpha(t) - \frac{1}{2}\alpha(u))\Gamma(-\frac{1}{2}\alpha(s) - \frac{1}{2}\alpha(u))\Gamma(-\frac{1}{2}\alpha(s) - \frac{1}{2}\alpha(t))},$$

which has similar virtues. Since this formula has total stu symmetry, it is only applicable to particles that are singlets of the Chan–Paton group.

Over the course of the next year or so, string theory (or *dual models*, as the subject was then called) underwent a sudden surge of popularity, marked by several remarkable discoveries. One was the discovery of an N-particle generalization of the Veneziano formula

[25, 26, 27, 28, 29]:

$$A_N(k) = g_{\text{open}}^{N-2} \int d\mu_N(y) \prod_{i<j} (y_i - y_j)^{\alpha' k_i \cdot k_j},$$

where $y_1, y_2, \ldots, y_N$ are real coordinates, any three of which are y_A, y_B, y_C, and

$$d\mu_N(y) = |(y_A - y_B)(y_B - y_C)(y_C - y_A)| \prod_{i=1}^{N-1} \theta(y_{i+1} - y_i)$$

$$\times \delta(y_A - y_A^0)\delta(y_B - y_B^0)\delta(y_C - y_C^0) \prod_{i=1}^{N} dy_i.$$

The formula is independent of y_A^0, y_B^0, y_C^0 due to its $SL(2, \mathbb{R})$ symmetry, which allows them to be mapped to arbitrary real values. This formula has cyclic symmetry in the N external lines.

Soon thereafter Shapiro formulated an N-particle generalization of the Virasoro formula [30]:

$$A_N(k_1, k_2, \ldots, k_N) = g_{\text{closed}}^{N-2} \int d\mu_N(z) \prod_{i<j} |z_i - z_j|^{\alpha' k_i \cdot k_j},$$

where $z_1, z_2, \ldots, z_N$ are complex coordinates, any three of which are z_A, z_B, z_C, and

$$d\mu_N(z) = |(z_A - z_B)(z_B - z_C)(z_C - z_A)|^2$$

$$\times \delta^2(z_A - z_A^0)\delta^2(z_B - z_B^0)\delta^2(z_C - z_C^0) \prod_{i=1}^{N} d^2 z_i.$$

The formula is independent of z_A^0, z_B^0, z_C^0 due to its $SL(2, \mathbb{C})$ symmetry, which allows them to be mapped to arbitrary complex values. This amplitude has total symmetry in the N external lines.

Both of these formulas were shown to have a consistent factorization on a spectrum of single-particle states described by an infinite number of harmonic oscillators [31, 32, 33, 34, 35]

$$\{a_m^\mu\} \qquad \mu = 0, 1, \ldots, d-1 \qquad m = 1, 2, \ldots$$

with one set of such oscillators in the Veneziano case and two sets in the Virasoro case. These results were interpreted as describing the scattering of modes of a relativistic string [35, 36, 37, 38, 39, 40]: open strings in the first case and closed strings in the second case. Amazingly, the formulas preceded the interpretation. Although, we did not propose a string interpretation, Gross, Neveu, Scherk, and I did realize that the relevant diagrams of the loop expansion were classified by the possible topologies of two-dimensional manifolds with boundaries [41].

Having found the factorization, it became possible to compute radiative corrections (loop amplitudes). This was initiated by Kikkawa, Sakita, and Virasoro [42] and followed up by many others. Let me describe my role in this. I was at Princeton, where I collaborated with Gross, Neveu, and Scherk in computing one-loop amplitudes. In particular, we discovered unanticipated singularities in the "nonplanar" open-string loop diagram [43]. The world sheet is a cylinder with two external particles attached to each boundary. Our computations showed that this diagram gives branch points that violate unitarity. This was a very disturbing conclusion, since it seemed to imply that the classical theory does not have a consistent quantum extension. This was also discovered by Frye and Susskind [44]. (The issue of quantum consistency turned out to be a recurring theme, which reappeared many years later, as discussed in Section 7.)

Soon thereafter Claude Lovelace pointed out [45] that these branch points become poles provided that

$$\alpha(0) = 1 \quad \text{and} \quad d = 26.$$

Until Lovelace's work, everyone assumed that the spacetime dimension was $d = 4$.[5] As we were not yet talking about gravity, there was no reason to consider anything else. Later, these poles were interpreted as closed-string modes in a one-loop open-string amplitude. Nowadays this is referred to as open string–closed string duality.

Lovelace's analysis also required there to be an infinite number of decoupling conditions. These turned out to be precisely the Virasoro constraints, which were discovered at about the same time [46, 47]. A couple of years later Brink and Olive constructed a physical-state projection operator [48], which they used to verify Lovelace's conjecture that the nonplanar loop amplitude actually contains closed-string poles when the decoupling conditions in the critical dimension are imposed [49].

Thus, quantum consistency was restored, but the price was high: a spectrum with a tachyon and 22 extra dimensions of space. In 1973, the origin of the critical dimension and the intercept condition were explained in terms of the light-cone gauge quantization of a fundamental string by Goddard, Goldstone, Rebbi, and Thorn [50]. Prior to this paper the string interpretation of dual models was only a curiosity. The GGRT approach was extended to interacting strings by Mandelstam [51].

[5]The idea of considering a higher dimension was suggested to Lovelace by David Olive.

4 1971 – 73: The RNS model

In January 1971 Pierre Ramond constructed a dual-resonance model generalization of the Dirac equation [52]. He reasoned as follows: just as the total momentum of a string, p^μ, is the zero mode of a momentum density $P^\mu(\sigma)$, so should the Dirac matrices γ^μ be the zero modes of densities $\Gamma^\mu(\sigma)$. Then he defined the modes of $\Gamma \cdot P$:

$$F_n = \int_0^{2\pi} e^{-in\sigma} \Gamma \cdot P d\sigma \qquad n \in \mathbb{Z}.$$

In particular,

$$F_0 = \gamma \cdot p + \text{oscillator terms}\,.$$

He proposed the wave equation

$$(F_0 + m)|\psi\rangle = 0,$$

which is now known as the *Dirac–Ramond Equation.* Its solutions give the spectrum of a noninteracting fermionic string.

Ramond also observed that the Virasoro algebra generalizes to[6]

$$\{F_m, F_n\} = 2L_{m+n} + \frac{c}{3} m^2 \delta_{m,-n}$$

$$[L_m, F_n] = (\frac{m}{2} - n) F_{m+n}$$

$$[L_m, L_n] = (m-n) L_{m+n} + \frac{c}{12} m^3 \delta_{m,-n}\,.$$

The free fermion spectrum should be restricted by the super-Virasoro constraints $F_n|\psi\rangle = L_n|\psi\rangle = 0$ for $n > 0$.

André Neveu and I proposed a new bosonic dual model, which we called the *dual pion model,* in March 1971 [53].[7] It has a similar structure to Ramond's free fermion theory, with the periodic density $\Gamma^\mu(\sigma)$ replaced by an antiperiodic one $H^\mu(\sigma)$. Then the modes

$$G_r = \int_0^{2\pi} e^{-ir\sigma} H \cdot P d\sigma \qquad r \in \mathbb{Z} + 1/2$$

satisfy a similar super-Virasoro algebra. The free particle spectrum is given by the wave equation $(L_0 - 1/2)|\psi\rangle = 0$ supplemented by the constraints $G_r|\psi\rangle = 0$ for $r > 0$. (These

[6] His paper does not include the central terms.

[7] We submitted another publication [54] one month earlier that contained some, but not all, of the right ingredients.

formulas are appropriate in the $\mathcal{F}_2$ picture discussed below.) We also constructed N-particle amplitudes analogous to those of the Veneziano model.

The $\pi + \pi \to \pi + \pi$ amplitude computed in the dual pion model turned out to have exactly the form that had been proposed earlier by Lovelace and Shapiro. However, the intercepts of the π and ρ Regge trajectories were $\alpha_\pi(0) = 1/2$ and $\alpha_\rho(0) = 1$. These were half a unit higher than was desired in each case. This implied that the pion was tachyonic and the rho was massless.

Soon after our paper appeared, Neveu traveled to Berkeley, where there was considerable interest in our results. This led to Charles Thorn (a student of Stanley Mandelstam at the time) joining us in a follow-up project in which we proved that the super-Virasoro constraints were fully implemented [55]. This required recasting the original description of the string spectrum (called the $\mathcal{F}_1$ picture) in a new form, which we called the $\mathcal{F}_2$ picture. The three of us then assembled these bosons together with Ramond's fermions into a unified interacting theory of bosons and fermions [56, 57], thereby obtaining an early version of what later came to be known as superstring theory.

The string world-sheet theory that gives this spectrum of bosons and fermions is

$$S = \int d\sigma d\tau \left(\partial_\alpha X^\mu \partial^\alpha X_\mu - i \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu \right),$$

where ψ^μ are two-dimensional Majorana spinors and ρ^α are two-dimensional Dirac matrices. Later in that same year (1971), Gervais and Sakita observed [58] that this action has *two-dimensional global supersymmetry* described by the infinitesimal fermionic transformations

$$\delta X^\mu = \bar{\varepsilon} \psi^\mu$$

$$\delta \psi^\mu = -i \rho^\alpha \varepsilon \partial_\alpha X^\mu.$$

There are two possible choices of boundary conditions for the fermi fields ψ^μ, one of which gives the boson spectrum (Neveu–Schwarz sector) and the other of which gives the fermion spectrum (Ramond sector). In fact, the boundary conditions are only compatible with global supersymmetry in the Ramond sector. Five years later, a more fundamental world-sheet action with local supersymmetry was discovered [60, 61]. It has the additional virtue of also accounting for the super-Virasoro constraints. The significance of this algebra is that the world-sheet theory, when properly gauge fixed and quantized, has *superconformal symmetry.*

Also in 1971, the four-dimensional super-Poincaré group was formulated by Golfand and Likhtman [62], who proposed constructing four-dimensional field theories with this symmetry. However, the celebrated Wess–Zumino work [63] on four-dimensional supersymmetric theories, a couple of years later, was motivated by the search for 4d interacting analogs of the 2d Gervais–Sakita world-sheet action. (They were unaware of the Golfand–Likhtman work at that time.)

The dual pion model has a manifest $\mathbb{Z}_2$ symmetry. Since the pion is odd and the rho is even, this symmetry was identified with G parity.[8] It was obvious that one could make a consistent truncation (at least at tree level) to the even G-parity sector and that then the model would be tachyon free. Because of the desired identification with physical hadrons, there was no motivation (at the time) to do that, however. Rather, considerable effort was expended in the following year attempting to modify the model so as to lower the intercepts by half a unit. An example is [64]. None of these constructions was entirely satisfactory, however.

One of the important questions in this period was whether all the physical string excitations have a positive norm. States of negative norm (called *ghosts*) would represent a breakdown of unitarity and causality, so it was essential that they not be present in the string spectrum. The first proof of the *no-ghost theorem* for the original bosonic string theory was achieved by Brower [65], building on earlier work by Del Giudice, Di Vecchia, and Fubini [66]. This work showed that a necessary condition for the absence of ghosts is $d \leq 26$, and that the critical value $d = 26$ has especially attractive features, as we already suspected based on the earlier observations of Lovelace.

I generalized Brower's proof of the no-ghost theorem to the RNS string theory and showed that $d = 10$ is the critical dimension and that the ground state fermion should be massless [67]. This was also done by Brower and Friedman a bit later [68]. An alternative, somewhat simpler, proof of the no-ghost theorem for both of the string theories was given by Goddard and Thorn at about the same time [69]. Other related work included [70, 71, 72].

Later in 1972, thanks to the fact that Murray Gell-Mann had become intrigued by my work with Neveu, I was offered a senior research appointment at Caltech. I think that the reason Gell-Mann became aware of our work was because he spent the academic year 1971-72 on a sabbatical at CERN, where there was an active dual models group. I felt very fortunate to receive such an offer, especially in view of the fact that the job market for theoretical physicists was extremely bad at the time. Throughout the subsequent years at Caltech, when my work was far from the mainstream, and therefore not widely appreciated, Gell-Mann was always very supportive. For example, he put funds at my disposal to invite

[8] G parity is a hadronic symmetry that is a consequence of charge conjugation invariance and isotopic spin symmetry.

visitors. This facilitated various collaborations with Lars Brink, Joël Scherk, and Michael Green among others.

One of the first things I did at Caltech was to study the fermion-fermion scattering amplitude. Using the physical-state projection operator [48], Olive and Scherk had derived a formula that involved the determinant of an infinite matrix [73]. C.C. Wu and I [74] discovered that this determinant is a simple function. We derived the result analytically in a certain limit and then verified numerically that it is exact everywhere. (The result was subsequently verified analytically [75].) To our surprise, the fermion-fermion scattering amplitude ended up looking very similar to the bosonic amplitudes. This might have been interpreted as a hint of spacetime supersymmetry, but this was before the Wess–Zumino paper, and that was not yet on my mind.

String theory is formulated as an on-shell S-matrix theory in keeping with its origins discussed earlier. However, the SLAC deep inelastic scattering experiments in the late 1960s made it clear that the hadronic component of the electromagnetic current is a physical off-shell quantity, and that its asymptotic properties imply that hadrons have hard pointlike constituents. With this motivation, I tried for the next year or so to construct off-shell amplitudes. Although some intriguing results were obtained [76, 77, 78], this was ultimately unsuccessful. Moreover, all indications were that strings were too soft to describe hadrons with their pointlike constituents.

At this point there were many good reasons to stop working on string theory: a successful and convincing theory of hadrons (QCD) was discovered, and string theory had many severe problems as a hadron theory. These included an unrealistic spacetime dimension, an unrealistic spectrum, and the absence of pointlike constituents. Also, convincing theoretical and experimental evidence for the standard model was rapidly falling into place. Understandably, given these successes and string theory's shortcomings, string theory rapidly fell out of favor. What had been a booming enterprise involving several hundred theorists rapidly came to a grinding halt.

Given that the world-sheet descriptions of the two known string theories have conformal invariance and superconformal invariance, it was a natural question whether one could obtain new string theories described by world-sheet theories with extended superconformal symmetry. The $N = 2$ case was worked out in [79]. The critical dimension is four, but the signature has to be $(2, 2)$. For a long time it was believed that the critical dimension of the $N = 4$ string is negative, but in 1992 Siegel argued that (due to the reducibility of the constraints) the $N = 4$ string is the same as the $N = 2$ string [80].

5 1974 – 75: Gravity and unification

Among the problems of the known string theories, as a theory of hadrons, was the fact that the spectrum of open strings contains massless spin 1 particles, and the spectrum of closed strings contains a massless spin 2 particle (as well as other massless particles). These particles lie on the leading Regge trajectories, and so the leading open-string Regge trajectory has intercept $\alpha(0) = 1$, and the leading closed-string Regge trajectory has intercept $\alpha(0) = 2$. In the attempts to reduce the open-string intercept to $\alpha(0) = 1/2$, mentioned earlier, it was expected that the closed-string intercept would be shifted to $\alpha(0) = 1$ at the same time. If this were to happen, then this trajectory could be identified with the *Pomeron* trajectory, which is responsible for the near constancy (up to logarithmic corrections) of hadronic total cross sections at high energy.

The alternative to modifying string theory to get what we wanted was to understand what the theory was giving without modification. String theories in the critical dimension clearly were beautiful theories, and they ought to be good for something. The fact that they were developed in an attempt to understand hadron physics did not guarantee that this was necessarily their appropriate physical application. Furthermore, the success of QCD made the effort to formulate a string theory of hadrons less pressing.

The first indication that such an agnostic attitude could prove worthwhile was a pioneering work by Neveu and Scherk [81], which studied the interactions of the massless spin 1 open-string particles at low energies (or, equivalently, in the *zero-slope limit*) and proved that their interactions agreed with those of Yang–Mills gauge particles in the adjoint representation of the Chan–Paton group. In other words, open-string theory was Yang–Mills gauge theory modified by higher dimension interactions at the string scale. This implies that the Chan–Paton group is actually a Yang–Mills gauge group.[9]

I arranged for Joël Scherk, with whom I had collaborated in Princeton, to visit Caltech in the winter and spring of 1974. Our interests and attitudes in physics were very similar, and so we were anxious to start a new collaboration. Each of us felt that string theory was too beautiful to be just a mathematical curiosity. It ought to have some physical relevance. We had frequently been struck by the fact that string theories exhibit unanticipated miraculous properties. What this means is that they have a very deep mathematical structure that is not fully understood. By digging deeper one could reasonably expect to find more surprises

[9] This work relating string theory and Yang–Mills theory followed an earlier study by Scherk describing how to obtain ϕ^3 field theory in the zero-slope limit [82].

and then learn new lessons. Therefore, despite the fact that the rest of the theoretical high energy physics community was drawn to the important project of exploring the standard model, we wanted to explore string theory.

Since my training was as an elementary particle physicist, gravity was far from my mind in early 1974. Traditionally, elementary particle physicists had ignored the gravitational force, which is entirely negligible under ordinary circumstances. For these reasons, we were not predisposed to interpret string theory as a physical theory of gravity. General relativists, the people who did study gravity, formed a completely different community. They attended different meetings, read different journals, and had no need for serious communication with particle physicists, just as particle physicists felt they had no need for relativists who studied topics such as black holes or the early universe.

Despite all this, we decided to do what could have been done two years earlier: we explored whether it is possible to interpret the massless spin 2 state in the closed-string spectrum as a graviton. This required carrying out an analysis analogous to the earlier one of Neveu and Scherk. This time one needed to decide whether the interactions of the massless spin 2 particle in string theory agree at low energy with those of the graviton in general relativity (GR). Success was inevitable, because GR is the only consistent possibility at low energies (*i.e.,* neglecting corrections due to higher-dimension operators), and string theory certainly is consistent. Therefore, the harder part of this work was forcing oneself to ask the right question rather than finding the right answer. In fact, by invoking certain general theorems, we were able to argue that string theory agrees with general relativity at low energies [83]. Although we were not aware of it at the time, Tamiaki Yoneya had obtained the same result somewhat earlier [84, 85].

In our paper, Scherk and I proposed to use string theory as a quantum theory of gravity, unified with the other forces. (Yoneya did not take this step.) This proposal had several advantages: 1) Gravity was required by the known string theories; 2) These string theories are free from the UV divergences that typically appear in point-particle theories of gravity. 3) Extra dimensions could be a good thing, since in a gravity theory their geometry would be determined by the dynamics. 4) Unification of gravity with other forces described by Yang–Mills theories was automatic when open strings are included.

We assumed that the size of the extra dimensions is comparable to the string scale. It then followed that the observed strength of gravity requires $\alpha' \sim 10^{-38}\,\mathrm{GeV}^{-2}$ instead of $\alpha' \sim 1\,\mathrm{GeV}^{-2}$, which is the hadronic value. Thus the change in interpretation meant that the tension of the strings suddenly increased by 38 orders of magnitude. This was a big conceptual leap, but the mathematics was unchanged.

Scherk and I were very excited by the possibility that string theory could be the Holy Grail of unified field theory, overcoming the problems that had stymied other approaches. In addition to publishing our work in scholarly journals we gave numerous lectures at conferences and physics departments all over the world. We even submitted a paper to the 1975 essay competition of the Gravity Research Foundation [86]. For the most part our work was received politely — as far as I know, no one accused us of being crackpots. Yet, for a decade, very few of the experts took the proposal seriously. Unfortunately, Scherk passed away midway through this 10-year period, though not before making some other important contributions that are discussed in the following sections.

6 1975 – 79: Supersymmetry and supergravity

Following the pioneering work of Wess and Zumino, discussed earlier, the study of supersymmetric quantum field theories became a major endeavor. One major step forward was the realization that supersymmetry can be realized as a local symmetry. This requires including a gauge field, called the *gravitino field*, which is vector-spinor. In four dimensions it describes a massless particle with spin 3/2, which is the supersymmetry partner of the graviton. Thus, local supersymmetry only appears in gravitational theories, which are called supergravity theories.

The first example of a supergravity theory was $\mathcal{N} = 1$, $d = 4$ supergravity. It was formulated in a second-order formalism by Ferrara, Freedman, and Van Nieuwenhuizen [87] and subsequently in a first-order formalism by Deser and Zumino [88]. The first-order formalism simplifies the analysis of terms that are quartic in fermi fields.

The two-dimensional locally supersymmetric and reparametrization invariant formulation of the RNS world-sheet action was constructed very soon thereafter [60, 61].[10] This construction was generalized to the $N = 2$ string of [79] by Brink and me [89]. Reparametrization-invariant world-sheet actions of this type are frequently associated with the name Polyakov, because he used them very skillfully five years later in constructing the path-integral formulation of string theory [90, 91]. Since neither Polyakov nor the authors of [60, 61] are happy with this usage, the new textbook [6] refers to this type of world-sheet action as a *string sigma-model action*.

The RNS closed-string spectrum contains a massless gravitino (in ten dimensions) in

[10]This generalized the one-dimensional result obtained a bit earlier for a spinning point particle [59].

addition to the graviton discussed in the previous section.[11] Since this is a gauge field, the only way the theory could be consistent is if the theory has local supersymmetry. This requires, in particular, that the spectrum should contain an equal number of bosonic and fermionic degrees of freedom at each mass level. However, as it stood, this was not the case. In particular, the bosonic sector contained a tachyon (the "pion"), which had no fermionic partner.

In 1976 Gliozzi, Scherk, Olive [92, 93] proposed a projection of the RNS spectrum – *the GSO Projection* – that removes roughly half of the states (including the tachyon). Specifically, in the bosonic (NS) sector they projected away the odd G-parity states, a possibility that was discussed earlier, and in the fermionic (R) sector they projected away half the states, keeping only certain definite chiralities. Then they counted the remaining physical degrees of freedom at each mass level. After the GSO projection the masses of open-string states, for both bosons and fermions, are given by $\alpha' M^2 = n$, where $n = 0, 1, \ldots$ Denoting the open-string degeneracies of states in the GSO-projected theory by $d_{\rm NS}(n)$ and $d_{\rm R}(n)$, they showed that these are encoded in the generating functions

$$f_{\rm NS}(w) = \sum_{n=0}^{\infty} d_{\rm NS}(n) w^n$$

$$= \frac{1}{2\sqrt{w}} \left[\prod_{m=1}^{\infty} \left(\frac{1 + w^{m-1/2}}{1 - w^m} \right)^8 - \prod_{m=1}^{\infty} \left(\frac{1 - w^{m-1/2}}{1 - w^m} \right)^8 \right].$$

and

$$f_{\rm R}(w) = \sum_{n=0}^{\infty} d_{\rm R}(n) w^n = 8 \prod_{m=1}^{\infty} \left(\frac{1 + w^m}{1 - w^m} \right)^8 .$$

In 1829, Jacobi proved the remarkable identity [94]

$$f_{\rm NS}(w) = f_{\rm R}(w),$$

though he used a different notation.[12] Thus, there are an equal number of bosons and fermions at every mass level, as required. This was compelling evidence (though not a proof) for *ten-dimensional spacetime supersymmetry* of the GSO-projected theory. Prior to this work, one knew that the RNS theory has world-sheet supersymmetry, but the realization that the theory should have spacetime supersymmetry was a major advance.

[11] More precisely, as was understood later, there are one or two gravitinos depending on whether one is describing a type I or type II superstring.

[12] Jacobi's paper acknowledges an assistant named Scherk!

Since a Majorana–Weyl spinor in ten dimensions has 16 real components, the minimal number of supercharges is 16. In particular, the massless modes of open superstrings at low energies are approximated by an $\mathcal{N} = 1$, $d = 10$ super Yang–Mills theory with 16 supersymmetries. This theory was constructed in [93, 95]. When this work was done, Brink and I were at Caltech and Scherk was in Paris. Brink and I wrote to Scherk informing him of our results and inviting him to join our collaboration, which he gladly accepted. Brink and I were unaware of the GSO collaboration, which was underway at that time, until their work appeared. Both papers pointed out that maximally supersymmetric Yang–Mills theories in less than ten dimensions could be deduced by dimensional reduction, and both of them constructed the $\mathcal{N} = 4$, $d = 4$ super Yang–Mills theory explicitly.

Having found the maximally supersymmetric Yang–Mills theories, it was an obvious problem to construct the maximally supersymmetric supergravity theories. Nahm showed [96] that the highest possible spacetime dimension for such a theory is $d = 11$. Soon thereafter, in a very impressive work, the Lagrangian for $\mathcal{N} = 1$, $d = 11$ supergravity was constructed by Cremmer, Julia, and Scherk [97]. The relevance of this beautiful theory to string theory remained mysterious for many years. This was finally understood a decade after the period covered by this article.

In 1978–79, I spent the academic year at the Ecole Normale Supérieure in Paris. There, I collaborated with Joël Scherk. Motivated by string theory considerations, we developed a scheme to break supersymmetry in the compactification of extra dimensions [98, 99, 100]. We only discussed our approach in a field theory context, because we were unable to decouple the supersymmetry breaking scale from the scale of the extra dimensions, which we believed to be necessary for at least one of the four-dimensional supersymmetries. In recent times others have applied these techniques to string theory compactifications, mostly in the context of large extra dimensions.

7 1979 – 84: Superstrings and Anomalies

Following Paris, I spent a month (July 1979) at CERN. There, Michael Green and I unexpectedly crossed paths. We had become acquainted in Princeton around 1970, when Green was at the IAS and I was at the University, but we had not collaborated before. In any case, following some discussions in the CERN cafeteria, we began a long and exciting collaboration. Our first goal was to understand better why the GSO-projected RNS string theory has spacetime supersymmetry.

Green, who worked at Queen Mary College London at the time, had several extended visits to Caltech in the 1980–85 period, and I had one to London in the fall of 1983. We also worked together several summers in Aspen. On several of these occasions we also collaborated with Lars Brink, who had visited Caltech and collaborated with me a few times previously.

After a year or so of unsuccessful efforts, Green and I discovered a new light-cone gauge formalism for the GSO-projected theory in which spacetime supersymmetry of the spectrum and interactions was easily proved. This was presented in three papers [101, 102, 103]. The first developed the formalism, while the next two used this light-cone gauge formalism to compute various tree and one-loop amplitudes and elucidate their properties. At this stage only open-string amplitudes were under consideration.

Our next project was to identify more precisely the possibilities for superstring theories. The GSO work had identified the proper projection for open strings, but it left unclear what one should do with the closed strings. Green and I realized that there are three distinct types of supersymmetry possible in ten dimensions and that all three of them could be realized by superstring theories. In [104] we formulated the type I, type IIA, and type IIB superstring theories. (We introduced these names a little later.) The type I theory is a theory of unoriented open and closed strings, whereas the the type II theories are theories of oriented closed strings only.

Brink, Green, and I formulated d-dimensional maximally supersymmetric Yang–Mills theories and supergravity theories as limits of superstring theory with $10 - d$ of the ten dimensions forming a torus. By computing one-loop string-theory amplitudes for massless gauge particles in the type I theory and gravitons in the type II theory and taking the appropriate limits, we showed that both the Yang–Mills and supergravity theories are ultraviolet finite at one loop for $d < 8$ [105]. The toroidally compactified string-loop formulas exhibited T-duality symmetry, though this was not pointed out explicitly in the article.

We also spent considerable effort formulating superstring field theory in the light-cone gauge [106, 107, 108]. This work became relevant about 20 years later, when the construction was generalized to the case of type IIB superstrings in a plane-wave background spacetime geometry.

The fact that our spacetime supersymmetric formalism was only defined in the light-cone gauge was a source of frustration. Brink and I had found a covariant world-line action for a massless superparticle earlier [109], so it was just a matter of finding the suitable superstring generalization. After a number of attempts, Green and I eventually found a covariant world-sheet action with manifest spacetime supersymmetry (and non-manifest kappa symmetry) [110, 111]. This covariant action reduces to our previous one in the light cone gauge, of course. It was natural to try to use it to define covariant quantization. However, due to a subtle combination of first-class and second-class constraints, it was immediately apparent that this action is extremely difficult to quantize covariantly. Numerous unsuccessful attempts over the years bear testimony to the truth of this assertion. More recently, Berkovits seems to have found a successful scheme. However, as far as I can tell, its logical foundations are not yet entirely clear.

Another problem of concern during this period was the formulation of ten-dimensional type IIB supergravity,[13] which is the leading low-energy approximation to type IIB superstring theory. Some partial results were obtained in separate collaborations with Green [112] and with Peter West [113]. A challenging aspect of the problem is the presence of a self-dual five-form field strength, which obstructs a straightforward construction of a manifestly covariant action. Therefore, I decided to focus on the equations of motion, instead, which I presented in [114]. Equivalent results were obtained in a superfield formalism by Howe and West [115].

Let me now turn to the issue of anomalies. Type I superstring theory is a well-defined ten-dimensional theory at tree level for any $SO(n)$ or $Sp(n)$ gauge group [116, 117]. However, in every case it is chiral (*i.e.,* parity violating) and the $d = 10$ super Yang–Mills sector is anomalous. Evaluation of a one-loop hexagon diagram exhibits explicit nonconservation of gauge currents of the schematic form

$$\partial_\mu J^\mu \sim \varepsilon^{\mu_1 \cdots \mu_{10}} F_{\mu_1\mu_2} \cdots F_{\mu_9\mu_{10}},$$

which is a fatal inconsistency.

Alvarez-Gaumé and Witten derived general formulas for gauge, gravitational, and mixed anomalies in an arbitrary spacetime dimension [118], and they discovered that the gravitational anomalies (nonconservation of the stress tensor) cancel in type IIB supergravity. This result was not really a surprise, since the one-loop type IIB superstring amplitudes are ultraviolet finite. It appeared likely that type I superstring theory is anomalous for any choice of the gauge group, but an explicit computation was required to decide for sure. In this case there are divergences that need to be regulated, so anomalies are definitely possible.

Green and I explored the anomaly problem for type I superstring theory off and on for almost two years until the crucial breakthroughs were made in August 1984 at the Aspen

[13]Type IIA supergravity can be obtained by dimensional reduction of 11-dimensional supergravity, but type IIB supergravity cannot be obtained in this way.

Center for Physics. That summer I was the organizer of a workshop entitled "Physics in Higher Dimensions" at the Aspen Center for Physics. This attracted many participants, even though string theory was not yet fashionable, because by that time there was considerable interest in supergravity theories in higher dimensions and Kaluza–Klein compactification. We benefitted from the presence of many leading experts including Bruno Zumino, Bill Bardeen, Dan Friedan, Steve Shenker, and others.

Green and I had tried unsuccessfully to compute the one-loop hexagon diagram in type I superstring theory using our supersymmetric light-cone gauge formalism, but this led to an impenetrable morass. In discussions with Friedan and Shenker the idea arose to carry out the computation using the covariant RNS formalism instead. At that point, Friedan and Shenker left Aspen, so Green and I continued on our own.

It soon became clear that both the cylinder and Möbius-strip diagrams contributed to the anomaly. Before a workshop seminar by one of the other workshop participants (I don't remember which one), I remarked to Green that there might be a gauge group for which the two contributions cancel. At the end of the seminar Green said to me "$SO(32)$," which was the correct result. Since this computation only showed the cancellation of the pure gauge part of the anomaly, we decided to explore the low-energy effective field theory to see whether the gravitational and mixed anomalies could also cancel. Before long, with the help of the results of Alvarez-Gaumé and Witten and useful comments by Bardeen and others, we were able to explain how this works. The effective field theory analysis was written up first [119], and the string loop analysis was written up somewhat later [120]. We also showed that the UV divergences of the cylinder and Möbius-strip diagrams cancel for $SO(32)$ [121]. Nowadays such cancellations are usually understood in terms of tadpole cancellations in a dual closed-string channel.

The effective field theory analysis showed that $E_8 \times E_8$ is a second gauge group for which the anomalies could cancel for a theory with $\mathcal{N} = 1$ supersymmetry in ten dimensions. In both cases, it is crucial for the result that the coupling to supergravity is included. The $SO(32)$ case could be accommodated by type I superstring theory, but we didn't know a superstring theory with gauge group $E_8 \times E_8$. We were aware of the article by Goddard and Olive that pointed out (among other things) that there are just two even-self-dual Euclidean lattices in 16 dimensions, and these are associated with precisely these two gauge groups [122]. However, we did not figure out how to exploit this fact before the problem was solved by others.

Before the end of 1984 there were two other major developments. The first one was the construction of the *heterotic string* by Gross, Harvey, Martinec, and Rohm [123, 124, 125]. Their construction actually accommodated both of the gauge groups. The second one was the demonstration by Candelas, Horowitz, Strominger, and Witten that *Calabi–Yau compactifications* of the $E_8 \times E_8$ heterotic string give supersymmetric four-dimensional effective theories with many realistic features [126].

By the beginning of 1985, superstring theory – with the goal of unification – had become a mainstream activity. In fact, there was a very sudden transition from benign neglect to unbounded euphoria, both of which seemed to me to be unwarranted. After a while, most string theorists developed a more realistic assessment of the problems and challenges that remained.

8 Postscript

The construction of a dual string theory description of QCD is still an actively pursued goal. It now appears likely that every well-defined (finite or asymptotically free) four-dimensional gauge theory has a string theory dual in a curved background geometry with five noncompact dimensions. The extra dimension corresponds to the energy scale of the gauge theory. The cleanest and best understood example of such a duality is the correspondence between $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with an $SU(N)$ gauge group and type IIB superstring theory in an $AdS_5 \times S^5$ spacetime with N units of five-form flux threading the sphere [127]. In particular, the (off-shell) energy–momentum tensor of the four-dimensional gauge theory corresponds to the (on-shell) graviton in five dimensions.

Such possibilities were not contemplated in the early years, so it understandable that success was not achieved. Moreover, the dual description of QCD is likely to be considerably more complicated than the example described above. For one thing, for realistic numbers of colors and flavors, the five-dimensional geometry is expected to have string-scale curvature, so that a supergravity approximation will not be helpful. However, it might still be possible to treat the inverse of the number of colors as small, so that a semiclassical string theory approximation (corresponding to the planar approximation to the gauge theory) can be used. If one is willing to sacrifice quantitative precision, one can already give constructions that have the correct qualitative features of QCD. One of their typical unrealistic features is that the Kaluza–Klein scale is comparable to the QCD scale. I remain optimistic that a correct construction of a string theory configuration that is dual to QCD exists. However, finding it and analyzing it might take a long time.

Acknowledgments

I am grateful to Lars Brink for reading the manuscript and making several helpful suggestions. I also wish to acknowledge the hospitality of the Galileo Galilei Institute and the Aspen Center for Physics. This work was partially supported in part by the U.S. Dept. of Energy under Grant No. DE-FG03-92-ER40701.

References

[1] J. H. Schwarz, "Reminiscences of Collaborations with Joël Scherk," arXiv:hep-th/0007117.

[2] J. H. Schwarz, "String Theory: The Early Years," arXiv:hep-th/0007118.

[3] J. H. Schwarz, "String Theory Origins of Supersymmetry," *Nucl. Phys. Proc. Suppl.*, **101**, 54 (2001) [arXiv:hep-th/0011078].

[4] J. H. Schwarz, *Superstrings – The First Fifteen Years of Superstring Theory*, Reprints and Commentary in 2 Volumes, World Scientific, 1985.

[5] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory* in 2 Volumes, Cambridge Univ. Press, 1987.

[6] K. Becker, M. Becker, and J. H. Schwarz, *String Theory and M-Theory: A Modern Introduction*, Cambridge Univ. Press, 2007.

[7] D. J. Gross and J. H. Schwarz, "Normal-Threshold Sheet Structure of Two-Particle Scattering Amplitudes" *Phys. Rev.*, **140**, B1054 (1965).

[8] G. F. Chew, *The S-Matrix Theory of Strong Interactions*, W. A. Benjamin and Co., 1961.

[9] G. F. Chew, *The Analytic S-Matrix: A Basis for Nuclear Democracy*, W. A. Benjamin and Co., 1966.

[10] G. F. Chew and S. C. Frautschi, "Principle of Equivalence for All Strongly Interacting Particles Within the S Matrix Framework," *Phys. Rev. Lett.*, **7**, 394 (1961).

[11] G. F. Chew and S. C. Frautschi, "Regge Trajectories and the Principle of Maximum Strength for Strong Interactions," *Phys. Rev. Lett.*, **8**, 41 (1962).

[12] R. Dolen, D. Horn, and C. Schmid, "Prediction of Regge Parameters of ρ Poles from Low-Energy πN Data," *Phys. Rev. Lett.*, **19**, 402 (1967).

[13] K. Igi and S. Matsuda, "New Sum Rules and Singularities in the Complex J Plane," *Phys. Rev. Lett.*, **18**, 625 (1967).

[14] K. Igi and S. Matsuda, "Some Consequences from Superconvergence for πN Scattering," *Phys. Rev.*, **163**, 1622 (1967).

[15] A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, "Dispersion Sum Rules and High Energy Scattering," *Phys. Lett.*, **24B**, 181 (1967).

[16] R. Dolen, D. Horn, and C. Schmid, "Finite-Energy Sum Rules and Their Application to πN Charge Exchange," *Phys. Rev.*, **166**, 1768 (1968).

[17] P. G. O. Freund, "Finite Energy Sum Rules and Bootstraps," *Phys. Rev. Lett.*, **20**, 235 (1968).

[18] H. Harari, "Duality Diagrams," *Phys. Rev. Lett.*, **22**, 562 (1969).

[19] J. L. Rosner, "Graphical Form of Duality," *Phys. Rev. Lett.*, **22**, 689 (1969).

[20] G. Veneziano, "Construction of a Crossing-Symmetric Regge-Behaved Amplitude for Linearly Rising Regge Trajectories," *Nuovo Cim.*, **57A**, 190 (1968).

[21] C. Lovelace, "A Novel Application of Regge Trajectories," *Phys. Lett.*, **28B**, 264 (1968).

[22] J. A. Shapiro, "Narrow Resonance Model with Regge Behavior for $\pi\pi$ scattering," *Phys. Rev.*, **179**, 1345 (1969).

[23] J. E. Paton and H. Chan, "Generalized Veneziano Model with Isospin," *Nucl. Phys.*, **B10**, 516 (1969).

[24] M. Virasoro, "Alternative Constructions of Crossing-Symmetric Amplitudes with Regge Behavior," *Phys. Rev.*, **177**, 2309 (1969).

[25] K. Bardakci and H. Ruegg, "Reggeized Resonance Model for Arbitrary Production Processes," *Phys. Rev.*, **181**, 1884 (1969).

[26] C. J. Goebel and B. Sakita, "Extension of the Veneziano Formula to N-Particle Amplitudes," *Phys. Rev. Lett.*, **22**, 257 (1969).

[27] H. M. Chan and T. S. Tsun, "Explicit Construction of the N-Point Function in the Generalized Veneziano Model," *Phys. Lett.*, **28B**, 485 (1969).

[28] Z. Koba and H. B. Nielsen, "Reaction Amplitudes for N Mesons, a Generalization of the Veneziano–Bardakci–Ruegg–Virasoro Model," *Nucl. Phys.*, **B10**, 633 (1969).

[29] Z. Koba and H. B. Nielsen, "Manifestly Crossing-Invariant Parametrization of the N-Meson Amplitude," *Nucl. Phys.*, **B12**, 517 (1969).

[30] J. A. Shapiro, "Electrostatic Analog for the Virasoro Model," *Phys. Lett.*, **33B**, 361 (1970).

[31] S. Fubini and G. Veneziano, "Level Structure of Dual Resonance Models," *Nuovo Cim.*, **64A**, 811 (1969).

[32] S. Fubini, D. Gordon, and G. Veneziano, "A General Treatment of Factorization in Dual Resonance Models," *Phys. Lett.*, **29B**, 679 (1969).

[33] K. Bardakci and S. Mandelstam, "Analytic Solution of the Linear-Trajectory Bootstrap," *Phys. Rev.*, **184**, 1640 (1969).

[34] S. Fubini and G. Veneziano, "Duality in Operator Formalism," *Nuovo Cim.*, **67A**, 29 (1970).

[35] Y. Nambu, "Quark Model and the Factorization of the Veneziano Model," p. 269 in Proc. Intern. Conf. on Symmetries and Quark Models, Wayne State Univ., 1969 (Gordon and Breach, NY 1970). Reprinted in *Broken Symmetry: selected papers of Y. Nambu*, eds. T. Eguchi and K. Nishijima, World Scientific (1995).

[36] Y. Nambu, "Duality and Hadrodynamics" Lectures at the Copenhagen Summer Symposium (1970). Reprinted in *Broken Symmetry: selected papers of Y. Nambu*, eds. T. Eguchi and K. Nishijima, World Scientific (1995).

[37] L. Susskind, "Dual-Symmetric Theory of Hadrons I," *Nuovo Cim.*, **69A**, 457 (1970).

[38] G. Frye, C. W. Lee, and L. Susskind, "Dual-Symmetric Theory of Hadrons. II. - Baryons," *Nuovo Cim.*, **69A**, 497 (1970).

[39] H. B. Nielsen, "An Almost Physical Interpretation of the N-Point Veneziano Model," submitted to Proc. of the XV Int. Conf. on High Energy Physics (Kiev, 1970), unpublished.

[40] D. B. Fairlie and H. B. Nielsen, "An Analogue Model for KSV Theory," *Nucl. Phys.*, **B20**, 637 (1970).

[41] D. J. Gross, A. Neveu, J. Scherk, and J. H. Schwarz, "The Primitive Graphs of Dual Resonance Models," *Phys. Lett.*, **31B**, 592 (1970).

[42] K. Kikkawa, B. Sakita and M. A. Virasoro, "Feynman-like Diagrams Compatible with Duality. I: Planar Diagrams," *Phys. Rev.*, **184**, 1701 (1969).

[43] D. J. Gross, A. Neveu, J. Scherk, and J. H. Schwarz, "Renormalization and Unitarity in the Dual Resonance Model," *Phys. Rev.*, **D2**, 697 (1970).

[44] G. Frye and L. Susskind, "Non-Planar Dual Symmetric Loop Graphs and the Pomeron," *Phys. Lett.*, **31B**, 589 (1970).

[45] C. Lovelace, "Pomeron Form Factors and Dual Regge Cuts," *Phys. Lett.*, **34B**, 500 (1971).

[46] M. Virasoro, "Subsidiary Conditions and Ghosts in Dual Resonance Models," *Phys. Rev.*, **D1**, 2933 (1970).

[47] S. Fubini and G. Veneziano, "Algebraic Treatment of Subsidiary Conditions in Dual Resonance Models," *Annals Phys.*, **63**, 12 (1971).

[48] L. Brink and D. I. Olive, "The Physical State Projection Operator in Dual Resonance Models for the Critical Dimension of Space-Time," *Nucl. Phys.*, **B56**, 253 (1973).

[49] L. Brink and D. I. Olive, "Recalculation of the Unitary Single Planar Dual Loop in the Critical Dimension of Space-Time," *Nucl. Phys.*, **B58**, 237 (1973).

[50] P. Goddard, J. Goldstone, C. Rebbi and C. B. Thorn, "Quantum Dynamics of a Massless Relativistic String," *Nucl. Phys.*, **B56**, 109 (1973).

[51] S. Mandelstam, "Interacting String Picture of Dual Resonance Models," *Nucl. Phys.*, **B64**, 205 (1973).

[52] P. Ramond, "Dual Theory for Free Fermions," *Phys. Rev.*, **D3**, 2415 (1971).

[53] A. Neveu and J. H. Schwarz, "Factorizable Dual Model of Pions," *Nucl. Phys.*, **B31**, 86 (1971).

[54] A. Neveu and J. H. Schwarz, "Tachyon-Free Dual Model with a Positive-Intercept Trajectory," *Phys. Lett.*, **3B4**, 517 (1971).

[55] A. Neveu, J. H. Schwarz, and C. B. Thorn, "Reformulation of the Dual Pion Model," *Phys. Lett.*, **35B**, 529 (1971).

[56] A. Neveu and J. H. Schwarz, "Quark Model of Dual Pions," *Phys. Rev.*, **D4**, 1109 (1971).

[57] C. B. Thorn, "Embryonic Dual Model for Pions and Fermions," *Phys. Rev.*, **D4**, 1112 (1971).

[58] J. L. Gervais and B. Sakita, "Field Theory Interpretation of Supergauges in Dual Models," *Nucl. Phys.*, **B34**, 632 (1971).

[59] L. Brink, S. Deser, B. Zumino, P. Di Vecchia and P. S. Howe, "Local Supersymmetry for Spinning Particles," *Phys. Lett.*, **B64**, 435 (1976).

[60] L. Brink, P. Di Vecchia, and P. Howe, "A Locally Supersymmetric and Reparametrization Invariant Action for the Spinning String," Phys. Lett. **65B**, 471 (1976).

[61] S. Deser and B. Zumino, "A Complete Action for the Spinning String," Phys. Lett. **65B**, 369 (1976).

[62] Yu. A. Golfand and E. P. Likhtman, "Extension of the Algebra of Poincaré Group Generators and Violation of P Invariance," *JETP Lett.* **13**, 323 (1971) [*Pisma Zh. Eksp. Teor. Fiz.* **13**, 452 (1971)].

[63] J. Wess and B. Zumino, "Supergauge Transformations in Four Dimensions," *Nucl. Phys.*, **B70**, 39 (1974).

[64] J. H. Schwarz, "Dual-Pion Model Satisfying Current-Algebra Constraints," *Phys. Rev.*, **D5**, 886 (1972).

[65] R. C. Brower, "Spectrum Generating Algebra and No-Ghost Theorem for the Dual Model," *Phys. Rev.*, **D6**, 1655 (1972).

[66] E. Del Giudice, P. Di Vecchia and S. Fubini, "General Properties of the Dual Resonance Model," *Annals Phys.*, **70**, 378 (1972).

[67] J. H. Schwarz, "Physical States and Pomeron Poles in the Dual Pion Model," *Nucl. Phys.*, **B46**, 61 (1972).

[68] R. C. Brower and K. A. Friedman, "Spectrum Generating Algebra and No Ghost Theorem for the Neveu–Schwarz Model," *Phys. Rev.*, **D7**, 535 (1973).

[69] P. Goddard and C. B. Thorn, "Compatibility of the Dual Pomeron with Unitarity and the Absence of Ghosts in the Dual Resonance Model," *Phys. Lett.*, **40B**, 235 (1972).

[70] J. L. Gervais and B. Sakita, "Ghost-free String Picture of Veneziano model," *Phys. Rev. Lett.*, **30**, 716 (1973).

[71] D. Olive and J. Scherk, "No-Ghost Theorem for the Pomeron Sector of the Dual Model," *Phys. Lett.*, **44B**, 296 (1973).

[72] E. F. Corrigan and P. Goddard, "The Absence of Ghosts in the Dual Fermion Model," *Nucl. Phys.*, **B68**, 189 (1974).

[73] D. I. Olive and J. Scherk, "Towards Satisfactory Scattering Amplitudes for Dual Fermions," *Nucl. Phys.*, **B64**, 334 (1973).

[74] J. H. Schwarz and C. C. Wu, "Evaluation of Dual Fermion Amplitudes," *Phys. Lett.*, **47B** (1973) 453.

[75] E. Corrigan, P. Goddard, R. A. Smith and D. I. Olive, "Evaluation of the Scattering Amplitude for Four Dual Fermions," *Nucl. Phys.*, **B67** (1973) 477.

[76] J. H. Schwarz, "Off-Mass-Shell Dual Amplitudes Without Ghosts," *Nucl. Phys.*, **B65** (1973) 131.

[77] J. H. Schwarz and C. C. Wu, "Off-Mass-Shell Dual Amplitudes. 2," *Nucl. Phys.*, B **B72**, 397 (1974).

[78] J. H. Schwarz, "Off-Mass-Shell Dual Amplitudes. 3," *Nucl. Phys.*, **B76**, 93 (1974).

[79] M. Ademollo *et al.*, "Dual String with U(1) Color Symmetry," *Nucl. Phys.*, **B111**, 77 (1976).

[80] W. Siegel, "The $N = 4$ String is the Same as the $N = 2$ String," *Phys. Rev. Lett.*, **69**, 1493 (1992) [arXiv:hep-th/9204005].

[81] A. Neveu and J. Scherk, "Connection Between Yang–Mills Fields and Dual Models," *Nucl. Phys.*, **B36**, 155 (1972).

[82] J. Scherk, "Zero-Slope Limit of the Dual Resonance Model," *Nucl. Phys.*, **B31**, 222 (1971).

[83] J. Scherk and J. H. Schwarz, "Dual Models for Non-Hadrons," *Nucl. Phys.*, **B81**, 118 (1974).

[84] T. Yoneya, "Quantum Gravity and the Zero Slope Limit of the Generalized Virasoro Model," *Nuovo Cim. Lett.*, **8**, 951 (1973).

[85] "Connection of Dual Models to Electrodynamics and Gravidynamics," *Prog. Theor. Phys.*, **51**, 1907 (1974).

[86] J. Scherk and J. H. Schwarz, "Dual Model Approach to a Renormalizable Theory of Gravitation," Submitted to the 1975 Gravitation Essay Contest of the Gravity Research Foundation. Reprinted in *Superstrings, Vol. 1*, ed. J. Schwarz, World Scientific (1985).

[87] D. Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, "Progress Toward a Theory of Supergravity," *Phys. Rev.*, **D13**, 3214 (1976).

[88] S. Deser and B. Zumino, "Consistent Supergravity," *Phys. Lett.*, **B62**, 335 (1976).

[89] L. Brink and J. H. Schwarz, "Local Complex Supersymmetry in Two Dimensions," *Nucl. Phys.*, **B121**, 285 (1977).

[90] A. M. Polyakov, "Quantum Geometry of Bosonic Strings," *Phys. Lett.*, **B103**, 207 (1981).

[91] A. M. Polyakov, "Quantum Geometry of Fermionic Strings," *Phys. Lett.*, **B103**, 211 (1981).

[92] F. Gliozzi, J. Scherk, and D. Olive, 'Supergravity and the Spinor Dual Model," *Phys. Lett.*, **65B**, 282 (1976).

[93] F. Gliozzi, J. Scherk, and D. Olive, "Supersymmetry, Supergravity Theories and the Dual Spinor Model," *Nucl. Phys.*, **B122**, 253 (1977).

[94] C. G. J. Jacobi, *Fundamenta*, Konigsberg, 1829.

[95] L. Brink, J. H. Schwarz, and J. Scherk , "Supersymmetric Yang–Mills Theories," *Nucl. Phys.*, **B121**, 77 (1977).

[96] W. Nahm, "Supersymmetries and Their Representations," *Nucl. Phys.*, **B135**, 149 (1978).

[97] E. Cremmer, B. Julia, and J. Scherk, "Supergravity Theory in 11 Dimensions," *Phys. Lett.*, **76B**, 409 (1978).

[98] J. Scherk and J. H. Schwarz, "Spontaneous Breaking of Supersymmetry Through Dimensional Reduction," *Phys. Lett.*, **B82**, 60 (1979).

[99] J. Scherk and J. H. Schwarz, "How to Get Masses from Extra Dimensions," *Nucl. Phys.*, **B153**, 61 (1979).

[100] E. Cremmer, J. Scherk and J. H. Schwarz, "Spontaneously Broken $N = 8$ Supergravity," *Phys. Lett.*, **B84**, 83 (1979).

[101] M. B. Green and J. H. Schwarz, "Supersymmetrical Dual String Theory," *Nucl. Phys.*, **B181**, 502 (1981).

[102] M. B. Green and J. H. Schwarz, "Supersymmetrical Dual String Theory. 2. Vertices And Trees," *Nucl. Phys.*, **B198**, 252 (1982).

[103] M. B. Green and J. H. Schwarz, "Supersymmetrical Dual String Theory. 3. Loops and Renormalization," *Nucl. Phys.*, **B198**, 441 (1982).

[104] M. B. Green and J. H. Schwarz, "Supersymmetrical String Theories," *Phys. Lett.*, **B109**, 444 (1982).

[105] M. B. Green, J. H. Schwarz and L. Brink, "$N = 4$ Yang–Mills and $N = 8$ Supergravity as Limits of String Theories," *Nucl. Phys.*, **B198**, 474 (1982).

[106] M. B. Green and J. H. Schwarz, "Superstring Interactions," *Nucl. Phys.*, **B218**, 43 (1983).

[107] M. B. Green, J. H. Schwarz and L. Brink, "Superfield Theory of Type II Superstrings," *Nucl. Phys.*, **B219**, 437 (1983).

[108] M. B. Green and J. H. Schwarz, "Superstring Field Theory," *Nucl. Phys.*, **B243**, 475 (1984).

[109] L. Brink and J. H. Schwarz, "Quantum Superspace," *Phys. Lett.*, **B100**, 310 (1981).

[110] M. B. Green and J. H. Schwarz, "Covariant Description of Superstrings," *Phys. Lett.*, **B136**, 367 (1984).

[111] M. B. Green and J. H. Schwarz, "Properties of the Covariant Formulation of Superstring Theories," *Nucl. Phys.*, **B243**, 285 (1984).

[112] M. B. Green and J. H. Schwarz, "Extended Supergravity in Ten Dimensions," *Phys. Lett.*, **B122**, 143 (1983).

[113] J. H. Schwarz and P. C. West, "Symmetries and Transformations of Chiral $N = 2$, $D = 10$ Supergravity," *Phys. Lett.*, **B126**, 301 (1983).

[114] J. H. Schwarz, "Covariant Field Equations of Chiral $N = 2$, $D = 10$ Supergravity," *Nucl. Phys.*, **B226**, 269 (1983).

[115] P. S. Howe and P. C. West, "The Complete $N = 2$, $D = 10$ Supergravity," *Nucl. Phys.*, **B238**, 181 (1984).

[116] J. H. Schwarz, "Gauge Groups for Type I Superstrings," p. 233 in Proc. Johns Hopkins Workshop (1982).

[117] N. Marcus and A. Sagnotti, "Tree Level Constraints on Gauge Groups for Type I Superstrings," *Phys. Lett.*, **B119**, 97 (1982).

[118] L. Alvarez-Gaume and E. Witten, "Gravitational Anomalies," *Nucl. Phys.*, **B234**, 269 (1984).

[119] M. B. Green and J. H. Schwarz, "Anomaly Cancellation in Supersymmetric $D = 10$ Gauge Theory and Superstring Theory," *Phys. Lett.*, **B149**, 117 (1984).

[120] M. B. Green and J. H. Schwarz, "The Hexagon Gauge Anomaly in Type I Superstring Theory," *Nucl. Phys.*, **B255**, 93 (1985).

[121] M. B. Green and J. H. Schwarz, "Infinity Cancellations in SO(32) Superstring Theory," *Phys. Lett.*, **B151**, 21 (1985).

[122] P. Goddard and D. I. Olive, "Algebras, Lattices and Strings," DAMTP-83/22, p. 19 in Proc. Marstrand Nobel Sympos. (1986).

[123] D. J. Gross, J. A. Harvey, E. J. Martinec and R. Rohm, "The Heterotic String," *Phys. Rev. Lett.*, **54**, 502 (1985).

[124] D. J. Gross, J. A. Harvey, E. J. Martinec and R. Rohm, "Heterotic String Theory. 1. The Free Heterotic String," *Nucl. Phys.*, **B256**, 253 (1985).

[125] D. J. Gross, J. A. Harvey, E. J. Martinec and R. Rohm, "Heterotic String Theory. 2. The Interacting Heterotic String," *Nucl. Phys.*, **B267**, 75 (1986).

[126] P. Candelas, G. T. Horowitz, A. Strominger and E. Witten, "Vacuum Configurations for Superstrings," *Nucl. Phys.*, **B258**, 46 (1985).

[127] J. M. Maldacena, "The Large N Limit of Superconformal Field Theories and Supergravity," *Adv. Theor. Math. Phys.*, **2**, 231 (1998) [arXiv:hep-th/9711200].